

# **Assignment-1**

EE24BTECH11052 - RONGALI CHARAN

## SECTION-A

### JEE ADVANCED

#### E - Subjective Problems

1. If  $f(x - y) = f(x).g(y) - f(y).g(x)$  and  $g(x - y) = g(x).g(y) - f(x).f(y)$  for all  $x, y \in \mathbb{R}$ . If right hand derivative at  $x = 0$  exists for  $f(x)$ . Find Derivative of  $g(x)$  at  $x = 0$ . (2005 - 4 Marks)

#### F. Match the Following

2. In this question there are entries in columns I and II. Each entry in **Column I** is related to exactly one entry in **Column II**. Write the correct letter from **Column II** against the entry number in **Column I** in your answer book. (2009 - 4 Marks)

##### Column I

- (A)  $\sin(\pi[x])$   
(B)  $\sin(\pi(x - [x]))$

##### Column II

- (p) differentiable everywhere  
(q) nowhere differentiable  
(r) not differentiable at 1 and -1

3. In the following  $[x]$  denotes the greatest integer less than or equal to  $x$ . Match the functions in Column I with the properties in column II and indicate your answer by darkening the appropriate bubbles in the  $4 \times 4$  matrix given in ORS. (2007 - 6 Marks)

##### Column I

- (A)  $x|x|$   
(B)  $\sqrt{|x|}$   
(C)  $x + [x]$   
(D)  $|x - 1| + |x + 1|$

##### Column II

- (p) continuous in  $(-1, 1)$   
(q) differentiable in  $(-1, 1)$   
(r) strictly increasing in  $(-1, 1)$   
(s) not differentiable atleast at one point in  $(-1, 1)$

4. Let  $f_1 : \mathbb{R} \rightarrow \mathbb{R}$   $f_2 : [0, \infty) \rightarrow \mathbb{R}$   $f_3 : \mathbb{R} \rightarrow \mathbb{R}$   $f_4 : [0, \infty) \rightarrow \mathbb{R}$  be defined by  $f_1(x) = \begin{cases} |x| & \text{if } x < 0 \\ e^x & \text{if } x \geq 0 \end{cases}$   
 $f_2(x) = x^2$  ;  $g(x) = \begin{cases} \sin(x) & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$  ;  $f_4(x) = \begin{cases} f_2(f_1(x)) & \text{if } x < 0 \\ f_2(f_1(x)) - 1 & \text{if } x \geq 0 \end{cases}$  (JEE Adv. 2014)

##### List-I

- P.  $f_4$  is  
Q.  $f_3$  is  
R.  $f_2 \circ f_1$  is  
S.  $f_2$  is

##### List-II

1. Onto but not one-one  
2. Neither continuous nor one-one  
3. Differentiable but not one-one  
4. Continuous and one-one

##### P Q R S

- (a) 3 1 4 2  
(c) 3 1 2 4

##### P Q R S

- (b) 1 3 4 2  
(a) 1 3 2 4

5. Let  $f_1 : \mathbb{R} \rightarrow \mathbb{R}$ ,  $f_2 : (-\frac{\pi}{2}, \frac{\pi}{2}) \rightarrow \mathbb{R}$ ,  $f_3 : (-1, e^{\frac{\pi}{2}} - 2) \rightarrow \mathbb{R}$  and  $f_4 : \mathbb{R} \rightarrow \mathbb{R}$  be defined by  
 1.  $f_1(x) = \sin(\sqrt{1 - e^{-x^2}})$ ,  
 2.  $f_2(x) = \begin{cases} \frac{|\sin x|}{\tan^{-1} x} & \text{if } x \neq 0 \\ e^x & \text{if } x = 0 \end{cases}$ , where the inverse trigonometric function  $\tan^{-1} x$  assumes value in  $(-\frac{\pi}{2}, \frac{\pi}{2})$ ,  
 3.  $f_3(x) = [\sin(\log_e(x + 2))]$ , where, for  $t \in \mathbb{R}$ ,  $[t]$  denotes the greatest integer less than or equal to  $t$ ,  
 4.  $f_4(x) = \begin{cases} x^2 \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ .

**List-I**

- P. The function  $f_1$  is  
Q. The function  $f_2$  is  
R. The function  $f_3$  is  
S. The function  $f_4$  is

- (a)  $P \rightarrow 2; Q \rightarrow 3; R \rightarrow 1; S \rightarrow 4$   
(c)  $P \rightarrow 4; Q \rightarrow 2; R \rightarrow 1; S \rightarrow 3$

**List-II**

1. NOT continuous at  $x = 0$
2. continuous at  $x = 0$  and NOT differentiable at  $x = 0$
3. differentiable at  $x = 0$  and its derivative is NOT continuous at  $x = 0$
4. differentiable at  $x = 0$  and its derivative is continuous at  $x = 0$

(JEE Adv. 2018)

- (b)  $P \rightarrow 4; Q \rightarrow 1; R \rightarrow 2; S \rightarrow 3$   
(a)  $P \rightarrow 2; Q \rightarrow 1; R \rightarrow 4; S \rightarrow 3$

*I - Integer Value Correct Type*

6. Let  $f : [1, \infty) \rightarrow [2, \infty)$  be a differentiable function such that  $f(1) = 2$ . If  $6 \int_1^x f(t) dt = 3xf(x) - x^3$  for all  $x \geq 1$ . Then the value of  $f(2)$  is (2011)
7. The largest value of non-negative integer  $a$  for which  $\lim_{x \rightarrow 1} \left\{ \frac{-ax + \sin(x-1) + a}{x + \sin(x-1) - 1} \right\}^{\frac{1-x}{1-\sqrt{x}}} = \frac{1}{4}$  (JEE Adv. 2014)
8. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  and  $g : \mathbb{R} \rightarrow \mathbb{R}$  be respectively given by  $f(x) = |x| + 1$  and  $g(x) = x^2 + 1$ . Define  $h : \mathbb{R} \rightarrow \mathbb{R}$  by
- $$h(x) = \begin{cases} \max\{f(x), g(x)\} & \text{if } x \leq 0 \\ \min\{f(x), g(x)\} & \text{if } x > 0 \end{cases}$$
- The number of points at which  $h(x)$  is not differentiable is (JEE Adv. 2014)
9. Let  $m$  and  $n$  be two positive integers greater than 1. If  $\lim_{\alpha \rightarrow 0} \left( \frac{e^{\cos(\alpha^n)} - e}{\alpha^m} \right) = -\left(\frac{e}{2}\right)$  then the value of  $\frac{m}{n}$  is (JEE Adv. 2015)
10. Let  $\alpha, \beta \in \mathbb{R}$  be such that  $\lim_{x \rightarrow 0} \frac{x^2 \sin(\beta x)}{\alpha x - \sin x} = 1$ . Then  $6(\alpha + \beta)$  equals. (JEE Adv. 2016)
15.  $\lim_{x \rightarrow 0} \frac{\log x^n - [x]}{[x]}, n \in \mathbb{N}, ([x] \text{ denotes greatest integer less than or equal to } x)$  [2002]
- (a) has value  $-1$   
 (b) has value  $0$   
 (c) has value  $1$   
 (d) does not exist

**SECTION-B**

**JEE MAIN/AIEEE**

11.  $\lim_{x \rightarrow 0} \frac{\sqrt{1 - \cos 2x}}{\sqrt{2}x}$  is [2002]
- (a) 1  
 (b)  $-1$   
 (c) 0  
 (d) does not exist
12.  $\lim_{x \rightarrow \infty} \left( \frac{x^2 + 5x + 3}{x^2 + x + 3} \right)^x$  [2002]
- (a)  $e^4$   
 (b)  $e^2$   
 (c)  $e^3$   
 (d) 1
13. Let  $f(x) = 4$  and  $f'(x) = 4$ . Then  $\lim_{x \rightarrow 2} \frac{xf(2) - 2f(x)}{x-2}$  is given by [2002]
- (a) 2  
 (b)  $-2$   
 (c)  $-4$   
 (d) 3
14.  $\lim_{n \rightarrow \infty} \frac{1^p + 2^p + 3^p + \dots + n^p}{n^{p+1}}$  is [2002]
- (a)  $\frac{1}{p+1}$   
 (b)  $\frac{1}{1-p}$   
 (c)  $\frac{1}{p} - \frac{1}{p-1}$   
 (d)  $\frac{1}{p+2}$