Mat Geo Presentation

CHARAN RONGALI
Electrical Engineering,
IIT Hyderabad.

November 6, 2024

Problem Statement

The area of the region bounded by the curve $y = \sqrt{16 - x^2}$ and x-axis is

Conic Representation

The equation of conic $g(x)$ is given by :

$$g(x) = \mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{3.1}$$

Matrix Equation

The parameters can be expxressed as

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ \mathbf{u} = \mathbf{0} \\ f = -16 \tag{3.2}$$

line equation can be represented as

$$L : \mathbf{x} = \mathbf{h} + k\mathbf{m} \tag{3.3}$$

as given line is x-axis,any point on it can be represented as

$$\mathbf{h} = \begin{pmatrix} x \\ 0 \end{pmatrix} \tag{3.4}$$

$$\mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{3.5}$$

$$\mathbf{x_i} = \mathbf{h} + k_i\mathbf{m} \tag{3.6}$$

Point of Intersection

By Substituting (3.6) in (3.1) we get two values of k_1 and k_2

$$k_1 = \frac{1}{\mathbf{m}^\top \mathbf{V}\mathbf{m}} \left(-m^\top \left(\mathbf{V}\mathbf{h} + \mathbf{u} \right) + \sqrt{\left[\mathbf{m}^\top \left(\mathbf{V}\mathbf{h} + \mathbf{u} \right) \right]^2 - g(\mathbf{h}) \left(\mathbf{m}^\top \mathbf{V}\mathbf{m} \right)} \right) \tag{3.7}$$

$$k_2 = \frac{1}{\mathbf{m}^\top \mathbf{V}\mathbf{m}} \left(-m^\top \left(\mathbf{V}\mathbf{h} + \mathbf{u} \right) - \sqrt{\left[\mathbf{m}^\top \left(\mathbf{V}\mathbf{h} + \mathbf{u} \right) \right]^2 - g(\mathbf{h}) \left(\mathbf{m}^\top \mathbf{V}\mathbf{m} \right)} \right) \tag{3.8}$$

so we get

$$k_1 = -x - 4 \tag{3.9}$$

$$k_2 = -x - 4 \tag{3.10}$$

by substituting in (3.6) we get

$$\mathbf{x_1} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{3.11}$$

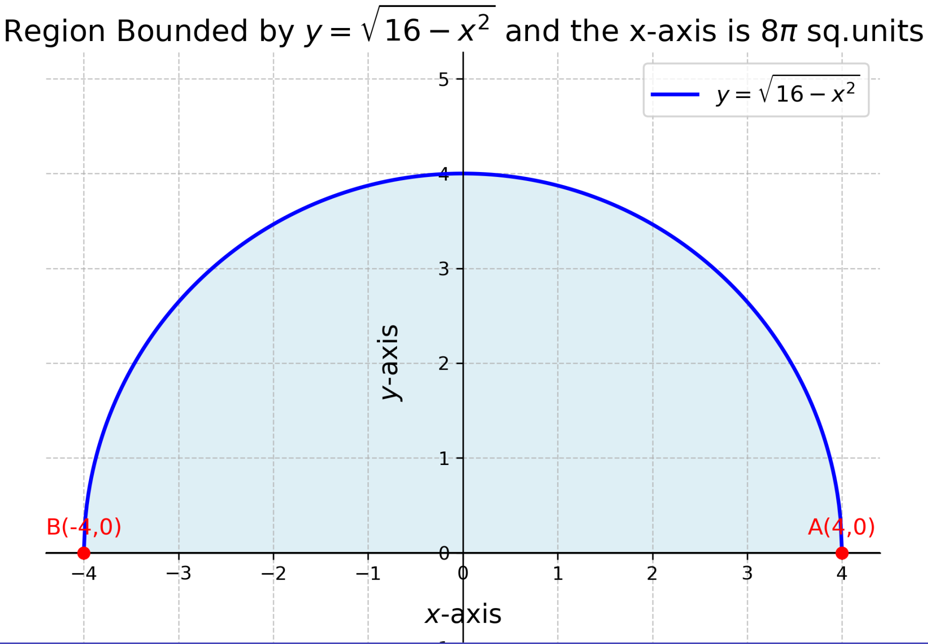
$$\mathbf{x_2} = \begin{pmatrix} -4 \\ 0 \end{pmatrix} \tag{3.12}$$

Area

The area bounded by the curve $y = \sqrt{16 - x^2}$ and x-axis is given by:

$$\int_{-4}^{4} \left(\sqrt{16 - x^2} \right) dx = 8\pi \tag{3.13}$$

Hence, the area bounded by the curve and the line is 8π sq units.



Region Bounded by $y = \sqrt{16 - x^2}$ and the x-axis is 8π sq.units
$y = \sqrt{16 - x^2}$
B(-4,0)
A(4,0)
y-axis
x-axis