

Gradient Ascent

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Problem Statement

Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius R is $\frac{4R}{3}$.

Geometric Analysis

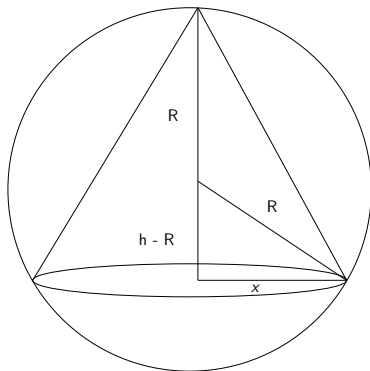
Let R be the radius of the sphere, and let h be the height (altitude) of the inscribed cone. Let x be the radius of the base of the cone.

By considering a cross-section of the sphere and the inscribed cone, we relate x , h , and R using the Pythagorean theorem. The center of the sphere lies on the axis of the cone.

$$x^2 + (h - R)^2 = R^2 \quad (3.1)$$

$$x^2 + h^2 - 2hR + R^2 = R^2 \quad (3.2)$$

$$x^2 = 2hR - h^2 \quad (3.3)$$



Volume Maximization

The volume V of the cone is given by:

$$V = \frac{1}{3}\pi x^2 h \quad (3.4)$$

$$V = \frac{1}{3}\pi(2hR - h^2)h \quad (3.5)$$

$$V = \frac{1}{3}\pi(2Rh^2 - h^3) \implies [\textit{CostFunction}] \quad (3.6)$$

To maximize the volume, we take $\frac{dV}{dh} = 0$:

$$\frac{dV}{dh} = \frac{1}{3}\pi(4Rh - 3h^2) \quad (3.7)$$

$$0 = h(4R - 3h) \implies h \neq 0 \quad (3.8)$$

$$4R - 3h = 0 \quad (3.9)$$

$$h = \frac{4R}{3} \quad (3.10)$$

Second Derivative Test

To confirm a maximum, we take the second derivative:

$$\frac{d^2V}{dh^2} = \frac{1}{3}\pi(4R - 6h) \quad (3.11)$$

Substituting $h = \frac{4R}{3}$:

$$\frac{d^2V}{dh^2} = \frac{1}{3}\pi(4R - 8R) \quad (3.12)$$

$$= -\frac{4\pi R}{3} \quad (3.13)$$

Since this is negative, $h = \frac{4R}{3}$ gives a maximum volume.

Computational Approach

Using Gradient Ascent, the update rule is:

$$h_{n+1} = h_n + \alpha \left. \frac{dV}{dh} \right|_{h=h_n} \quad (3.14)$$

For our case:

$$h_{n+1} = h_n + \alpha \frac{\pi}{3} (4Rh_n - 3h_n^2) \quad (3.15)$$

Choosing:

$$R = 1, \quad \alpha = 0.0001, \quad h_0 = 0.5 \quad (3.16)$$

This converges to $h = \frac{4}{3}$.

Graphical Representation

After a few iterations, the value of h will converge to approximately $\frac{4}{3} \approx 1.333$.

