MIE 1622H Assignment 1 - Mean-Variance Portfolio Selection Strategies

1. Implement portfolio re-balancing investment strategies in Python

1. "Buy and hold" strategy: hold initial portfolio for the entire investment horizon of 2 years

portfolio weight $w_{20\times1} = [0, 0, 0, 0, 0, 0, 0, 0, 0, 980, 0, 0, 0, 0, 0, 0, 0, 0, 0, 20000]^T$

2. "Equally weighted" (also known as $\frac{1}{n}$) portfolio strategy: asset weights are selected as $w_i^t = \frac{1}{n}$, where n is the number of assets.

portfolio weight $w_{20\times1} = \left[\frac{1}{20}\ \frac{1}{20}\ \dots\ \frac{1}{20}\right]^T$

3. "Minimum variance" portfolio strategy: minimum variance portfolio

$$\min_{w}\ w^T Q w$$
$$s.t. \sum_i w_i = 1, \quad w_i \geq 0$$

Estimate portfolio weight $w_{20\times1} = [w_1\ w_2\ \dots\ w_{20}]^T$
Define CPLEX objects:

1. Linear part of objective function: $c_{20\times1} = [0\ 0\ \dots\ 0]^T$
2. Quadratic part of objective function: $Qmat_{20\times20} = 2Q$
3. Bounds on variables: $lb_{20\times1} = [0\ 0\ \dots\ 0]^T, ub_{20\times1} = [\infty\ \infty\ \dots\ \infty]^T$
4. Linear constrain: $A_{1\times20} = [1\ 1\ \dots\ 1]\ s.t.\ Aw = 1$

4. "Maximum Sharpe ratio" portfolio strategy: maximizes Sharpe ratio

$$\min_{y\in R^n, k\in R}\ y^T Q y$$
$$s.t. \sum_i(\mu_i - rf)y_i = 1,\ \sum_i y_i - k = 0\ (derived\ from\ \sum_i y_i = k)$$
$$x \geq 0\ (derived\ from\ lk \leq Ay \leq ul,\ k \geq 0)$$

Estimate parameter $x_{21\times1} = [y_1\ y_2\ \dots\ y_{20}\ \ k]^T$
Define CPLEX objects:

1. Linear part of objective function: $c_{21\times1} = [0\ 0\ \dots\ 0\ 0]^T$
2. Quadratic part of objective function: $Qmat_{21\times21} = \begin{bmatrix} 2Q & & 0 \\ & \dots & \\ 0 & & 0 \end{bmatrix}$
3. Bounds on variables: $lb_{21\times1} = [0\ 0\ \dots\ 0\ \ 0]^T, ub_{21\times1} = [\infty\ \infty\ \dots\ \infty]^T$
4. Linear constrain: $A = \begin{bmatrix} \mu_1 - rf, \dots, \mu_{20} - rf, & 0 \\ 1 \quad \dots \quad 1 & -1 \end{bmatrix}\ s.t.\ \begin{matrix} \sum_i(\mu_i - rf)y_i = 1 \\ \sum_i y_i - k = 0 \end{matrix}$

Computing portfolio weight $w = \frac{y}{k}$

Above steps illustrate how to compute the portfolio weights, and you could use that information to calculate transaction cost and cash account.

Design and implement a rounding procedure, so that you always trade (buy or sell) an integer number of shares.

"Buy and hold" strategy already has integer position since it holds initial portfolio ("IBM" = 980 shares, "BK" = 20000 shares) during all investment horizons. We have to design a rounding procedure for the "Equally weighted", "Minimum variance" and "Maximum Sharpe ratio" portfolio strategy. All strategy functions return positions and cash accounts after rebalancing. Using **if statement,** if cash account is nonnegative, round up the position after rebalancing and update transaction cost and cash account. Using **while statement**, while cash account is negative, decrease positions of all assets by 0.05 % (using 0.5% for Maximum Sharpe ratio to save computational time) and round up, then update the transaction cost and cash account is each iteration. In this way, investors always trade an integer number of shares and will not go over the budget.

Design and implement a validation procedure to test each of your strategies is feasible (you have enough budget to re-balance portfolio, you correctly compute transaction costs, funds in your cash account are non-negative).

Using an **if statement**, if cash account is negative, validation procedure will print out a string indicates negative cash account with specific strategy followed specific period. Example would look like:

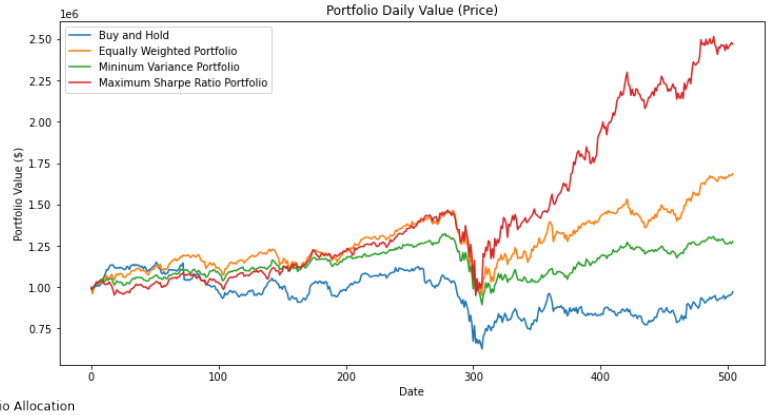
> Period 1: start date 01/02/2019, end date 02/28/2019
> Strategy "Buy and Hold", cash account is negative

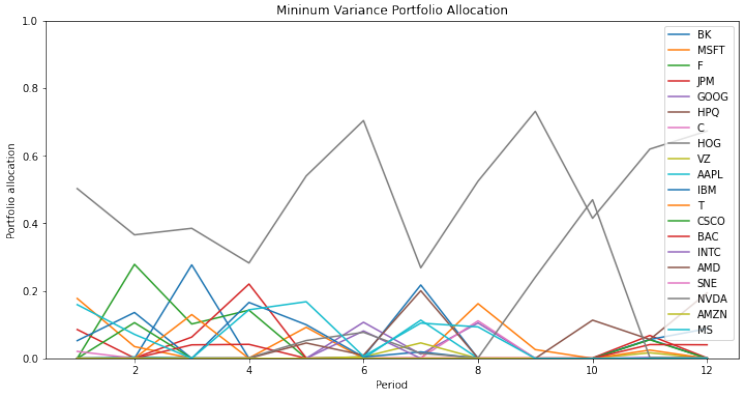
2. Analyze your results

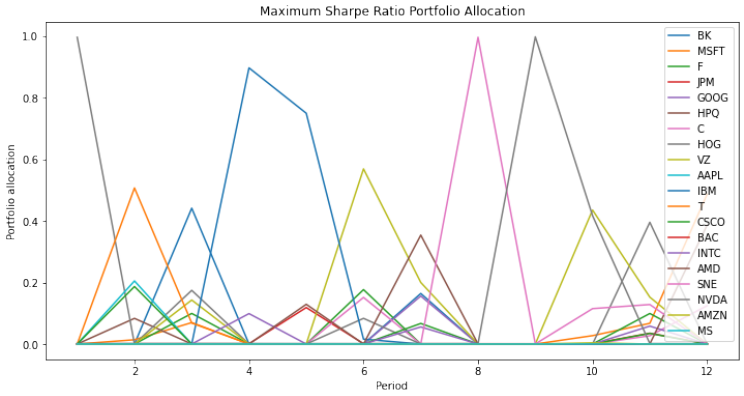
Produce the output for the 12 periods (include the results in the appendix)

> Period 1: start date 01/02/2019, end date 02/28/2019
> Strategy "Buy and Hold", value begin = $ 1000070.06, value end = $ 1121179.83
> Strategy "Equally Weighted Portfolio", value begin = ..., value end = ...

Plot one chart illustrates the daily value of your portfolio over the years 2019 and 2020 using daily prices provided. Plot two charts for strategy 3 and 4 to show dynamic changes in portfolio allocations. In each chart, x-axis represents the rolling up time horizon, y-axis denotes portfolio weights between 0 and 1, and distinct lines display the position of selected assets over time periods.







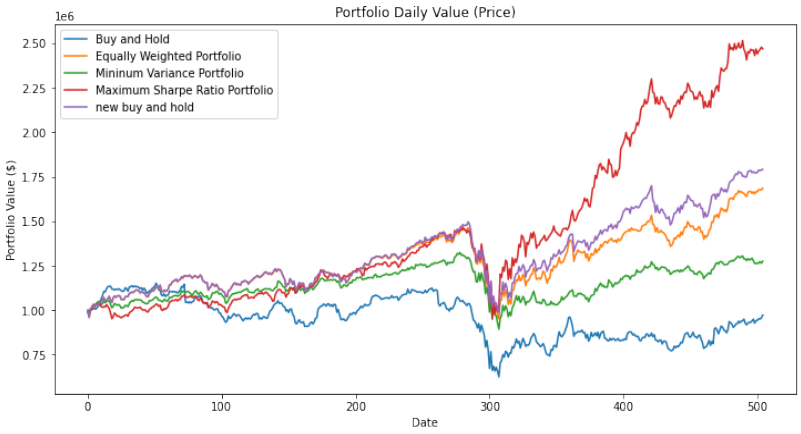
Compare your trading strategies and discuss their performance relative to each other. Which strategy would you select for managing your own portfolio and why?

"**Maximum Sharpe ratio**" portfolio strategy achieved highest value among all strategies at the end to trading period. The CPLEX optimization solver finds the wights that have the maximum excess return per unit of risk. The portfolio with maximum Sharpe ratio is efficient, has highest return but high risk. "**Equally weighted**" portfolio strategy gives the second performance (meant highest portfolio values). The portfolio adopts this strategy is not efficient, has lower return and higher risk compare to maximum Sharpe ratio portfolio. Notice this strategy has better performance than minimum variance strategy, I think one reason is it not required frequent buy and sell, so the transaction cost is lower, and this amount could be invested to gain more profit. "**Minimum variance**" portfolio strategy gives the third performance. The portfolio with minimum variance is also efficient, has lowest risk but low return. "**Buy and hold**" strategy has the worst performance. The initial portfolio allocates most money in "BK" and its end price is lower than initial price. Not analysing the market's behavior would lead to the failure of investment. The portfolio uses this strategy is not efficient, has lower return and higher risk compare to minimum variance portfolio.

Although the "Maximum Sharpe ratio" portfolio is also efficient, I would select "Minimum variance" portfolio strategy as a student with lower risk tolerance. And might consider use commission-free stock investing platform like Robinhood and wealthsimple to avoid high transaction cost.

3. Discuss possible improvements to your trading strategies

Test your Python program for different variations of your strategies, e.g., select 1/n portfolio at the beginning of period 1 and hold it till the end of period 12 (as if the re-balancing strategy required large transaction costs).



Implement new strategy **"new buy and hold"**: only rebalance first period using "Equally weighted" portfolio strategy and hold this portfolio for the entire investment horizon of 2 years, where portfolio weight $w_{20\times1}$ =[507, 6495, 542, 48, 2477, 999, 1552, 981, 1296, 479, 1956, 1248, 1999, 1116, 2533, 1026, 369, 33, 1312, 1114]^T. The plot in the left illustrates this new strategy could achieve better performance than previous rebalance strategy except "Maximum Sharpe ratio" portfolio strategy.

Can you suggest any improvements of the trading strategies that you have implemented?

1. Try rebalancing first period using "Minimum variance" and "Maximum Sharpe ratio" portfolio strategy and hold this portfolio for the entire investment horizon of 2 years. No buy and sell could avoid transaction cost.
2. Try variation based on your expectation of the assets, for example invest more on AMAZON if you expect its value would grow a lot in the future. And hold this portfolio for the entire investment horizon of 2 years.
3. Try different round procedures. For example, I decrease positions of all assets by 0.05 % in part 2, try decrease less percentage in each step. In this way, we could buy more assets and use our money more efficient, but it is more computational expensive.

Appendix

Period 1: start date 01/02/2019, end date 02/28/2019
Strategy "Buy and Hold", value begin = $ 1000070.06, value end = $ 1121179.83
Strategy "Equally Weighted Portfolio", value begin = $ 991114.55, value end = $ 1097016.56
Strategy "Minimum Variance Portfolio", value begin = $ 991691.72, value end = $ 1057825.46
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 990122.24, value end = $ 1016707.93
Strategy "new buy and hold", value begin = $ 991114.64, value end = $ 1097015.62

Period 2: start date 03/01/2019, end date 04/30/2019
Strategy "Buy and Hold", value begin = $ 1126131.27, value end = $ 1075001.89
Strategy "Equally Weighted Portfolio", value begin = $ 1103380.21, value end = $ 1188831.89
Strategy "Minimum Variance Portfolio", value begin = $ 1055521.17, value end = $ 1108271.91
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1007073.46, value end = $ 1076734.88
Strategy "new buy and hold", value begin = $ 1103662.39, value end = $ 1189909.60

Period 3: start date 05/01/2019, end date 06/28/2019
Strategy "Buy and Hold", value begin = $ 1070867.54, value end = $ 969057.81
Strategy "Equally Weighted Portfolio", value begin = $ 1181339.19, value end = $ 1169072.49
Strategy "Minimum Variance Portfolio", value begin = $ 1092052.28, value end = $ 1099289.29
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1058911.83, value end = $ 1071597.10
Strategy "new buy and hold", value begin = $ 1182525.66, value end = $ 1170399.84

Period 4: start date 07/01/2019, end date 08/30/2019
Strategy "Buy and Hold", value begin = $ 976973.31, value end = $ 933721.61
Strategy "Equally Weighted Portfolio", value begin = $ 1179615.54, value end = $ 1149906.92
Strategy "Minimum Variance Portfolio", value begin = $ 1097136.27, value end = $ 1129028.57
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1068069.23, value end = $ 1136445.34
Strategy "new buy and hold", value begin = $ 1181570.67, value end = $ 1151899.24

Period 5: start date 09/03/2019, end date 10/31/2019
Strategy "Buy and Hold", value begin = $ 922211.42, value end = $ 1028337.74
Strategy "Equally Weighted Portfolio", value begin = $ 1138200.36, value end = $ 1252905.58
Strategy "Minimum Variance Portfolio", value begin = $ 1115053.20, value end = $ 1181762.87
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1133179.08, value end = $ 1239935.00
Strategy "new buy and hold", value begin = $ 1140339.87, value end = $ 1254444.28

Period 6: start date 11/01/2019, end date 12/31/2019
Strategy "Buy and Hold", value begin = $ 1037933.42, value end = $ 1099403.03
Strategy "Equally Weighted Portfolio", value begin = $ 1270628.78, value end = $ 1373667.98
Strategy "Minimum Variance Portfolio", value begin = $ 1183566.33, value end = $ 1255090.18
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1242029.90, value end = $ 1362483.58
Strategy "new buy and hold", value begin = $ 1273113.55, value end = $ 1384602.11

Period 7: start date 01/02/2020, end date 02/28/2020
Strategy "Buy and Hold", value begin = $ 1112112.69, value end = $ 900207.54
Strategy "Equally Weighted Portfolio", value begin = $ 1396558.16, value end = $ 1258843.62
Strategy "Minimum Variance Portfolio", value begin = $ 1255114.09, value end = $ 1158897.38
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1371868.85, value end = $ 1278000.44
Strategy "new buy and hold", value begin = $ 1410262.68, value end = $ 1278401.61

Period 8: start date 03/02/2020, end date 04/30/2020
Strategy "Buy and Hold", value begin = $ 924774.25, value end = $ 856285.51
Strategy "Equally Weighted Portfolio", value begin = $ 1312828.96, value end = $ 1216883.59
Strategy "Minimum Variance Portfolio", value begin = $ 1209172.34, value end = $ 1077306.45
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1333275.52, value end = $ 1409744.90
Strategy "new buy and hold", value begin = $ 1335491.54, value end = $ 1264553.17

Period 9: start date 05/01/2020, end date 06/30/2020
Strategy "Buy and Hold", value begin = $ 822532.65, value end = $ 875128.45

Strategy "Equally Weighted Portfolio", value begin = $ 1172552.44, value end = $ 1318118.18
Strategy "Minimum Variance Portfolio", value begin = $ 1046238.06, value end = $ 1082083.61
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1348086.72, value end = $ 1626363.23
Strategy "new buy and hold", value begin = $ 1219882.06, value end = $ 1380984.30

Period 10: start date 07/01/2020, end date 08/31/2020
Strategy "Buy and Hold", value begin = $ 852159.31, value end = $ 852474.32
Strategy "Equally Weighted Portfolio", value begin = $ 1309241.68, value end = $ 1497197.90
Strategy "Minimum Variance Portfolio", value begin = $ 1085041.40, value end = $ 1244520.11
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1685655.52, value end = $ 2219117.69
Strategy "new buy and hold", value begin = $ 1377433.89, value end = $ 1659397.41

Period 11: start date 09/01/2020, end date 10/30/2020
Strategy "Buy and Hold", value begin = $ 857122.42, value end = $ 795062.75
Strategy "Equally Weighted Portfolio", value begin = $ 1508009.71, value end = $ 1409869.38
Strategy "Minimum Variance Portfolio", value begin = $ 1246719.68, value end = $ 1194768.57
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 2264846.97, value end = $ 2144738.58
Strategy "new buy and hold", value begin = $ 1680199.49, value end = $ 1524215.91

Period 12: start date 11/02/2020, end date 12/31/2020
Strategy "Buy and Hold", value begin = $ 811070.20, value end = $ 972162.37
Strategy "Equally Weighted Portfolio", value begin = $ 1422281.51, value end = $ 1685227.91
Strategy "Minimum Variance Portfolio", value begin = $ 1205256.86, value end = $ 1274965.89
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 2142217.61, value end = $ 2469352.97
Strategy "new buy and hold", value begin = $ 1532796.81, value end = $ 1791360.54