11.16.2.2.5

EE24BTECH11052 - Rongali Charan

Question: A Die is thrown.Describe the following events:
E: an even number greater than 4

Solution:

1 Theoretical Solution

1) **Total Number of Possible Outcomes**
   Let X be the random variable representing the outcome of a single die roll. The sample space is $S = \{1, 2, 3, 4, 5, 6\}$. We are interested in the event E where the outcome is an even number greater than 4. The only outcome satisfying this condition is 6.
2) **Probability of Success**
   The probability of event E is the number of favorable outcomes divided by the total number of possible outcomes:
   $$P(E) = \frac{\text{Number of outcomes in E}}{\text{Total number of outcomes in S}} = \frac{1}{6}$$
3) **Defining the Random Variable**
   We model this problem using Bernoulli random variables,
   Let X be the random variable that represents the die turn up to be a 6:
   $$X = 1, \text{ If the number is 6, } \left(\text{With probability } p = \frac{1}{6}\right) \tag{3.1}$$
   $$X = 0, \text{ if number gets in } \{1, 2, 3, 4, 5\}, \left(\text{With probability } 1 - p = \frac{5}{6}\right) \tag{3.2}$$
4) **Probability Mass Function (PMF):**
   The PMF of a Bernoulli random variable X is given by:
   $$P(X = x) = p^x (1 - p)^{1-x}, x \in \{0, 1\} \tag{4.1}$$
   substituting $p = \frac{1}{6}$,
   $$P(X = 1) = 0.166666, P(X = 0) = 0.833333 \tag{4.2}$$
   $$P(X = x) = \begin{cases} 0.166666, & x = 1 \\ 0.833333, & x = 0 \\ 0, & \text{otherwise} \end{cases} \tag{4.3}$$

5) **Cumulative Distribution function (CDF):**
The CDF of a discrete random variable is defined as:

$$F_X(x) = P(X \leq x) \tag{5.1}$$

$$F_X(x) = \begin{cases} 0, & x < 0 \\ \frac{5}{6}, & 0 \leq x < 1 \\ 1, & x \geq 1 \end{cases} \tag{5.2}$$

2 Numerical Solution (Monte Carlo)

We can estimate the probability using the Monte Carlo method. We simulate a large number of die rolls and count how many times we get a 6.

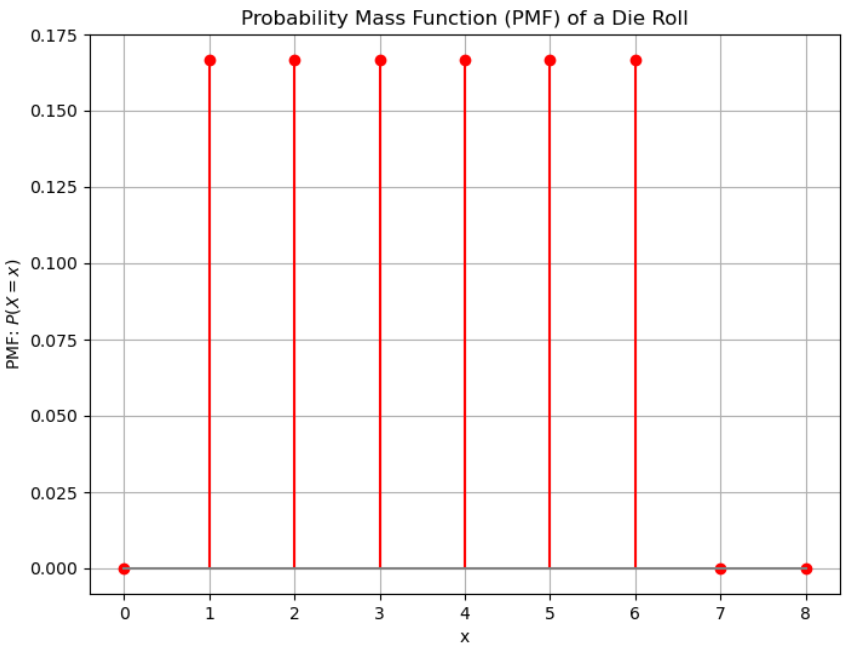

Fig. 5.1: PMF of the Random Variable

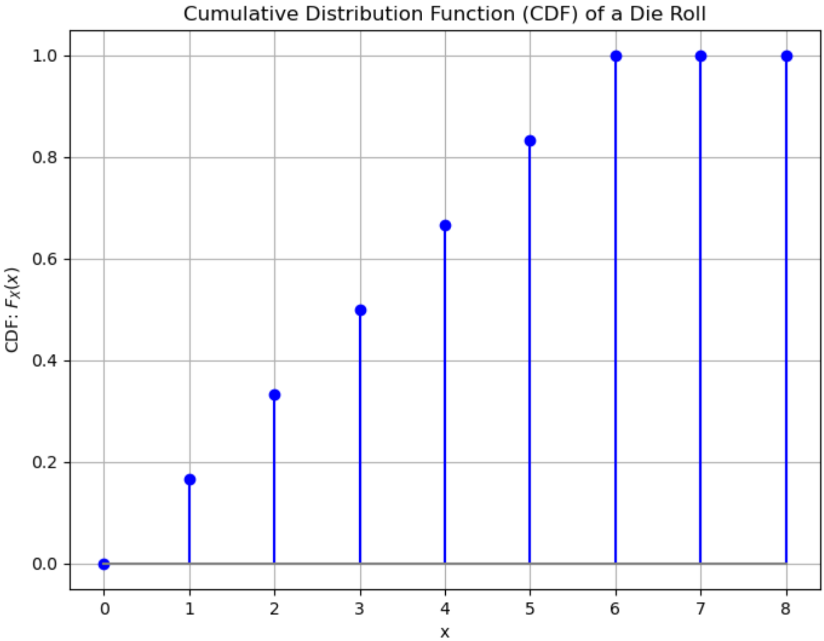

Fig. 5.2: CDF of the Random Variable