REPORT

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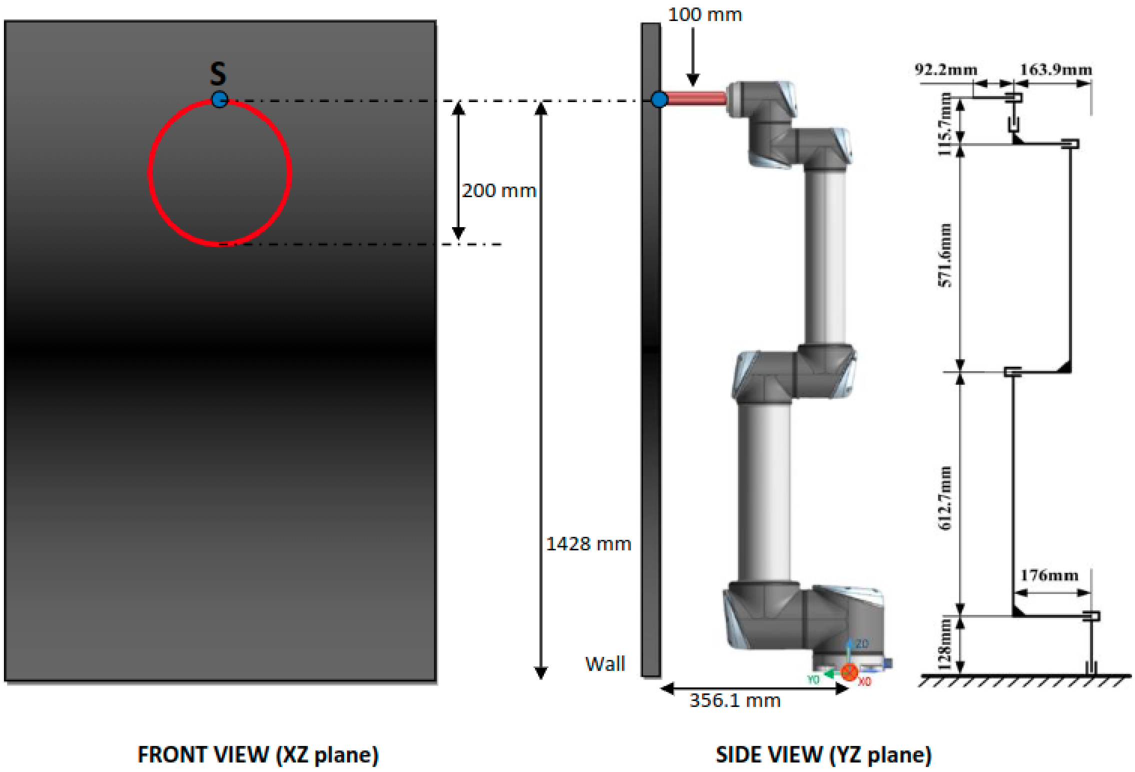

The length of the pen is 100mm. This can be considered as the length of the last link so we have to add it to the length of the last link.

DH Table:

Links	a_i	α_i	d_i	θ_i
0-1	0	-90	128	θ_1
1-2	612.7	180	0	θ_2-90
2-3	571.6	180	0	θ_3
3-4	0	90	163.9	θ_4+90
4-5	0	-90	115.7	θ_5
5-6	0	0	192.2	θ_6

Jacobian Matrix:

- The jacobian is found using the first method

Step 1: Calculate ${}^{0}_{i}T$
Step 2: Calculate O_i
Step 3: Calculate Z_i
Step 4: Calculate J_i
Step 5: Write J

- The J_i depends on the type of the joint.

  If it the joint is revolute then

$$J_i = \begin{bmatrix} Z_{i-1} x (O_n - O_{i-1}) \\ Z_{i-1} \end{bmatrix}$$

  If the joint is prismatic then

$$J_i = \begin{bmatrix} Z_{i-1} \\ 0 \end{bmatrix}$$

- The J_i vectors of size 6x1 for all the joint are calculated and $J=[J_1\ J_2 \ldots\ldots\ldots\ldots J_n]$, where J is the Jacobian.
- The origin Vectors found are:

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The origin vectors are:
O0:
⎡0⎤
⎢0⎥
⎣0⎦
O1:
⎡ 0 ⎤
⎢ 0 ⎥
⎣128⎦
O2:
⎡612.7·sin(θ2(t))·cos(θ1(t))⎤
⎢612.7·sin(θ1(t))·sin(θ2(t))⎥
⎣  612.7·cos(θ2(t)) + 128   ⎦
O3:
⎡571.6·sin(θ2(t))·cos(θ1(t))·cos(θ3(t)) + 612.7·sin(θ2(t))·cos(θ1(t)) - 571.6·sin(θ3(t))·cos(θ1(t))·cos(θ2(t))⎤
⎢571.6·sin(θ1(t))·sin(θ2(t))·cos(θ3(t)) + 612.7·sin(θ1(t))·sin(θ2(t)) - 571.6·sin(θ1(t))·sin(θ3(t))·cos(θ2(t))⎥
⎣             571.6·sin(θ2(t))·sin(θ3(t)) + 571.6·cos(θ2(t))·cos(θ3(t)) + 612.7·cos(θ2(t)) + 128               ⎦
O4:
⎡-163.9·sin(θ1(t)) + 571.6·sin(θ2(t))·cos(θ1(t))·cos(θ3(t)) + 612.7·sin(θ2(t))·cos(θ1(t)) - 571.6·sin(θ3(t))·cos(θ1(t))·cos(θ2(t))⎤
⎢571.6·sin(θ1(t))·sin(θ2(t))·cos(θ3(t)) + 612.7·sin(θ1(t))·sin(θ2(t)) - 571.6·sin(θ1(t))·sin(θ3(t))·cos(θ2(t)) + 163.9·cos(θ1(t))⎥
⎣                        571.6·sin(θ2(t))·sin(θ3(t)) + 571.6·cos(θ2(t))·cos(θ3(t)) + 612.7·cos(θ2(t)) + 128                        ⎦
O5:
⎡115.7·(sin(θ2(t))·sin(θ3(t))·cos(θ1(t)) + cos(θ1(t))·cos(θ2(t))·cos(θ3(t)))·sin(θ4(t)) + 115.7·(sin(θ2(t))·cos(θ1(t))·cos(θ3(t)) - sin(θ3(t))·cos(θ1(t))·cos(θ2(t)))·cos(θ4(t)) - 163.9·sin(θ1(t)) + 571.6
⎢115.7·(sin(θ1(t))·sin(θ2(t))·sin(θ3(t)) + sin(θ1(t))·cos(θ2(t))·cos(θ3(t)))·sin(θ4(t)) + 115.7·(sin(θ1(t))·sin(θ2(t))·cos(θ3(t)) - sin(θ1(t))·sin(θ3(t))·cos(θ2(t)))·cos(θ4(t)) + 571.6·sin(θ1(t))·sin(θ2(
⎣                                   115.7·(sin(θ2(t))·sin(θ3(t)) + cos(θ2(t))·cos(θ3(t)))·cos(θ4(t)) + 115.7·(-sin(θ2(t))·cos(θ3(t)) + sin(θ3(t))·cos(θ2(t)))·sin(θ4(t)) + 571.6·sin(θ2(t))·sin(θ3

·sin(θ2(t))·cos(θ1(t))·cos(θ3(t)) + 612.7·sin(θ2(t))·cos(θ1(t)) - 571.6·sin(θ3(t))·cos(θ1(t))·cos(θ2(t))⎤
t))·cos(θ3(t)) + 612.7·sin(θ1(t))·sin(θ2(t)) - 571.6·sin(θ1(t))·sin(θ3(t))·cos(θ2(t)) + 163.9·cos(θ1(t))⎥
(t)) + 571.6·cos(θ2(t))·cos(θ3(t)) + 612.7·cos(θ2(t)) + 128                                             ⎦
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O6:
⎡-192.2·((sin(θ2(t))·sin(θ3(t))·cos(θ1(t)) + cos(θ1(t))·cos(θ2(t))·cos(θ3(t)))·cos(θ4(t)) - (sin(θ2(t))·cos(θ1(t))·cos(θ3(t)) - sin(θ3(t))·cos(θ1(t))·cos(θ2(t)))·sin(θ4(t)))·sin(θ5(t)) + 115.7·(sin(θ2(t)
⎢-192.2·((sin(θ1(t))·sin(θ2(t))·sin(θ3(t)) + sin(θ1(t))·cos(θ2(t))·cos(θ3(t)))·cos(θ4(t)) - (sin(θ1(t))·sin(θ2(t))·cos(θ3(t)) - sin(θ1(t))·sin(θ3(t))·cos(θ2(t)))·sin(θ4(t)))·sin(θ5(t)) + 115.7·(sin(θ1(t)
⎣                                                    -192.2·(-(sin(θ2(t))·sin(θ3(t)) + cos(θ2(t))·cos(θ3(t)))·sin(θ4(t)) + (-sin(θ2(t))·cos(θ3(t)) + sin(θ3(t))·cos(θ2(t)))·cos

)·sin(θ3(t))·cos(θ1(t)) + cos(θ1(t))·cos(θ2(t))·cos(θ3(t)))·sin(θ4(t)) + 115.7·(sin(θ2(t))·cos(θ1(t))·cos(θ3(t)) - sin(θ3(t))·cos(θ1(t))·cos(θ2(t)))·cos(θ4(t)) - 192.2·sin(θ1(t))·cos(θ5(t)) - 163.9·sin(θ
)·sin(θ2(t))·sin(θ3(t)) + sin(θ1(t))·cos(θ2(t))·cos(θ3(t)))·sin(θ4(t)) + 115.7·(sin(θ1(t))·sin(θ2(t))·cos(θ3(t)) - sin(θ1(t))·sin(θ3(t))·cos(θ2(t)))·cos(θ4(t)) + 571.6·sin(θ1(t))·sin(θ2(t))·cos(θ3(t)) +
(θ4(t)))·sin(θ5(t)) + 115.7·(sin(θ2(t))·sin(θ3(t)) + cos(θ2(t))·cos(θ3(t)))·cos(θ4(t)) + 115.7·(-sin(θ2(t))·cos(θ3(t)) + sin(θ3(t))·cos(θ2(t)))·sin(θ4(t)) + 571.6·sin(θ2(t))·sin(θ3(t)) + 571.6·cos(θ2(t))

1(t)) + 571.6·sin(θ2(t))·cos(θ1(t))·cos(θ3(t)) + 612.7·sin(θ2(t))·cos(θ1(t)) - 571.6·sin(θ3(t))·cos(θ1(t))·cos(θ2(t))⎤
612.7·sin(θ1(t))·sin(θ2(t)) - 571.6·sin(θ1(t))·sin(θ3(t))·cos(θ2(t)) + 192.2·cos(θ1(t))·cos(θ5(t)) + 163.9·cos(θ1(t))⎥
·cos(θ3(t)) + 612.7·cos(θ2(t)) + 128                                                                                  ⎦
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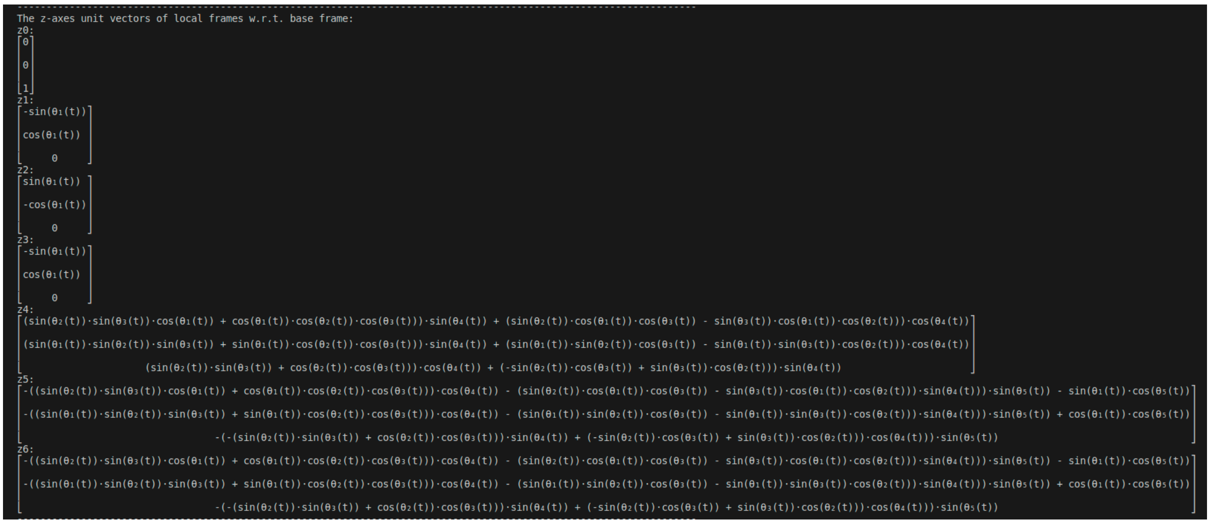
- The Z-axis unit vectors were found to be:



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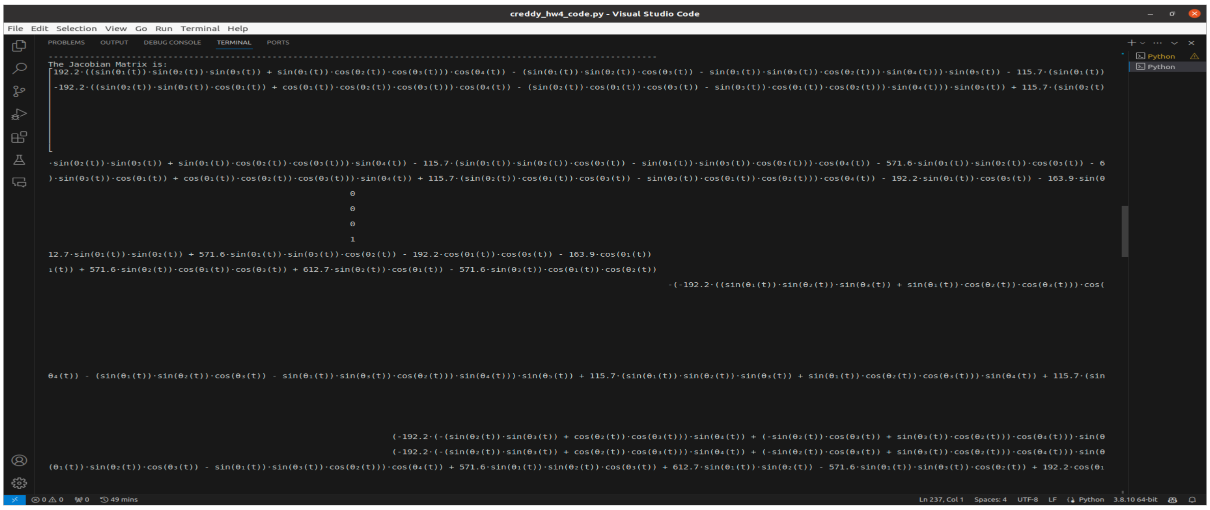
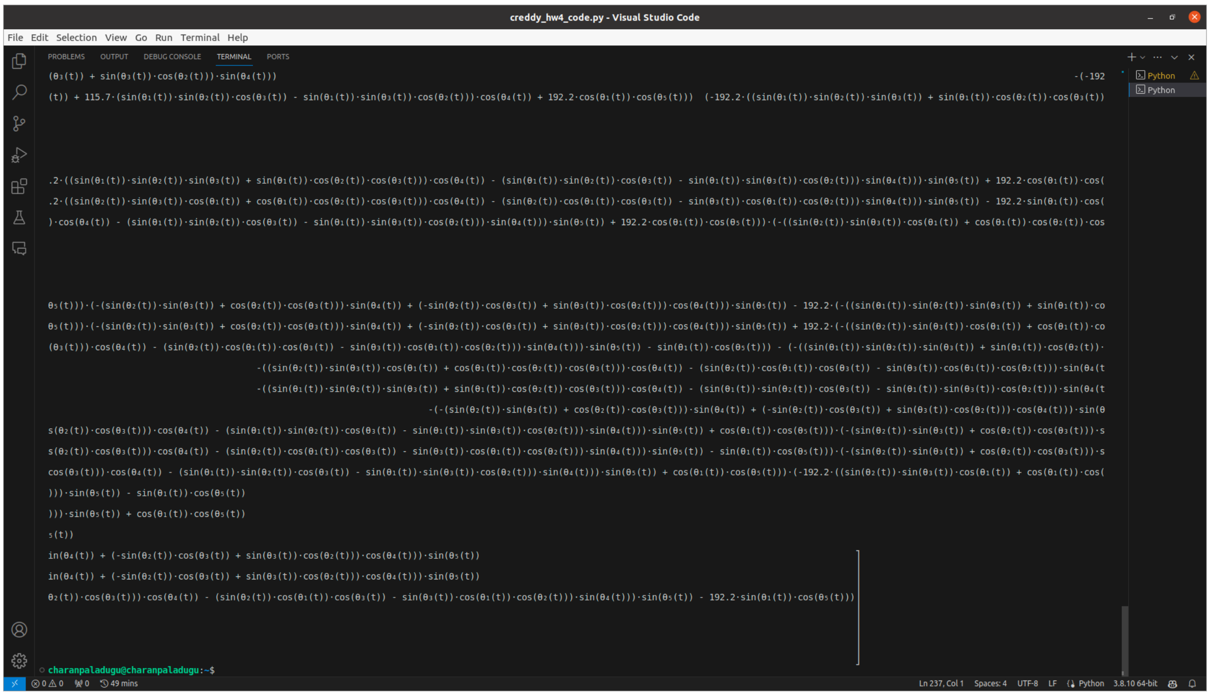
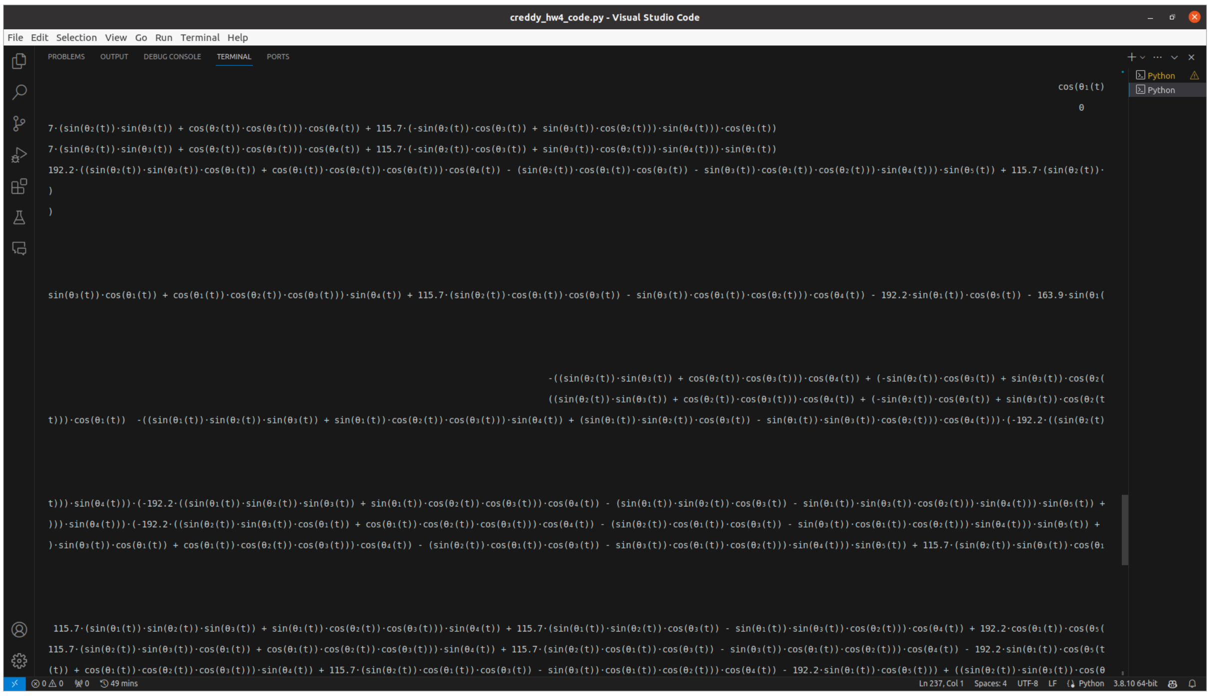
The z-axes unit vectors of local frames w.r.t. base frame:
z0:
⎡0⎤
⎢ ⎥
⎢0⎥
⎢ ⎥
⎣1⎦
z1:
⎡-sin(θ₁(t))⎤
⎢           ⎥
⎢cos(θ₁(t)) ⎥
⎢           ⎥
⎣     0     ⎦
z2:
⎡sin(θ₁(t)) ⎤
⎢           ⎥
⎢-cos(θ₁(t))⎥
⎢           ⎥
⎣     0     ⎦
z3:
⎡-sin(θ₁(t))⎤
⎢           ⎥
⎢cos(θ₁(t)) ⎥
⎢           ⎥
⎣     0     ⎦
z4:
⎡(sin(θ₂(t))⋅sin(θ₃(t))⋅cos(θ₁(t)) + cos(θ₁(t))⋅cos(θ₂(t))⋅cos(θ₃(t)))⋅sin(θ₄(t)) + (sin(θ₂(t))⋅cos(θ₁(t))⋅cos(θ₃(t)) - sin(θ₃(t))⋅cos(θ₁(t))⋅cos(θ₂(t)))⋅cos(θ₄(t))⎤
⎢(sin(θ₁(t))⋅sin(θ₂(t))⋅sin(θ₃(t)) + sin(θ₁(t))⋅cos(θ₂(t))⋅cos(θ₃(t)))⋅sin(θ₄(t)) + (sin(θ₁(t))⋅sin(θ₂(t))⋅cos(θ₃(t)) - sin(θ₁(t))⋅sin(θ₃(t))⋅cos(θ₂(t)))⋅cos(θ₄(t))⎥
⎣(sin(θ₂(t))⋅sin(θ₃(t)) + cos(θ₂(t))⋅cos(θ₃(t)))⋅cos(θ₄(t)) + (-sin(θ₂(t))⋅cos(θ₃(t)) + sin(θ₃(t))⋅cos(θ₂(t)))⋅sin(θ₄(t))⎦
z5:
⎡-((sin(θ₂(t))⋅sin(θ₃(t))⋅cos(θ₁(t)) + cos(θ₁(t))⋅cos(θ₂(t))⋅cos(θ₃(t)))⋅cos(θ₄(t)) - (sin(θ₂(t))⋅cos(θ₁(t))⋅cos(θ₃(t)) - sin(θ₃(t))⋅cos(θ₁(t))⋅cos(θ₂(t)))⋅sin(θ₄(t)))⋅sin(θ₅(t)) - sin(θ₁(t))⋅cos(θ₅(t))⎤
⎢-((sin(θ₁(t))⋅sin(θ₂(t))⋅sin(θ₃(t)) + sin(θ₁(t))⋅cos(θ₂(t))⋅cos(θ₃(t)))⋅cos(θ₄(t)) - (sin(θ₁(t))⋅sin(θ₂(t))⋅cos(θ₃(t)) - sin(θ₁(t))⋅sin(θ₃(t))⋅cos(θ₂(t)))⋅sin(θ₄(t)))⋅sin(θ₅(t)) + cos(θ₁(t))⋅cos(θ₅(t))⎥
⎣-(-(sin(θ₂(t))⋅sin(θ₃(t)) + cos(θ₂(t))⋅cos(θ₃(t)))⋅sin(θ₄(t)) + (-sin(θ₂(t))⋅cos(θ₃(t)) + sin(θ₃(t))⋅cos(θ₂(t)))⋅cos(θ₄(t)))⋅sin(θ₅(t))⎦
z6:
⎡-((sin(θ₂(t))⋅sin(θ₃(t))⋅cos(θ₁(t)) + cos(θ₁(t))⋅cos(θ₂(t))⋅cos(θ₃(t)))⋅cos(θ₄(t)) - (sin(θ₂(t))⋅cos(θ₁(t))⋅cos(θ₃(t)) - sin(θ₃(t))⋅cos(θ₁(t))⋅cos(θ₂(t)))⋅sin(θ₄(t)))⋅sin(θ₅(t)) - sin(θ₁(t))⋅cos(θ₅(t))⎤
⎢-((sin(θ₁(t))⋅sin(θ₂(t))⋅sin(θ₃(t)) + sin(θ₁(t))⋅cos(θ₂(t))⋅cos(θ₃(t)))⋅cos(θ₄(t)) - (sin(θ₁(t))⋅sin(θ₂(t))⋅cos(θ₃(t)) - sin(θ₁(t))⋅sin(θ₃(t))⋅cos(θ₂(t)))⋅sin(θ₄(t)))⋅sin(θ₅(t)) + cos(θ₁(t))⋅cos(θ₅(t))⎥
⎣-(-(sin(θ₂(t))⋅sin(θ₃(t)) + cos(θ₂(t))⋅cos(θ₃(t)))⋅sin(θ₄(t)) + (-sin(θ₂(t))⋅cos(θ₃(t)) + sin(θ₃(t))⋅cos(θ₂(t)))⋅cos(θ₄(t)))⋅sin(θ₅(t))⎦
```

- The Jacobian matrix found out was:




creddy_hw4_code.py - Visual Studio Code
File Edit Selection View Go Run Terminal Help
PROBLEMS OUTPUT DEBUG CONSOLE TERMINAL PORTS
charanpaladugu@charanpaladugu:~$
Ln 237, Col 1 Spaces: 4 UTF-8 LF Python 3.8.10 64-bit


creddy_hw4_code.py - Visual Studio Code
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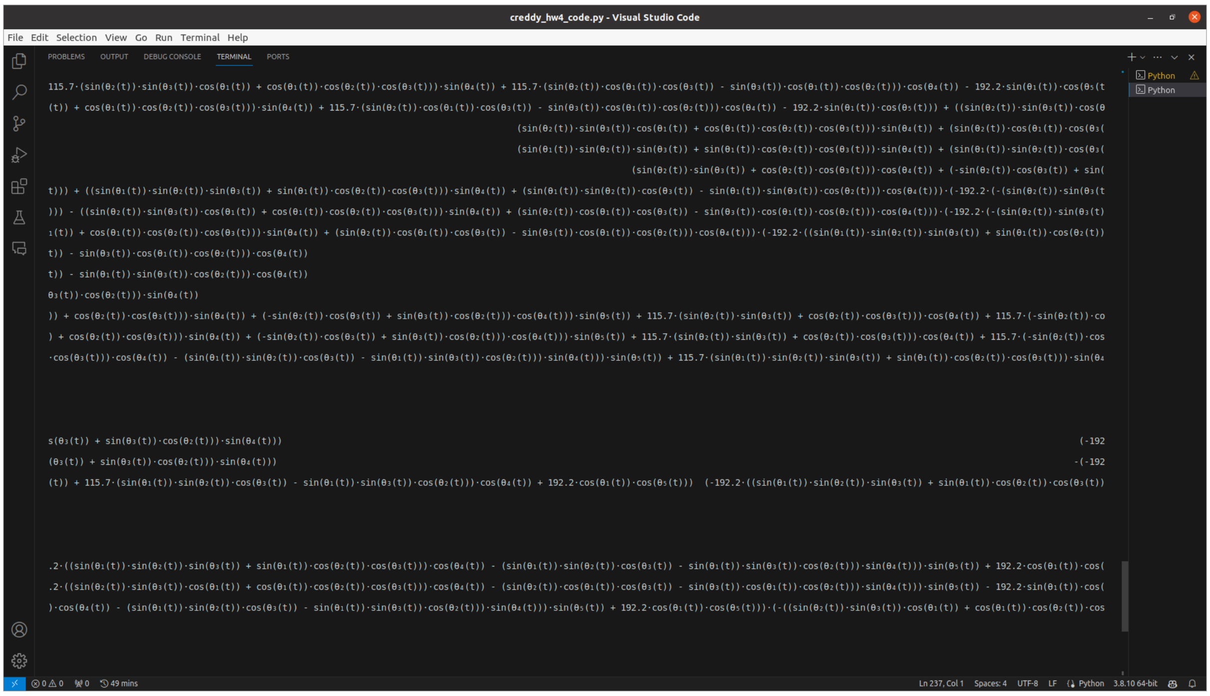
PROBLEMS OUTPUT DEBUG CONSOLE TERMINAL PORTS
Ln 237, Col 1 Spaces: 4 UTF-8 LF Python 3.8.10 64-bit



Jacobian Matrix at T=0

- We know that at t=0 all the values of theta are 0. For this initial condition we get the manipulator at the erect position.
- At this initial condition values for theta and values shown in the DH table we can build the intermediary transformation matrices.
- Using these matrices we can get

$$T_2^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 128 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 740.7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1312.3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_5^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 163.9 \\ 0 & -1 & 0 & 1428.0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_6^0 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 356.1 \\ 0 & 1 & 0 & 1428.0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- From the above matrices we can write the z-axes unit vector frames with respect to the base frame and origins of all the frames as follows:

$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$O_1 = \begin{bmatrix} 0 \\ 0 \\ 128 \end{bmatrix}$$

$$O_2 = \begin{bmatrix} 0 \\ 0 \\ 740.7 \end{bmatrix}$$

$$O_3 = \begin{bmatrix} 0 \\ 0 \\ 1312.3 \end{bmatrix}$$

$$O_4 = \begin{bmatrix} 0 \\ 163.9 \\ 1312.3 \end{bmatrix}$$

$$O_5 = \begin{bmatrix} 0 \\ 163.9 \\ 1428.0 \end{bmatrix}$$

$$O_6 = \begin{bmatrix} 0 \\ 356.1 \\ 1428.0 \end{bmatrix}$$

Similarly th Z unit vectors are

$$Z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$Z_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$Z_2 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

$$Z_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$Z_4 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$Z_5 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$Z_6 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

- The Jacobian matrix would be

$$\begin{bmatrix} -356.1 & 1300.0 & -687.3 & 115.7 & -192.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

- These steps have to be followed to calculate matrices for other instances of time.

Circle Trajectory:

We know that polar coordinates of the circle that needs to be traced by the manipulator are:

$$x = r \text{ x } \cos(\theta+90)$$
$$z = r \text{ x } \sin(\theta+90))$$
$$y = 0$$

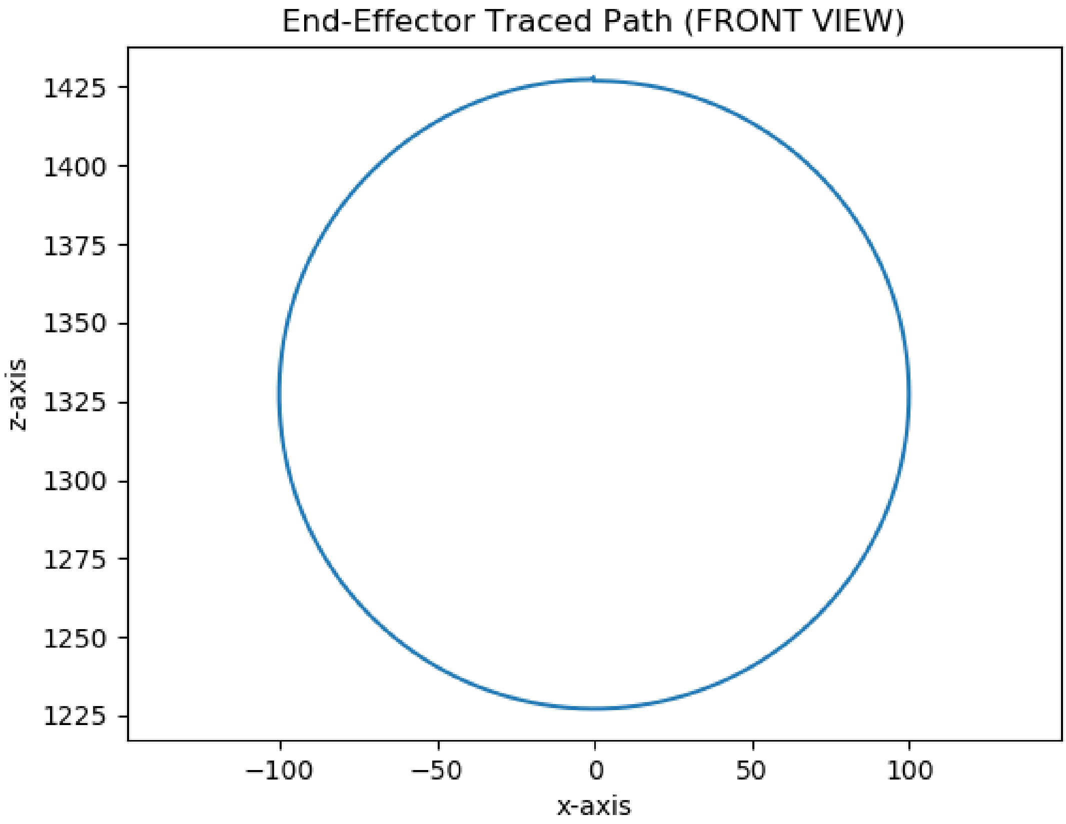
A phase of 90 is added for the manipulator to trace the way.
Differentiate the above equations we get

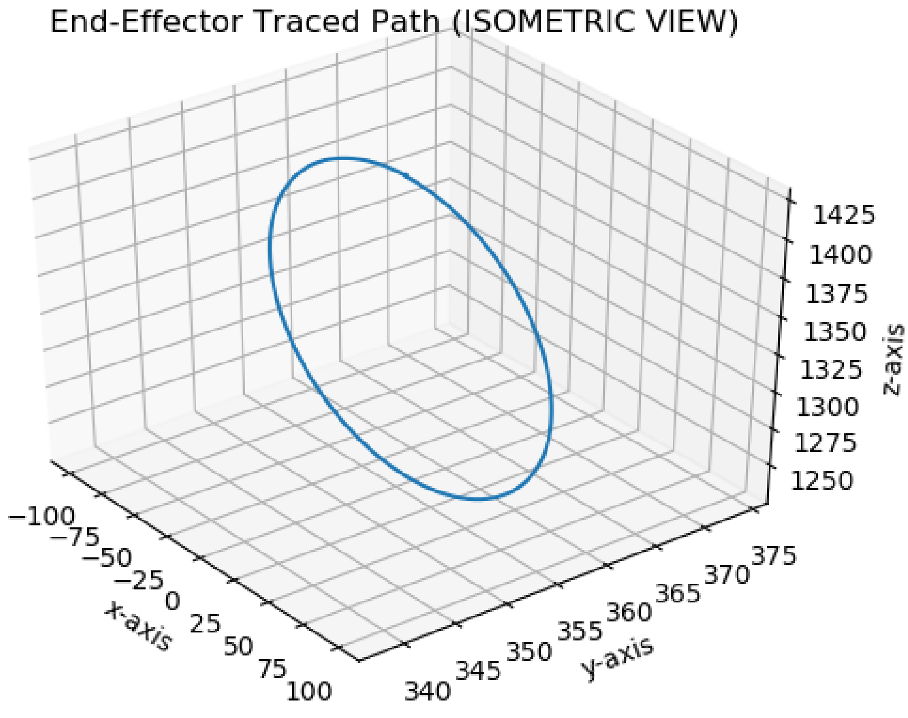
$$\dot{x} =- r \times sin(\theta + 90) \times \dot{\theta}$$
$$\dot{z} = r \times cos(\theta + 90) \times \dot{\theta}$$
$$\dot{y} = 0$$

The relationship between joint velocity and end-effector values are given as

$$\dot{q} = J^{-1} \times \epsilon$$

Where ε represents the end-effector velocities.
We can now write it by integrating q= q+$\dot{q}\Delta t$
Now as time passes, we get new q values which has its own Jacobian matrix, and correspondingly the new (x,y,z) position vector of the end-effector.
Plotting these positions we get:


End-Effector Traced Path (FRONT VIEW)
1425
1400
1375
1350
1325
1300
1275
1250
1225
z-axis
−100
−50
0
50
100
x-axis


End-Effector Traced Path (ISOMETRIC VIEW)
1425
1400
1375
1350
1325
1300
1275
1250
z-axis
−100
−75
−50
−25
0
25
50
75
100
x-axis
340
345
350
355
360
365
370
375
y-axis