GATE LEVEL MINIMIZATION

Authors

(Joint work of Group13)

INDIAN INSTITUTE OF TECHNOLOGY HYDERABAD

Digital Systems
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Paper Short Title

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Map Metho

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Conditions

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Introduction

 Gate-level minimization is the design task of finding an optimal gate-level implementation of the Boolean functions describing a digital circuit.

- It is too difficult to execute it by manual method when the logic has more number of inputs.
- This problem has been solved by computer-based logic synthesis tools that minimize a large set of Boolean equations efficiently and quickly.

Paper Short Title

Introduction

Map Metho

2 Variable K-M

3 Variable K-M

4 Variable K-Ma

Prime Implica

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Map Method

- The Map Method is the simple, straight forward procedure for minimizing Boolean functions Known as Karnaugh Maps(K-Maps)
- K-Maps are a visual representation of truth tables.
- Adjacent cells differ by one variable, enabling simplification.
- K-Maps exist for 2, 3, 4, and 5 variables.

Note

The main assumption in the K-map method:

The simplest algebraic expression is one that has a minimum number of terms with the smallest possible number of literals in each term. Paper Short Title

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Map Method

2 Variable K-Ma

3 Variable K-M

Variable K-Ma

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Product Of Sum

Don't Care Conditions

2 Variable K-Map

- Four squares representing minterms.
- Simplified expressions minimize gates and inputs.

m_0	m_1	
m_2	m_3	



• Example: F(x,y) = xy and F(x,y) = x + y





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Introduction

Map Method

2 Variable K-Map

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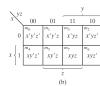
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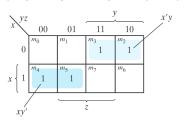
3 Variable K-Map

- 8 squares arranged in Gray code sequence.
- Adjacent squares differ by only one variable.

	m_0	m_1	m_3	m_2
	m_4	m_5	m_7	m_6
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• Example 1: $F(x, y, z) = \Sigma(2, 3, 4, 5) \rightarrow x'y + xy'$



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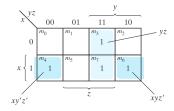
3 Variable K-Map



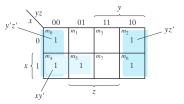


3 Variable K-Map examples

• Example 2: $F(x,y,z) = \Sigma(3,4,6,7) \rightarrow yz + xz'$



• Example 3: $F(x, y, z) = \Sigma(0, 2, 4, 5, 6) \rightarrow z' + xy'$



Paper Short Title

Introduction

Map Method

2 Variable K-

3 Variable K-Map

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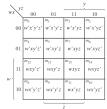
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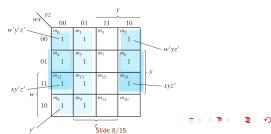
4 Variable K-Map

- 16 squares, organized using Gray code.
- Allows simplifications of larger functions.

m_0	m_1	m_3	m_2	
m_4	m_5	m_7	m_6	
m ₁₂	m_{13}	m ₁₅	m_{14}	
m_8	m_9	m_{11}	m_{10}	



• $F(w, x, y, z) = \Sigma(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14) \rightarrow y' + w'z' + xz'$



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Map Method

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3 Variable N

4 Variable K-Map

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Product Of Sum

Conditions

Prime Implicants

- A prime implicant is a product term obtained by combining the maximum possible number of adjacent squares in a Karnaugh map (K-map). An implicant is prime if no other implicant with fewer literals covers it.
- A prime implicant is essential if a minterm is covered only by that implicant. Essential prime implicants must be included in the simplified function.
- Procedure for Finding Prime Implicants:
 - A single 1 in the K-map is a prime implicant if it has no adjacent 1s.
 - Two adjacent 1s form a prime implicant unless they are part of a larger group.
 - Four adjacent 1s form a prime implicant unless they are part of a group of eight.

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Map Method

2 Variable K-Ma

Variable K-M

Variable K-Map

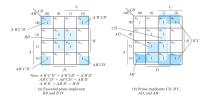
Prime Implicants

Product Of Sums

Don't Care Conditions

Example

 $\bullet \ \, \mathsf{Example:} \ \, F(A,B,C,D) = \Sigma(0,2,3,5,7,8,9,10,11,13,15)$



- \bullet F = BD + B'D + CD + AD
- $\bullet \ F = BD + B'D + CD + AB'$
- $\bullet \ F = BD + B'D + B'C + AD$
- F = BD + B'D + B'C + AB'

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4 Variable K-I

Prime Implicants

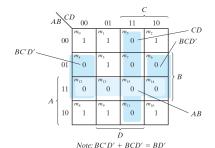
Product Of Sum

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Sum-of-Products (SOP) and Product-of-Sums (POS)

- SOP: OR-ing product terms (AND gates to OR gate).
- POS: AND-ing sum terms (OR gates to AND gate).
- Example: $F(A, B, C, D) = \Sigma(0, 1, 2, 5, 8, 9, 10)$ F' = AB + CD + BD' (SOP), F = (A' + B')(C' + D')(B' + D) (POS)



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Map Method

8 Variable K-N

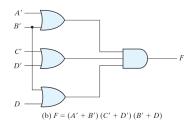
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Prime Impli

Product Of Sums

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B' D' C' A' D (a) F = B'D' + B'C' + A'C'D



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Introduction

Map Method

2 Variable K-Ma

5 Variable K-IVI

4 Variable K-Ma

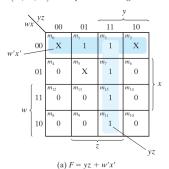
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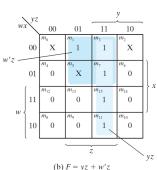
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Don't Care Conditions

- Unused input combinations can be treated as 0 or 1 for simplification.
- Helps minimize logic further.
- Example: $F(w,x,y,z) = \Sigma(1,3,7,11,15)$ with don't-cares $\Sigma(0,2,5)$ simplifies to yz+w'x'





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Introduction

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Don't Care Conditions

Conclusion

- Gate minimization reduces hardware cost and improves efficiency.
- K-Maps provide a systematic simplification method.
- Understanding simplification techniques aids in modern digital design.

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Once Again...

THANK YOU!

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introduction

Map Method

2 Variable K-Map

variable K-IVIap

4 Variable K-Map

Prime Implica

Product Of Sums

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