APS 502 Computational Project 2

PART 1

a. Use yahoo (e.g. yahoo.com or yahoo.ca) finance to get the monthly adjusted closing prices of SPY, GOVT, and EEMV from Jan 2014 to end of Feb 2020 and compute the expected returns of the three assets, the standard deviations of the three assets as well as the co-variances between all assets over this time period. Show these parameters in your report but you don't have to show the monthly prices or the computations that you did to get the parameters.

For the given period, 74 historical monthly return (r_{it}) for each asset could be derived by using the following formular.

$$r_{it} = \frac{B - A}{A}, where$$

$$A = adjusted\ closing\ price\ of\ the\ first\ trading\ day\ of\ the\ month$$
$$B = adjusted\ closing\ price\ of\ the\ last\ trading\ day\ of\ the\ month$$

Using monthly return (r_{it}) to compute the expected returns and standard deviations of the three assets as well as the covariances between all assets. Noted that MVO use the geometric returns.

	Arithmetic mean	**Expected return (Geometric mean)**	**Standard deviation**
SPY	0.795162%	0.738002%	3.366410%
GOVT	0.304937%	0.300033%	0.992467%
EEMV	0.154443%	0.100985%	3.259540%

Variance and covariance matrix.

	SPY	**GOVT**	**EEMV**
SPY	0.113327%	-0.012307%	0.068210%
GOVT	-0.012307%	0.009850%	-0.002484%
EEMV	0.068210%	-0.002484%	0.106246%

b. Use the mean-variance optimization model to generate an efficient frontier of the three assets. Create a table where for each expected return goal R show the optimal weights of the assets as well as the portfolio variance value. Also, plot the efficient frontier. Note: the range of R can be the smallest positive expected return among the three assets to the largest expected return among the assets. You are free to choose the points in the range to use for the optimizations, but they should be at least 10 return points equally spaced out.

Let $w = [w_1, w_2, w_3]$ *be the portfolio weight,*
where w_1, w_2, w_3 *is the weight for SPY, GOVT, EEMV correspondingly.*

Markowitz optimization objective:

$$\min_x \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij}$$
$$= \frac{1}{2}[0.113327\%\ w_1^2 + 0.009850\%\ w_2^2 + 0.106246\%\ w_3^2 + 2(-0.012307\%)\ w_1 w_2 + 2(0.068210\%)\ w_1 w_3 + 2(-0.002484\%) w_2 w_3]$$

Constraints:

$$0.738002\%\ w_1 + 0.300033\%\ w_2 + 0.100985\%\ w_3 \geq R, where\ R\ is\ the\ expected\ return\ goal.$$

$$w_1 + w_2 + w_3 = 1$$
$$(w_1, w_2, w_3 \geq 0, constrain\ that\ does\ not\ allow\ short\ selling)$$

Create a table where for each expected return goal R show the optimal weights of the assets as well as the portfolio variance value.

With short selling:

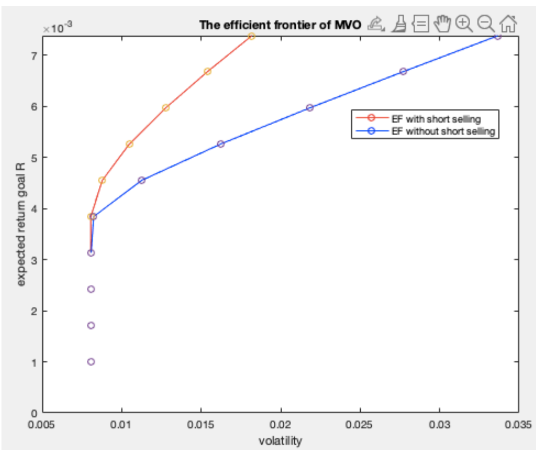
R	w_1	w_2	w_3	Variance
0.001010	0.1658	0.8595	-0.0253	0.00006488080
0.001718	0.1658	0.8595	-0.0253	0.00006488080
0.002425	0.1658	0.8595	-0.0253	0.00006488080
0.003133	0.1659	0.8595	-0.0254	0.00006488081
0.003841	0.1760	0.8593	-0.0353	0.00006496576
0.004549	0.2878	0.8571	-0.1448	0.00007709118
0.005257	0.3996	0.8548	-0.2544	0. 00010972777
0.005964	0.5114	0.8526	-0.3640	0. 00016289099
0.006672	0.6232	0.8504	-0.4736	0. 00023657280
0.007380	0.7350	0.8482	-0.5831	0. 00033077610

Without short selling:

R	w_1	w_2	w_3	Variance
0.001010	0.1495	0.8499	0.0006	0.00006530
0.001718	0.1499	0.8501	0.0000	0.00006528
0.002425	0.1499	0.8500	0.0001	0.00006529
0.003133	0.1498	0.8500	0.0003	0.00006529
0.003841	0.1920	0.8079	0.0001	0.00006791
0.004549	0.3536	0.6464	0.0000	0.00012660
0.005257	0.5152	0.4848	0.0000	0.00026245
0.005964	0.6768	0.3232	0.0000	0.00047552
0.006672	0.8384	0.1616	0.0000	0.00076580
0.007380	1.0000	0.0000	0.0000	0.00113327

Plot the efficient frontier.

Efficient frontier for both situations does not include the portfolios with expected return goal R smaller than 0.003133 approximately, because those portfolios have almost same variance but smaller expected return.
Short selling enlarges the efficient frontier. Portfolios with short selling have better performance than portfolios without short selling, because it has smaller volatility for same expected return.



PART 1 (b) MATLAB CODE

```
%Q: Symmetric matrix represents the quadratic term in objective
Q = [0.00113327, -0.00012307, 0.00068210;
    -0.00012307, 0.00009850, -0.00002484;
    0.00068210, -0.00002484, 0.00106246;];
%c: Coefficient vector for the linear term in objective
c = [0 0 0]';
%A, b: Inequality constraints (<)
A = -[0.00738002, 0.00300033, 0.00100985];

%Aeq, beq: Equality constraints
Aeq = [1 1 1];
beq = [1];

%lb, ub: Variable bounds
ub = [inf; inf; inf;];
% lb without constrain (allow short selling)
lb_1 = [-inf; -inf; -inf;];
% lb with constrain (not allow short selling)
lb_2 = [0; 0; 0;];


R = linspace(0.00100985,0.00738002,10); %expected return goal
var_1 = []; %variance of protfolio with short selling
var_2 = []; %variance of protfolio without short selling
vol_1 = []; %volatility of protfolio with short selling
vol_2= [];  %volatility of protfolio without short selling
weight_1 = []; %portfolio weight with short selling
weight_2 = []; %portfolio weight without short selling

for i=1:10
    b = -R(i);
    %without constrain (allow short selling)
    [x_1, fval_1, exitflag_1, output_1, lambda_1] = quadprog(Q, c, A,
b, Aeq, beq, lb_1, ub);
    weight_1= [weight_1;x_1'];
    var_1 = [var_1;(fval_1*2)];
    vol_1 = [vol_1;sqrt(fval_1*2)];

    % lb with constrain (not allow short selling)
    [x_2, fval_2, exitflag_2, output_2, lambda_2] = quadprog(Q, c, A,
b, Aeq, beq, lb_2, ub);
    weight_2= [weight_2;x_2'];
    var_2 = [var_2;(fval_2*2)];
    vol_2 = [vol_2;sqrt(fval_2*2)]
end

% plot efficient frontier and points correspond to R
plot(vol_1(4:10),R(4:10),'r-o',vol_2(4:10),R(4:10),'b-
o',vol_1,R,'o',vol_2,R,'o')
legend('EF with short selling','EF without short selling')
ylim([0,0.00738002])
ylabel('expected return goal R')

xlabel('volatility')
title('The efficient frontier of MVO')

format long
fprintf('portfolio allow short selling:\n')
fprintf('portfolio weight:\n')
weight_1
fprintf('portfolio variance:\n')
var_1

fprintf('portfolio not allow short selling:\n')
fprintf('portfolio weight:\n')
weight_2
fprintf('portfolio variance:\n')
var_2
```

Since our objective is to min ½ portfolio variance, so we have to double the fval_1 (and fval_2) to get the variance of portfolio.

PART 1 (b) MATLAB OUTPUT

```
portfolio allow short selling:
portfolio weight:

weight_1 =

   0.165810494909766   0.859479722883079  -0.025290217792846
   0.165821122082911   0.859480109064936  -0.025301231147847
   0.165809819811053   0.859479664003260  -0.025289483814313
   0.165923097090641   0.859479220608413  -0.025402317699054
   0.175982732091837   0.859277450888229  -0.035260182980067
   0.287777062146016   0.857056226640234  -0.144833288786250
   0.399557302286757   0.854835202537419  -0.254392504824176
   0.511364363198854   0.852613716513777  -0.363978079712631
   0.623167860397005   0.850392262728890  -0.473560123125895
   0.734973019147791   0.848170790624727  -0.583143809772518

portfolio variance:

var_1 =

   1.0e-03 *

   0.064880807869400
   0.064880807996382
   0.064880807867819
   0.064880818698996
   0.064965767005796
   0.077091183486610
   0.109727773098264
   0.162890998207502
   0.236572800079828
   0.330776107609999

portfolio not allow short selling:
portfolio weight:

weight_2 =

   0.149536952596833   0.849850377077081   0.000612670326086
   0.149920510165697   0.850078435031525   0.000001054802778
   0.149865531868423   0.850045774234048   0.000088693897529
   0.149753243837778   0.849978781203879   0.000267974958343
   0.192002518291598   0.807909869108022   0.000087612600380
   0.353587831149282   0.646389180897831   0.000022987952889
   0.515174115339148   0.484824598950858   0.000001285710002
   0.676782349997918   0.323217630579042   0.000000019423040
   0.838391208881686   0.161608677224247   0.000000113894068
   0.999999999909355   0.000000000092142   0.000000000000424

portfolio variance:

var_2 =

   0.000065301639238
   0.000065281999283
   0.000065284784708
   0.000065290512802
   0.000067908742978
   0.000126595256336
   0.000262450426555
   0.000475524234169
   0.000765798140614
   0.001133269999772
```

c. Take the minimum variance portfolio from (b) (this is the portfolio in the efficient frontier with the lowest variance). Using monthly returns from only March 2020 compare the minimum variance portfolio with the equal weighted portfolio and a portfolio that has 60% in SPY, 30% in GOVT, and 10% in EEMV. Rank the 3 portfolios in terms of returns. Explain the relative performance of the portfolios.

Monthly returns from only March 2020:

	SPY	GOVT	EEMV
monthly returns	-0.1611968	0.03243652	-0.1280432

Use previous calculated variance and covariance:

	SPY	GOVT	EEMV
SPY	0.113327%	-0.012307%	0.068210%
GOVT	-0.012307%	0.009850%	-0.002484%
EEMV	0.068210%	-0.002484%	0.106246%

1. Minimum variance portfolio (choose the one with short selling):

$$w_1 = 0.1659, w_2 = 0.8595, w_3 = -0.0254$$

$\overline{r_{p1}} = \sum_{i=1}^{n} w_i \overline{r_i} = 0.1659\,(-0.1611968) + 0.8595(0.03243652) - 0.0254\,(-0.1280432) = 0.00438894$

$\sigma_{p1}^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij} = 0.1662^2\,(0.113327\%) + 0.8595^2\,(0.009850\%) + (-0.0256)^2\,(0.106246\%) +$
$2\,(0.1662)(0.8595)(-0.012307\%) + 2\,(0.1662)(-0.0256)(0.068210\%) +$
$2(0.8595)(-0.0256)(-0.002484\%) = 0.000064880$

$\sigma_{p1} = 0.0080548$

2. Equal weighted portfolio: $w_1 = \frac{1}{3}, w_2 = \frac{1}{3}, w_3 = \frac{1}{3}$

$\overline{r_{p2}} = \sum_{i=1}^{n} w_i \overline{r_i} = \frac{1}{3}\,(-0.1611968) + \frac{1}{3}(0.03243652) - \frac{1}{3}(-0.1280432) = -0.0856012$

$\sigma_{p2}^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij} = \frac{1}{3}^2\,(0.113327\%) + \frac{1}{3}^2\,(0.009850\%) + \frac{1}{3}^2\,(0.106246\%) +$
$2\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)(-0.012307\%) + 2\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)(0.068210\%) + 2\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)(-0.002484\%) = 0.000373624$

$\sigma_{p2} = 0.0193294$

3. Portfolio with 60% in SPY, 30% in GOVT, and 10% in EEMV: $w_1 = 0.6, w_2 = 0.3, w_3 = 0.1$

$\overline{r_{p3}} = \sum_{i=1}^{n} w_i \overline{r_i} = 0.6\,(-0.1611968) + 0.3(0.03243652) + 0.1(-0.1280432) = -0.0997914$

$\sigma_{p3}^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij} = 0.6^2\,(0.113327\%) + 0.3^2\,(0.009850\%) + 0.1^2\,(0.106246\%) +$
$2\,(0.6)(0.3)(-0.012307\%) + 2\,(0.6)(0.1)(0.068210\%) + 2(0.3)(0.1)(-0.002484\%) = 0.000463523$

$\sigma_{p3} = 0.0215296$

Relative performance of the portfolios:

Rank the 3 portfolios in terms of returns: $\overline{r_{p3}} < \overline{r_{p2}} < \overline{r_{p1}}$
Rank the 3 portfolios in terms of volatility: $\sigma_{p1} < \sigma_{p2} < \sigma_{p3}$

Among three portfolios, minimum variance portfolio has the best performance since it has greatest expected return and smallest volatility. Portfolio with 60% in SPY, 30% in GOVT, and 10% in EEMV has the worst performance since it has lowest expected return and greatest volatility. And equal weighted portfolio has the mediate performance.

PART 2

Repeat (b) of PART 1 using the stocks SPY, GOV, EEMV as well as the stocks below (so portfolios will have 8 assets now) that have heavy involvement and connection to development or use of blockchain technology (some people think that these stocks are going to do well in the future)
(4) CME Group (CME)
(5) Broadridge Financial Solutions (BR)
(6) Cboe Global Markets (CBOE)
(7) Intercontinental Exchange (ICE)
(8) Accenture (ACN)

Compute the expected returns and standard deviations of assets. Noted that MVO use the geometric returns.

	Arithmetic mean	**Expected return (Geometric mean)**	**Standard deviation**
SPY	0.795162%	0.738002%	3.366410%
GOVT	0.304937%	0.300033%	0.992467%
EEMV	0.154443%	0.100985%	3.259540%
CME	1.499107%	1.401204%	4.466907%
BR	1.553257%	1.411464%	5.375145%
CBOE	1.224894%	1.057789%	5.761642%
ICE	1.159854%	1.048744%	4.727548%
ACN	1.401445%	1.283322%	4.813903%

Variance and covariance matrix.

	SPY	**GOVT**	**EEMV**	**CME**	**BR**	**CBOE**	**ICE**	**ACN**
SPY	**0.113327%**	-0.012307%	0.068210%	0.024939%	0.094254%	0.026041%	0.058589%	0.122228%
GOVT	-0.012307%	**0.009850%**	-0.002484%	-0.006244%	0.003185%	0.002323%	-0.010802%	-0.008293%
EEMV	0.068210%	-0.002484%	**0.106246%**	-0.020499%	0.065582%	-0.008895%	-0.000082%	0.057814%
CME	0.024939%	-0.006244%	-0.020499%	**0.199533%**	0.088056%	0.114209%	0.112676%	0.037369%
BR	0.094254%	0.003185%	0.065582%	0.088056%	**0.288922%**	0.052664%	0.070973%	0.135153%
CBOE	0.026041%	0.002323%	-0.008895%	0.114209%	0.052664%	**0.331965%**	0.088286%	0.044015%
ICE	0.058589%	-0.010802%	-0.000082%	0.112676%	0.070973%	0.088286%	**0.223497%**	0.094864%
ACN	0.122228%	-0.008293%	0.057814%	0.037369%	0.135153%	0.044015%	0.094864%	**0.231737%**

Let $w = [w_1, w_2, w_3, w_4, w_5, w_6, w_7, w_8]$ *be the portfolio weight,*
where $w_1, w_2, w_3, w_4, w_5, w_6, w_7, w_8$ *is the weight for* $SPY, GOVT, EEMV, CME, BR, CBOE, ICE, ACN$ *correspondingly.*

Markowitz optimization objective:

$$
\begin{aligned}
\min_x \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij}
&= \frac{1}{2}[0.113327\%\, w_1^2 + 0.009850\%\, w_2^2 + 0.106246\%\, w_3^2 + 0.199533\% w_4^2 \\
&+ 0.288922\%\, w_5^2 + 0.331965\% w_6^2 + 0.223497\%\, w_7^2 + 0.231737\%\, w_8^2 \\
&+ 2(-0.012307\%)\, w_1 w_2 + 2(0.068210\%)\, w_1 w_3 + 2(0.024939\%) w_1 w_4 \\
&+ 2(0.094254\%)\, w_1 w_5 + 2(0.026041\%)\, w_1 w_6 + 2(0.058589\%) w_1 w_7 \\
&+ 2(0.122228\%)\, w_1 w_8 + 2(-0.002484\%)\, w_2 w_3 + 2(-0.006244\%) w_2 w_4 \\
&+ 2(0.003185\%)\, w_2 w_5 + 2(0.002323\%)\, w_2 w_6 + 2(-0.010802\%) w_2 w_7 \\
&+ 2(-0.008293\%)\, w_2 w_8 + 2(-0.020499\%) w_3 w_4 + 2(0.065582\%)\, w_3 w_5 \\
&+ 2(-0.008895\%)\, w_3 w_6 + 2(-0.000082\%) w_3 w_7 + 2(0.057814\%)\, w_3 w_8 \\
&+ 2(0.088056\%)\, w_4 w_5 + 2(0.114209\%)\, w_4 w_6 + 2(0.112676\%) w_4 w_7 \\
&+ 2(0.037369\%)\, w_4 w_8 + 2(0.052664\%)\, w_5 w_6 + 2(0.070973\%) w_5 w_7 \\
&+ 2(0.135153\%)\, w_5 w_8 + 2(0.088286\%) w_6 w_7 + 2(0.044015\%)\, w_6 w_8 \\
&+ 2(0.094864\%)\, w_7 w_8]
\end{aligned}
$$

Constraints:

$$0.738002\%\, \mathrm{w}_1 + 0.300033\%\, \mathrm{w}_2 + 0.100985\%\, \mathrm{w}_3 + 1.401204\% \mathrm{w}_4 + 1.411464\%\, \mathrm{w}_5 + 1.057789\% \mathrm{w}_6 + 1.048744\% \mathrm{w}_7 + 1.283322\% \mathrm{w}_8 \geq R, where\ R\ is\ the\ expected\ return\ goal.$$

$$w_1 + w_2 + w_3 + w_4 + w_5 + w_6 + w_7 + w_8 = 1$$
$$(w_1, w_2, w_3, w_4, w_5, w_6, w_7, w_8 \geq 0, constrain\ that\ does\ not\ allow\ short\ selling)$$

Create a table where for each expected return goal R show the optimal weights of the assets as well as the portfolio variance value.

With short selling:

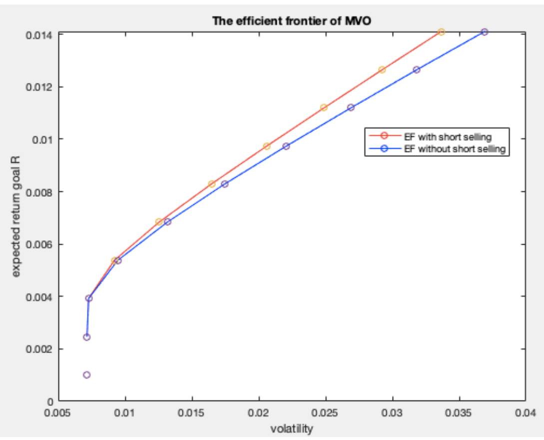
R	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	Variance
0.001010	0.19896146	0.8273147	0.0063595	0.0585253	-0.06200477	-0.01751675	0.0222259	-0.03386552	0.000051217920135
0.002466	0.19896149	0.8273144	0.0063590	0.0585257	- 0.06200454	-0.01751673	0.0222257	-0.03386515	0.000051217920137
0.003922	0.1994	0.8143	-0.0144	0.0718	-0.0527	-0.0164	0.0175	-0.0195	0.000052813
0.005378	0.2011	0.7676	-0.0886	0.1191	-0.0196	-0.0122	0.0006	0.0320	0.000084640
0.006834	0.2028	0.7209	-0.1628	0.1665	0.0135	-0.0081	-0.0162	0.0836	0.000157290
0.008290	0.2044	0.6742	-0.2370	0.2138	0.0466	-0.0039	-0.0331	0.1351	0.000270771
0.009746	0.2061	0.6275	-0.3112	0.2612	0.0797	0.0002	-0.0500	0.1866	0.000425078
0.011202	0.2078	0.5808	-0.3855	0.3085	0.1128	0.0044	-0.0669	0.2381	0.000620213
0.012659	0.2094	0.5341	-0.4597	0.3559	0.1459	0.0085	-0.0838	0.2896	0.000856181
0.014115	0.2111	0.4874	-0.5339	0.4033	0.1790	0.0127	-0.1006	0.3411	0.001132967

Without short selling:

R	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	Variance
0.001010	0.1978	0.8267	0.0077	0.0587	-0.0621	-0.0175	0.0223	-0.0336	0.000051219
0.002466	0.1974	0.8264	0.0082	0.0588	-0.0622	-0.0175	0.0224	-0.0335	0.000051220
0.003922	0.1851	0.8045	0.0004	0.0759	-0.0534	-0.0159	0.0181	-0.0148	0.000052943
0.005378	0.1154	0.7094	0.0001	0.1439	-0.0234	-0.0097	0.0044	0.0598	0.000089298
0.006834	0.0456	0.6141	0.0000	0.2121	0.0065	-0.0035	-0.0093	0.1345	0.000173005
0.008290	0.0000	0.5060	0.0000	0.2832	0.0347	0.0028	-0.0258	0.1992	0.000304422
0.009746	0.0001	0.3740	0.0000	0.3598	0.0595	0.0094	-0.0477	0.2450	0.000487833
0.011202	0.0000	0.2421	0.0000	0.4364	0.0843	0.0160	-0.0696	0.2908	0.000724449
0.012659	0.0000	0.1101	0.0000	0.5130	0.1092	0.0225	-0.0915	0.3366	0.001014294
0.014115	0.0000	0.0000	0.0000	0.6300	0.1359	0.0044	-0.1597	0.3894	0.001362624

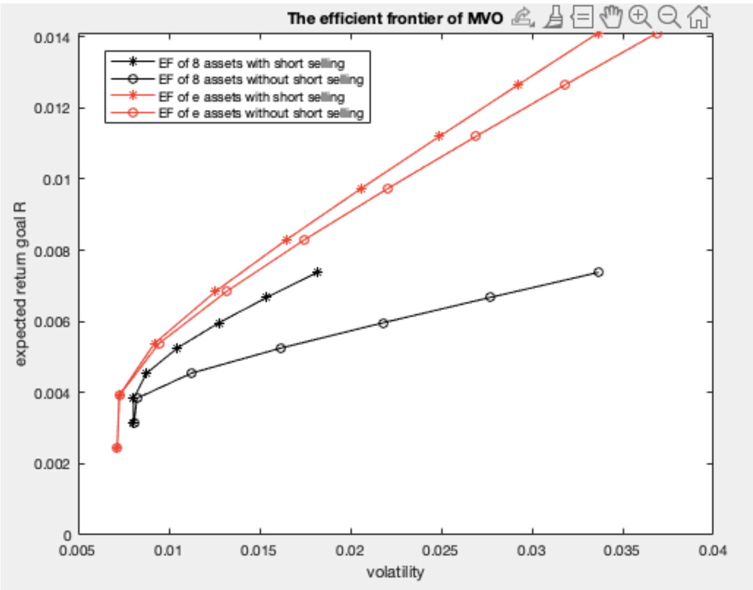
Plot the efficient frontier.

Efficient frontier for both situations does not include the portfolios with expected return goal R smaller than 0.002466 because those portfolios have almost same variance but smaller expected return.



Compare the efficient frontiers from PART 1 and PART 2. Does including the stocks in PART 2 lead to better portfolios? Discuss.

The following plot contains the four efficient frontiers from part 1 and 2, which could illustrate that including the stocks in part 2 lead to better portfolios. The efficient frontiers for 8 assets are in the upper left side of the efficient frontiers for 3 assets, whether with short selling or not. For given expected return, the portfolios of 8 assets has smaller volatility. This generalizes the idea that diversification could reduce portfolio volatility without sacrificing the expected return.



CODE FOR ABLOVE PLOT

```
% data from part 1
part1_R =
 [0.00313324000000,0.003841036666667,0.004548833333333,0.005256630000000,0.00596
 0.007380020000000];
% with short selling
part1_x1 =
 [0.008054863046570,0.008060134428519,0.008780158511474,0.010475102534022,0.01276
% without short selling
part1_x2 =
 [0.008080254501056,0.008240676123823,0.011251455742977,0.016200321804065,0.02180

% data from part 1
part2_R =
 [0.002465937777778,0.003922025555556,0.005378113333333,0.006834201111111,0.00829
% with short selling
part2_x1 =
 [0.007156669626109,0.007267262061494,0.009200017273912,0.012541531523071,0.01645
% without short selling
part2_x2 =
 [0.007156795638398,0.007276194078780,0.009449776283009,0.013153129829349,0.01744

% plot efficient frontier of 4 cases
plot(part1_x1,part1_R,'black-*',part1_x2,part1_R,'black-
o',part2_x1,part2_R,'red-*',part2_x2,part2_R,'red-o')
legend('EF of 8 assets with short selling','EF of 8 assets without
 short selling','EF of e assets with short selling','EF of e assets
 without short selling')
ylim([0,0.01411464])
ylabel('expected return goal R')
xlabel('volatility')
title('The efficient frontier of MVO')
```

PART 2 MATLAB CODE

```
%Q: Symmetric matrix represents the quadratic term in objective
Q = [0.00113327, -0.00012307, 0.00068210, 0.00024939,   0.00094254,
 0.00026041, 0.00058589, 0.00122228;
    -0.00012307, 0.00009850,  -0.00002484, -0.00006244,  0.00003185,
 0.00002323, -0.00010802,-0.00008293;
     0.00068210, -0.00002484, 0.00106246, -0.00020499,  0.00065582,
 -0.00008895,-0.00000082,0.00057814;
     0.00024939, -0.00006244, -0.00020499, 0.00199533,   0.00088056,
 0.00114209, 0.00112676, 0.00037369;
     0.00094254, 0.00003185,  0.00065582, 0.00088056,   0.00288922,
 0.00052664, 0.00070973, 0.00135153;
     0.00026041, 0.00002323,  -0.00008895, 0.00114209,   0.00052664,
 0.00331965, 0.00088286, 0.00044015;
     0.00058589, -0.00010802, -0.00000082,  0.00112676,   0.00070973,
 0.00088286, 0.00223497, 0.00094864;
     0.00122228, -0.00008293, 0.00057814, 0.00037369,   0.00135153,
 0.00044015, 0.00094864, 0.00231737;];

%c: Coefficient vector for the linear term in objective
c = [0 0 0 0 0 0 0 0]';
%A, b: Inequality constraints (<)
A = -
[0.00738002,0.00300033,0.00100985,0.01401204,0.01411464,0.01057789,0.01048744,0.01

%Aeq, beq: Equality constraints
Aeq = [1 1 1 1 1 1 1 1];
beq = [1];

%lb, ub: Variable bounds
ub = [inf; inf; inf;];
% lb without constrain (allow short selling)
lb_1 = [-inf; -inf; -inf;];
% lb with constrain (not allow short selling)
lb_2 = [0; 0; 0;];


R = linspace(0.00100985,0.01411464,10); %expected return goal
var_1 = []; %variance of protfolio with short selling
var_2 = []; %variance of protfolio without short selling
vol_1 = []; %volatility of protfolio with short selling
vol_2= [];  %volatility of protfolio without short selling
weight_1 = []; %portfolio weight with short selling
weight_2 = []; %portfolio weight without short selling

for i=1:10
    b = -R(i);
    %without constrain (allow short selling)
    [x_1, fval_1, exitflag_1, output_1, lambda_1] = quadprog(Q, c, A,
b, Aeq, beq, lb_1, ub);
    weight_1= [weight_1;x_1'];
    var_1 = [var_1;(fval_1*2)];
    vol_1 = [vol_1;sqrt(fval_1*2)];

   % lb with constrain (not allow short selling)
   [x_2, fval_2, exitflag_2, output_2, lambda_2] = quadprog(Q, c, A,
b, Aeq, beq, lb_2, ub);
   weight_2= [weight_2;x_2'];
   var_2 = [var_2;(fval_2*2)];
   vol_2 = [vol_2;sqrt(fval_2*2)];
end
```

```
% plot efficient frontier
plot(vol_1(2:10),R(2:10),'r-o',vol_2(2:10),R(2:10),'b-
o',vol_1,R,'o',vol_2,R,'o')
legend('EF with short selling','EF without short selling')
ylim([0,0.01411464])
ylabel('expected return goal R')
xlabel('volatility')
title('The efficient frontier of MVO')

format long
fprintf('portfolio allow short selling:\n')
fprintf('portfolio weight:\n')
weight_1
fprintf('portfolio variance:\n')
var_1

fprintf('portfolio not allow short selling:\n')
fprintf('portfolio weight:\n')
weight_2
fprintf('portfolio variance:\n')
var_2
```

PART 2 MATLAB OUTPUT

```
portfolio allow short selling:
portfolio weight:

weight_1 =

  Columns 1 through 3

   0.198961459294491   0.827314736960316   0.006359592527478
   0.198961491735537   0.827314414661926   0.006359052989499
   0.199426825101451   0.814260295008740  -0.014386190017745
   0.201091945301601   0.767559238560815  -0.088602713134251
   0.202756902525831   0.720861128541536  -0.162814478758626
   0.204422018370892   0.674160157970507  -0.237030863722609
   0.206087069487040   0.627459933536465  -0.311246013526550
   0.207752147620981   0.580759363689153  -0.385461731315102
   0.209417311323185   0.534057809447193  -0.459679078889547
   0.211082336890049   0.487357998938989  -0.533893556899015

  Columns 4 through 6

   0.058525373761296  -0.062004768019933  -0.017516754826553
   0.058525715673298  -0.062004541105956  -0.017516728087762
   0.071762969582065  -0.052749067596389  -0.016357046077475
   0.119119447418565  -0.019637667924675  -0.012208319965353
   0.166472897925809   0.013471645878894  -0.008059850761510
   0.213829287786722   0.046582984740305  -0.003911132168307
   0.261184895035826   0.079693796655670   0.000237523321700
   0.308540862595283   0.112804852660515   0.004386208266321
   0.355897862755215   0.145916603885927   0.008534976456005
   0.403253042887346   0.179027122923576   0.012683596078779

  Columns 7 through 8

   0.022225884331304  -0.033865524028400
   0.022225749196648  -0.033865155063189
   0.017508325701563  -0.019466111702210
   0.000631445155222   0.032046624588075
  -0.016244321913715   0.083556076561782
  -0.033121170324106   0.135068717346597
  -0.049997717082995   0.186580512572843
  -0.066874401041598   0.238092697524446
  -0.083751483075210   0.289605998097233
  -0.100627871159050   0.341117330339327

portfolio variance:

var_1 =

    0.000051217920135
    0.000051217920137
    0.000052813097870
    0.000084640317840
    0.000157290012944
    0.000270771176945
    0.000425077734150
    0.000620212824657
    0.000856180958377
    0.001132967080016
```

```
portfolio not allow short selling:
portfolio weight:

weight_2 =

  Columns 1 through 3

   0.197840862607479   0.826672804559739   0.007701573232407
   0.197391692332286   0.826414709554285   0.008239021004183
   0.185105465274723   0.804545192671392   0.000442855656924
   0.115425360384968   0.709442386312566   0.000088377909348
   0.045573293728373   0.614077069920554   0.000036537194648
   0.000006317002415   0.505996278416583   0.000000004244820
   0.000054326518344   0.374021301885113   0.000001053405133
   0.000002648565237   0.242100721980657   0.000000185474196
   0.000014249626791   0.110144300626345   0.000001667494361
   0.000000770567209   0.000009255385346   0.000000270149631

  Columns 4 through 6

   0.058731107471298  -0.062136729785869  -0.017494986201322
   0.058814184345091  -0.062189334012329  -0.017486217074043
   0.075912505867091  -0.053381777770570  -0.015945394677357
   0.143946385229900  -0.023417640651646  -0.009745408865869
   0.212080320016519   0.006525092331337  -0.003536309428937
   0.283165510163178   0.034688089487771   0.002801783804603
   0.359798095576560   0.059507281472032   0.009382422101652
   0.436417814126911   0.084333985213625   0.015962473193563
   0.513046268914826   0.109155794219570   0.022542913624700
   0.630021477817327   0.135891771002582   0.004406599834469

  Columns 7 through 8

   0.022317571044982  -0.033632202928714
   0.022354102960605  -0.033538159110078
   0.018145037695508  -0.014823884717711
   0.004438002048476   0.059822537632257
  -0.009270335002814   0.134514331240319
  -0.025823779237564   0.199165796118193
  -0.047722050753567   0.244957569794733
  -0.069608498603012   0.290790670048823
  -0.091502592071722   0.336597397565129

  -0.159708239132840   0.389378094376276

portfolio variance:

var_2 =

   0.000051218839560
   0.000051219723810
   0.000052943000272
   0.000089298271799
   0.000173004824308
   0.000304421535408
   0.000487832668110
   0.000724449165119
   0.001014294417785
   0.001362624035710
```