

Assignment-1

EE24BTECH11052 - RONGALI CHARAN

SECTION-A

JEE ADVANCED

E - Subjective Problems

- 1) If $f(x - y) = f(x).g(y) - f(y).g(x)$ and $g(x - y) = g(x).g(y) - f(x).f(y)$ for all $x, y \in \mathbb{R}$. If right hand derivative at $x = 0$ exists for $f(x)$. Find Derivative of $g(x)$ at $x = 0$ (2005 - 4 Marks)

F - Match the Following

- 1) In this question there are entries in columns I and II. Each entry in **Column I** is related to exactly one entry in **Column II**. Write the correct letter from **Column II** against the entry number in **Column I** in your answer book. (2009 - 4 Marks)

Column I

- a) $\sin(\pi[x])$
b) $\sin(\pi(x - [x]))$

Column II

- a) differentiable everywhere
b) nowhere differentiable
c) not differentiable at 1 and -1

- 2) In the following $[x]$ denotes the greatest integer less than or equal to x . Match the functions in Column I with the properties in column II and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in ORS. (2007 - 6 Marks)

Column I

- a) $x|x|$
b) $\sqrt{|x|}$
c) $x + [x]$
d) $|x - 1| + |x + 1|$

Column II

- a) continuous in $(-1, 1)$
b) differentiable in $(-1, 1)$
c) strictly increasing in $(-1, 1)$
d) not differentiable atleast at one point in $(-1, 1)$

- 3) Let $f_1 : \mathbb{R} \rightarrow \mathbb{R}$ $f_2 : [0, \infty) \rightarrow \mathbb{R}$ $f_3 : \mathbb{R} \rightarrow \mathbb{R}$ $f_4 : [0, \infty) \rightarrow \mathbb{R}$ be defined by $f_1(x) = \begin{cases} |x| & \text{if } x < 0 \\ e^x & \text{if } x \geq 0 \end{cases}$
 $f_2(x) = x^2$; $g(x) = \begin{cases} \sin x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$; $f_4(x) = \begin{cases} f_2(f_1(x)) & \text{if } x < 0 \\ f_2(f_1(x)) - 1 & \text{if } x \geq 0 \end{cases}$ (JEE Adv. 2014)

List-I

- P. f_4 is
Q. f_3 is
R. $f_2 \circ f_1$ is
S. f_2 is

List-II

1. Onto but not one-one
2. Neither continuous nor one-one
3. Differentiable but not one-one
4. Continuous and one-one

P Q R S

- (a) 3142
(c) 3124

P Q R S

- (b) 1342
(a) 1324

- 4) Let $f_1 : \mathbb{R} \rightarrow \mathbb{R}$, $f_2 : (-\frac{\pi}{2}, \frac{\pi}{2}) \rightarrow \mathbb{R}$, $f_3 : (-1, e^{\frac{\pi}{2}} - 2) \rightarrow \mathbb{R}$ and $f_4 : \mathbb{R} \rightarrow \mathbb{R}$ be defined by
 1. $f_1(x) = \sin(\sqrt{1 - e^{-x^2}})$,
 2. $f_2(x) = \begin{cases} \frac{|\sin x|}{\tan^{-1} x} & \text{if } x \neq 0 \\ e^x & \text{if } x = 0 \end{cases}$, where the inverse trigonometric function $\tan^{-1} x$ assumes value in $(-\frac{\pi}{2}, \frac{\pi}{2})$,
 3. $f_3(x) = [\sin(\log_e(x + 2))]$, where, for $t \in \mathbb{R}$, $[t]$ denotes the greatest integer less than or equal to t ,
 4. $f_4(x) = \begin{cases} x^2 \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$.

List-I

- P. The function f_1 is
 Q. The function f_2 is
 R. The function f_3 is
 S. The function f_4 is

List-II

1. NOT continuous at $x = 0$
2. continuous at $x = 0$ and NOT differentiable at $x = 0$
3. differentiable at $x = 0$ and its derivative is NOT continuous at $x = 0$
4. differentiable at $x = 0$ and its derivative is continuous at $x = 0$

(JEE Adv. 2018)

- a) $P \rightarrow 2; Q \rightarrow 3; R \rightarrow 1; S \rightarrow 4$
 a) $P \rightarrow 4; Q \rightarrow 2; R \rightarrow 1; S \rightarrow 3$

- a) $P \rightarrow 4; Q \rightarrow 1; R \rightarrow 2; S \rightarrow 3$
 a) $P \rightarrow 2; Q \rightarrow 1; R \rightarrow 4; S \rightarrow 3$

I - Integer Value Correct Type

- 1) Let $f : [1, \infty) \rightarrow [2, \infty)$ be a differentiable function such that $f(1) = 2$. If $6 \int_1^x f(t) dt = 3xf(x) - x^3$ for all $x \geq 1$. Then the value of $f(2)$ is (2011)
- 2) The largest value of non-negative integer a for which $\lim_{x \rightarrow 1} \left\{ \frac{-ax + \sin(x-1) + a}{x + \sin(x-1) - 1} \right\}^{\frac{1-x}{1-\sqrt{x}}} = \frac{1}{4}$ (JEE Adv. 2014)
- 3) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$ be respectively given by $f(x) = |x| + 1$ and $g(x) = x^2 + 1$. Define $h : \mathbb{R} \rightarrow \mathbb{R}$ by

$$h(x) = \begin{cases} \max\{f(x), g(x)\} & \text{if } x \leq 0 \\ \min\{f(x), g(x)\} & \text{if } x > 0 \end{cases}$$
 The number of points at which $h(x)$ is not differentiable is (JEE Adv. 2014)
- 4) Let m and n be two positive integers greater than 1. If $\lim_{\alpha \rightarrow 0} \left(\frac{e^{\cos(\alpha^n)} - e}{\alpha^m} \right) = -\left(\frac{e}{2}\right)$ then the value of $\frac{m}{n}$ is (JEE Adv. 2015)
- 5) Let $\alpha, \beta \in \mathbb{R}$ be such that $\lim_{x \rightarrow 0} \frac{x^2 \sin(\beta x)}{\alpha x - \sin x} = 1$. Then $6(\alpha + \beta)$ equals. (JEE Adv. 2016)

SECTION-B
JEE MAIN/AIEEE

- 1) $\lim_{x \rightarrow 0} \frac{\sqrt{1 - \cos 2x}}{\sqrt{2x}}$ is [2002]
 - (a) 1
 - (b) -1
 - (c) 0
 - (d) does not exist
- 2) $\lim_{x \rightarrow \infty} \left(\frac{x^2 + 5x + 3}{x^2 + x + 3} \right)^x$ [2002]
 - (a) e^4
 - (b) e^2
 - (c) e^3
 - (d) 1
- 3) Let $f(x) = 4$ and $f'(x) = 4$. Then $\lim_{x \rightarrow 2} \frac{xf(2) - 2f(x)}{x-2}$ is given by [2002]
 - (a) 2
 - (b) -2
 - (c) -4
 - (d) 3
- 4) $\lim_{n \rightarrow \infty} \frac{1^p + 2^p + 3^p + \dots + n^p}{n^{p+1}}$ is [2002]
 - (a) $\frac{1}{p+1}$
 - (b) $\frac{1}{1-p}$

(c) $\frac{1}{p} - \frac{1}{p-1}$

(d) $\frac{1}{p+2}$

5) $\lim_{x \rightarrow 0} \frac{\log x^n - [x]}{[x]}, n \in N, ([x] \text{ denotes greatest integer less than or equal to } x)$

[2002]

(a) has value -1

(b) has value 0

(c) has value 1

(d) does not exist