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# Reasoning under Uncertainty

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# Reasoning under uncertainty

- Agents in the real world need to handle uncertainty, whether due to partial observability, nondeterminism, or adversaries.
- An agent may never know for sure what state it is in now or where it will end up after a sequence of actions.

# Nature of Uncertain Knowledge

- Let us try to write rules for dental diagnosis using propositional logic, so that we can see how the logical approach breaks down. Consider the following simple rule:

$$\text{Toothache} \Rightarrow \text{Cavity.}$$

- The problem is that this rule is wrong.
- Not all patients with toothaches have cavities; some of them have gum disease, swelling, or one of several other problems:

$$\text{Toothache} \Rightarrow \text{Cavity} \vee \text{GumProblem} \vee \text{Swelling} \vee \ldots\ldots..$$

# Nature of Uncertain Knowledge

- In order to make the rule true, we have to add an almost unlimited list of possible problems. We could try turning the rule into a causal rule:

$$\text{Cavity} \Rightarrow \text{Toothache}$$

But this rule is also not right; not all cavities cause pain. Toothache and a Cavity are always not connected, so the judgement may go wrong.

# Nature of Uncertain Knowledge

- This is typical of the medical domain, as well as most other judgmental domains: law, business, design, automobile repair, gardening, dating, and so on.
- The agent's knowledge can at best provide only a degree of belief in the relevant sentences.
- Our main tool for dealing with degrees of belief is probability theory.
- A logical agent believes each sentence to be true or false or has no opinion, whereas a probabilistic agent may have a numerical degree of belief between 0 (for sentences that are certainly false) and 1 (certainly true).

# Basic Probability Notation

- Random variables are typically divided into three kinds, depending on the type of the domain:
- **Boolean random variables**, such as Cavity, have the domain (true, false) or (1,0)
- **Discrete random variables**, take on values from a countable domain. For example, the domain of Weather might be (sunny, rainy, cloudy, snow).
- **Continuous random variables** (bounded or unbounded) take on values from the real numbers. Ex: $temp=21.4$; $temp<21.4$ or $temp< 1$.

# Atomic events or sample points

- Atomic event: A complete specification of the state of the world about which the agent is uncertain
- E.g., if the world consists of only two Boolean variables Cavity and Toothache, then there are 4 distinct atomic events:

$$Cavity = false \wedge Toothache = false$$

$$Cavity = false \wedge Toothache = true$$

$$Cavity = true \wedge Toothache = false$$

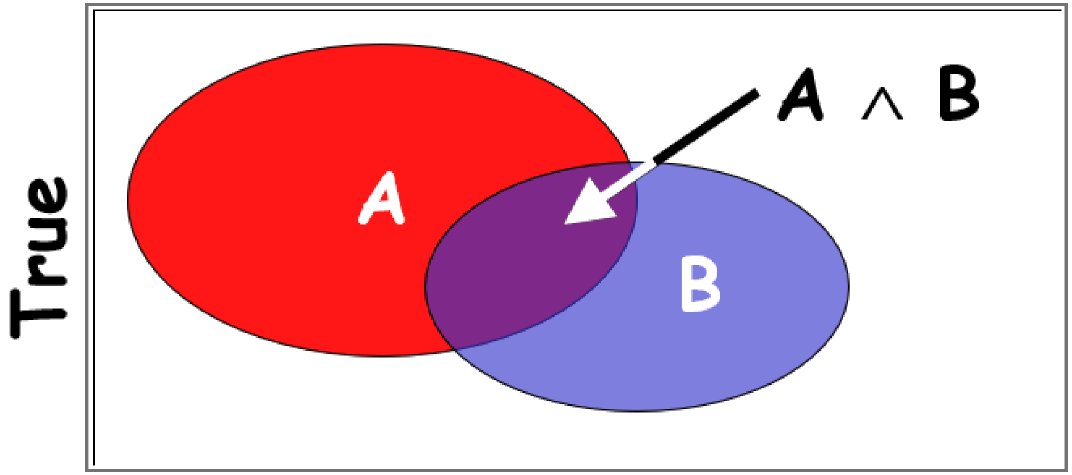
$$Cavity = true \wedge Toothache = true$$

- Atomic events are mutually exclusive and exhaustive
- When two events are mutually exclusive, it means they cannot both occur at the same time.
- When two events are exhaustive, it means that one of them must occur.

# Axioms of Probability Theory

- All probabilities between 0 and 1
  - $0 \le P(A) \le 1$
  - $P(true) = 1$
  - $P(false) = 0$.
- The probability of disjunction is:

$$P(A \vee B) = P(A) + P(B) - P(A \wedge B)$$



# Prior probability

- The unconditional or prior probability associated with a proposition **A** is the degree of belief according to the absence of any other information;
- It is written as P ( A ).
- For example, if the prior probability that I have a cavity is 0.1, then we would write

**P ( Cavity= true ) = 0.1 or P ( cavity ) = 0.1**

- P ( A ) can be used only when there is no other information.
- As soon as some new information is known, we must reason with the conditional probability of a given that new information.

# Prior probability…

- Sometimes, we will want to talk about the probabilities of all the possible values of a random variable.
- In that case, we will use an expression such as P(Weather), which denotes a vector of values for the probabilities of each individual state of the weather.
- Instead of writing these four equations

P ( Weather = sunny) = 0.7

P ( Weather= rain) = 0.2

P ( Weather= cloudy) = 0.08

P(Weather = snow) = 0.02

we may simply write: P( Weather) = (0.7,0.2,0.08,0.02) (Note that the probabilities sum to 1 )

- This statement defines a prior probability distribution for the random variable Weather.

# Prior probability…

- Joint probability distribution for a set of random variables gives the probability of every atomic event on those random variables
- P(Weather, Cavity) = a 4 × 2 matrix of values:

| Weather = | sunny | rainy | cloudy | snow |
|---|---|---|---|---|
| Cavity = true | 0.144 | 0.02 | 0.016 | 0.02 |
| Cavity = false | 0.576 | 0.08 | 0.064 | 0.08 |

- A full joint distribution specifies the probability of every atomic event and is therefore a complete specification of one's uncertainty about the world in question.

# Conditional or posterior probability

- The notation used is P(a l b),where a and b are any proposition.

This is read as "the probability of a, given that all we know is b." For example,

P(cavity l toothache) = 0.8

“indicates that if a patient is observed to have a toothache and no other information is yet available, then the probability of the patient's having a cavity will be 0.8.”

# Conditional or posterior probability

- Conditional probabilities can be defined in terms of unconditional probabilities.

$$P(a|b) = \frac{P(a \wedge b)}{P(b)}$$

holds whenever P(b)>0

This equation can be written as

P(a^b) = P(a|b) * P(b) (which is called product rule)

Alternative way:

P(a^b) = P(b|a) * P(a)

# Chain Rule/Product Rule

$$P(X_1, \ldots, X_n) = P(X_n | X_1..X_{n-1})P(X_{n-1} | X_1..X_{n-2}) \ldots P(X_1)$$

$$= \prod P(X_i | X_1, ..X_{i-1})$$