Assignment-1

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SECTION-A

JEE ADVANCED

E - Subjective Problems

1) If $f(x-y) = f(x).g(y) - f(y).g(x)$ and $g(x-y) = g(x).g(y) - f(x).f(y)$ for all $x, y \in R$. If right hand derivative at $x = 0$ exists for $f(x)$. Find Derivative of $g(x)$ at $x = 0$ (2005 - 4 Marks)

F - Match the Following

1) In this question there are entries in columns I and II. Each entry in Column I is related to exactly one entry in Column II. Write the correct letter from Column II against the entry number in Column Iin your answer book. (2009 - 4 Marks)

Column I

a) $\sin(\pi[x])$
b) $\sin(\pi(x - [x]))$

Column II

a) differentiable everywhere
b) nowhere differentiable
c) not differentiable at 1 and -1

2) In the following $[x]$ denotes the greatest integer less than or equal to x. Match the functions in Column I with the properties in column II and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in ORS. (2007 - 6 Marks)

Column I

a) $x|x|$
b) $\sqrt{|x|}$
c) $x + [x]$
d) $|x-1| + |x+1|$

Column II

a) continuous in $(-1, 1)$
b) differentiable in $(-1, 1)$
c) strictly increasing in $(-1, 1)$
d) not differentiable atleast at one point in $(-1, 1)$

3) Let $f_1 : R \to R$ $f_2 : [0, \infty) \to R$ $f_3 : R \to R$ $f_4 : [0, \infty) \to R$ be defined by

$$f_1(x) = \begin{cases} |x| & \text{if } x < 0 \\ e^x & \text{if } x \geq 0 \end{cases};$$

$$f_2(x) = x^2;$$

$$f_3(x) = \begin{cases} \sin x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases};$$

$$f_4(x) = \begin{cases} f_2(f_1(x)) & \text{if } x < 0 \\ f_2(f_1(x)) - 1 & \text{if } x \geq 0 \end{cases}.$$

(JEE Adv. 2014)

List-I

P. f_4 is
Q. f_3 is
R. $f_2 o f_1$ is
S. f_2 is

List-II

1. Onto but not one-one
2. Neither continuous nor one-one
3. Differentiable but not one-one
4. Continuous and one-one

P Q R S

(a) 3142

(b) 3124
(c) 1342
(d) 1324

4) Let $f_1 : \mathbf{R} \to \mathbf{R}, f_2 : (-\frac{\pi}{2}, \frac{\pi}{2}) \to \mathbf{R}, f_3 : (-1, e^{\frac{\pi}{2}} - 2) \to \mathbf{R}$ and $f_4 : \mathbf{R} \to \mathbf{R}$ be defined by
1. $f_1(x) = \sin(\sqrt{1 - e^{-x^2}})$,
2. $f_2(x) = \begin{cases} \frac{|\sin x|}{\tan^{-1} x} & \text{if } x \neq 0 \\ e^x & \text{if } x = 0 \end{cases}$,where the inverse trigonometric function $\tan^{-1} x$ assumes value in $(-\frac{\pi}{2}, \frac{\pi}{2})$,
3. $f_3(x) = [\sin(log_e(x + 2))]$,where, for $t \in \mathbf{R}$, $[t]$ denotes the greatest integer less than or equal to t,
4. $f_4(x) = \begin{cases} x^2 \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$.

List-I	**List-II**
P. The function f_1 is	1. NOT continuous at $x = 0$
Q. The function f_2 is	2. continuous at $x = 0$ and NOT differentiable at $x = 0$
R. The function f_3 is	3. differentiable at $x = 0$ and its derivative is NOT continuous at $x = 0$
S. The function f_4 is	4. differentiable at $x = 0$ and its derivative is continuous at $x = 0$

(JEE Adv. 2018)

a) $P \to 2; Q \to 3; R \to 1; S \to 4$
b) $P \to 4; Q \to 2; R \to 1; S \to 3$
c) $P \to 4; Q \to 1; R \to 2; S \to 3$
d) $P \to 2; Q \to 1; R \to 4; S \to 3$

I - Integer Value Correct Type

1) Let $f : [1, \infty) \to [2, \infty)$ be a differentiable function such that $f(1) = 2$. If $6\int_1^x f(t)dt = 3xf(x) - x^3$ for all $x \geq 1$. Then the value of $f(2)$ is (2011)

2) The largest value of non-negative integer a for which

$$\lim_{x \to 1} \{\frac{-ax + \sin(x - 1) + a}{x + \sin(x - 1) - 1}\}^{\frac{1-x}{1-\sqrt{x}}} = \frac{1}{4}$$

(JEE Adv. 2014)

3) Let $f : R \to R$ and $g : R \to R$ be respectively given by $f(x) = |x| + 1$ and $g(x) = x^2 + 1$. Define $h : R \to R$ by

$$h(x) = \begin{cases} \text{maxf(x),g(x)} & \text{if } x \leq 0 \\ \text{minf(x),g(x)} & \text{if } x > 0 \end{cases}.$$

The number of points at which $h(x)$ is not differentiable is (JEE Adv. 2014)

4) Let m and n be two positive integers greater than 1. If

$$\lim_{\alpha \to 0} (\frac{e^{\cos(\alpha^n)} - e}{\alpha^m}) = -(\frac{e}{2})$$

then the value of $\frac{m}{n}$ is (JEE Adv. 2015)

5) Let $\alpha, \beta \in \mathbf{R}$ be such that

$$\lim_{x \to 0} \frac{x^2 \sin(\beta x)}{\alpha x - sinx} = 1$$

. Then $6(\alpha + \beta)$ equals. (JEE Adv. 2016)

SECTION-B

JEE MAIN/AIEEE

1) $\lim_{x\to 0} \frac{\sqrt{1-\cos 2x}}{\sqrt{2}x}$ is [2002]
   a) 1
   b) -1
   c) 0
   d) does not exist

2) $\lim_{x\to\infty} (\frac{x^2+5x+3}{x^2+x+3})^x$ [2002]
   a) e^4
   b) e^2
   c) e^3
   d) 1

3) Let $f(x) = 4$ and $f'(x) = 4$. Then $\lim_{x\to 2} \frac{xf(2)-2f(x)}{x-2}$ is given by [2002]
   a) 2
   b) -2
   c) -4
   d) 3

4) $\lim_{n\to\infty} \frac{1^p+2^p+3^p+\cdots+n^p}{n^{p+1}}$ is [2002]
   a) $\frac{1}{p+1}$
   b) $\frac{1}{1-p}$
   c) $\frac{1}{p} - \frac{1}{p-1}$
   d) $\frac{1}{p+2}$

5) $\lim_{x\to 0} \frac{\log x^n - [x]}{[x]}, n \in N$, ($[x]$ denotes greatest integer less than or equal to x) [2002]
   a) has value -1
   b) has value 0
   c) has value 1
   d) does not exist