Dynamic Asset Allocation Project

The goal of the optimization effort is to create a tool allows the investors to make regular decisions about re-balancing their portfolio and compare different investment strategies.

Part 1 Implement Financial Optimization Algorithm

1. "**Buy and hold**" strategy: hold initial portfolio for the entire investment horizon of 2 years

   portfolio weights $w_{20\times1} = [0, 0, 0, 0\ , 0, 0, 0, 0, 0, 980, 0, 0, 0, 0, 0, 0, 0, 0, 0, 20000]^T$

2. "**Equally weighted**" (also known as $\frac{1}{n}$) portfolio strategy: asset weights are selected as $w_i^t = \frac{1}{n}$, where $n = 20$ is the number of assets.

   portfolio weights $w_{20\times1} = \left[\frac{1}{20}\ \ \frac{1}{20}\ \ ...\ \ \frac{1}{20}\right]^T$

3. "**Minimum variance**" portfolio strategy: minimum variance portfolio

$$\begin{aligned} \min_{w} \quad & w^T Q w \\ s.t. \quad & \sum_{i=1}^{n} w_i = 1 \\ & w_i \geq 0 \end{aligned}$$

   Estimate portfolio weights $w_{20\times1} = [w_1\ \ w_2\ \ ...w_{20}]^T$

   Define CPLEX objects:

   1. Linear part of objective $c_{20\times1} = [0\ \ 0\ \ \ ...\ \ 0]^T$
   2. Quadratic part of objective $Q_{mat} = 2Q$
   3. Bounds on variables $lb_{20\times1} = [0\ \ 0\ \ ...0]^T$
      $ub_{20\times1} = [\infty\ \ \infty ... \infty]^T$
   4. Linear constraints $A_{1\times20} = [1\ \ \ 1\ \ \ \ ...\ \ \ 1]\ s.t.\ Aw = 1$

4. "**Maximum Sharpe ratio**" portfolio strategy: maximizes Sharpe ratio

$$\begin{aligned} \min_{y\in R^n, k\in R} \quad & y^T Q y \\ s.t. \quad & \sum_{i=1}^{n} (\mu_i - r_f) y_i = 1 \\ & \sum_{i=1}^{n} y_i = k \\ & y \geq 0, k \geq 0 \end{aligned}$$

   Estimate portfolio weights $x_{21\times1} = [y_1\ \ y_2\ \ ...y_{20}\ \ \ k]^T$

   Define CPLEX objects:

   1. Linear part of objective $c_{21\times1} = [0\ \ 0\ \ \ ...\ \ 0\ \ 0]^T$
   2. Quadratic part of objective $Q_{mat_{21\times21}} = \begin{bmatrix} 2Q & 0 \\ 0 & 0 \end{bmatrix}$
   3. Bounds on variables $lb_{21\times1} = [0\ \ 0\ \ ...0]^T$
      $ub_{21\times1} = [\infty\ \ \infty ... \infty]^T$
   4. Linear constraints $A_{1\times20} = \begin{bmatrix} \mu_1 - r_f & ... & \mu_{20} - r_f & 0 \\ 1 & ... & 1 & -1 \end{bmatrix}\ s.t.$
      $\sum_{i=1}^{n} (\mu_i - r_f) y_i = 1,\ \ \sum_{i=1}^{n} y_i = k$

   Computing portfolio weight $w = \frac{y}{k}$

5. "**Equal risk contributions**" portfolio strategy: compute a portfolio that has equal risk contributions to standard deviation for each period and re-balance accordingly.

$$\begin{aligned} \min_{w} \quad & \sum_{i=1}^{n}\sum_{j=1}^{n}\left(w_i(Qw)_i - w_j(Qw)_j\right)^2 \\ s.t. \quad & \sum_{i=1}^{n} w_i = 1 \\ & w_i \geq 0 \end{aligned}$$

Estimate portfolio weights $w_{20\times1} = [w_1 \ w_2 \ \dots w_{20}]^T$

Define IPOPT objects:

1. Starting point $w_{20\times1} = \left[\frac{1}{20} \ \frac{1}{20} \ \dots \ \frac{1}{20}\right]^T$
2. Bounds on variable weights $lb_{20\times1} = [0 \ 0 \ \dots 0]^T$, $ub_{20\times1} = [1 \ 1 \ \dots 1]^T$
3. Bounds on constraints $cl = 1$, $cu = 1$
4. Gradient of objective $\frac{f(x)}{\partial x_i} = 8(Qw)_i\sum_{j=1}^{n}(w_i(Qw)_i - w_j(Qw)_j)$

6. "**Leveraged equal risk contributions**" portfolio strategy: take long 200% position in equal risk contributions portfolio and short risk-free asset for each period and re-balance accordingly.

1. Borrow money (long 200% position) at risk-free interest rate in the beginning of current period
2. Use same portfolio weights of "Equal risk contributions" to allocate the money on assets
3. Return money include interest in the beginning of next period
4. Repeat step 1-3 total 12 times.

7. "**Robust mean-variance optimization**" portfolio strategy: compute a robust mean-variance portfolio for each period and re-balance accordingly.

$$\begin{aligned} \min_{w} \quad & w^T Q w \\ s.t. \quad & \sum_{i=1}^{n} w_i = 1 \\ & w_i \geq 0 \\ & \mu^T w \geq \varepsilon_{ret} \\ & w\Theta^T w \leq \tilde{\varepsilon}_{rob} \end{aligned}$$

Estimate portfolio weights $w_{20\times1} = [w_1 \ w_2 \ \dots w_{20}]^T$

Select targets in order to derive a portfolio have return greater than "Minimum variance" portfolio, and standard deviation smaller than "Equally weighted" portfolio.

1. Target risk estimation error $\tilde{\varepsilon}_{rob}$ — variance of "Equally weighted" portfolio
2. Target return ε_{ret} — return of "Minimum variance" portfolio

Define CPLEX objects

1. Linear part of objective $c_{20\times1} = [0 \ 0 \ \dots 0]^T$
2. Quadratic part of objective $Q_{mat} = 2Q$
3. Bounds on variables $lb_{20\times1} = [0 \ 0 \ \dots 0]^T$, $ub_{20\times1} = [1 \ 1 \ \dots 1]^T$
4. Linear constraints $A = \begin{bmatrix} 1 & 1 & \dots & 1 \\ \mu_1 & \mu_2 & \dots & \mu_{20} \end{bmatrix} s.t. \ Aw = 1, and \ \mu^T w \geq \varepsilon_{ret}$
5. Quadratic constraints $\Theta = \mathrm{diag(Q)} \ \ s.t. \ w\Theta^T w \leq \tilde{\varepsilon}_{rob}$

Above steps illustrate how to compute the portfolio weights, and you could use that information to calculate transaction cost and cash account.

Part 2 Re-balance portfolio (years 2019 and 2020)

Re-balance portfolio according to an investment strategy at the first trading day of each 2-month holding period (up to 12 re-balances for 2 years): given a current portfolio, the market prices on that day, and the estimates of the mean and covariance of the daily returns, make a decision about what to buy and sell according to a strategy. Need to take into account the trading costs.

Design a rounding procedure (round the number of shares traded to integer values)

If cash account is nonnegative, round down the position after rebalancing and update transaction cost and cash account. Otherwise, first reduce position by allocation negative cash account into assets according to portfolio weights, then round down the position and update transaction cost and cash account. In this way, investors always trade an integer number of shares and will not go over the budget.

Design a validation procedure (verify if strategie is feasible, have enough budget to re-balance portfolio)

Using an if statement, if cash account is negative, validation procedure will print out a string indicates negative cash account with specific strategy followed specific period.

For "Leveraged equal risk contributions" strategy, the money supposed to be returned in the beginning of next period. However, the algorithm subtracts the borrowed money plus the interest in the end of current period. As a result, the validation procedure would indicate cash account is negative. Have to manually check whether the cash account is negative or not before return money.

$cash\ account_{2019-2020} = [2722.22, 1869.98, 3239.97, 2224.37, 2915.67, 2596.23, 2773.02, 1770.1, 1185.35, 2685.14, 1267.66, 1964.95]$

$cash\ account_{2008-2009} = [815.24, 318.86, 397.68, 443.58, 400.96, 440.64, 318.37, 156.45, 275.81, 336.48, 335, 240.09]$

Compute portfolio values at the first and last trading day for the 12 periods (complete results in appendix)

Period 1: start date 01/02/2019, end date 02/28/2019

Strategy "Buy and Hold", value begin = $ 1000070.06, value end = $ 1121179.83

Strategy "Equally Weighted Portfolio", value begin = $ 991110.53, value end = $ 1097139.05

Strategy "Minimum Variance Portfolio", value begin = $ 991694.21, value end = $ 1057724.38

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 990119.39, value end = $ 1016524.41

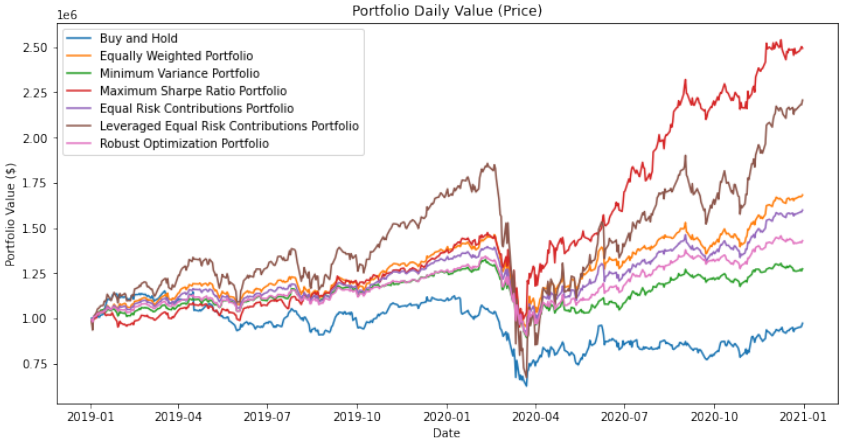
Strategy "Equal Risk Contributions Portfolio", value begin = $ 991350.69, value end = $ 1086730.66

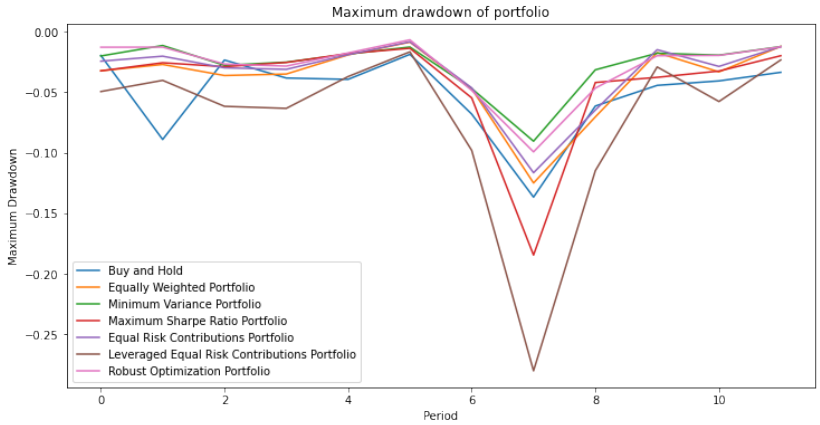
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 983446.28, value end = $ 1174676.92

Strategy "Robust Optimization Portfolio", value begin = $ 992339.32, value end = $ 1074547.25

…

Plot **daily value of portfolio** (for each of the seven trading strategies).

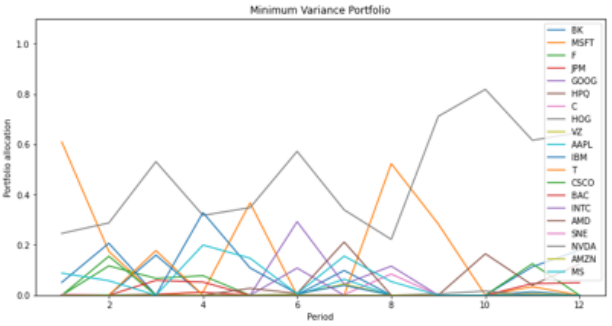


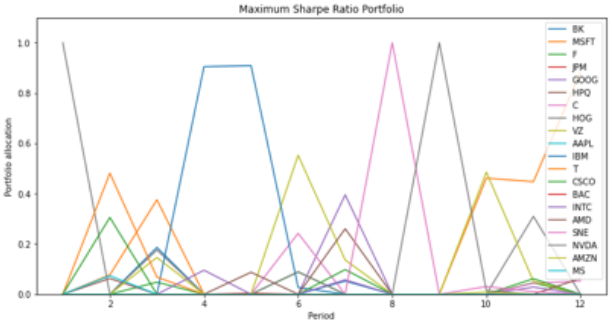


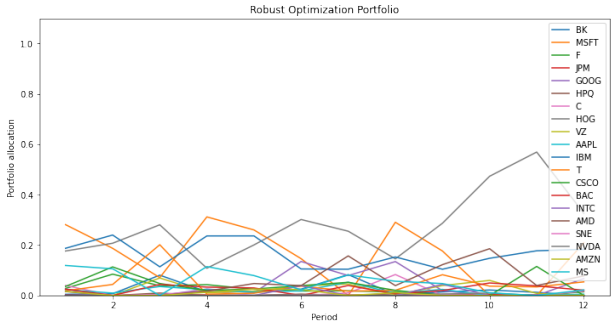
Plot **maximum drawdown of portfolio** (for each of the seven trading strategies) for each of the 12 periods.

Plot **dynamic changes in portfolio allocations** under strategy 3, 4 and 7. In each chart, x-axis represents the rolling up time horizon, y-axis denotes portfolio weights between 0 and 1, and distinct lines display the position of selected assets over time periods.

Robust portfolio selection strategy reduces trading strategy compared with strategies 3 and 4. All portfolios constructed are on the efficient frontier. Strategies 3 and 4 are more sensitive to the change of expected return. Hence, the portfolio weights will change more than robust portfolio each period after rebalancing.







Compare trading strategies and discuss their performance relative to each other.

"Leveraged equal risk contributions" takes long 200% position, it has high return and high risk (standard deviation). This is a dangerous trading strategy for new investor due to high leverage and there's a chance to get into debt. ***"Equal risk contributions"*** has moderate return and risk. This strategy diversify risk by allocation risk equally into asset. Both strategies are more depend on the market than "Robust mean-variance optimization", and they perform well in good market. ***"Robust mean-variance optimization"*** is efficient and has the best performance among last three strategies. It will have highest return among three strategies if they have same standard deviations.

From plots of *daily value* and *maximum drawdown*, classify expected return and standard deviation from high to low.

	Expected Return (2019 – 2020)	Risk (standard deviation)
high	"Maximum Sharpe ratio" "Leveraged equal risk contributions"	"Leveraged equal risk contributions" "Maximum Sharpe ratio" "Buy and hold"
moderate	"Equally weighted" "Equal risk contributions" "Robust mean-variance optimization" "Minimum variance"	"Equally weighted" "Equal risk contributions"
low	"Buy and hold"	"Robust mean-variance optimization" "Minimum variance"

"Equally weighted" has moderate return and risk. The portfolio is diversified at some level by allocation money equally into assets. And it not required frequent buy and sell unless stock price change dramatically, so the transaction cost is lower, and this amount could be invested to gain more profit. Noted the risk is higher than "Robust mean-variance optimization", which is set by the CPLEX optimization program of RMV. ***"Buy and hold"*** has low return but high risk. The portfolio value only increases when the prices of initial holding assets rise. Not analyzing the market's behavior would lead to investment failure.

"Maximum Sharpe ratio", ***"Minimum variance"*** and ***"Robust mean-variance optimization"*** strategies return efficient portfolios, they are different points along the efficient frontier correspond to different risk preference. The higher the return, the larger the risk.

I would invest by using ***"Robust mean-variance optimization"*** since it would allow me to earn great return without take too much risk.

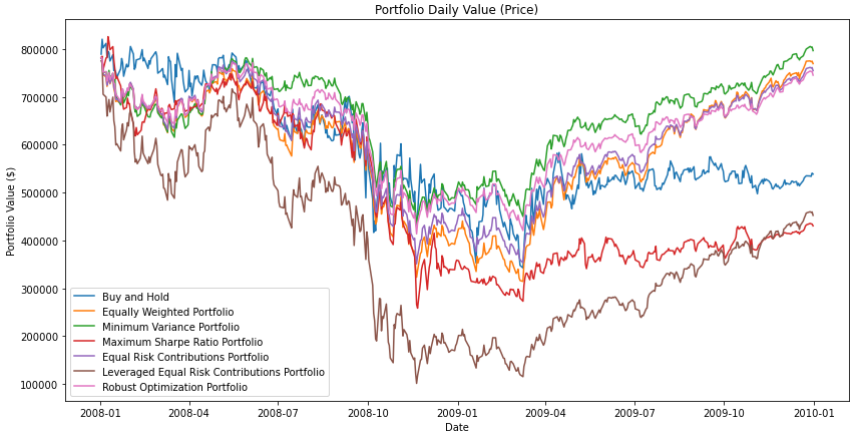
Part 3 Re-balance portfolio (years 2008 and 2009)

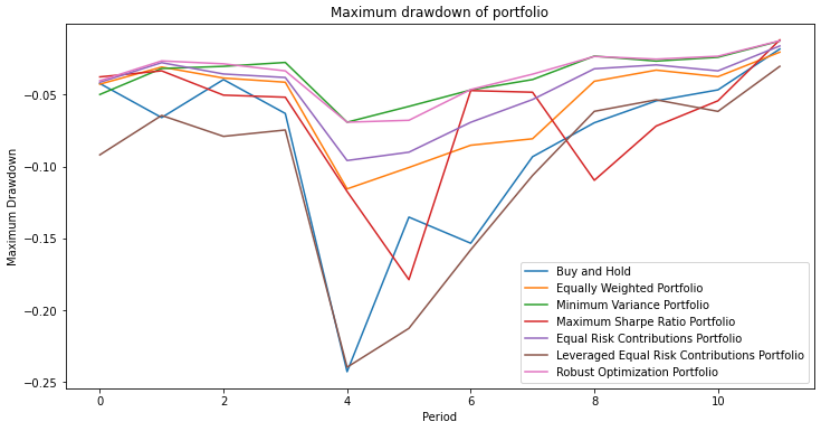
Compute portfolio values at the first and last trading day for the 12 periods (complete results in appendix)

Period 1: start date 01/02/2008, end date 02/29/2008
Strategy "Buy and Hold", value begin = $ 789230.94, value end = $ 749509.71
Strategy "Equally Weighted Portfolio", value begin = $ 782158.10, value end = $ 669692.42
Strategy "Minimum Variance Portfolio", value begin = $ 781379.45, value end = $ 666830.57
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 781378.15, value end = $ 681744.86
Strategy "Equal Risk Contributions Portfolio", value begin = $ 782129.34, value end = $ 674108.17
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 775567.54, value end = $ 558995.70
Strategy "Robust Optimization Portfolio", value begin = $ 782106.92, value end = $ 679233.31
…

For strategy ***"Maximum Sharpe ratio"***, all assets had negative expected return μ for some period during 2008 financial crisis. There is no feasible solution for optimal weights since it violated the constraint $\sum_i(\mu_i - rf)y_i = 1$. Keep the same portfolio in this case.

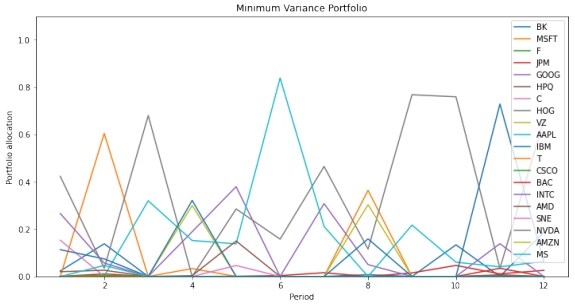
Plot **daily value of portfolio** (for each of the seven trading strategies).

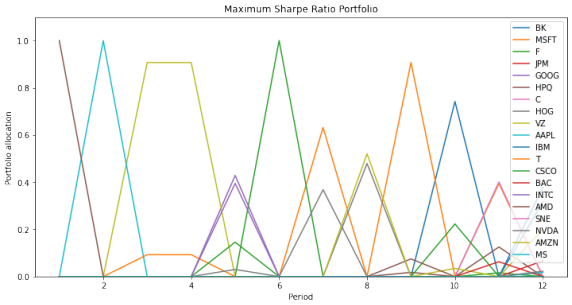


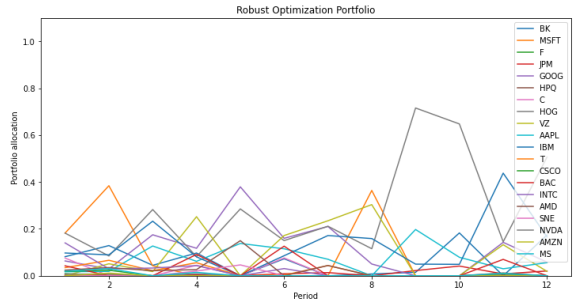


Plot **maximum drawdown of portfolio** (for each of the seven trading strategies) for each of the 12 periods.

Plot **dynamic changes in portfolio allocations** under strategy 3, 4 and 7.







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Compare trading strategies and discuss their performance relative to each other.

From plots of *daily value* and *maximum drawdown*, classify expected return and standard deviation from high to low.

	Expected Return (2008 – 2009)	Risk (standard deviation)
high	"Minimum variance" "Equally weighted" "Equal risk contributions" "Robust mean-variance optimization"	"Leveraged equal risk contributions" "Maximum Sharpe ratio" "Buy and hold"
moderate	"Buy and hold"	"Equally weighted" "Equal risk contributions"
low	"Maximum Sharpe ratio" "Leveraged equal risk contributions"	"Robust mean-variance optimization" "Minimum variance"

The market crashed during 2008 and covid-19 crisis. The plot of daily value of portfolio shows 2008 financial crisis are more severe, the portfolio value dropped more in percentage and the market takes more time to recover. The strategies with high risk like "**Maximum Sharpe ratio**" and "**Leveraged equal risk contributions**" heavily influenced by the change of the market. They performed worse during 2008-2009 than during 2019-2020, since they perform badly when the market experienced prolonged price drop, but perform well when the market expect price rising. On the other hand, strategies with low risk like ***"Minimum variance", "Equally weighted", "Equal risk contributions"*** and ***"Robust mean-variance optimization"*** were influenced less by the volatility of the market. They performed better during 2008-2009 than during 2019-2020, since they are able to tolerant some risk. ***"Buy and hold"*** has moderate return, its performance relies on the allocation of initial portfolio which remains same and not adjustable during whole trading periods.

Appendix

For years 2019 and 2020

Period 1: start date 01/02/2019, end date 02/28/2019
Strategy "Buy and Hold", value begin = $ 1000070.06, value end = $ 1121179.83
Strategy "Equally Weighted Portfolio", value begin = $ 991110.53, value end = $ 1097139.05
Strategy "Minimum Variance Portfolio", value begin = $ 991694.21, value end = $ 1057724.38
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 990119.39, value end = $ 1016524.41
Strategy "Equal Risk Contributions Portfolio", value begin = $ 991350.69, value end = $ 1086730.66
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 983446.28, value end = $ 1174676.92
Strategy "Robust Optimization Portfolio", value begin = $ 992339.32, value end = $ 1074547.25

Period 2: start date 03/01/2019, end date 04/30/2019
Strategy "Buy and Hold", value begin = $ 1126131.27, value end = $ 1075001.89
Strategy "Equally Weighted Portfolio", value begin = $ 1103425.44, value end = $ 1188889.41
Strategy "Minimum Variance Portfolio", value begin = $ 1055665.23, value end = $ 1108286.33
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1007117.74, value end = $ 1076765.61
Strategy "Equal Risk Contributions Portfolio", value begin = $ 1090630.16, value end = $ 1157845.60
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1177450.15, value end = $ 1323589.17
Strategy "Robust Optimization Portfolio", value begin = $ 1074340.83, value end = $ 1122847.09

Period 3: start date 05/01/2019, end date 06/28/2019
Strategy "Buy and Hold", value begin = $ 1070867.54, value end = $ 969057.81
Strategy "Equally Weighted Portfolio", value begin = $ 1181393.74, value end = $ 1169301.49
Strategy "Minimum Variance Portfolio", value begin = $ 1092250.70, value end = $ 1099713.54
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1060451.78, value end = $ 1073388.89
Strategy "Equal Risk Contributions Portfolio", value begin = $ 1148388.20, value end = $ 1137079.10
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1297459.60, value end = $ 1271719.60
Strategy "Robust Optimization Portfolio", value begin = $ 1107850.57, value end = $ 1108871.34

Period 4: start date 07/01/2019, end date 08/30/2019
Strategy "Buy and Hold", value begin = $ 976973.31, value end = $ 933721.61
Strategy "Equally Weighted Portfolio", value begin = $ 1179796.24, value end = $ 1150034.55
Strategy "Minimum Variance Portfolio", value begin = $ 1097559.87, value end = $ 1129571.96
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1071209.32, value end = $ 1140264.81
Strategy "Equal Risk Contributions Portfolio", value begin = $ 1143098.02, value end = $ 1126352.25
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1280061.65, value end = $ 1242431.39
Strategy "Robust Optimization Portfolio", value begin = $ 1109294.23, value end = $ 1110363.07

Period 5: start date 09/03/2019, end date 10/31/2019
Strategy "Buy and Hold", value begin = $ 922211.42, value end = $ 1028337.74
Strategy "Equally Weighted Portfolio", value begin = $ 1138330.91, value end = $ 1252907.51
Strategy "Minimum Variance Portfolio", value begin = $ 1115840.21, value end = $ 1182581.22
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1139080.87, value end = $ 1246475.50
Strategy "Equal Risk Contributions Portfolio", value begin = $ 1116150.65, value end = $ 1217011.46
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1214362.18, value end = $ 1435049.27
Strategy "Robust Optimization Portfolio", value begin = $ 1100551.07, value end = $ 1183461.31
Period 6: start date 11/01/2019, end date 12/31/2019
Strategy "Buy and Hold", value begin = $ 1037933.42, value end = $ 1099403.03
Strategy "Equally Weighted Portfolio", value begin = $ 1270619.43, value end = $ 1373612.58
Strategy "Minimum Variance Portfolio", value begin = $ 1184635.69, value end = $ 1256110.15
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1248551.92, value end = $ 1369849.65
Strategy "Equal Risk Contributions Portfolio", value begin = $ 1231427.79, value end = $ 1323342.21
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1460359.77, value end = $ 1679664.37
Strategy "Robust Optimization Portfolio", value begin = $ 1191246.37, value end = $ 1260434.80

Period 7: start date 01/02/2020, end date 02/28/2020
Strategy "Buy and Hold", value begin = $ 1112112.69, value end = $ 900207.54
Strategy "Equally Weighted Portfolio", value begin = $ 1396423.77, value end = $ 1258476.44
Strategy "Minimum Variance Portfolio", value begin = $ 1256482.80, value end = $ 1159523.89
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1379227.69, value end = $ 1284635.75

Strategy "Equal Risk Contributions Portfolio", value begin = $ 1341597.82, value end = $ 1217151.26
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1715721.02, value end = $ 1395728.68
Strategy "Robust Optimization Portfolio", value begin = $ 1269042.95, value end = $ 1173571.68

Period 8: start date 03/02/2020, end date 04/30/2020
Strategy "Buy and Hold", value begin = $ 924774.25, value end = $ 856285.51
Strategy "Equally Weighted Portfolio", value begin = $ 1312362.36, value end = $ 1215384.78
Strategy "Minimum Variance Portfolio", value begin = $ 1209874.83, value end = $ 1076960.66
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1340311.96, value end = $ 1417151.32
Strategy "Equal Risk Contributions Portfolio", value begin = $ 1270623.86, value end = $ 1156052.01
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1526902.07, value end = $ 1250002.75
Strategy "Robust Optimization Portfolio", value begin = $ 1226968.13, value end = $ 1107208.22

Period 9: start date 05/01/2020, end date 06/30/2020
Strategy "Buy and Hold", value begin = $ 822532.65, value end = $ 875128.45
Strategy "Equally Weighted Portfolio", value begin = $ 1171234.18, value end = $ 1316237.52
Strategy "Minimum Variance Portfolio", value begin = $ 1046176.45, value end = $ 1081152.03
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1357463.54, value end = $ 1637822.10
Strategy "Equal Risk Contributions Portfolio", value begin = $ 1115040.01, value end = $ 1244225.78
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1145706.82, value end = $ 1412945.97
Strategy "Robust Optimization Portfolio", value begin = $ 1070465.12, value end = $ 1173877.59

Period 10: start date 07/01/2020, end date 08/31/2020
Strategy "Buy and Hold", value begin = $ 852159.31, value end = $ 852474.32
Strategy "Equally Weighted Portfolio", value begin = $ 1307182.90, value end = $ 1494129.26
Strategy "Minimum Variance Portfolio", value begin = $ 1084288.34, value end = $ 1242755.18
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1698911.14, value end = $ 2238261.54
Strategy "Equal Risk Contributions Portfolio", value begin = $ 1239262.11, value end = $ 1427723.00
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1396575.55, value end = $ 1824158.77
Strategy "Robust Optimization Portfolio", value begin = $ 1182537.77, value end = $ 1345541.04

Period 11: start date 09/01/2020, end date 10/30/2020
Strategy "Buy and Hold", value begin = $ 857122.42, value end = $ 795062.75
Strategy "Equally Weighted Portfolio", value begin = $ 1504824.50, value end = $ 1407554.23
Strategy "Minimum Variance Portfolio", value begin = $ 1244968.45, value end = $ 1193594.76
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 2285546.99, value end = $ 2164732.78
Strategy "Equal Risk Contributions Portfolio", value begin = $ 1438427.98, value end = $ 1350040.62
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1840037.59, value end = $ 1611698.27
Strategy "Robust Optimization Portfolio", value begin = $ 1351729.90, value end = $ 1294543.81

Period 12: start date 11/02/2020, end date 12/31/2020
Strategy "Buy and Hold", value begin = $ 811070.20, value end = $ 972162.37
Strategy "Equally Weighted Portfolio", value begin = $ 1419989.30, value end = $ 1682329.90
Strategy "Minimum Variance Portfolio", value begin = $ 1204236.99, value end = $ 1273585.40
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 2162771.27, value end = $ 2493427.99
Strategy "Equal Risk Contributions Portfolio", value begin = $ 1361171.18, value end = $ 1598524.48
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1633325.23, value end = $ 2206617.10
Strategy "Robust Optimization Portfolio", value begin = $ 1304500.36, value end = $ 1429517.46

For years 2008 and 2009

Period 1: start date 01/02/2008, end date 02/29/2008
Strategy "Buy and Hold", value begin = $ 789230.94, value end = $ 749509.71
Strategy "Equally Weighted Portfolio", value begin = $ 782158.10, value end = $ 669692.42
Strategy "Minimum Variance Portfolio", value begin = $ 781379.45, value end = $ 666830.57
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 781378.15, value end = $ 681744.86
Strategy "Equal Risk Contributions Portfolio", value begin = $ 782129.34, value end = $ 674108.17
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 775567.54, value end = $ 558995.70
Strategy "Robust Optimization Portfolio", value begin = $ 782106.92, value end = $ 679233.31

Period 2: start date 03/03/2008, end date 04/30/2008
Strategy "Buy and Hold", value begin = $ 754361.26, value end = $ 752687.89
Strategy "Equally Weighted Portfolio", value begin = $ 659705.67, value end = $ 726740.28
Strategy "Minimum Variance Portfolio", value begin = $ 657758.18, value end = $ 745536.40
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 667671.09, value end = $ 705483.22
Strategy "Equal Risk Contributions Portfolio", value begin = $ 664197.28, value end = $ 739831.98
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 536648.31, value end = $ 659706.17
Strategy "Robust Optimization Portfolio", value begin = $ 670983.30, value end = $ 740765.79

Period 3: start date 05/01/2008, end date 06/30/2008
Strategy "Buy and Hold", value begin = $ 779329.50, value end = $ 663602.44
Strategy "Equally Weighted Portfolio", value begin = $ 750503.35, value end = $ 632811.20
Strategy "Minimum Variance Portfolio", value begin = $ 760638.52, value end = $ 716542.62
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 715301.68, value end = $ 640487.94
Strategy "Equal Risk Contributions Portfolio", value begin = $ 761938.53, value end = $ 660674.23
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 692628.46, value end = $ 507434.63
Strategy "Robust Optimization Portfolio", value begin = $ 758127.19, value end = $ 695802.53

Period 4: start date 07/01/2008, end date 08/29/2008
Strategy "Buy and Hold", value begin = $ 674748.24, value end = $ 619979.82
Strategy "Equally Weighted Portfolio", value begin = $ 633743.28, value end = $ 647028.72
Strategy "Minimum Variance Portfolio", value begin = $ 717240.72, value end = $ 726244.06
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 665688.61, value end = $ 645164.69
Strategy "Equal Risk Contributions Portfolio", value begin = $ 659955.31, value end = $ 668388.60
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 503828.80, value end = $ 516813.67
Strategy "Robust Optimization Portfolio", value begin = $ 692571.01, value end = $ 695321.09

Period 5: start date 09/02/2008, end date 10/31/2008
Strategy "Buy and Hold", value begin = $ 621151.79, value end = $ 579282.75
Strategy "Equally Weighted Portfolio", value begin = $ 648620.38, value end = $ 473989.94
Strategy "Minimum Variance Portfolio", value begin = $ 710132.36, value end = $ 548418.30
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 627266.68, value end = $ 469277.32
Strategy "Equal Risk Contributions Portfolio", value begin = $ 667920.34, value end = $ 495917.70
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 513920.21, value end = $ 247640.58
Strategy "Robust Optimization Portfolio", value begin = $ 685674.98, value end = $ 529525.82

Period 6: start date 11/03/2008, end date 12/31/2008
Strategy "Buy and Hold", value begin = $ 576738.59, value end = $ 500698.25
Strategy "Equally Weighted Portfolio", value begin = $ 471107.93, value end = $ 407311.95
Strategy "Minimum Variance Portfolio", value begin = $ 549181.59, value end = $ 505940.31
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 463706.03, value end = $ 358974.74
Strategy "Equal Risk Contributions Portfolio", value begin = $ 494211.31, value end = $ 434761.28
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 243499.46, value end = $ 184471.28
Strategy "Robust Optimization Portfolio", value begin = $ 531210.64, value end = $ 493103.33

Period 7: start date 01/02/2009, end date 02/27/2009
Strategy "Buy and Hold", value begin = $ 505855.81, value end = $ 414408.26
Strategy "Equally Weighted Portfolio", value begin = $ 424687.35, value end = $ 348038.49
Strategy "Minimum Variance Portfolio", value begin = $ 522354.74, value end = $ 497715.59
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 357684.81, value end = $ 297790.07
Strategy "Equal Risk Contributions Portfolio", value begin = $ 452958.80, value end = $ 386744.36
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 201674.65, value end = $ 142396.48

Strategy "Robust Optimization Portfolio", value begin = $ 509631.04, value end = $ 463400.24

Period 8: start date 03/02/2009, end date 04/30/2009
Strategy "Buy and Hold", value begin = $ 400004.61, value end = $ 475987.18
Strategy "Equally Weighted Portfolio", value begin = $ 331192.37, value end = $ 532505.63
Strategy "Minimum Variance Portfolio", value begin = $ 478086.28, value end = $ 652574.59
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 283420.05, value end = $ 369416.73
Strategy "Equal Risk Contributions Portfolio", value begin = $ 370568.83, value end = $ 555789.33
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 127267.64, value end = $ 255296.22
Strategy "Robust Optimization Portfolio", value begin = $ 446863.11, value end = $ 609956.40

Period 9: start date 05/01/2009, end date 06/30/2009
Strategy "Buy and Hold", value begin = $ 483627.06, value end = $ 538125.39
Strategy "Equally Weighted Portfolio", value begin = $ 531618.60, value end = $ 559588.83
Strategy "Minimum Variance Portfolio", value begin = $ 647362.22, value end = $ 654335.24
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 367170.82, value end = $ 379049.22
Strategy "Equal Risk Contributions Portfolio", value begin = $ 554804.30, value end = $ 576360.79
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 253118.94, value end = $ 272910.26
Strategy "Robust Optimization Portfolio", value begin = $ 605256.89, value end = $ 613734.77

Period 10: start date 07/01/2009, end date 08/31/2009
Strategy "Buy and Hold", value begin = $ 528549.59, value end = $ 554215.70
Strategy "Equally Weighted Portfolio", value begin = $ 558946.53, value end = $ 658850.60
Strategy "Minimum Variance Portfolio", value begin = $ 653725.90, value end = $ 697226.01
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 369815.56, value end = $ 399956.53
Strategy "Equal Risk Contributions Portfolio", value begin = $ 576360.04, value end = $ 655667.99
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 271767.96, value end = $ 346934.86
Strategy "Robust Optimization Portfolio", value begin = $ 613535.81, value end = $ 656268.07

Period 11: start date 09/01/2009, end date 10/30/2009
Strategy "Buy and Hold", value begin = $ 529171.54, value end = $ 510275.57
Strategy "Equally Weighted Portfolio", value begin = $ 635151.26, value end = $ 672310.43
Strategy "Minimum Variance Portfolio", value begin = $ 680548.84, value end = $ 714749.12
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 382682.34, value end = $ 382316.48
Strategy "Equal Risk Contributions Portfolio", value begin = $ 635726.72, value end = $ 674288.74
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 326590.05, value end = $ 366446.95
Strategy "Robust Optimization Portfolio", value begin = $ 641054.56, value end = $ 670450.25

Period 12: start date 11/02/2009, end date 12/31/2009
Strategy "Buy and Hold", value begin = $ 515205.91, value end = $ 538768.59
Strategy "Equally Weighted Portfolio", value begin = $ 675078.62, value end = $ 769442.27
Strategy "Minimum Variance Portfolio", value begin = $ 709456.59, value end = $ 796664.36
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 380383.65, value end = $ 430687.15
Strategy "Equal Risk Contributions Portfolio", value begin = $ 675604.41, value end = $ 754770.04
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 366241.93, value end = $ 452596.04
Strategy "Robust Optimization Portfolio", value begin = $ 666694.94, value end = $ 745409.97
Daily value of portfolio