Types of Learning

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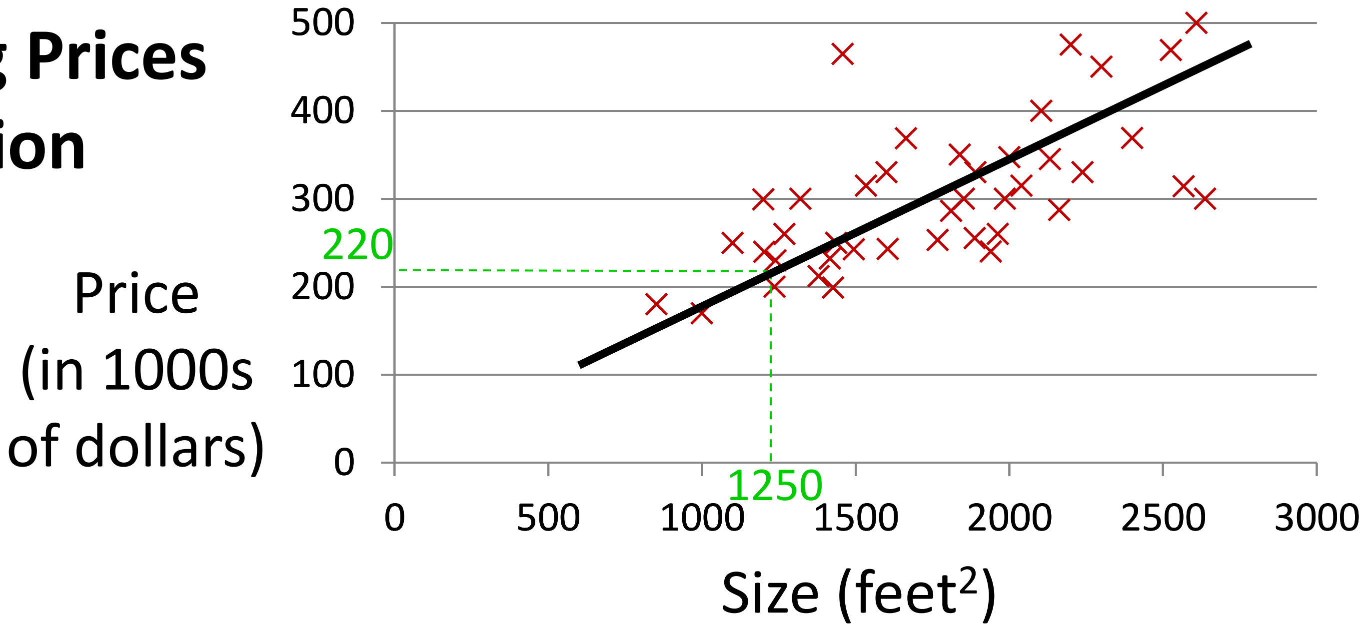
Linear regression with one variable

Ex. Housing Prices prediction



Supervised Learning

Given the "right answer" for each example in the data.

Regression Problem

Predict real-valued output

Classification : Discrete-valued output

Training set of housing prices

Size in feet^2 (x)	Price ($) in 1000's (y)
2104	460
1416	232
1534	315
852	178
…	…

m

Notation:

m = Number of training examples

x's = "input" variable / features

y's = "output" variable / "target" variable

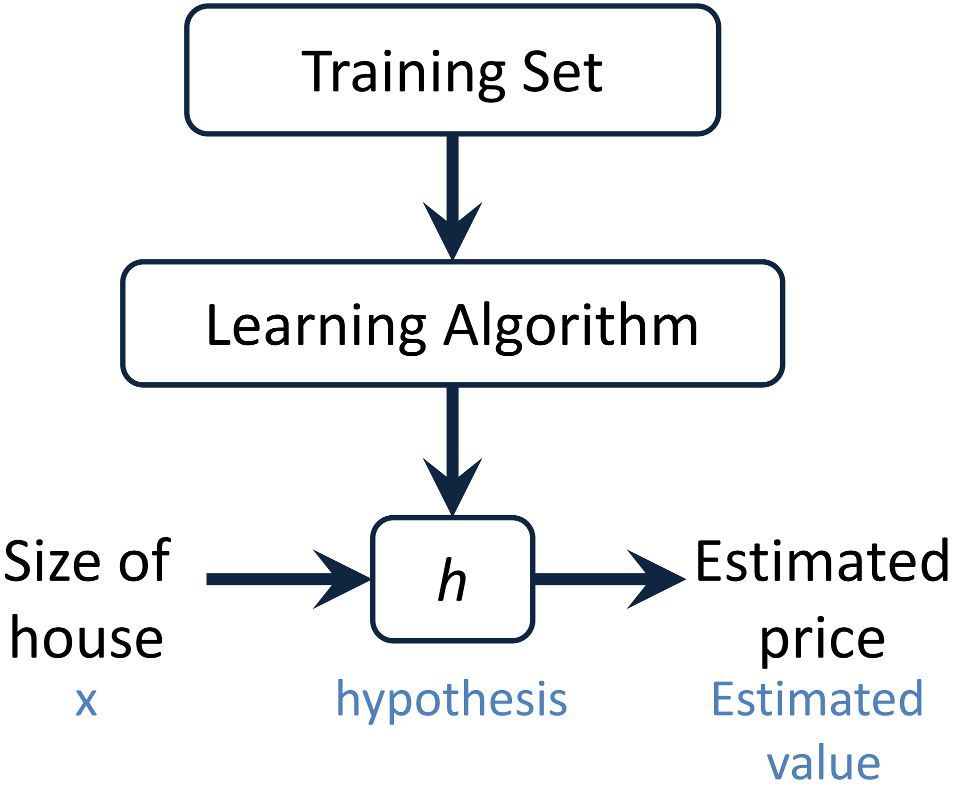
(x, y) – one training example

$(x^{(i)}, y^{(i)})$ – ith training example

$x^{(1)} = 2104$

$x^{(2)} = 1416$

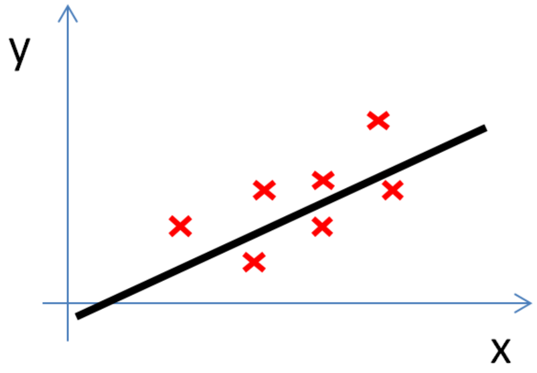
$y^{(1)} = 460$



h maps from x's to y's

How do we represent *h* ?

$h_\Theta(x) = \Theta_0 + \Theta_1 x$



Linear regression with one variable.
Univariate linear regression.

One variable

Linear regression with one variable

Cost function

Training Set

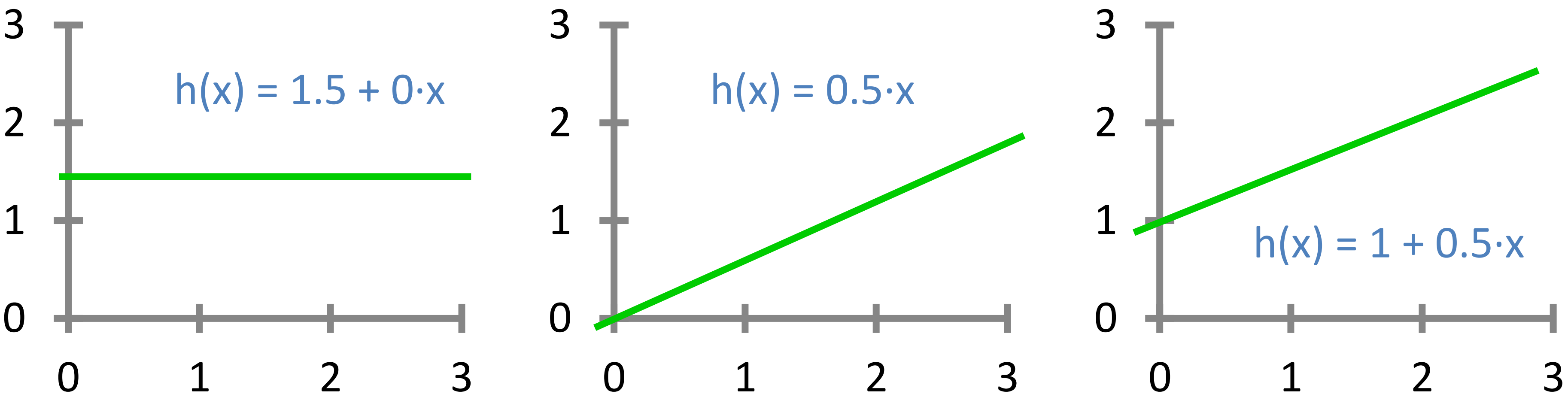
Size in feet^2 (x)	**Price ($) in 1000's (y)**
2104	460
1416	232
1534	315
852	178
…	…

Hypothesis: $h_\theta(x) = \theta_0 + \theta_1 x$

θ_i's: Parameters

How to choose θ_i's ?

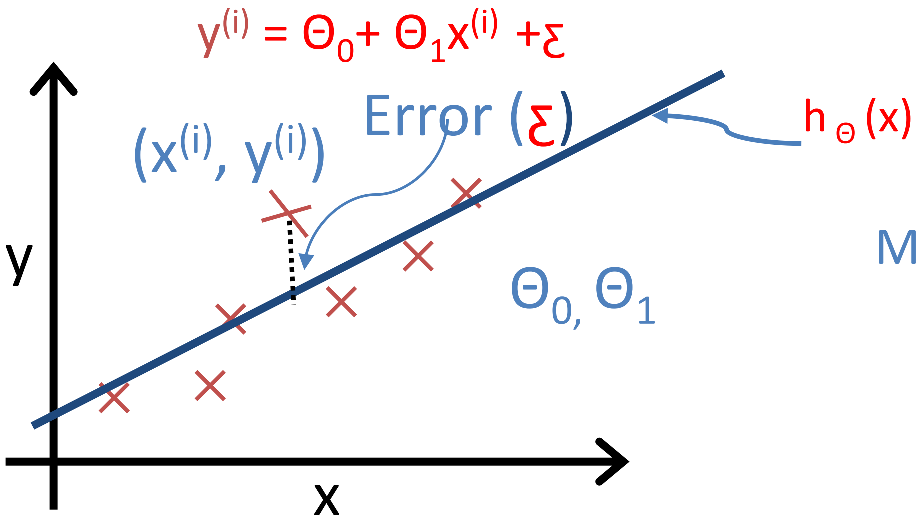
$$h_\theta(x) = \theta_0 + \theta_1 x$$



$\theta_0 = 1.5$
$\theta_1 = 0$

$\theta_0 = 0$
$\theta_1 = 0.5$

$\theta_0 = 1$
$\theta_1 = 0.5$



$$\underset{\Theta_0\,\Theta_1}{\text{Minimize}} \ \frac{1}{2m}\sum_{i=1}^{m}\left(h_\Theta(x^{(i)}) - y^{(i)}\right)^2$$

$$h_\Theta(x) = \Theta_0 + \Theta_1 x^{(i)}$$

$$J(\Theta_0, \Theta_1) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\Theta(x^{(i)}) - y^{(i)}\right)^2$$

Idea: Choose θ_0, θ_1 so that $h_\theta(x)$ is close to y for our training examples (x, y)

$\underset{\Theta_0\,\Theta_1}{\text{Minimize}}\ J(\Theta_0, \Theta_1)$: Cost Function

Squared error function

Simplified

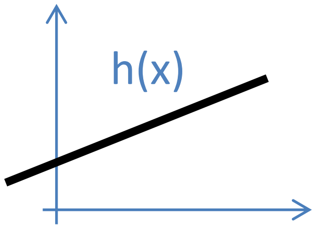
Hypothesis:

$$h_\theta(x) = \theta_0 + \theta_1 x$$

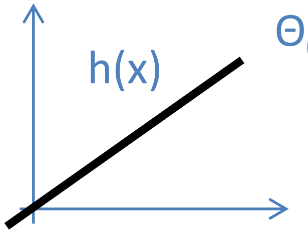
$$h_\theta(x) = \theta_1 x$$

Parameters:

$$\theta_0, \theta_1$$



$$\theta_1$$



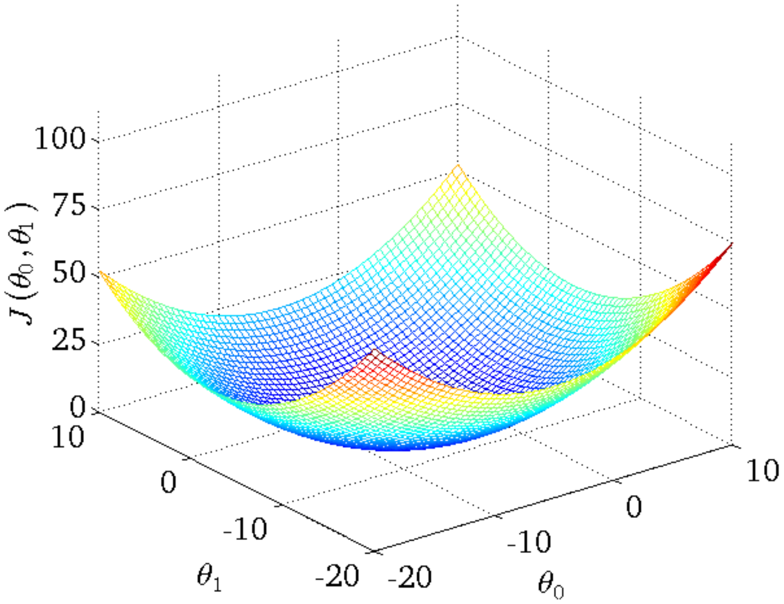
Cost Function:

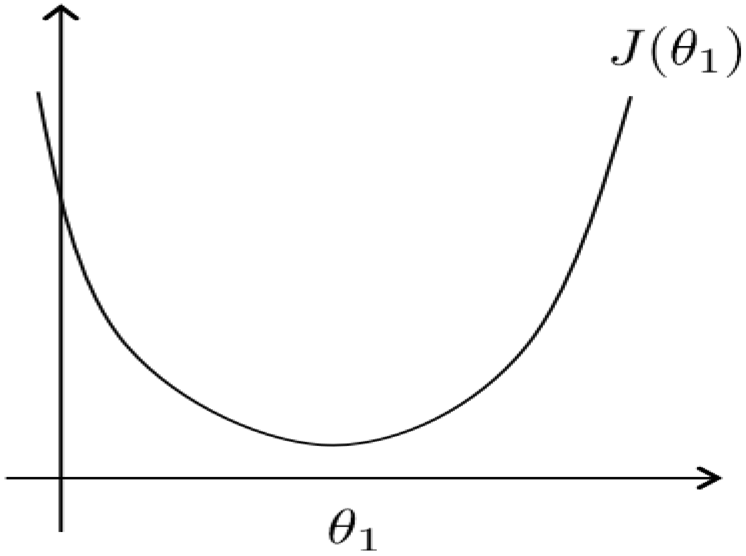
$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

$$J(\theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

Goal: $\underset{\theta_0, \theta_1}{\text{minimize}}\ J(\theta_0, \theta_1)$

$$\underset{\theta_1}{\text{minimize}}\ J(\theta_1)$$


100
75
50
25
0
$J(\theta_0,\theta_1)$
10
0
-10
-20
θ_1
-20
-10
0
10
θ_0


$J(\theta_1)$
θ_1

Hypothesis: $h_\theta(x) = \theta_0 + \theta_1 x$

Parameters: θ_0, θ_1

Cost Function: $J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$

Goal: $\underset{\theta_0, \theta_1}{\text{minimize}}\ J(\theta_0, \theta_1)$

Linear regression with one variable

Gradient descent

Have some function $J(\theta_0, \theta_1)$

Want $\min_{\theta_0, \theta_1} J(\theta_0, \theta_1)$

Outline:

- Start with some θ_0, θ_1
- Keep changing θ_0, θ_1 to reduce $J(\theta_0, \theta_1)$

  until we hopefully end up at a minimum

Gradient descent algorithm

$$\text{repeat until convergence } \{$$
$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) \qquad (\text{for } j = 0 \text{ and } j = 1)$$
$$\}$$

Learning rate

Simultaneously update
Θ_0 & Θ_1

Correct: Simultaneous update

$$\text{temp0} := \theta_0 - \alpha \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$$
$$\text{temp1} := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$$
$$\theta_0 := \text{temp0}$$
$$\theta_1 := \text{temp1}$$

Incorrect:

$$\text{temp0} := \theta_0 - \alpha \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$$
$$\theta_0 := \text{temp0}$$
$$\text{temp1} := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$$
$$\theta_1 := \text{temp1}$$

Gradient descent algorithm

repeat until convergence {

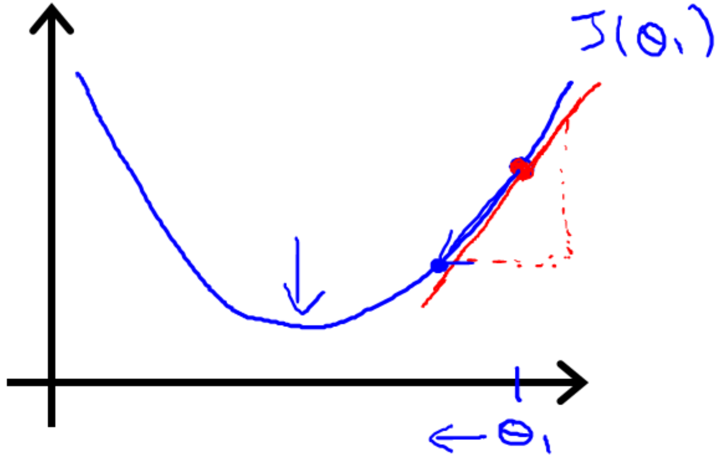
$$\theta_j := \theta_j - \alpha \boxed{\frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)}$$ (simultaneously update $j = 0$ and $j = 1$)

}

Learning rate

derivative

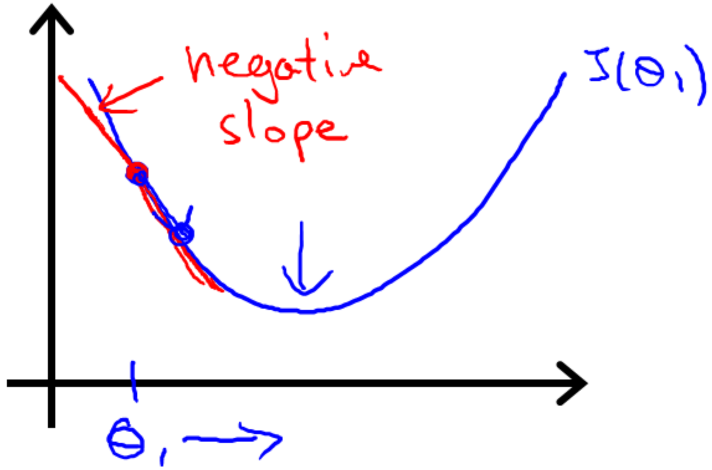
The partial derivative is the slop of the tangent line to this curve at that point.



$$\Theta_1 := \Theta_1 - \alpha \boxed{\frac{\partial}{\partial \theta_1} J(\theta_1)}$$

≥ 0

$$\Theta_1 := \Theta_1 - \alpha(positive\ number)$$



$$\Theta_1 := \Theta_1 - \alpha \boxed{\frac{\partial}{\partial \theta_1} J(\theta_1)}$$

≤ 0

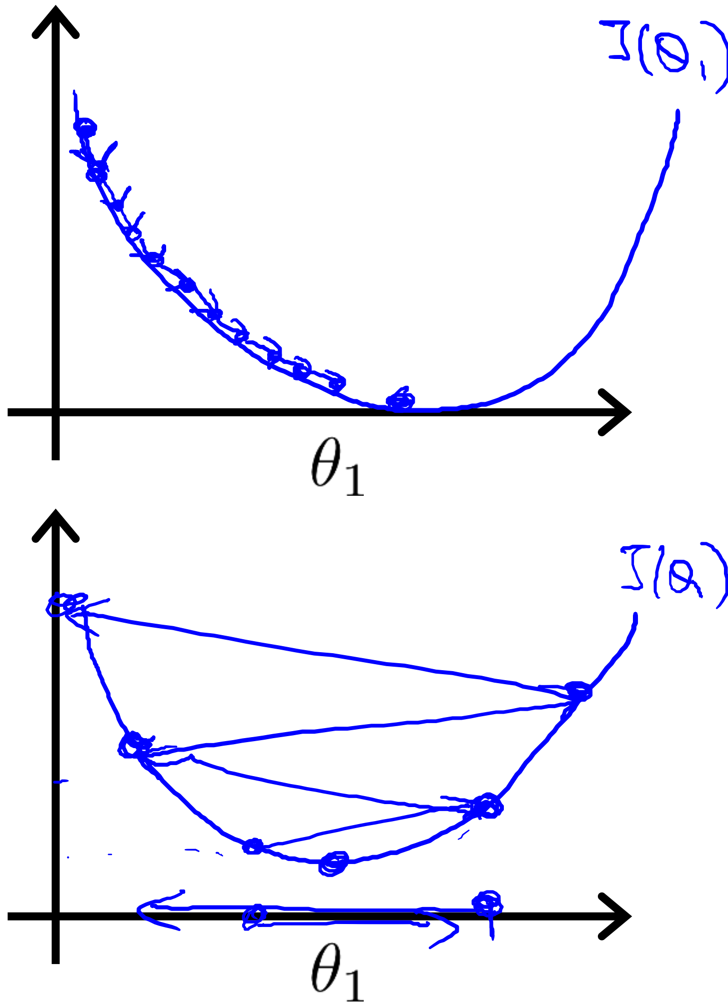
$$\Theta_1 := \Theta_1 - \alpha(negative\ number)$$

Role of Learning Rate

$$\theta_1 := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_1)$$

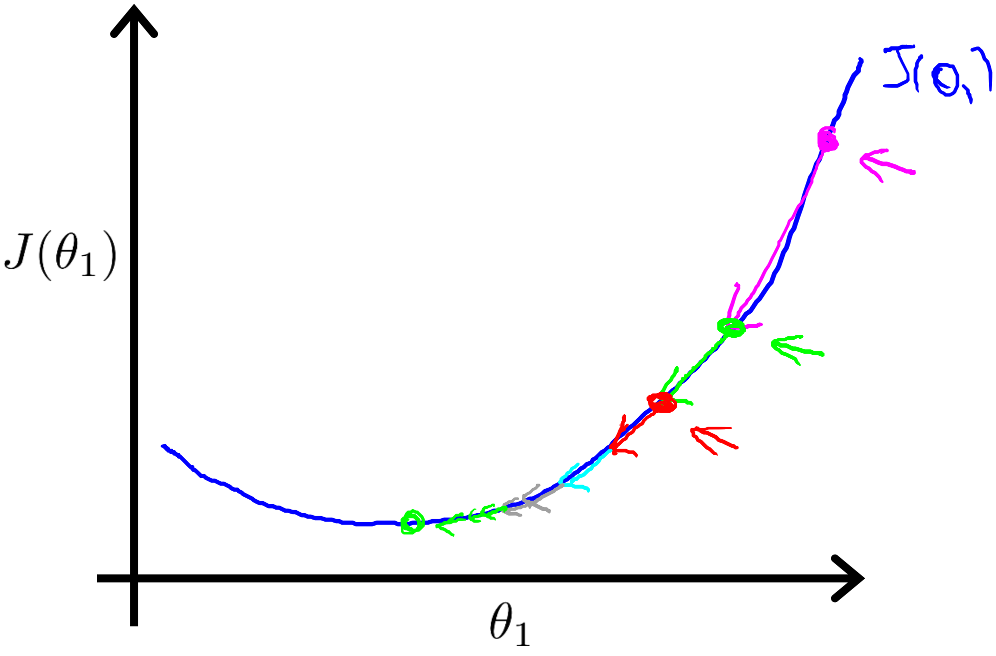
If α is too small, gradient descent can be slow.

If α is too large, gradient descent can overshoot the minimum. It may fail to converge, or even diverge.



Gradient descent can converge to a local minimum, even with the learning rate α fixed.

$$\theta_1 := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_1)$$



As we approach a local minimum, gradient descent will automatically take smaller steps. So, no need to decrease α over time.

Linear regression with one variable

Gradient descent for linear regression

Gradient descent algorithm

repeat until convergence {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)$$

$$(\text{for } j = 1 \text{ and } j = 0)$$

}

Linear Regression Model

$$h_\theta(x) = \theta_0 + \theta_1 x$$

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

$$\frac{\partial}{\partial\theta_j} J(\theta_0, \theta_1) = \frac{\partial}{\partial\theta_j} \cdot \frac{1}{2m} \sum_{i=1}^{m} (h_\Theta(x^{(i)}) - y^{(i)})^2$$

$$= \frac{\partial}{\partial\theta_j} \cdot \frac{1}{2m} \sum_{i=1}^{m} (\theta_0 + \theta_1 \cdot x^{(i)} - y^{(i)})^2$$

$$j = 0 : \frac{\partial}{\partial\theta_0} J(\theta_0, \theta_1) = \frac{1}{m} \sum_{i=1}^{m} (h_\Theta(x^{(i)}) - y^{(i)})$$

$$j = 1 : \frac{\partial}{\partial\theta_1} J(\theta_0, \theta_1) = \frac{1}{m} \sum_{i=1}^{m} (h_\Theta(x^{(i)}) - y^{(i)}) \cdot x^{(i)}$$

Gradient descent algorithm

$$\text{repeat until convergence } \{$$

$$\frac{d}{d\theta_0} \cdot J(\theta_0, \theta_1)$$

$$\theta_0 := \theta_0 - \alpha \boxed{\frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)}$$

$$\theta_1 := \theta_1 - \alpha \boxed{\frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right) \cdot x^{(i)}}$$

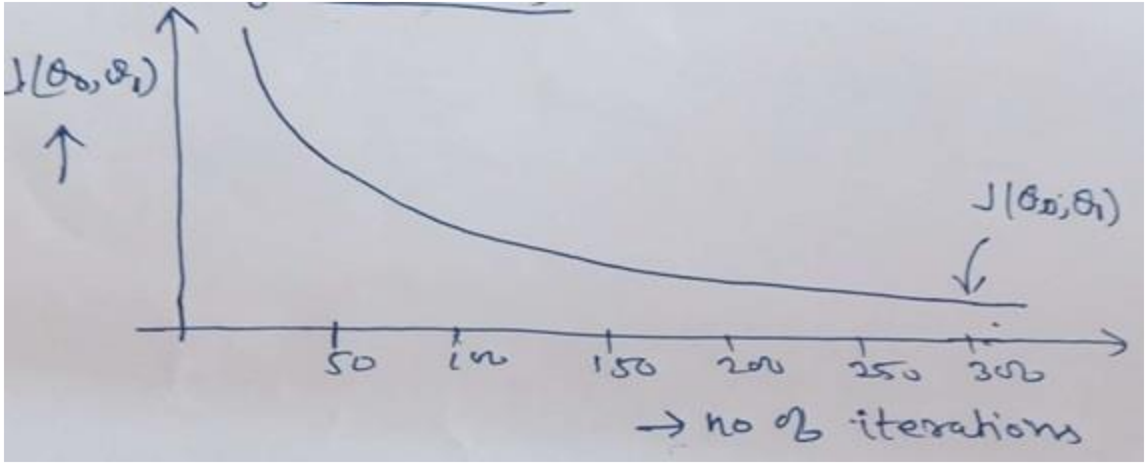
$$\}$$

$$\frac{d}{d\theta_1} \cdot J(\theta_0, \theta_1)$$

update θ_0 and θ_1 simultaneously

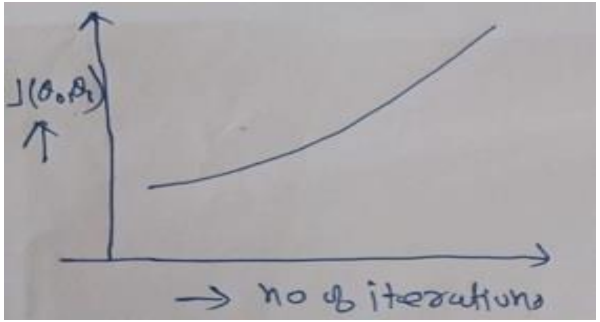
How to choose learning rate (α)

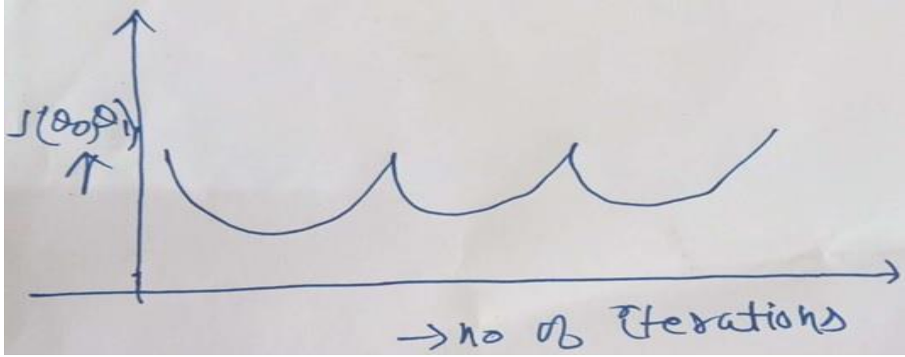
If gradient decent is working correctly then $J(\theta_0, \theta_1)$ should decrease after every iteration.



How to choose learning rate (α)

If gradient decent is not working correctly then..





- **Cause:** Large α
- **Solution:** use small α
- **To choose α, Try as,**

0.001, 0.003, 0.01, 0.03, 0.1, …….., 1.0

Linear Regression with Multiple Variable

- Ex. House Price Prediction

Size in feet^2	No. of bed rooms	No. of floors	Age of House in Years	Price ($) in 1000's (y)
2104	5	1	45	450
1416	3	2	40	232
1534	3	2	30	315
….	…	-----		…
…..	….	----		….

Linear Regression with Multiple Variable

Ex. House Price Prediction

Notation:

n = number of features

$x^{(i)}$ = input (features) of i^{th} training example

$x_j^{(i)}$ =value of feature j in i^{th} training example.

Hypothesis

Simple Linear Regression: $h_\theta(x) = \theta_0 + \theta_1 x$

For multi-variate linear regression:

$$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \cdots + \theta_n x_n$$

- For convenience of notation, define $x_0 = 1$
- Hypothesis:

$$h_\theta(x) = \theta^T x = \theta_0 x_0 + \theta_1 x_1 + \theta_2 x_2 + \cdots + \theta_n x_n$$

Parameters: $\theta_0, \theta_1, \ldots, \theta_n$

Cost function:

$$J(\theta_0, \theta_1, \ldots, \theta_n) = \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2$$

Gradient descent

Repeat $\{$

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \ldots, \theta_n)$$

$\}$ (simultaneously update for every $j = 0, \ldots, n$)

Gradient Descent

Previously (n=1):

Repeat $\{$

$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})$$

$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x^{(i)}$$

(simultaneously update θ_0, θ_1)

$\}$

New algorithm $(n \geq 1)$:

Repeat $\{$

$$\theta_j := \theta_j - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)}$$

simultaneously update θ_j for $j = 0, \ldots, n$

$\}$

Gradient Descent

Previously (n=1):

Repeat {

$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})$$

$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x^{(i)}$$

(simultaneously update θ_0, θ_1)

}

New algorithm $(n \geq 1)$:

Repeat {

$$\theta_j := \theta_j - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)}$$

simultaneously update θ_j for $j = 0, \ldots, n$

}

$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_0^{(i)}$$

$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_1^{(i)}$$

$$\theta_2 := \theta_2 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_2^{(i)}$$

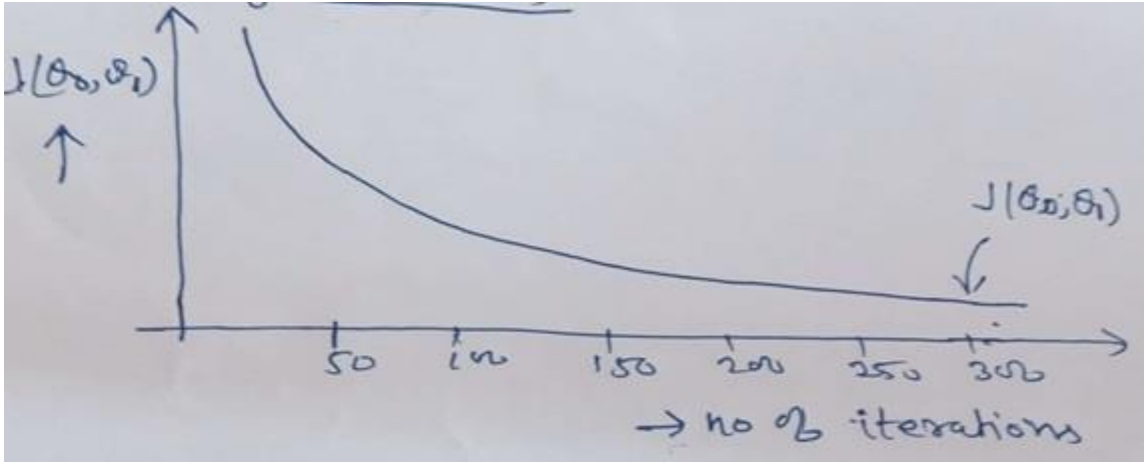
...

Is gradient descent working properly?

- Plot how $J(\theta)$ changes with every iteration of gradient descent
- For sufficiently small learning rate, $J(\theta)$ should decrease with every iteration
  - If not, learning rate needs to be reduced
- However, too small learning rate means slow convergence

How to choose learning rate (α)

If gradient decent is working correctly then J(θ) should decrease after every iteration.



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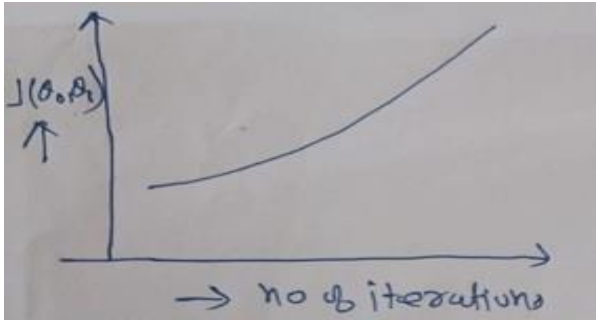

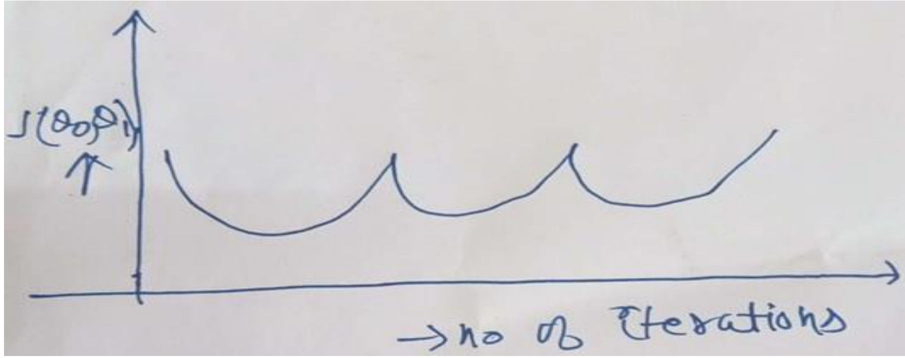

- **Cause:** Large α
- **Solution:** use small α
- **To choose α, Try as,**

0.001, 0.003, 0.01, 0.03, 0.1, 1.0


Thank You
Gracias
Danke
Merci
Grazie