9-9.2-40

EE24BTECH11052 - RONGALI CHARAN

Question: The area of the region bounded by the curve $y = \sqrt{16 - x^2}$ and x-axis is

1) 8π sq units
2) 20π sq units
3) 16π sq units
4) 256π sq units

Solution: The equation of conic is $g(x)$

$$g(x) = \mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{4.1}$$

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{4.2}$$

$$\mathbf{u} = 0 \tag{4.3}$$

$$f = -16 \tag{4.4}$$

as we know x-axis is representes as

$$\mathbf{h} = \begin{pmatrix} x \\ 0 \end{pmatrix} \tag{4.5}$$

for finding point of intersection of conic with the line $g(\mathbf{h}) = 0$
by solving we get two values as $\mathbf{x_1}$ and $\mathbf{x_2}$ as

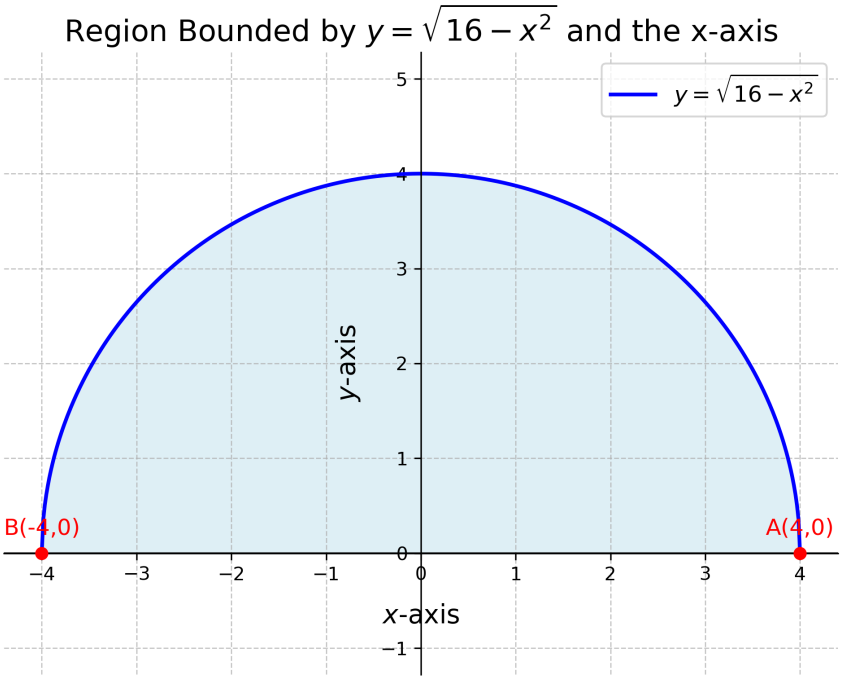
$$\mathbf{x_1} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{4.6}$$

$$\mathbf{x_2} = \begin{pmatrix} -4 \\ 0 \end{pmatrix} \tag{4.7}$$

The area bounded by the curve $y = 16 - x^2$ and x-axis is given by:

$$\int_{-4}^{4} \left(\sqrt{16 - x^2} \right) dx = 8\pi \tag{4.8}$$

Hence, the area bounded by the curve and the line is 8π sq units.


Region Bounded by $y = \sqrt{16 - x^2}$ and the x-axis
$y = \sqrt{16 - x^2}$
B(-4,0)
A(4,0)
y-axis
x-axis