

11.16.2.2.5

EE24BTECH11052 - Rongali Charan

Question: A Die is thrown. Describe the following events:

E: an even number greater than 4

Solution:

1) Total Number of Possible Outcomes

Let X be the random variable representing the outcome of a single die roll. The sample space is $S = \{1, 2, 3, 4, 5, 6\}$. We are interested in the event E where the outcome is an even number greater than 4. The only outcome satisfying this condition is 6.

2) Probability of Success

The probability of event E is the number of favorable outcomes divided by the total number of possible outcomes:

$$P(E) = \frac{\text{Number of outcomes in E}}{\text{Total number of outcomes in S}} = \frac{1}{6}$$

3) Defining the Random Variable

The PMF of the random variable X is given by:

$$P_X(x) = \begin{cases} \frac{1}{6}, & x \in \{1, 2, 3, 4, 5, 6\} \\ 0, & \text{otherwise} \end{cases}$$

For the event E (rolling a 6), we can define a new random variable Y such that:

Let Y be the random variable that represents the die turn up to be a 6:

$$Y = 1, \text{ If the number is 6, } \left(\text{With probability } p = \frac{1}{6} \right) \quad (3.1)$$

$$Y = 0, \text{ if number gets in } \{1, 2, 3, 4, 5\}, \left(\text{With probability } 1 - p = \frac{5}{6} \right) \quad (3.2)$$

4) Probability Mass Function (PMF):

The PMF of a Bernoulli random variable Y is given by:

$$P(Y = y) = p^y (1 - p)^{1-y}, y \in \{0, 1\} \quad (4.1)$$

substituting $p = \frac{1}{6}$,

$$P(Y = 1) = 0.166666, P(Y = 0) = 0.833333 \quad (4.2)$$

$$P(Y = y) = \begin{cases} 0.166666, & y = 1 \\ 0.833333, & y = 0 \\ 0, & \text{otherwise} \end{cases} \quad (4.3)$$

5) Cumulative Distribution function (CDF):

The CDF of a discrete uniform random variable X is:

$$F_X(k) = P(X \leq k) = \frac{k}{6}, \quad k \in \{1, 2, 3, 4, 5, 6\} \quad (5.1)$$

$$F_X(k) = \begin{cases} 0, & k < 1 \\ \frac{k}{6}, & 1 \leq k \leq 6 \\ 1, & k > 6 \end{cases} \quad (5.2)$$

The CDF of a Bernoulli random variable Y is defined as:

$$F_Y(y) = P(Y \leq y) \quad (5.3)$$

$$F_Y(y) = \begin{cases} 0, & y < 0 \\ \frac{5}{6}, & 0 \leq y < 1 \\ 1, & y \geq 1 \end{cases} \quad (5.4)$$

6) Numerical Solution (Monte Carlo)

We can estimate the probability using the Monte Carlo method. We simulate a large number of die rolls and count how many times we get a 6.

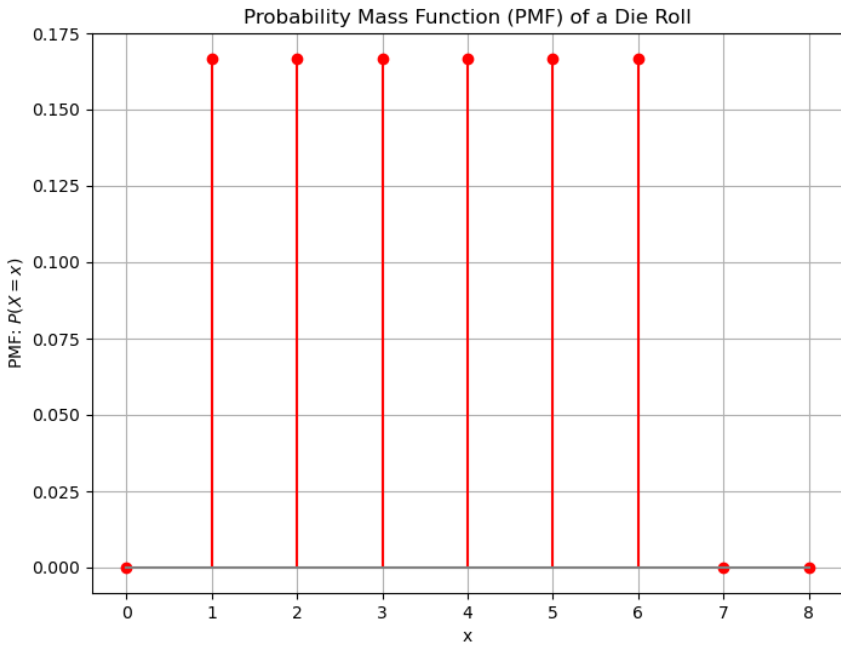


Fig. 6.1: PMF of the Random Variable

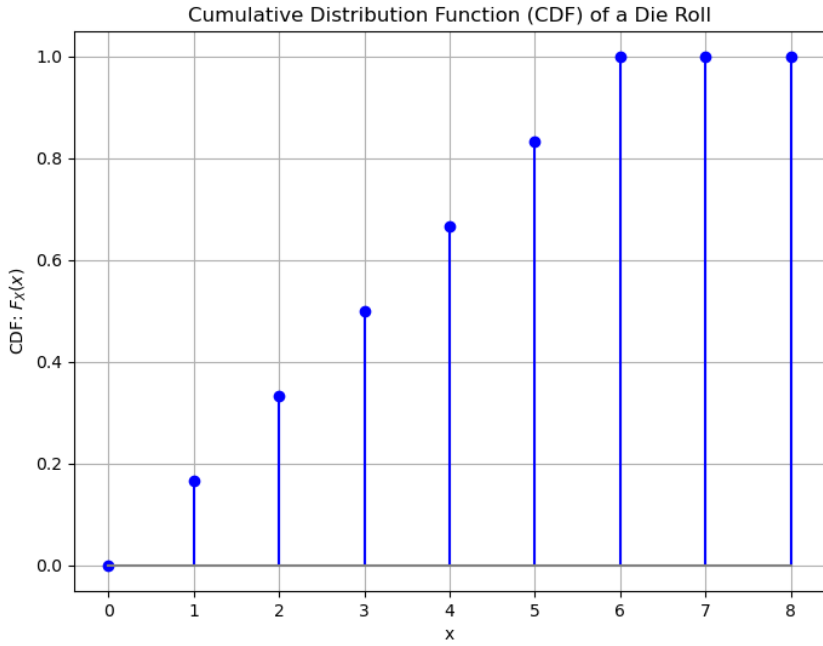


Fig. 6.2: CDF of the Random Variable