CS 3011: Artificial Intelligence

Solving Problems by Searching

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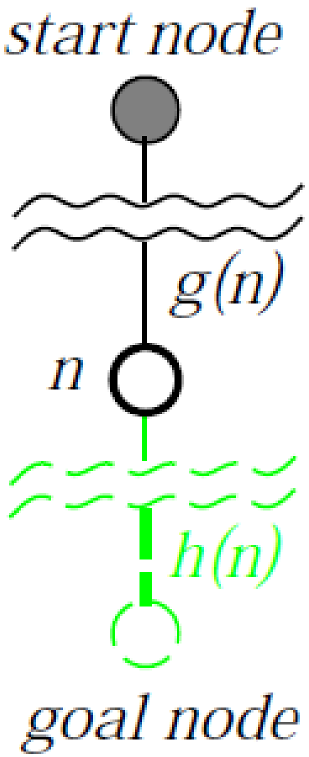
A* Search

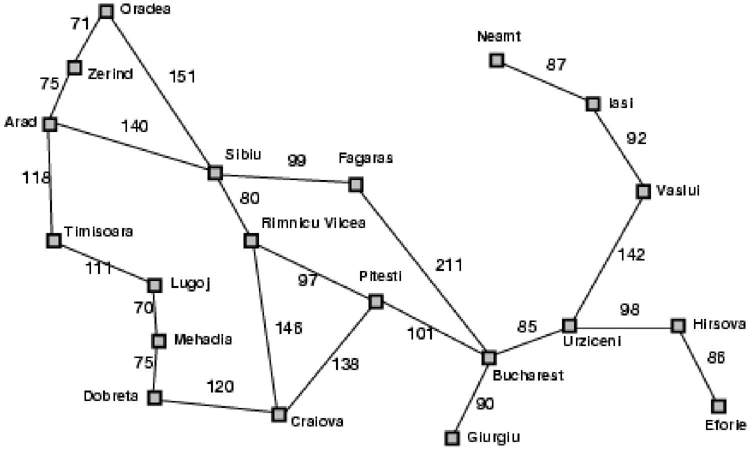
- Idea: Avoid expanding paths that are already expensive
- Evaluation function $f(n) = g(n) + h(n)$
  - $g(n)$ = exact cost so far to reach n
  - $h(n)$ = estimated cost to goal from n
  - $f(n)$ = estimated total cost of cheapest path through n to goal
- A* search uses an admissible heuristic:
  - $h(n) \leq h^*(n)$ where $h^*(n)$ is the true cost from n
  - Also $h(n) \geq 0$, and $h(G)=0$ for any goal G
  - E.g., $h_{SLD}(n)$ is an admissible heuristic because it doesn't overestimate the actual road distance.



A* Search

- If we are trying to find the cheapest solution, a reasonable thing to try first is the node with the lowest value of $g(n) + h(n)$
- This strategy is more than just reasonable
  - Provided that $h(n)$ satisfies certain conditions, A* using TREE search is both complete and optimal.

A* search example



Straight–line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	10
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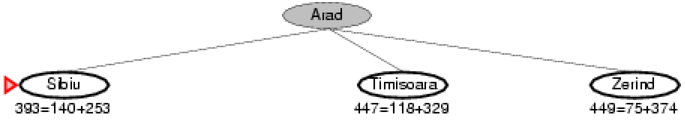
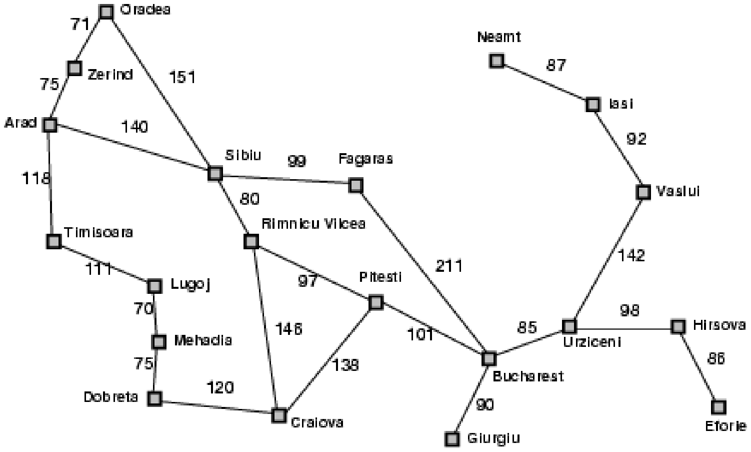
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

A* Tree search example





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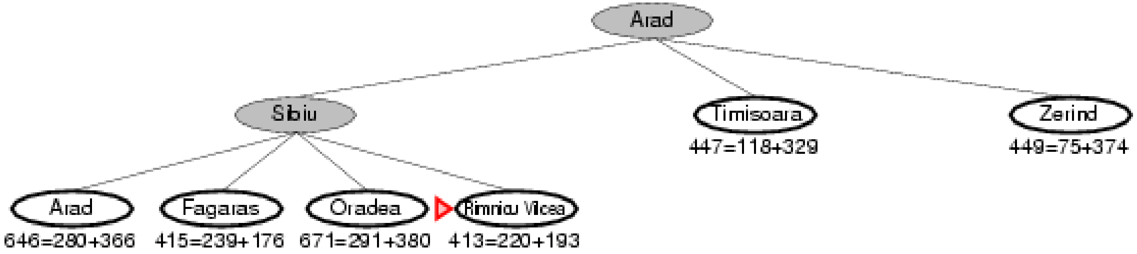

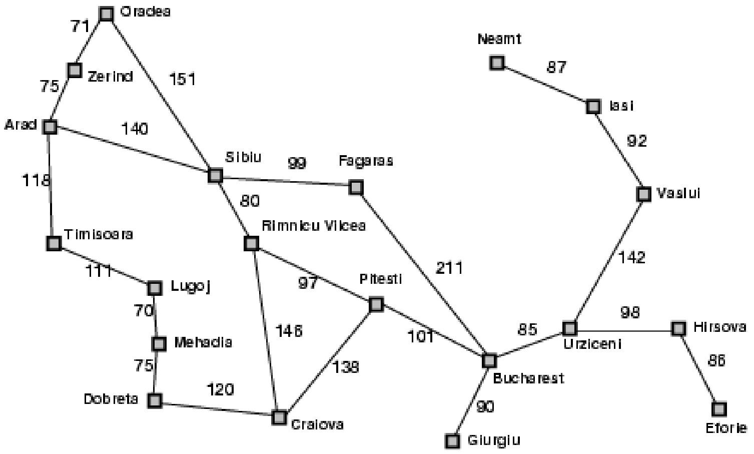

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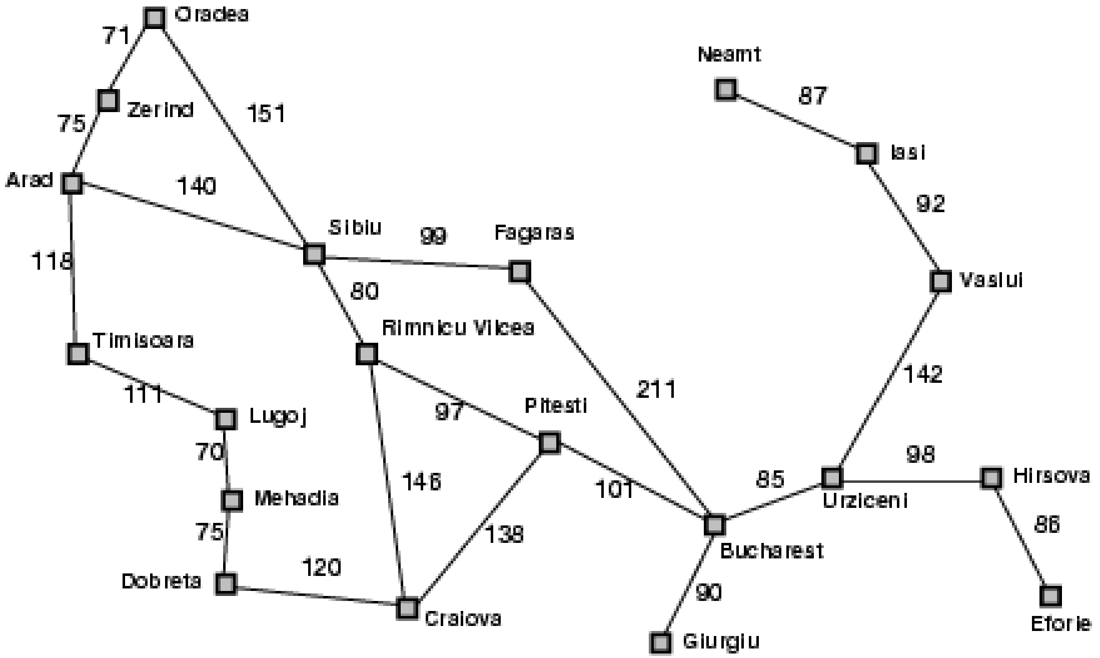

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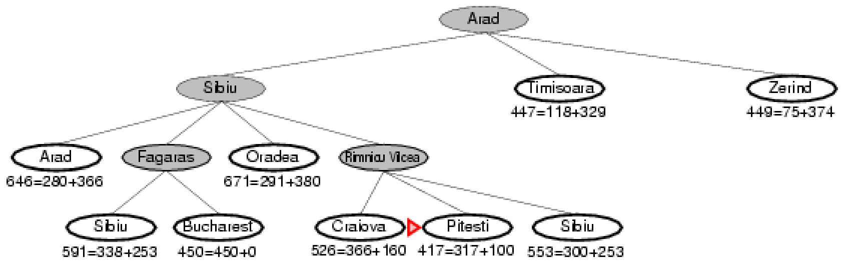

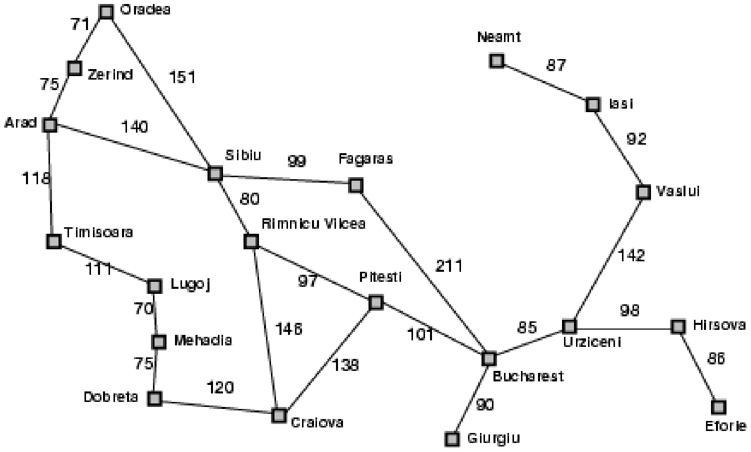

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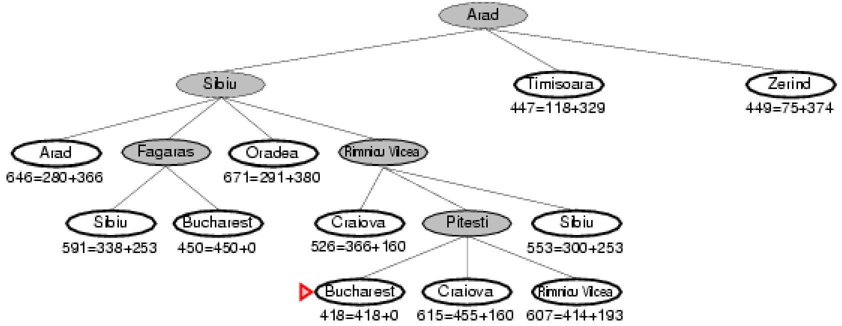

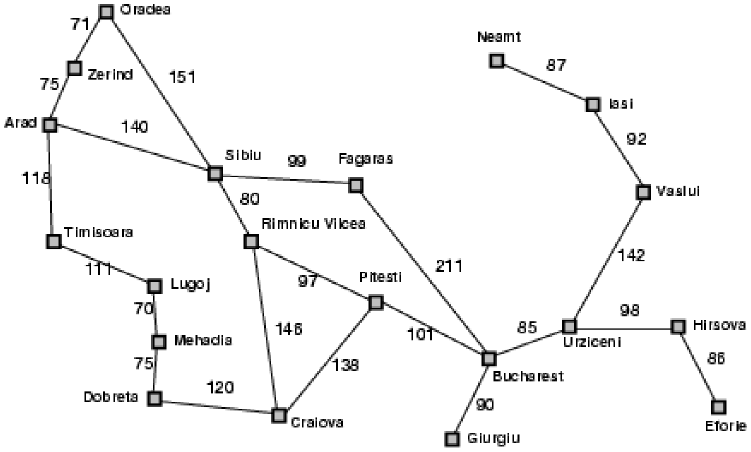

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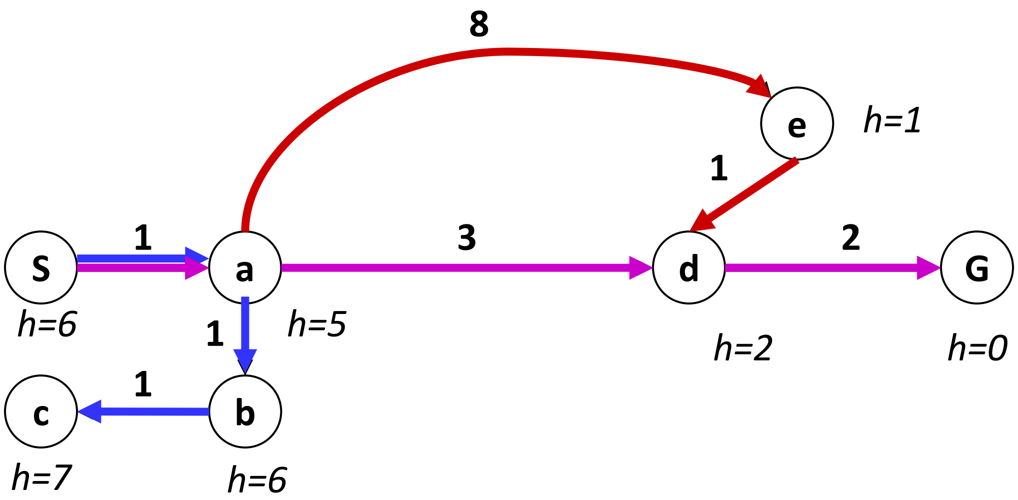
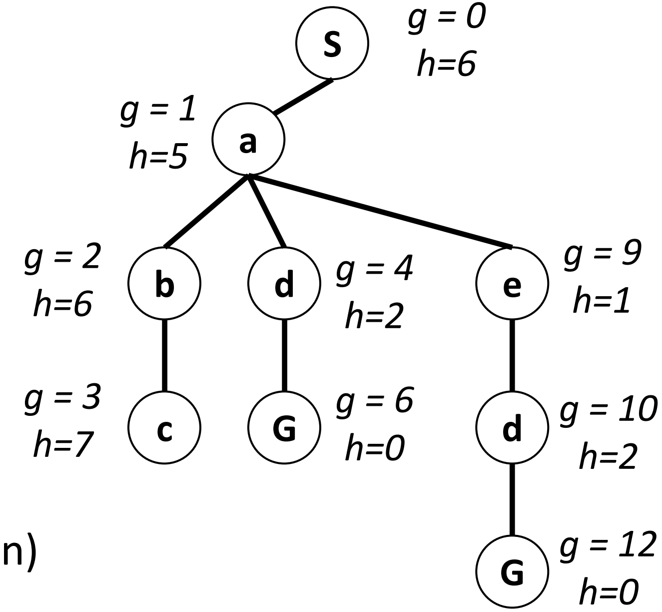
Combining UCS and Greedy

- Uniform-cost orders by path cost, g(n)
- Greedy orders by heuristic value, h(n)

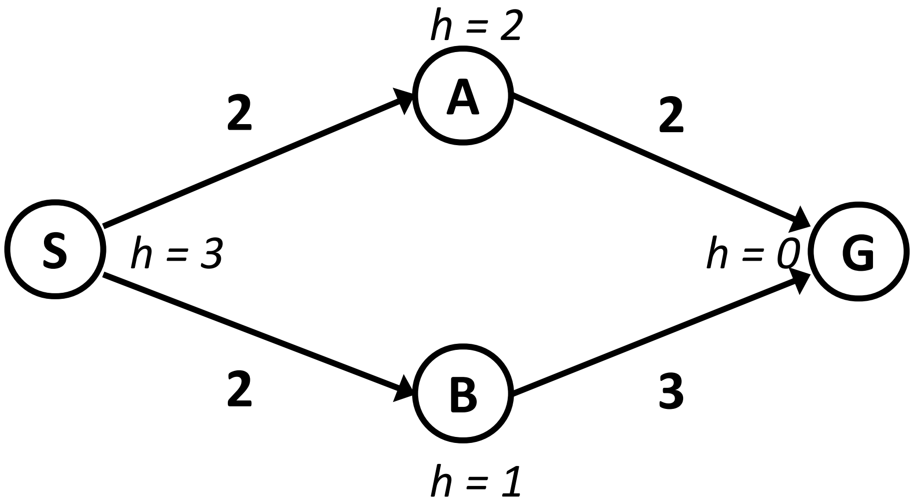




- A* Search orders by the sum: f(n) = g(n) + h(n)

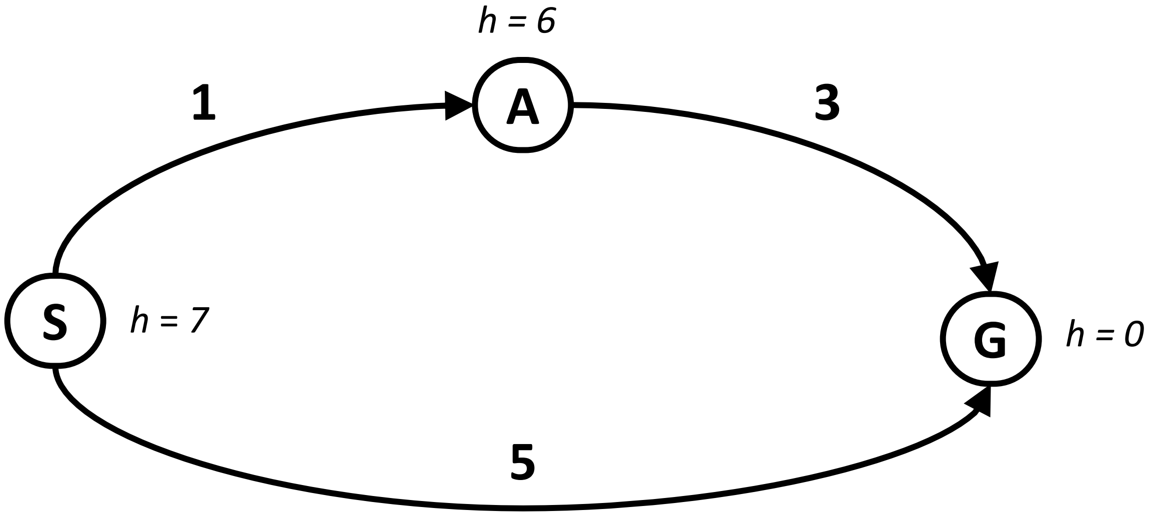
When should A* terminate?

- Should we stop when we enqueue a goal?



- No: only stop when we dequeue a goal

Is A* Optimal?



- What went wrong?
- We need estimates to be less than actual costs!

Admissible Heuristics

- A heuristic *h* is *admissible* (optimistic) if:

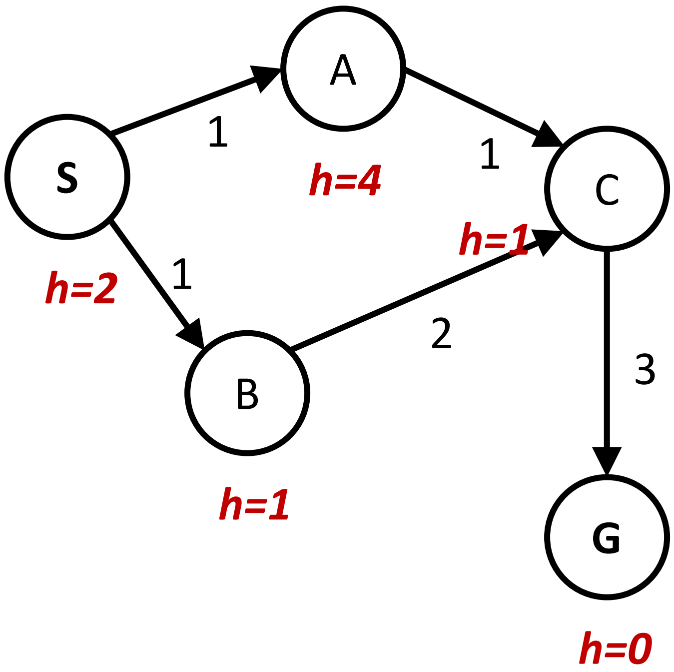
$$0 \leq h(n) \leq h^*(n)$$

  where $h^*(n)$ is the true cost to a nearest goal

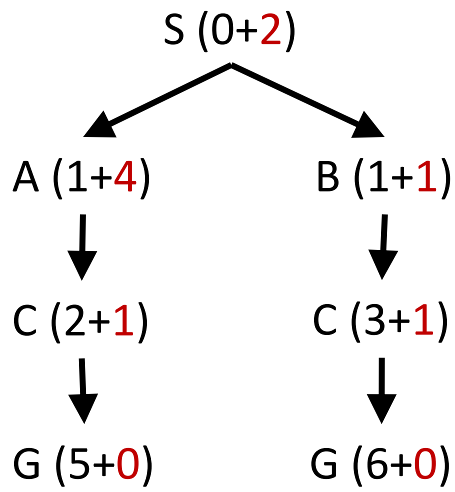
- Theorem: If *h(n)* is admissible, A* using `TREE-SEARCH` is optimal

A* Graph Search Gone Wrong?

State space graph

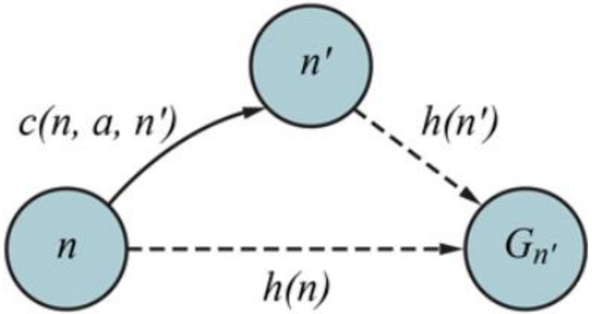


Search tree



Consistent heuristics

- A heuristic is consistent if for every node n, every successor n' of n generated by any action a,
- $h(n) \le c(n,a,n') + h(n')$
- `If h is consistent, we have
- $f(n') = g(n') + h(n')$
- $= g(n) + c(n,a,n') + h(n')$
- $\ge g(n) + h(n)$
- $= f(n)$
- i.e., f(n) is non-decreasing along any path.
- Theorem: If h(n) is consistent, A* using GRAPH-SEARCH is optimal



Dominance

Definition: If $h_2(n) \geq h_1(n)$ for all n (both admissible) then h_2 dominates h_1

- **Essentially, domination translates directly into efficiency:**
- h2 is better for search.
- A* using h2 will never expand more nodes than A* using h1 .

Properties of A*

- Complete? Yes
- Time? Exponential: O(b^m)
- Space? O(b^m):Keeps all nodes in memory
- Optimal? Yes