MIE 1622H Assignment 2 - Risk-Based and Robust Portfolio Selection Strategies

1. Implement investment strategies in Python

1. "Equal risk contributions" portfolio strategy: compute a portfolio that has equal risk contributions to standard deviation for each period and re-balance accordingly.

$$\min_{w} \sum_{i=1}^{n} \sum_{j=1}^{n} \left(w_i (Qw)_i - w_j (Qw)_j \right)^2$$
$$s.t. \sum_{i=1}^{n} w_i = 1, \ w_i \geq 0$$

Estimate portfolio weights $w_{20 \times 1} = [w_1 \ w_2 \ ... w_{20}]^T$
Define IPOPT objects:

1. Starting point: $w_{20 \times 1} = \left[\frac{1}{20} \ \frac{1}{20} \ ... \ \frac{1}{20} \right]^T$
2. Bounds on variable weights: $lb_{20 \times 1} = [0 \ 0 \ ... 0]^T, ub_{20 \times 1} = [1 \ 1 \ ... 1]^T$
3. Bounds on constraints: $cl = 1, cu = 1$
4. Gradient of objective function: $\frac{f(x)}{\partial x_i} = 8(Qw)_i \sum_{j=1}^{n}(w_i(Qw)_i - w_j(Qw)_j)$

2. "Leveraged equal risk contributions" portfolio strategy: take long 200% position in equal risk contributions portfolio and short risk-free asset for each period and re-balance accordingly.

1. Borrow money (long 200% position) at risk-free interest rate in the beginning of current period
2. Use same portfolio weights of "Equal risk contributions" to allocate the money on assets
3. Return money include interest in the beginning of next period
4. Repeat step 1-3 total 12 times.

3. "Robust mean-variance optimization" portfolio strategy: compute a robust mean-variance portfolio for each period and re-balance accordingly.

$$\min_{w} w^T Q w$$
$$s.t. \sum_{i}^{n} w_i = 1, \ w_i \geq 0$$
$$\mu^T w \geq \varepsilon_{ret}$$
$$w\Theta^T w \leq \tilde{\varepsilon}_{rob}$$

Estimate portfolio weights $w_{20 \times 1} = [w_1 \ w_2 \ ... w_{20}]^T$

Select targets in order to derive a portfolio have return greater than "Minimum variance" portfolio, and standard deviation smaller than "Equally weighted" portfolio.
1. Target risk estimation error $\tilde{\varepsilon}_{rob}$: variance of "Equally weighted" portfolio
2. Target return ε_{ret}: return of "Minimum variance" portfolio

Define CPLEX objects:
1. Linear part of objective function: $c_{20 \times 1} = [0 \ 0 \ ... 0]^T$
2. Quadratic part of objective function: $Q_{mat} = 2Q$
3. Bounds on variables: $lb_{20 \times 1} = [0 \ 0 \ ... 0]^T, ub_{20 \times 1} = [1 \ 1 \ ... 1]^T$
4. Linear constraints: $A = \begin{bmatrix} 1 & 1 & ... & 1 \\ \mu_1 & \mu_2 & ... & \mu_{20} \end{bmatrix} s.t. \ Aw = 1, and \ \mu^T w \geq \varepsilon_{ret}$
5. Quadratic constraints $\Theta = \text{diag}(Q) \ s.t. \ w\Theta^T w \leq \tilde{\varepsilon}_{rob}$

Above steps illustrate how to compute the portfolio weights, and you could use that information to calculate transaction cost and cash account.

Design and implement a rounding procedure, so that you always trade (buy or sell) an integer number of shares.

If cash account is nonnegative, round down the position after rebalancing and update transaction cost and cash account. Otherwise, first reduce position by allocation negative cash account into assets according to portfolio weights, then round down the position and update transaction cost and cash account. In this way, investors always trade an integer number of shares and will not go over the budget.

Design and implement a validation procedure to test each of your strategies is feasible (you have enough budget to re-balance portfolio, you correctly compute transaction costs, funds in your cash account are non-negative).

Using an if statement, if cash account is negative, validation procedure will print out a string indicates negative cash account with specific strategy followed specific period.

For "Leveraged equal risk contributions" strategy, the money supposed to be returned in the beginning of next period. However, the algorithm subtracts the borrowed money plus the interest in the end of current period. As a result, the validation procedure would indicate cash account is negative. Have to manually check whether the cash account is negative or not before return money.

$$cash\ account_{2019-2020} = [2722.22, 1869.98, 3239.97, 2224.37, 2915.67, 2596.23, 2773.02, 1770.1, 1185.35, 2685.14, 1267.66, 1964.95]$$

$$cash\ account_{2008-2009} = [815.24, 318.86, 397.68, 443.58, 400.96, 440.64, 318.37, 156.45, 275.81, 336.48, 335, 240.09]$$

2. Analyze your results (2019 - 2020)

Produce the output for the 12 periods (include the results in the appendix).

Period 1: start date 01/02/2019, end date 02/28/2019

Strategy "Buy and Hold", value begin = $ 1000070.06, value end = $ 1121179.83

Strategy "Equally Weighted Portfolio", value begin = $ 991110.53, value end = $ 1097139.05

Strategy "Minimum Variance Portfolio", value begin = $ 991694.21, value end = $ 1057724.38

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 990119.39, value end = $ 1016524.41

Strategy "Equal Risk Contributions Portfolio", value begin = $ 991350.69, value end = $ 1086730.66

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 983446.28, value end = $ 1174676.92

Strategy "Robust Optimization Portfolio", value begin = $ 992339.32, value end = $ 1074547.25

Plot daily value of portfolio (for each of the seven trading strategies).
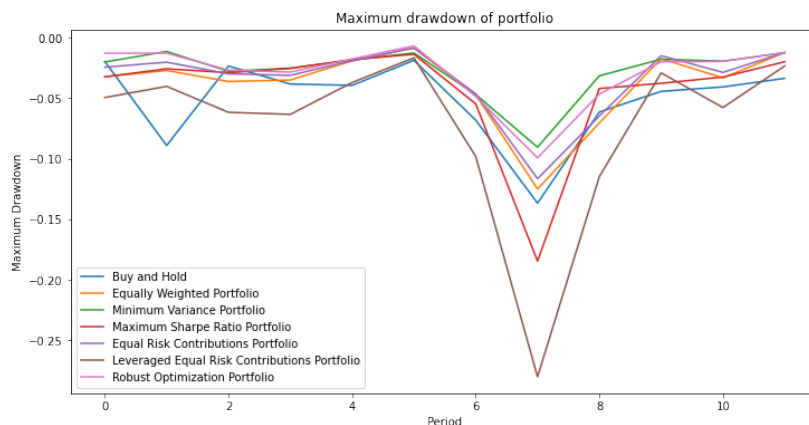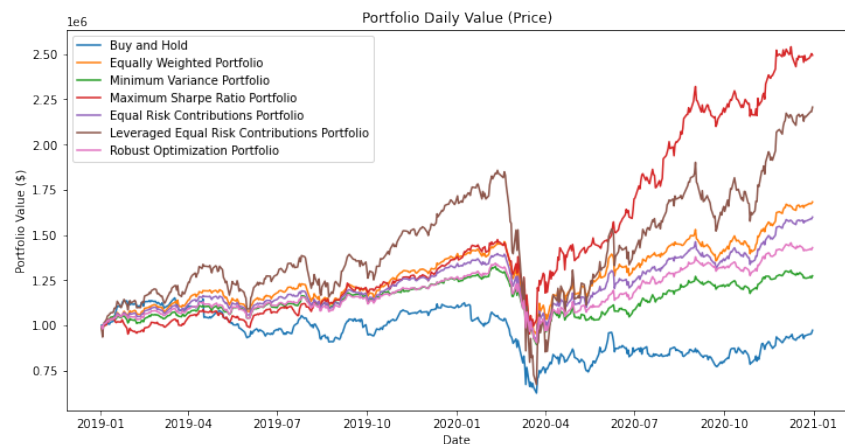
Plot maximum drawdown of portfolio (for each of the seven trading strategies) for each of the 12 periods.

Plot dynamic changes in portfolio allocations under strategy 7.

Robust portfolio selection strategy reduces trading strategy compared with strategies 3 and 4. All portfolios constructed are on the efficient frontier. Strategies 3 and 4 are more sensitive to the change of expected return. Hence, the portfolio weights will change more than robust portfolio each period after rebalancing.
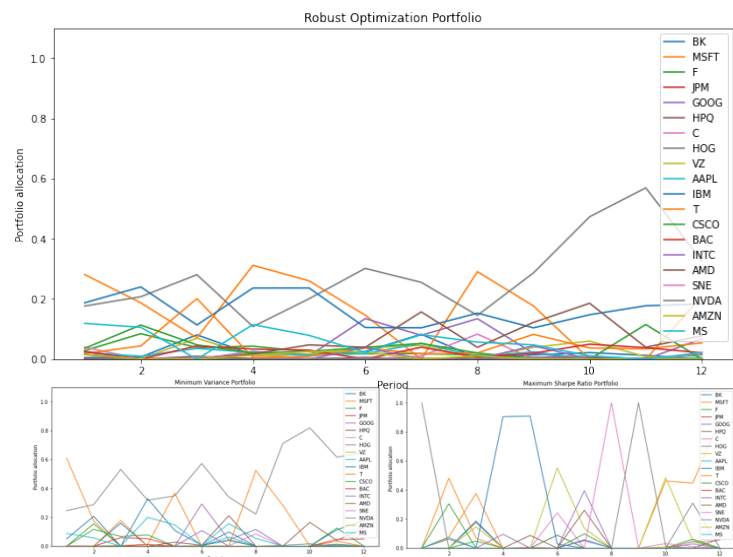
Compare "equal risk contributions", "leveraged equal risk contributions" and "robust mean variance optimization" trading strategies between each other and to four strategies implemented in Assignment 1 and discuss their performance relative to each other. Which strategy would you select for managing your own portfolio and why?

"Leveraged equal risk contributions" takes long 200% position, it has high return and high risk (standard deviation). This is a dangerous trading strategy for new investor due to high leverage and there's a chance to get into debt. *"Equal risk contributions"* has moderate return and risk. This strategy diversify risk by allocation risk equally into asset. Both strategies are more depend on the market than "Robust mean-variance optimization", and they perform well in good market. *"Robust mean-variance optimization"* is efficient and has the best performance among three strategies implemented in Assignment 2. It will have highest return among three strategies if they have same standard deviations.

From plots of *daily value* and *maximum drawdown*, classify expected return and standard deviation from high to low.

	Expected Return (2019 – 2020)	Risk (standard deviation)
high	"Maximum Sharpe ratio" "Leveraged equal risk contributions"	"Leveraged equal risk contributions" "Maximum Sharpe ratio" "Buy and hold"
moderate	"Equally weighted" "Equal risk contributions" "Robust mean-variance optimization" "Minimum variance"	"Equally weighted" "Equal risk contributions"
low	"Buy and hold"	"Robust mean-variance optimization" "Minimum variance"

"Equally weighted" has moderate return and risk. The portfolio is diversified at some level by allocation money equally into assets. And it not required frequent buy and sell unless stock price change dramatically, so the transaction cost is lower, and this amount could be invested to gain more profit. Noted the risk is higher than "Robust mean-variance optimization", which is set by the CPLEX optimization program of RMV. *"Buy and hold"* has low return but high risk. The portfolio value only increases when the prices of initial holding assets rise. Not analyzing the market's behavior would lead to investment failure.

"Maximum Sharpe ratio", *"Minimum variance"* and *"Robust mean-variance optimization"* strategies return efficient portfolios, they are different points along the efficient frontier correspond to different risk preference. The higher the return, the larger the risk. I would invest by using *"Robust mean-variance optimization"* since it would allow me to earn great return without take too much risk.

3. Test trading strategies (2008 - 2009)

Produce the output for the 12 periods (include the results in the appendix).

> Period 1: start date 01/02/2008, end date 02/29/2008
> Strategy "Buy and Hold", value begin = $ 789230.94, value end = $ 749509.71
> Strategy "Equally Weighted Portfolio", value begin = $ 782158.10, value end = $ 669692.42
> Strategy "Minimum Variance Portfolio", value begin = $ 781379.45, value end = $ 666830.57
> Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 781378.15, value end = $ 681744.86
> Strategy "Equal Risk Contributions Portfolio", value begin = $ 782129.34, value end = $ 674108.17
> Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 775567.54, value end = $ 558995.70
> Strategy "Robust Optimization Portfolio", value begin = $ 782106.92, value end = $ 679233.31

For strategy **"Maximum Sharpe ratio"**, all assets had negative expected return μ for some period during 2008 financial crisis. There is no feasible solution for optimal weights since it violated the constraint $\sum_i (\mu_i - rf) y_i = 1$. Keep the same portfolio in this case.
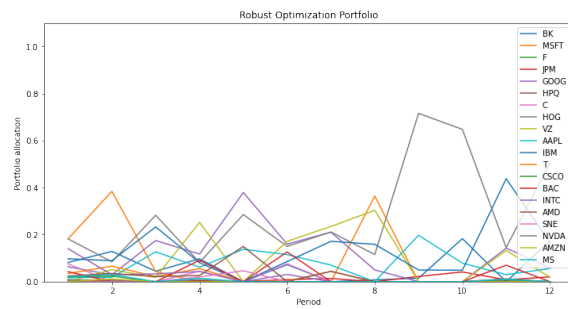
Plot daily value of portfolio (for each of the seven trading strategies).
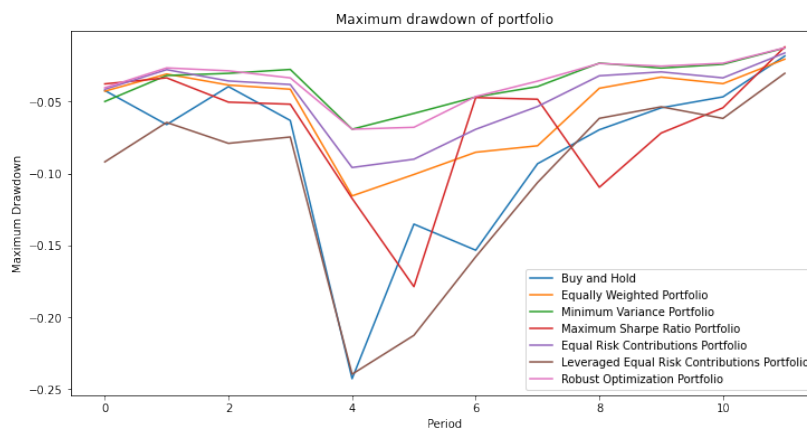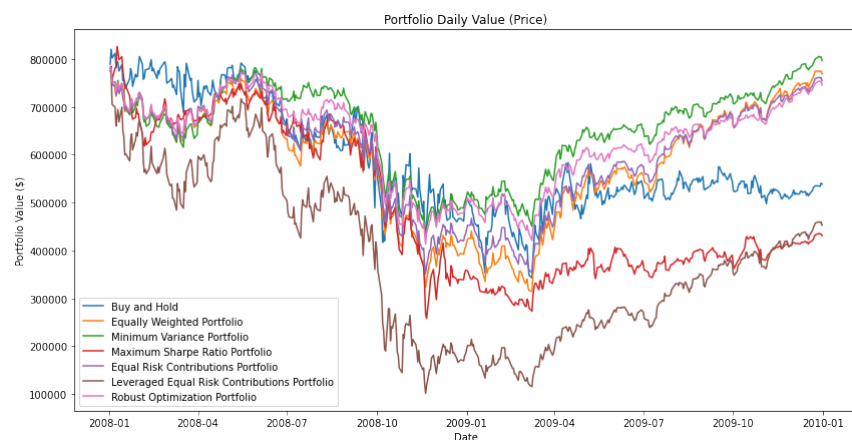
Plot maximum drawdown of portfolio (for each of the seven trading strategies) for each of the 12 periods.

Plot dynamic changes in portfolio allocations under strategy 3, 4 and 7. Does your robust portfolio selection strategy reduce trading as compared with the strategies 3 and 4?
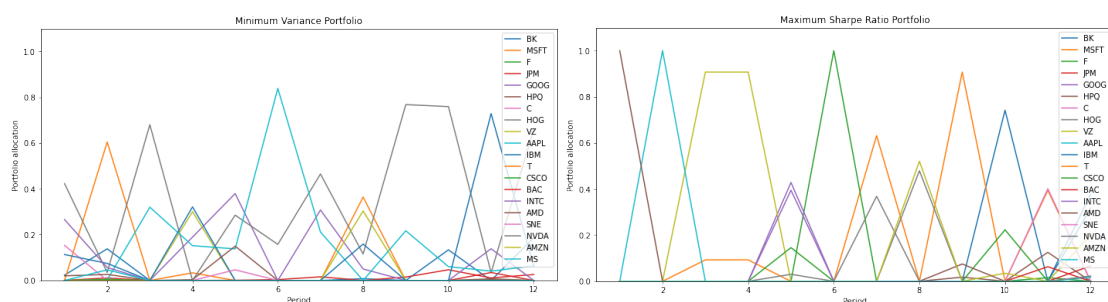
Robust portfolio selection strategy reduces trading strategy compared with strategies 3 and 4. All portfolios constructed are on the efficient frontier. Strategies 3 and 4 are more sensitive to the change of expected return. Hence, the portfolio weights will change more than robust portfolio each period after rebalancing.

Compare and discuss relative performance of seven trading strategies during 2019-2020 and 2008-2009 time periods. Which strategy would you select for managing your own portfolio during 2008-2009 time period, why?

From plots of *daily value* and *maximum drawdown*, classify expected return and standard deviation from high to low.

	Expected Return (2008 – 2009)	Risk (standard deviation)
high	"Minimum variance" "Equally weighted" "Equal risk contributions" "Robust mean-variance optimization"	"Leveraged equal risk contributions" "Maximum Sharpe ratio" "Buy and hold"
moderate	"Buy and hold"	"Equally weighted" "Equal risk contributions"
low	"Maximum Sharpe ratio" "Leveraged equal risk contributions"	"Robust mean-variance optimization" "Minimum variance"

The market crashed during 2008 and covid-19 crisis. The plot of daily value of portfolio shows 2008 financial crisis are more severe, the portfolio value dropped more in percentage and the market takes more time to recover. The strategies with high risk like **"Maximum Sharpe ratio"** and **"Leveraged equal risk contributions"** heavily influenced by the change of the market. They performed worse during 2008-2009 than during 2019-2020, since they perform badly when the market experienced prolonged price drop, but perform well when the market expect price rising. On the other hand, strategies with low risk like ***"Minimum variance", "Equally weighted", "Equal risk contributions"*** and ***"Robust mean-variance optimization"*** were influenced less by the volatility of the market. They performed better during 2008-2009 than during 2019-2020, since they are able to tolerant some risk. ***"Buy and hold"*** has moderate return, its performance relies on the allocation of initial portfolio which remains same and not adjustable during whole trading periods.

Appendix

Period 1: start date 01/02/2019, end date 02/28/2019
Strategy "Buy and Hold", value begin = $ 1000070.06, value end = $ 1121179.83
Strategy "Equally Weighted Portfolio", value begin = $ 991110.53, value end = $ 1097139.05
Strategy "Minimum Variance Portfolio", value begin = $ 991694.21, value end = $ 1057724.38
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 990119.39, value end = $ 1016524.41
Strategy "Equal Risk Contributions Portfolio", value begin = $ 991350.69, value end = $ 1086730.66
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 983446.28, value end = $ 1174676.92
Strategy "Robust Optimization Portfolio", value begin = $ 992339.32, value end = $ 1074547.25

Period 2: start date 03/01/2019, end date 04/30/2019
Strategy "Buy and Hold", value begin = $ 1126131.27, value end = $ 1075001.89
Strategy "Equally Weighted Portfolio", value begin = $ 1103425.44, value end = $ 1188889.41
Strategy "Minimum Variance Portfolio", value begin = $ 1055665.23, value end = $ 1108286.33
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1007117.74, value end = $ 1076765.61
Strategy "Equal Risk Contributions Portfolio", value begin = $ 1090630.16, value end = $ 1157845.60
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1177450.15, value end = $ 1323589.17
Strategy "Robust Optimization Portfolio", value begin = $ 1074340.83, value end = $ 1122847.09

Period 3: start date 05/01/2019, end date 06/28/2019
Strategy "Buy and Hold", value begin = $ 1070867.54, value end = $ 969057.81
Strategy "Equally Weighted Portfolio", value begin = $ 1181393.74, value end = $ 1169301.49
Strategy "Minimum Variance Portfolio", value begin = $ 1092250.70, value end = $ 1099713.54
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1060451.78, value end = $ 1073388.89
Strategy "Equal Risk Contributions Portfolio", value begin = $ 1148388.20, value end = $ 1137079.10
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1297459.60, value end = $ 1271719.60
Strategy "Robust Optimization Portfolio", value begin = $ 1107850.57, value end = $ 1108871.34

Period 4: start date 07/01/2019, end date 08/30/2019
Strategy "Buy and Hold", value begin = $ 976973.31, value end = $ 933721.61
Strategy "Equally Weighted Portfolio", value begin = $ 1179796.24, value end = $ 1150034.55
Strategy "Minimum Variance Portfolio", value begin = $ 1097559.87, value end = $ 1129571.96
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1071209.32, value end = $ 1140264.81
Strategy "Equal Risk Contributions Portfolio", value begin = $ 1143098.02, value end = $ 1126352.25
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1280061.65, value end = $ 1242431.39
Strategy "Robust Optimization Portfolio", value begin = $ 1109294.23, value end = $ 1110363.07

Period 5: start date 09/03/2019, end date 10/31/2019
Strategy "Buy and Hold", value begin = $ 922211.42, value end = $ 1028337.74
Strategy "Equally Weighted Portfolio", value begin = $ 1138330.91, value end = $ 1252907.51
Strategy "Minimum Variance Portfolio", value begin = $ 1115840.21, value end = $ 1182581.22
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1139080.87, value end = $ 1246475.50
Strategy "Equal Risk Contributions Portfolio", value begin = $ 1116150.65, value end = $ 1217011.46
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1214362.18, value end = $ 1435049.27
Strategy "Robust Optimization Portfolio", value begin = $ 1100551.07, value end = $ 1183461.31
Period 6: start date 11/01/2019, end date 12/31/2019
Strategy "Buy and Hold", value begin = $ 1037933.42, value end = $ 1099403.03
Strategy "Equally Weighted Portfolio", value begin = $ 1270619.43, value end = $ 1373612.58
Strategy "Minimum Variance Portfolio", value begin = $ 1184635.69, value end = $ 1256110.15
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1248551.92, value end = $ 1369849.65
Strategy "Equal Risk Contributions Portfolio", value begin = $ 1231427.79, value end = $ 1323342.21
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1460359.77, value end = $ 1679664.37
Strategy "Robust Optimization Portfolio", value begin = $ 1191246.37, value end = $ 1260434.80

Period 7: start date 01/02/2020, end date 02/28/2020
Strategy "Buy and Hold", value begin = $ 1112112.69, value end = $ 900207.54
Strategy "Equally Weighted Portfolio", value begin = $ 1396423.77, value end = $ 1258476.44
Strategy "Minimum Variance Portfolio", value begin = $ 1256482.80, value end = $ 1159523.89
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1379227.69, value end = $ 1284635.75

Strategy "Equal Risk Contributions Portfolio", value begin = $ 1341597.82, value end = $ 1217151.26
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1715721.02, value end = $ 1395728.68
Strategy "Robust Optimization Portfolio", value begin = $ 1269042.95, value end = $ 1173571.68

Period 8: start date 03/02/2020, end date 04/30/2020
Strategy "Buy and Hold", value begin = $ 924774.25, value end = $ 856285.51
Strategy "Equally Weighted Portfolio", value begin = $ 1312362.36, value end = $ 1215384.78
Strategy "Minimum Variance Portfolio", value begin = $ 1209874.83, value end = $ 1076960.66
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1340311.96, value end = $ 1417151.32
Strategy "Equal Risk Contributions Portfolio", value begin = $ 1270623.86, value end = $ 1156052.01
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1526902.07, value end = $ 1250002.75
Strategy "Robust Optimization Portfolio", value begin = $ 1226968.13, value end = $ 1107208.22

Period 9: start date 05/01/2020, end date 06/30/2020
Strategy "Buy and Hold", value begin = $ 822532.65, value end = $ 875128.45
Strategy "Equally Weighted Portfolio", value begin = $ 1171234.18, value end = $ 1316237.52
Strategy "Minimum Variance Portfolio", value begin = $ 1046176.45, value end = $ 1081152.03
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1357463.54, value end = $ 1637822.10
Strategy "Equal Risk Contributions Portfolio", value begin = $ 1115040.01, value end = $ 1244225.78
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1145706.82, value end = $ 1412945.97
Strategy "Robust Optimization Portfolio", value begin = $ 1070465.12, value end = $ 1173877.59

Period 10: start date 07/01/2020, end date 08/31/2020
Strategy "Buy and Hold", value begin = $ 852159.31, value end = $ 852474.32
Strategy "Equally Weighted Portfolio", value begin = $ 1307182.90, value end = $ 1494129.26
Strategy "Minimum Variance Portfolio", value begin = $ 1084288.34, value end = $ 1242755.18
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1698911.14, value end = $ 2238261.54
Strategy "Equal Risk Contributions Portfolio", value begin = $ 1239262.11, value end = $ 1427723.00
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1396575.55, value end = $ 1824158.77
Strategy "Robust Optimization Portfolio", value begin = $ 1182537.77, value end = $ 1345541.04

Period 11: start date 09/01/2020, end date 10/30/2020
Strategy "Buy and Hold", value begin = $ 857122.42, value end = $ 795062.75
Strategy "Equally Weighted Portfolio", value begin = $ 1504824.50, value end = $ 1407554.23
Strategy "Minimum Variance Portfolio", value begin = $ 1244968.45, value end = $ 1193594.76
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 2285546.99, value end = $ 2164732.78
Strategy "Equal Risk Contributions Portfolio", value begin = $ 1438427.98, value end = $ 1350040.62
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1840037.59, value end = $ 1611698.27
Strategy "Robust Optimization Portfolio", value begin = $ 1351729.90, value end = $ 1294543.81

Period 12: start date 11/02/2020, end date 12/31/2020
Strategy "Buy and Hold", value begin = $ 811070.20, value end = $ 972162.37
Strategy "Equally Weighted Portfolio", value begin = $ 1419989.30, value end = $ 1682329.90
Strategy "Minimum Variance Portfolio", value begin = $ 1204236.99, value end = $ 1273585.40
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 2162771.27, value end = $ 2493427.99
Strategy "Equal Risk Contributions Portfolio", value begin = $ 1361171.18, value end = $ 1598524.48
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1633325.23, value end = $ 2206617.10
Strategy "Robust Optimization Portfolio", value begin = $ 1304500.36, value end = $ 1429517.46

Period 1: start date 01/02/2008, end date 02/29/2008
Strategy "Buy and Hold", value begin = $ 789230.94, value end = $ 749509.71
Strategy "Equally Weighted Portfolio", value begin = $ 782158.10, value end = $ 669692.42
Strategy "Minimum Variance Portfolio", value begin = $ 781379.45, value end = $ 666830.57
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 781378.15, value end = $ 681744.86
Strategy "Equal Risk Contributions Portfolio", value begin = $ 782129.34, value end = $ 674108.17
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 775567.54, value end = $ 558995.70
Strategy "Robust Optimization Portfolio", value begin = $ 782106.92, value end = $ 679233.31

Period 2: start date 03/03/2008, end date 04/30/2008
Strategy "Buy and Hold", value begin = $ 754361.26, value end = $ 752687.89
Strategy "Equally Weighted Portfolio", value begin = $ 659705.67, value end = $ 726740.28
Strategy "Minimum Variance Portfolio", value begin = $ 657758.18, value end = $ 745536.40
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 667671.09, value end = $ 705483.22
Strategy "Equal Risk Contributions Portfolio", value begin = $ 664197.28, value end = $ 739831.98
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 536648.31, value end = $ 659706.17
Strategy "Robust Optimization Portfolio", value begin = $ 670983.30, value end = $ 740765.79

Period 3: start date 05/01/2008, end date 06/30/2008
Strategy "Buy and Hold", value begin = $ 779329.50, value end = $ 663602.44
Strategy "Equally Weighted Portfolio", value begin = $ 750503.35, value end = $ 632811.20
Strategy "Minimum Variance Portfolio", value begin = $ 760638.52, value end = $ 716542.62
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 715301.68, value end = $ 640487.94
Strategy "Equal Risk Contributions Portfolio", value begin = $ 761938.53, value end = $ 660674.23
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 692628.46, value end = $ 507434.63
Strategy "Robust Optimization Portfolio", value begin = $ 758127.19, value end = $ 695802.53

Period 4: start date 07/01/2008, end date 08/29/2008
Strategy "Buy and Hold", value begin = $ 674748.24, value end = $ 619979.82
Strategy "Equally Weighted Portfolio", value begin = $ 633743.28, value end = $ 647028.72
Strategy "Minimum Variance Portfolio", value begin = $ 717240.72, value end = $ 726244.06
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 665688.61, value end = $ 645164.69
Strategy "Equal Risk Contributions Portfolio", value begin = $ 659955.31, value end = $ 668388.60
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 503828.80, value end = $ 516813.67
Strategy "Robust Optimization Portfolio", value begin = $ 692571.01, value end = $ 695321.09

Period 5: start date 09/02/2008, end date 10/31/2008
Strategy "Buy and Hold", value begin = $ 621151.79, value end = $ 579282.75
Strategy "Equally Weighted Portfolio", value begin = $ 648620.38, value end = $ 473989.94
Strategy "Minimum Variance Portfolio", value begin = $ 710132.36, value end = $ 548418.30
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 627266.68, value end = $ 469277.32
Strategy "Equal Risk Contributions Portfolio", value begin = $ 667920.34, value end = $ 495917.70
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 513920.21, value end = $ 247640.58
Strategy "Robust Optimization Portfolio", value begin = $ 685674.98, value end = $ 529525.82

Period 6: start date 11/03/2008, end date 12/31/2008
Strategy "Buy and Hold", value begin = $ 576738.59, value end = $ 500698.25
Strategy "Equally Weighted Portfolio", value begin = $ 471107.93, value end = $ 407311.95
Strategy "Minimum Variance Portfolio", value begin = $ 549181.59, value end = $ 505940.31
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 463706.03, value end = $ 358974.74
Strategy "Equal Risk Contributions Portfolio", value begin = $ 494211.31, value end = $ 434761.28
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 243499.46, value end = $ 184471.28
Strategy "Robust Optimization Portfolio", value begin = $ 531210.64, value end = $ 493103.33

Period 7: start date 01/02/2009, end date 02/27/2009
Strategy "Buy and Hold", value begin = $ 505855.81, value end = $ 414408.26
Strategy "Equally Weighted Portfolio", value begin = $ 424687.35, value end = $ 348038.49
Strategy "Minimum Variance Portfolio", value begin = $ 522354.74, value end = $ 497715.59
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 357684.81, value end = $ 297790.07
Strategy "Equal Risk Contributions Portfolio", value begin = $ 452958.80, value end = $ 386744.36
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 201674.65, value end = $ 142396.48

Strategy "Robust Optimization Portfolio", value begin = $ 509631.04, value end = $ 463400.24

Period 8: start date 03/02/2009, end date 04/30/2009
Strategy "Buy and Hold", value begin = $ 400004.61, value end = $ 475987.18
Strategy "Equally Weighted Portfolio", value begin = $ 331192.37, value end = $ 532505.63
Strategy "Minimum Variance Portfolio", value begin = $ 478086.28, value end = $ 652574.59
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 283420.05, value end = $ 369416.73
Strategy "Equal Risk Contributions Portfolio", value begin = $ 370568.83, value end = $ 555789.33
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 127267.64, value end = $ 255296.22
Strategy "Robust Optimization Portfolio", value begin = $ 446863.11, value end = $ 609956.40

Period 9: start date 05/01/2009, end date 06/30/2009
Strategy "Buy and Hold", value begin = $ 483627.06, value end = $ 538125.39
Strategy "Equally Weighted Portfolio", value begin = $ 531618.60, value end = $ 559588.83
Strategy "Minimum Variance Portfolio", value begin = $ 647362.22, value end = $ 654335.24
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 367170.82, value end = $ 379049.22
Strategy "Equal Risk Contributions Portfolio", value begin = $ 554804.30, value end = $ 576360.79
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 253118.94, value end = $ 272910.26
Strategy "Robust Optimization Portfolio", value begin = $ 605256.89, value end = $ 613734.77

Period 10: start date 07/01/2009, end date 08/31/2009
Strategy "Buy and Hold", value begin = $ 528549.59, value end = $ 554215.70
Strategy "Equally Weighted Portfolio", value begin = $ 558946.53, value end = $ 658850.60
Strategy "Minimum Variance Portfolio", value begin = $ 653725.90, value end = $ 697226.01
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 369815.56, value end = $ 399956.53
Strategy "Equal Risk Contributions Portfolio", value begin = $ 576360.04, value end = $ 655667.99
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 271767.96, value end = $ 346934.86
Strategy "Robust Optimization Portfolio", value begin = $ 613535.81, value end = $ 656268.07

Period 11: start date 09/01/2009, end date 10/30/2009
Strategy "Buy and Hold", value begin = $ 529171.54, value end = $ 510275.57
Strategy "Equally Weighted Portfolio", value begin = $ 635151.26, value end = $ 672310.43
Strategy "Minimum Variance Portfolio", value begin = $ 680548.84, value end = $ 714749.12
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 382682.34, value end = $ 382316.48
Strategy "Equal Risk Contributions Portfolio", value begin = $ 635726.72, value end = $ 674288.74
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 326590.05, value end = $ 366446.95
Strategy "Robust Optimization Portfolio", value begin = $ 641054.56, value end = $ 670450.25

Period 12: start date 11/02/2009, end date 12/31/2009
Strategy "Buy and Hold", value begin = $ 515205.91, value end = $ 538768.59
Strategy "Equally Weighted Portfolio", value begin = $ 675078.62, value end = $ 769442.27
Strategy "Minimum Variance Portfolio", value begin = $ 709456.59, value end = $ 796664.36
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 380383.65, value end = $ 430687.15
Strategy "Equal Risk Contributions Portfolio", value begin = $ 675604.41, value end = $ 754770.04
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 366241.93, value end = $ 452596.04
Strategy "Robust Optimization Portfolio", value begin = $ 666694.94, value end = $ 745409.97
Daily value of portfolio