APS 502 Computational Project #1

Problem 1

Let $x = \begin{bmatrix} x_c \\ x_g \end{bmatrix}$ be the bond portfolio, where $x_c = amount\ of\ corporate\ bond\ purchased$, and $x_g = amount\ of\ government\ bond\ purchased.\ c = cash\ amout\ not\ invested\ in\ the\ bonds.$

maximize yield $0.04x_c + 0.03x_g$

subject to
$$x_c + x_g + c \leq 100000$$
$$\frac{2x_c}{100000} + \frac{x_g}{100000} \leq 1.5$$
$$\frac{3x_c}{100000} + \frac{4x_g}{100000} \leq 3.6$$
$$x_c, x_g, c \geq 0$$

Optimal bond portfolio: $x = \begin{bmatrix} 50000 \\ 50000 \end{bmatrix}, c = 0$

Maximum yield: $0.04\ (50000) + 0.03\ (50000) = 3500$

In order to maximize yield, manager should allocate \$50000 to corporate bond and \$50000 to government bond.

Problem 2, Part 1

Let $x_i = amount\ of\ bond\ i\ \ purchased\ (i = 1,2 \ldots 13)$,

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \ldots \\ x_{13} \end{bmatrix}$$ *is the bond portfolio*

$$z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \end{bmatrix}, where\ z_i = amount\ of\ cash\ to\ be\ carried\ on$$

minimize cost

$$108x_1 + 94x_2 + 99x_3 + 92.7x_4 + 96.6x_5 + 95.9x_6 + 92.9x_7 + 110x_8 + 104x_9 + 101x_{10} + 107x_{11} + 102x_{12} + 95.2x_{13}$$

subject to

$$10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 6x_6 + 5x_7 + 10x_8 + 8x_9 + 6x_{10} + 10x_{11} + 7x_{12} + 100x_{13} - z_1 \geq 500$$
$$10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 6x_6 + 5x_7 + 10x_8 + 8x_9 + 6x_{10} + 110x_{11} + 107x_{12} + z_1 - z_2 \geq 200$$
$$10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 6x_6 + 5x_7 + 110x_8 + 108x_9 + 106x_{10} + z_2 - z_3 \geq 800$$
$$10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 6x_6 + 5x_7 + 110x_8 + 108x_9 + 106x_{10} + z_3 - z_4 \geq 400$$
$$10x_1 + 7x_2 + 8x_3 + 106x_4 + 107x_5 + z_4 - z_5 \geq 700$$
$$110x_1 + 107x_2 + 108x_3 + z_5 \geq 900$$
$$x_1, x_2, \ldots x_{13} \geq 0$$
$$z_1, z_2, z_3, z_4, z_5 \geq 0$$

Optimal bond portfolio:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \end{bmatrix} = \begin{bmatrix} 8.1818 \\ 0 \\ 0 \\ 0 \\ 5.7774 \\ 2.6202 \\ 0 \\ 0 \\ 6.1298 \\ 0 \\ 0.1180 \\ 0 \\ 3.1180 \end{bmatrix}, z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Lowest cost that covers the streams of liabilities:

$$x_1p_1 + x_5p_5 + x_6p_6 + x_9p_9 + x_{11}p_{11} + x_{13}p_{13} = 2640$$

Problem 2, Part 2

Add one more constrain, and other condition remains the same as part 1:

V_{total}*: bond portfolio's value (in dollars)*

V_B*: bond portfolio's value in bonds rated B*

$$\frac{V_B}{V_{total}} \leq \frac{1}{2}$$

$$\Rightarrow 2(108x_1 + 94x_2 + 99x_3 + 92.7x_4 + 96.6x_5 + 95.9x_6) \leq 108x_1 + 94x_2 + 99x_3 + 92.7x_4 + 96.6x_5 + 95.9x_6 + 92.9x_7 + 110x_8 + 104x_9 + 101x_{10} + 107x_{11} + 102x_{12} + 95.2x_{13}$$

$$\Rightarrow 108x_1 + 94x_2 + 99x_3 + 92.7x_4 + 96.6x_5 + 95.9x_6 - 92.9x_7 - 110x_8 - 104x_9 - 101x_{10} - 107x_{11} - 102x_{12} - 95.2x_{13} \leq 0$$

Optimal bond portfolio:

$$x_2 = 8.4112,\ x_4 = 5.7422,\ x_7 = 3.2297, x_9 = 6.3937, x_{11} = 0.3579, x_{13} = 3.3579$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \end{bmatrix} = \begin{bmatrix} 0 \\ 8.4112 \\ 0 \\ 5.7422 \\ 0 \\ 0 \\ 3.2297 \\ 0 \\ 6.3937 \\ 0 \\ 0.3579 \\ 0 \\ 3.3579 \end{bmatrix}, z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 32.4504 \\ 0 \end{bmatrix}$$

Lowest cost that covers the streams of liabilities, and satisfy that at most 50% of bond portfolio's value in bonds rated B:

$$x_2p_2 + x_4p_4 + x_7p_7 + x_9p_9 + x_{11}p_{11} + x_{13}p_{13} = 2645.9$$

Compared to optimal bond portfolio from part 1, part 2 model has higher cost and higher overall rating, because the price for higher rating bond is more expensive.

Part 1 model: $\frac{V_B}{V_{total}} = \frac{x_1p_1 + x_5p_5 + x_6p_6}{2640} = \frac{8.1818\ (108) + 5.7774\ (96.6) + 2.6202\ (95.9)}{2640} = 0.6413$

Part 2 model: $\frac{V_B}{V_{total}} = \frac{x_2p_2 + x_4p_4}{2702.70} = \frac{8.4112\ (94) + 5.7422\ (92.7)}{2645.9} = 0.5$