CO-4

Syllabus

1) Introduction to queues
2) measures of System performance.
3) characteristics of queueing system.
4) stachastic processes overview
5) discrete-time
6) Markov chains
7) continuous time Markov chain
8) birth-death processes
9) poisson process & exponential distribution
10) Birth-death queuing systems,
11) Non-birth-death Markovian queuing system
12) Queuing networks.

M/M/1/∞/FCFS — M1 → mean for
M/M/1/n/FCFS — M2 →
M/M/S/∞/FCFS — M3
M/M/s/n/FCFS — M4

M₁ = mean for arrival rate
M₂ = mean for service rate
1 = single server
∞ = Queue length infinite
n = capacity
M1 → when queue length is not mentioned
M2 → when queue length is mentioned.

Session-21

Introduction to queues & measures of system performance.

Characterstics of queuing system

1) Arrival process:

The arrival process describe the arrival pattern at the queue.

2) ~~Several tim~~ Service process:

The service process represents the time taken to serve customers reffered to as service time.

3) Queue discipline:

The que discipline specifies the order in which the customers in the queue are served.

In syllabus one first come first serve 4 models only.

first come first server (FCFS)
last come first server (LCFS)
service in random order (SIRO)

4) No. of serveers: As there a single server or multiple servers. Is there a single queue that feeds all servers. Seperate queus at each server.

5) calling population: The population of potential customers, reffered to as the calling population may be assumed to be finite or infinite.

6) System capacity: In many queuing systems, there may be in a limit to the no. of customers that may be in the waiting lines or system.

A|B|C|N|K (or) M|M|1|Q|FCFS

A → ~~Represent~~s

Transient sate : A system is said to be in transient state when the behaviour of the system is dependent on time

Steady state

A system is said to be in steady state when the behaviour of the system is independents of time.

In this topic we study only the steady state analysis.

A list of symbols

n = no-of customers in the queing system

$P_n(t)$ = steady state probability of having 'n' customers in the system

$P(n)$ = transient state probability that exactly n customers are in the system at time t.

λ = mean arrival rate ($\frac{1}{\lambda}$ is the interarrival time)

μ = mean service rate ($\frac{1}{\mu}$ is the mean service time)

s = no-of parallel service stations.

ρ = $\lambda/(\mu s)$ = traffic intensity (or utilization factor) for the service facility, that is, the expected fraction of the time the server is busy.

L_s = Expected system length, that is, expected member of customers in the system.

(number of customers waiting in the queu + no. of customers in service).

L_q = Expected queue length, that is, expected no. of customers waiting in the queue

W_s = Expected waiting time of an arriving customer in the system.

W_q = Expected waiting time of an arriving customer in the queue
(Expected w.t in the system - expected service time).

$(W|W>0)$ = Expected waiting time of a customer who has to wait.

$(L|L>0)$ = Expected length of a non-empty queue

$P(W>0)$ = probability of an arriving customer has to wait.

Ex:1

let arrival rate is λ and service rate is μ.

No. of customers in the system =

$\frac{1}{\lambda} = 10\,min$

$\frac{1}{\lambda} = \frac{10}{60} = \frac{1}{6}$

$\lambda = 6$ per 1 hr

$\frac{1}{\mu} = 6\,min$

$\frac{1}{\mu} = \frac{6}{60}$

$\frac{1}{\mu} = \frac{1}{10}$

$\mu = 10$ / per hour.

$$L_s = \frac{\lambda}{\mu - \lambda} = \frac{6^2}{10(10-6)} = \frac{36}{10(4)} = \frac{36}{40}$$

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} =$$

Ex-2

Average rate (λ) = 10 machines/ hours.

Repair Rate:

Machine A : Average repair ~~rat~~ time =

$B = \frac{1}{\mu} = 4 \text{min} = \frac{4}{60} = \frac{1}{15}$

$\mu = 15/\text{hor}$

Ⓐ → $10 + 1.67 = 11.67$

Ⓑ $15 + 1.33 = 16.3.$

Mechanic A is cheaper than Mechanic B.

Mechanic A:

1. wages per hour (wa) = Rs 10

2. Idle cost per repair (ca) = $\frac{20}{12} = 1.67.$

Total cost per repair = wa + ca

$= 10 + 1.67 = 11.67$

Machanic B:

1. wages per hour (wb) = 15

2. Idle cost per repair (cb) = $\frac{20}{15} = 1.33.$

total cost per repair = wb + ca

$= 15 + 1.33 = 16.33$

2/11/24

Model-2

M/M/1/N/FCFS

1. $\rho = \frac{\lambda}{\mu}$

2. $\rho = \frac{\lambda}{\mu} \neq 1$ } two cases if $\rho = 1$, $\rho \neq 1$
   $\frac{\lambda}{\mu} = 1$

3. ~~P_0~~

~~$P_0 = \begin{cases} \frac{1-\rho}{1-\rho^{N+1}} \\ \frac{1}{N+1} \end{cases}$~~

3. $P_0 = \begin{cases} \frac{1-\rho}{1-\rho^{N+1}} & \text{if } \rho \neq 1 \\ \frac{1}{N+1} & \text{if } \rho = 1 \end{cases}$

ρ = utilization factor server busy

λ_{eff} = effective arrival rate

L_s = system length

L_q = queue length

W_q = how much waiting for queue

W_s = system waiting time

4. $\lambda_{eff} = \lambda(1 - P_N)$

5. $P_N = P_0 \cdot \rho^n$

6. $L_s = P_0 \sum_{n=0}^{n} n \rho^n$

7. $L_q = L_s - \frac{\lambda_{effect}}{\mu}$

8. $W_q = \frac{L_q}{\lambda_{eff}}$

9. $W_s = \frac{L_s}{\lambda_{eff}}$

10) In a railway marshaling yard, goods and services arrive at a rate of 30 trains per day..
Serveice time 30 mins capacity of yard is to admit 9 trains

$n = 9$
$\lambda = 30/\text{day}$
$\frac{1}{\mu} = 30\text{min} = \cancel{\frac{30}{60}} = \frac{1}{2} = \mu =$
$\mu = \frac{1}{30}$ ~~10~~ 5 8
$\mu = \frac{1}{\cancel{30}} \times 60 \times \cancel{24}$
$\mu = 40/\text{day}$
$\rho = \frac{\lambda}{\mu} = \frac{30}{40} = 0.75 \neq 1$

(1). $P_0 = \frac{1-\rho}{1-\rho^{N+1}} = \frac{1-0.75}{1-0.75^{(9+1)}}$

$= \frac{0.25}{(0.25)^{10}}$

$= 0.264$

(2) What is propability that a newly coming trains yard will

$P_N = P_0 \cdot \rho^N$
$= 0.264 \times (0.75)^9$
$= 0.019.$

3) Avg no. of waiting trans in the que in service

$$L_s = P_0 \sum_{n=0}^{n} n \cdot \rho^n$$

$$0.264 \sum_{0}^{9} n \cdot (0.75)^n$$

$$0.264 \times [0 \times (0.75)^0 + 1 \times (0.75)^1 + (2 \times (0.75)^2 + \dots + 9(0.75)^9$$

$L_s = 3$ approx

4. effective arrival rate.

$$\lambda_{eff} = \lambda(1 - P_N).$$

$$= 30(1 - 0.019)$$

$$= \cancel{30} \; 29.4.$$

5. $$W_s = \frac{L_s}{\lambda_{eff}}$$

$$= \frac{3}{29.4}$$

$$= 0.10.$$

---

10) $\lambda = 30 / hr$

$n = 14.$

$\mu = 20 / hr$

$$\rho = \frac{\lambda}{\mu} = \frac{3\cancel{0}}{2\cancel{0}} = 1.5$$

$$P_0 = \frac{1 - \rho}{1 - \rho^{N+1}}$$

$$\frac{1 - 1.5}{(1 - 1.5)^{14+1}}$$

$$= \frac{-0.5}{(-0.5)^{15}}$$

$$= 0.0014$$

① $\lambda_{eff} = \lambda(1 - P_N).$

$P_N = 0.0014 \times (1.5)^{14}$

$P_N = 0.4087$

$\lambda_{eff} = 30(1 - 0.4087)$

$= 0.5913 \times 30 = 17.739$

3) $1 - P_N$

$W_s = \frac{L_s}{\lambda_{eff}} = \frac{14.441}{17.739}$ $W_s = 0.814 hor.$

$L_s = P_0 \sum_{n=0}^{n} n \cdot \rho^n$

$0.0014 \times \sum_{n=0}^{14} (n \cdot (1.5)^n)$

$0.0014 (0(1.5)^0 + 1(1.5)^1 + 2(1.5)^2 + --- 14(1.5)^{14}$

$= 14.441$

2) $1 - P_N$

$1 - 0.4087$

$= 0.5913$

3(a) $n = 5$

$\lambda = 10/hor$

$\mu = 5/hor$

$\rho = \frac{\lambda}{\mu} = \frac{10}{5}$ 2

$\rho = 2 \neq 1.$

① empty

$P_0 = \frac{1-\rho}{1-\rho^{(N+1)}} = \frac{1-2}{(1-2)^{5+1}} = \frac{-1}{(-1)^6} = \frac{\cancel{-1}}{\cancel{+1}} = -1$

$P_0 = -1$

② full

$P_N = P_0 \rho^n$

$= -1 \cdot (-1)^5$

$= -1 \cdot (-1)$

$= 1.$

④ $\lambda_{eff} = \lambda(1 - P_N)$

$= 10(1-1)$

$= 10.$

③ $L_s = P_0 \sum_{n=0}^{n} n \cdot \rho^n$

$= 1 \sum_{n=0}^{5} n (1)^n$

$= 1 (0(1)^0 + 1(1)^1 + 1(2)^1 + 1(3)^1 + 1(4)^1 + 1($

$= 1 (0 + 1 + 2 + 3 + 4 + 5)$

$= 1(15) = -15.$

$\frac{1}{\mu} = 5 min$

$\mu = \frac{1}{\cancel{5}} \times \cancel{60}^{12}$

$\mu = 12$ hr.

$\rho = \frac{\lambda}{\mu} = \frac{\cancel{10}^{5}}{\cancel{12}_{6}} = \frac{5}{6} = 0.8 \neq 1.$

$P_0 = \frac{1-\rho}{(1-\rho)^{5+1}} = \frac{1-0.8}{(1-0.8)^6} = \frac{\cancel{0.2}}{(0.2)^{\cancel{6}}} = \frac{1}{(0.2)^5}$

$= 0.25.$

$P_N = P_0 \cdot \rho^n$

$= 0.25 \times (0.2)^5$

$=$

$n = 10$ cars

$\lambda = 20/hr$

$\frac{1}{\mu} = 10$ min

$\mu = \frac{1}{10} \times 60$

$\mu = 6$

$\rho = \frac{\lambda}{\mu} = \frac{\cancel{20}^{10}}{\cancel{6}_{3}} = 3.33 \neq 1.$

$P_0 = \frac{1-\rho}{1-\rho^{10+1}} = \frac{1-3.33}{(1-3.33)^{11}} =$

$P_N = P_0 \cdot \rho^N$

$= \cdot (3.33)^{10}$

find the probability that an arrival ~~time~~ finds the car parking system.

Model-3

M/M/S/∞/FCFS

→ utilization factor

$\rho = \frac{\lambda}{S\mu}$

2) $P_0 = \sum_{n=0}^{S-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{S!}\cdot\left(\frac{\lambda}{\mu}\right)^S \cdot \frac{S\mu}{S\mu-\lambda}$

3) $P_N = \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n P_0$

4) $L_q = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^S}{(S-1)!\,(S\mu-\lambda)^2} \times P_0$ queue length

5) $L_S = L_q + \frac{\lambda}{\mu}$ System length

6) $W_q = \frac{L_q}{\lambda}$ waiting time of queue

7) $W_S = W_q + \frac{1}{\mu}$ System waiting time

8) $L/L>0 = \frac{1}{1-S}$

9) $(W/W>0) = \frac{1}{S\mu-\lambda}$

city hospitals eye clinic offers three vision test every wednesday evening. they are three doctors on duty. A test-atelkey on Avg 20.min and the actual time is found to be approximately exponentley distributed. Clients arrived according to a poision process with a mean of 6/hr and patiance are taken FCFS baises. The hospital planner are intrested in knowing

1) what is the avg no. of people waiting
2) The avg amount of time in patiance spends at the clinic
3) The avg percentage idle time for each of the doctors.

Sol: $\lambda = 6/hr$

$\frac{1}{\mu} = 20 min$

$\mu = \frac{1}{\cancel{20}} \times \cancel{60}^{3}$

$\mu = 3/hr$

$S = 3$

1) $\rho = \frac{\lambda}{S\mu} = \frac{\cancel{6}^{2}}{\cancel{3} \times 3} = \frac{2}{3}$

$$P_0 = \frac{1}{\sum_{n=0}^{S-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{S!}\left(\frac{\lambda}{\mu}\right)^S \cdot \frac{S\mu}{S\mu - \lambda}}$$

$$= \frac{1}{\left[\sum_{n=0}^{2} \frac{1}{n!}\left(\frac{6}{3}\right)^n + \frac{1}{3!}\left(\frac{6}{3}\right)^3 \cdot \frac{3 \cdot 3}{3 \cdot 3 - 6}\right]}$$

$$= \frac{1}{\frac{1}{0!}2^0 + \frac{1}{1!}2^1 + \frac{1}{2!}2^2 + \frac{1}{\cancel{3}}(8)^4 \cdot \frac{\cancel{9}\cancel{3}}{9-6\cancel{3}}}$$

$$= \frac{1}{1+2+\frac{\cancel{4}^2}{\cancel{2}}+4}$$

$$= \frac{1}{1+2+2+4}$$

$$\boxed{P_0 = \frac{1}{9}}$$

1) avg no. of people waiting = Lq

$$L_q = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^s}{(s-1)!(s\mu-\lambda)^2} \times P_0$$

$$= \frac{6 \cdot 3\left(\frac{\cancel{6}}{\cancel{3}}\right)^3}{(3-1)!(3 \cdot 3-6)^2} \times \frac{1}{9}$$

$$= \frac{\cancel{18} \times 8}{\cancel{2} \times \cancel{9}} \times \frac{1}{9}$$

$$\boxed{L_q = \frac{8}{9}}$$

2) $L_s = L_q + \frac{\lambda}{\mu}$

$$= \frac{8}{9} + \frac{6}{3} \quad \frac{8+18}{9} = \frac{26}{9}$$

$$W_q = \frac{L_q}{\lambda} = \frac{\frac{8}{9}}{6} = \frac{\cancel{8}^4}{9} \times \frac{1}{\cancel{6}_3} = \frac{8}{54}$$

b) $W_s = W_q + \frac{1}{\mu}$

$= \frac{8}{54} + \frac{1}{3} = \frac{8+18}{54} = \frac{\cancel{26}^{\cancel{13}}}{\cancel{54}_{27}} = 0.4$

c). $P_0 + P_1 + P_2$

$\cancel{P_1 = \frac{1}{\frac{1}{1!}\cdot\left(\frac{6}{3}\right)^1 +}}$

$P_1 = \frac{1}{1!}\left(\frac{\cancel{6}^2}{\cancel{3}}\right)^1 \cdot \frac{1}{9}$

$= 2/9$

$P_2 = \frac{1}{2!}\left(\frac{\cancel{6}}{\cancel{3}}\right)^2 \cdot \frac{1}{9}$

$= \frac{2}{9}$

$P_0 + P_1 + P_2 = \frac{1}{9} + \frac{2}{9} + \frac{2}{9} = \frac{5}{9}$

$\frac{5}{9} \times 100 =$

A telephone exchange have two long distance the telephone during the faction at avg rate of 15/hr the length mean of 5 mins

1) what is the probability that A subscriber will have to wait for long distant on the peak hours of the day.

2) what is the average waiting time of the customer

Sol $\lambda = 15$ ~~arr~~/hr

$\frac{1}{\mu} = \frac{\cancel{1}}{5}$

$\mu = \frac{1}{\cancel{5}} \times \cancel{60}$ 12/hr

$S = 2.$

(1) $\rho = \frac{\lambda}{S\mu} = \frac{15}{12 \times 2} = \frac{\cancel{15}^{5}}{2 \times \cancel{12}_{4}} = \frac{5}{8}.$

(2) $$P_0 = \frac{1}{\sum_{n=0}^{S-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{S!}\cdot\left(\frac{\lambda}{\mu}\right)^S \cdot \frac{S\mu}{S\mu-\lambda}}$$

$$= \frac{1}{\frac{1}{0!}\left(\frac{15}{12}\right)^0 + \frac{1}{1!}\left(\frac{15}{12}\right)^1 + \frac{1}{2!}\left(\frac{\cancel{15}^{5}}{\cancel{12}_{4}}\right)^2 \frac{2 \cdot 12}{2 \cdot 12 - 15}}$$

$$= \frac{1}{1 + \frac{15}{12} + \frac{1}{2}\left(\frac{5}{4}\right)^2 \cdot \frac{24}{24-15}}$$

$$= \frac{1}{1 + \frac{15}{12} + \frac{1}{2}\left(\frac{5}{4}\right)^2 \cdot \frac{\cancel{24}^{8}}{\cancel{9}_{3}}}$$

$$\frac{1}{1+\frac{5}{4}+\frac{1}{\cancel{2}}\left(\frac{25}{\cancel{16}_{4}}\right)\cdot\frac{\cancel{8}\,4}{3}}$$

$$\frac{1}{1+\frac{5}{4}+\frac{25}{12}} = \frac{2}{7} = 0.2$$

① $1 - P_0 - P_1$

$1 - 0.2 - \frac{1}{1!}\left(\frac{\cancel{15}\,5}{\cancel{12}\,4}\right)^1 \cdot 0.2$

$1 - 0.2 - \frac{1}{1}\left(\frac{5}{4}\right)\cdot 0.2$

$0.8 - \frac{5}{4}(0.2) = 0.8 - (1.25)(0.2)$

$= 0.55$

② $W_q = \frac{L_q}{\lambda}$

$$L_q = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^s}{(s-1)!\,(s\mu-\lambda)^2} \times P_0$$

$$= \frac{15\times 12\left(\frac{\cancel{15}}{\cancel{12}}\right)^2}{(2-1)!\,(2\times 12-15)^2} \times 0.2$$

$$= \frac{15\times 12\left(\frac{25}{16}\right)}{1(24-15)^2} = \frac{\cancel{15}\,5 \times \cancel{12}\,\cancel{3}\,1 \times \frac{25}{\cancel{16}\,4}}{\cancel{9}\,\cancel{3} \times 9}$$

$= 3.47.$

2) 1). $\lambda = \frac{20}{8} hr = 2.5 / hr$

$\frac{1}{\mu} = 40$

$\mu = \frac{1}{\cancel{40}_{2}} \times \cancel{60}^{3} = \frac{3}{2} = 1.5 hr$

$S = 3.$

① $\delta = \frac{\lambda}{s\mu} = \frac{2.5}{3(1.5)} = \frac{\cancel{2.5}^{0.5}}{3 \times \cancel{1.5}_{0.3}} = 1.25.$

$$P_0 = \frac{1}{\sum_{n=0}^{s-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s \cdot \frac{s\mu}{s\mu - \lambda}}$$

$$= \frac{1}{\sum_{n=0}^{3-1} \frac{1}{n!}\left(\frac{2.5}{1.5}\right)^n + \frac{1}{3!}\left(\frac{2.5}{1.5}\right)^3 \cdot \frac{3(1.5)}{(3(1.5)-2.5)}}$$

0.173

$= \delta \times 5 \times 8 =$

② $w_s = w_q + \frac{1}{\mu} = 0.816.$

$w_q = \frac{L_q}{\lambda}$

$$L_q = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^s}{(s-1)(s\mu-\lambda)^2} P_0$$

$$= \frac{2.5 \times 1.5\left(\frac{2.5}{1.5}\right)^3}{2(3 \times 1.5 - 2.5)^2}$$

3) $s = 3$

$\lambda = \cancel{\frac{48}{8}}^{6} / hr$

$\frac{1}{\mu} = 1.5$

$\mu = \frac{1}{\cancel{1.5}} \times \cancel{60}$

$\mu = 4 / hr.$

$s = 3,\ \lambda = 6/hr,\ \mu = 4/hr.$

$\rho = \frac{\lambda}{s\mu} = \frac{6}{3 \times 4} = \frac{6}{\cancel{12}} = 0.5$

c) Lq, D) wq, E) $= \rho \times 5 \times 8$

$= 1 - P_0 - P_1 - P_2 - P_3.$

1). $L_s = Lq + \frac{\lambda}{\mu}$

$$Lq = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^s}{(s-1)!\,(s\mu-\lambda)^2} \times P_0$$

$$= \frac{6 \times 4\left(\frac{6}{4}\right)^3}{(2)!\,(3(4)-6)^2} \times 0.210$$

$$= \frac{24 \times \frac{27}{64}}{2(12-6)}$$

$$= \frac{\cancel{24} \times \frac{27}{\cancel{64}\,8}}{2(36 \times 36)} =$$

$$P_0 = \frac{1}{\sum_{n=0}^{2} \frac{1}{n!}\left(\frac{6}{4}\right)^n + \frac{1}{3!}\cdot\left(\frac{6}{4}\right)^3 \cdot \frac{3(4)}{3(4)-6}}$$

$$\frac{1}{\frac{1}{0!}\cdot 0 + \frac{1}{1!}\left(\frac{3}{2}\right) + \frac{1}{6}\left(\frac{3}{2}\right)^3 \cdot \frac{\cancel{12}^{2}}{\cancel{6}}}$$

$$\frac{1}{1 + \frac{3}{2} + \frac{1}{\cancel{6}_{\cancel{3}}}\left(\frac{\cancel{27}^{9}}{8}\right)\cancel{2}}$$

$$= \frac{1}{1+\frac{3}{2}+\frac{9}{8}}$$

$$= \frac{1}{\frac{8+12+9}{8}}$$

$$= \frac{1}{\frac{29}{8}} = \frac{8}{29} = 0.210.$$

Model-4

1) $P_n = \begin{cases} \dfrac{(s\rho)^n \cdot P_0}{n!} & 0 \leq n \leq s \\ \dfrac{s^s \rho^n P_0}{s!} & s \leq n < N \\ 0 & n > N. \end{cases}$

$\hookrightarrow$ System is full.

2) $\rho = \dfrac{\lambda}{s\mu}$

3) $P_0 = \begin{cases} \left[\sum_{n=0}^{s-1} \dfrac{(s\rho)^n}{n!} + \dfrac{(s\rho)^s}{s!(1-\rho)} \left(1-\rho^{N-s+1}\right)\right]^{-1} & \rho \neq 1 \\ \left[\sum_{n=0}^{s-1} \dfrac{(s\rho)^n}{n!} + \dfrac{(s\rho)^n}{s!} (N-s+1)\right]^{-1} & \rho = 1. \end{cases}$

4) $L_q = \sum_{n=s+1}^{N} (n-s) P_n$

5) Avg. no. of service facility: $L_s - L_q =$

$L_s - L_q = s + \sum_{n=s}^{N} (n-s) \dfrac{(\rho s)^n}{n!}$

6) Avg waiting time in the system.

$W_s = \dfrac{L_s}{\lambda_{eff}}$

$\boxed{\lambda_{eff} = \lambda(1 - P_N)}$

$$W_q = W_s - \frac{1}{\mu}$$

$$L_s = L_q + \frac{\lambda_{eff}}{\mu}$$

9/11/24

STOCHASTIC PROCESS:

MARCO PROCESS:

If set of $\{x(t), t \in T\}$

$t_1 < t_2 < t_3 < \ldots t_n < t$ probability of

$$pr\{(a \le x(t) \le b \mid x(t_1) = x_1, x(t_2) = x_2 \ldots x(t_n) = x_n\}$$

$$= pr\{a \le x(t) \le b \mid x(t_n) = x_n\}$$

The process $\{x(t) \mid t = T\}$ is a marco process

Marco chain

A discrite pramenter marco process is known as marco chain:

Transistion probability: P_{JK} is called the transistion probability and represents the probabilty of transistion from state J from the ath the nth prime to the state K at the $(n+1)$th prime

Homo-Geneous Marco chain:

→ If the Transistion probability P_{JK} is independent of n, The marco chain said to be Homo Geneous.

→ If it is dependent on n, is said to non-Homo-generous.

$$\begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix} = \begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 1 & 0 & 0 \\ 0.7 & 0.3 & 0 \end{bmatrix} \begin{matrix} =1 \\ =1 \\ =1 \end{matrix}$$

ne step transistion probability:

The transistion probability P_{JK} refer to the states (J, K) at two sucessive trails $(n)^{th}$ trail and $(n+1)^{th}$ trail. the transistion is one step transistion probability.

if we are constant with the pairs of state (J, K) at two non-sucessive trails say J at n^{th} trail, K at $n+1^{th}$ trail the corresponding probability is then called m_step transistion probability

$$pr = \{ x_{n+m} = K / x_n = J \}$$

$\Sigma P_{JK} = 1$ for all J these probabilities return the raw total

matrix form that is called transistion probability matrix

$$\Sigma P_{JK} = 1 \quad p = \begin{bmatrix} P_{11} & P_{12} & P_{13} & \cdots & P_{1n} \\ P_{21} & P_{22} & P_{23} & \cdots & P_{2n} \\ \vdots & & & & \\ P_{n1} & P_{n2} & P_{n3} & \cdots & P_{nn} \end{bmatrix}_{m \times m}$$

This is called the transistion probability matrix of the marco chain p is a stochastic matrix.

The transistion matrix is a square matrix with non negative matrix a unit raw sum.

customers strength to exhibit lolery to protect brands but may be persuaded through clean marketing and advertising to switch brang consider A, B, C and advertising to switch brang: A, B, & C customers "unyielding the case of three brands: A, B, & C customers loyalty to a given brand is estimated at 75% giving the competitors only a 25% margin to realise a switch. Competitors launch their advertising campaigns once a year. For brand A customers, the probabilities of switching to brands B and C are 0.1 & 0.5 respectively. Customers of Brand B are likely to switch to A & C with probabilities 0.2 & 0.05 respectively. Brand C customers can switch can switch to brands A & B with equal probabilities.

$$P = \begin{bmatrix} 0.75 & 0.1 & 0.15 \\ 0.2 & 0.75 & 0.05 \\ 0.125 & 0.125 & 0.75 \end{bmatrix}$$

In the long run the market share for each brand is

$[x\ y\ z]\ P = [x\ y\ z]$

$$[x\ y\ z] \begin{bmatrix} 0.75 & 0.1 & 0.15 \\ 0.2 & 0.75 & 0.05 \\ 0.125 & 0.125 & 0.75 \end{bmatrix} = [x\ y\ z]$$

$0.75x + 0.2y + 0.125z = x$

$0.1x + 0.75y + 0.125z = y$

$0.15x + 0.05y + 0.75z = z$

$((0.75-1)x) + 0.2y + 0.125z = 0$

$0.1x + (0.75-1)y + 0.125z = 0$

$0.15x + 0.05y + (0.75-1)z = 0.$

$-0.25x + 0.2y + 0.125z = 0$

$0.1x + 0.25y + 0.125z = 0$

$0.15x + 0.05y - 0.25z = 0.$

$x = \cancel{0.34}\ 0.253$

$y = \cancel{0.30}\ 0.316$

$z = \cancel{0.29}\ 0.430$

$x + y + z = 1.$

$0.25x + 0.2y + 0.125z$

$0.15x + 0.05 \cancel{+} 0.25$

A traffic Signal can be is one of three states green yellow Y, Red R, The transistion probabilities is b/w the states are as ~~follows~~ as

1. If the signal is green, is remains Green with 0.7 probability of turns yellow with 0.2 and turns Red with 0.1.
2. If the signal is yellow it remains yellow prob with 0.4 turns Green with 0.4 and turns red is prob of 0.3, stays yellow with 0.4 0.3

3. If the signal is red, it remains the prob of 0.5, turn yellow with 0.2 and stated red 0.3.

Task construct the transistion prob matrix and find the signal is yellow after two time steps if it starts as with green.

Sol:

$$P = \begin{array}{c} \\ G \\ Y \\ R \end{array} \begin{array}{c} \begin{array}{ccc} G & Y & R \end{array} \\ \begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.3 & 0.4 & 0.3 \\ 0.5 & 0.2 & 0.3 \end{bmatrix} \end{array}$$

initial state vecter (starting at Green) $[1, 0, 0]$

Step: 1 TP

$$\pi_1 = \pi_0 P = [1, 0, 0] \cdot \begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.3 & 0.4 & 0.3 \\ 0.5 & 0.2 & 0.3 \end{bmatrix}$$

π_1: $[0.7 \quad 0.4 \quad 0.3]$

Step: 2 T.P

$$\pi_2 = \pi_1 P.$$

$$[0.7 \; 0.4 \; 0.3] \begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.3 & 0.4 & 0.3 \\ 0.5 & 0.2 & 0.3 \end{bmatrix}$$

$$\pi_2 = \begin{bmatrix} 0.49 & 0.14 & 0.07 \\ 0.12 & 0.16 & 0.12 \\ 0.15 & 0.06 & 0.09 \end{bmatrix}$$

$$\pi_2 = [0.64 \quad 0.26 \quad 0.1]$$

The prob of is that after two steps is yellow

0.26.

Sol:

$$P = \begin{bmatrix} 0.8 & 0.2 \\ 0.6 & 0.4 \end{bmatrix}$$

$$\pi_0 = [0.6 \quad 0.4]$$

step 1. $\pi = \pi_0 P = [0.6 \quad 0.4] \begin{bmatrix} 0.8 & 0.2 \\ 0.6 & 0.4 \end{bmatrix}$

$$= 0.6\times0.8 + 0.4 * 0.6 \quad 0.6\times0.2+0.4\times0.4)$$

$$= [0.72 \quad 0.28]$$

after two years

$\pi_2 = \pi_1 . P (or) \pi_0 . P^2$

$$\pi_2 = [0.72 \quad 0.28] \begin{bmatrix} 0.8 & 0.2 \\ 0.6 & 0.4 \end{bmatrix}$$

$$[0.744 \quad 0.256]$$

$$\text{from} \quad \begin{array}{c|cc} & \text{A} & \text{B} \\ \text{A} & 0.8 & 0.2 \\ \text{B} & 0.6 & 0.4 \end{array}$$

to

$$[x \quad y] P = [x \quad y]$$

$$[x \quad y] \begin{bmatrix} 0.8 & 0.2 \\ 0.6 & 0.4 \end{bmatrix} = [x \quad y]$$

$0.8x + 0.6y = x \Rightarrow -0.2x + 0.6y = 0 \text{ ---(1)}$

$0.2x + 0.4y = y \Rightarrow 0.2x - 0.6y = 0 \text{ ---(2)}$

$$x + y = 1.$$

$\pi_0 = \left[\frac{1}{4} \ \frac{1}{3} \ \frac{5}{15}\right]$

$\pi_1 = \pi_0 P$

$\pi_2 = \pi_1 P (\text{or}) \ \pi_0 P^2$

$[x \ y \ z] P = [x, y, z]$.

$x + y + z = 1$.

Recurrent state:

A state J is said recurrent(or) presistent $F_{JJ} = 1$ and transistent if $F_{JJ} < 1$. A recurrance state is said to be NULL Recurrent(or) persistent (or) Persistent NULL if $\mu_{JJ} = \infty$ that is if the mean reccurrance time is infinite and μ non-null Recurrent

if $\mu_{JJ} < \infty$

→ Hence the states of can be classified as consistent and persistent.

persistent can be seperated as Non-NULL and NULL presistent.

A Non-NULL periodic

periodic and A-periodic

A Recurrent state is said to be a periodic if its ~~speed~~ period $d_i = 1$ periodic $d_i > 1$

communicating classes:

state (i) and (j) communicates each is accessible from other

Transistion state:

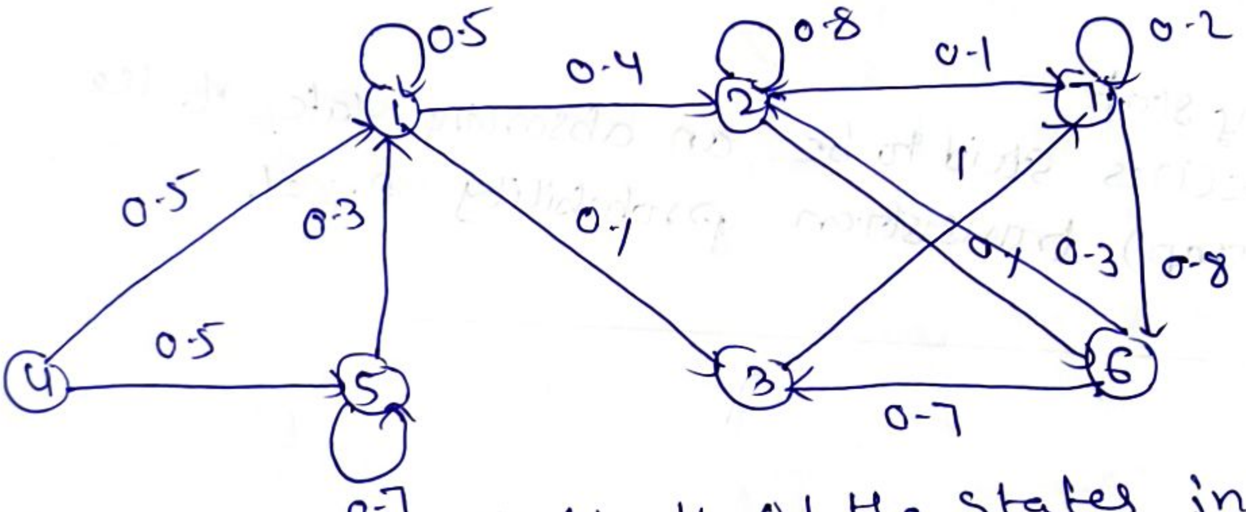
once the process with in the state(i), there is a positive probability that it will never return to state(i).

Absorbing state:

A state(i) is said to be an absorbing state if the (one step) transistion probability $P_{ii} = 1$.

Consider a Markov chain on $S = \{1,2,3,\cdots 7\}$ with transition probabilities.

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{bmatrix} 0.5 & 0.4 & 0.1 & 0 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 & 0 & 0.1 & 0.1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0.5 & 0 & 0 & 0 & 0.5 & 0 & 0 \\ 0.3 & 0 & 0 & 0 & 0.7 & 0 & 0 \\ 0 & 0.3 & 0.7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.8 & 0.2 \end{bmatrix} \end{array}$$



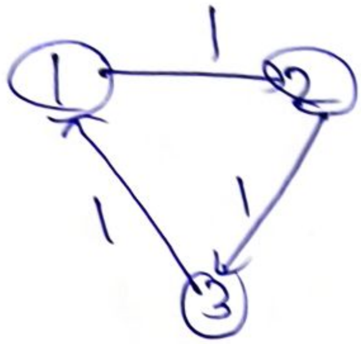
The major results that all of the states in an ~~The states~~ irreducible set are of same type, and they have the same period.

Terminal

If a state is periodic, determine its period classify the states of the following markov chain

a)

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$



$$P^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$P^3 = P \cdot P^2$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$P^4 = P^3 \cdot P$

$\underline{I} \cdot P$

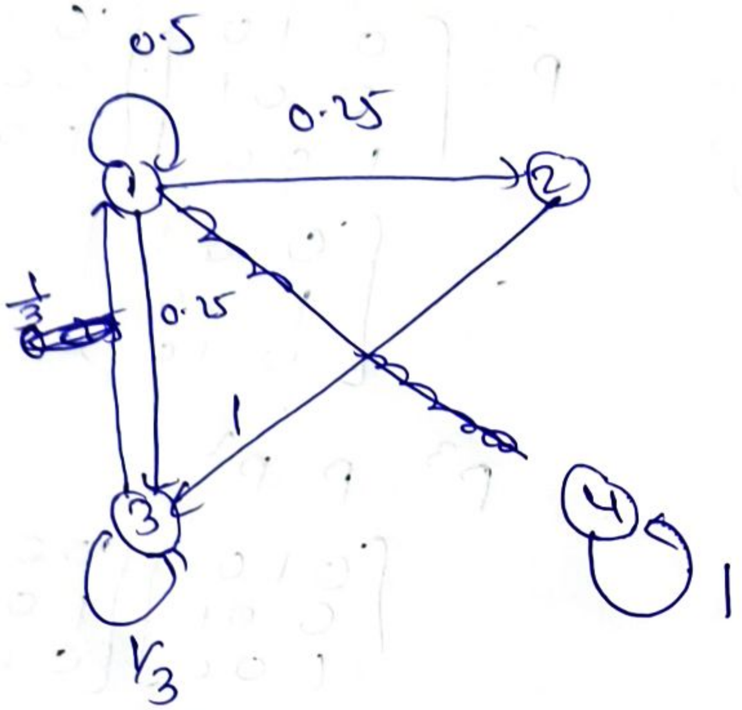
$P^4 = P$

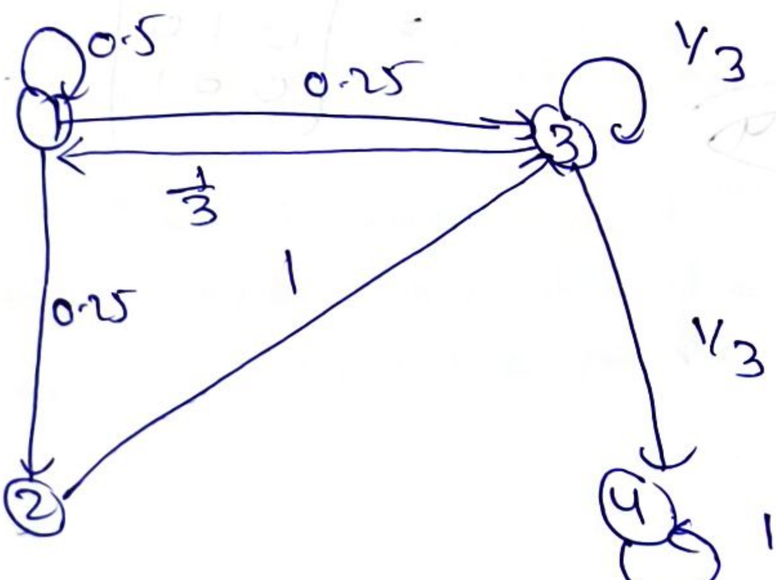
period = 4

(b)

b)

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0.5 & 0.25 & 0.25 & 0 \\ 2 & 0 & 0 & 1 & 0 \\ 3 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ 4 & 0 & 0 & 0 & 1 \end{array}$$





It is nonclosed and it is also irreducible
and they have the same period.