

9-9.2-40

EE24BTECH11052 - RONGALI CHARAN

Question: The area of the region bounded by the curve $y = \sqrt{16 - x^2}$ and x -axis is

- 1) 8π sq units
- 2) 20π sq units
- 3) 16π sq units
- 4) 256π sq units

Solution: The equation of conic $g(x)$ is given by :

$$g(x) = \mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (4.1)$$

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (4.2)$$

$$\mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (4.3)$$

$$f = -16 \quad (4.4)$$

$$L : \mathbf{x} = \mathbf{h} + k\mathbf{m} \quad (4.5)$$

$$\mathbf{h} = \begin{pmatrix} x \\ 0 \end{pmatrix} \quad (4.6)$$

$$\mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (4.7)$$

$$\mathbf{x}_i = \mathbf{h} + k_i \mathbf{m} \quad (4.8)$$

$$k_1 = \frac{1}{\mathbf{m}^T \mathbf{V} \mathbf{m}} \left(-\mathbf{m}^T (\mathbf{V} \mathbf{h} + \mathbf{u}) + \sqrt{[\mathbf{m}^T (\mathbf{V} \mathbf{h} + \mathbf{u})]^2 - g(\mathbf{h}) (\mathbf{m}^T \mathbf{V} \mathbf{m})} \right) = -x + 4 \quad (4.9)$$

$$k_2 = \frac{1}{\mathbf{m}^T \mathbf{V} \mathbf{m}} \left(-\mathbf{m}^T (\mathbf{V} \mathbf{h} + \mathbf{u}) - \sqrt{[\mathbf{m}^T (\mathbf{V} \mathbf{h} + \mathbf{u})]^2 - g(\mathbf{h}) (\mathbf{m}^T \mathbf{V} \mathbf{m})} \right) = -x - 4 \quad (4.10)$$

$$\mathbf{x}_1 = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \quad (4.11)$$

$$\mathbf{x}_2 = \begin{pmatrix} -4 \\ 0 \end{pmatrix} \quad (4.12)$$

The area bounded by the curve $y = \sqrt{16 - x^2}$ and x -axis is given by:

$$\int_{-4}^4 \left(\sqrt{16 - x^2} \right) dx = 8\pi \quad (4.13)$$

Hence, the area bounded by the curve and the line is 8π sq units.

Region Bounded by $y = \sqrt{16 - x^2}$ and the x-axis is 8π sq.units

