

# Thermal Physics

## Module III

### MODULE 3 THERMAL PHYSICS

12 Hrs

Kinetic Theory of Gases and Maxwell Boltzmann Distribution, Zeroth Law of Thermodynamics and Concept of Temperature, First Law of Thermodynamics and Conservation of Energy: Carnot's Engine, Second Law of Thermodynamics, and concept of Entropy

## Assumptions of kinetic theory

1. A gas consists of a large number of identical molecules moving randomly.
2. The volume of individual molecules is negligible compared to the total volume of gas.
3. Molecules interact only through elastic collisions.
4. The time of collision is much smaller than the time between collisions.
5. No external forces (except container walls) act on the molecules

## Introduction

In 1765, James Watt invented a superior steam engine with a separate condenser

The laws of thermodynamics comprise the essence of 200 years of experimentation and theoretical interpretation.

From the Kinetic Theory of Gases, we know that:

$$\text{where: } PV = \frac{2}{3}N\left[\frac{1}{2}m\langle v^2 \rangle\right]$$

- $P$  is the pressure,
- $N$  is the number of molecules,
- $V$  is the volume,
- $m$  is the mass of a single molecule,
- $\langle v^2 \rangle$  is the mean square velocity of the molecules.

For an ideal gas, the ideal gas law is given by:

$$PV = nRT$$

where:

- $n$  is the number of moles,
- $R$  is the gas constant,
- $T$  is the temperature.

Substituting this into the expression for pressure:

$$\text{So, we can equate: } \frac{2}{3}N\left[\frac{1}{2}m\langle v^2 \rangle\right] = nRT$$

$$T = \frac{2}{3} \frac{N}{nR} \left[\frac{1}{2}m\langle v^2 \rangle\right] \quad \text{or} \quad T = \frac{2}{3} \frac{1}{k} \left[\frac{1}{2}m\langle v^2 \rangle\right]$$

$$\frac{1}{2}m\langle v^2 \rangle = \frac{3}{2}kT$$

Since the average kinetic energy is  $\langle E_k \rangle = \frac{1}{2}m\langle v^2 \rangle$

We have the relation between the average kinetic energy and temperature:

$$\langle E_k \rangle = \frac{3}{2}kT$$

## Thermodynamics

### Introduction - An overview

Thermodynamics deals with the study of heat, work, and energy, as well as how they interact with each other in different systems

It focuses on understanding the relationships between various forms of energy and how energy is transferred or transformed in physical processes

#### Energy

the most fundamental and universal concepts

It's not a "thing" that exists by itself, but is rather an attribute of matter or radiation

generally accepted "the capacity to do work"

Energy are basically two kinds; KE & PE

The "microscopic" form of KE, *thermal energy*, is completely random in the kinds of motions it exhibits and in its direction.

It causes a rise in the temperature of the surroundings.

So it's measured in terms of its ability to perform work or to transfer heat

**Work** the transfer of energy by some means

like energy, it can take various forms.

Mechanical work  $\Delta x$

Electrical work  $\Delta V$

Thermodynamical work  $\Delta T$

**Heat** the energy in transit/transfer (between hot to cold body)

Heat and work are both measured in energy units, so they must both represent energy

**There are four fundamental laws of thermodynamics**

**Zeroth Law of Thermodynamics**

**First Law of Thermodynamics (Conservation of Energy)**

**Second Law of Thermodynamics**

**Third Law of Thermodynamics**

#### Quasi-static Process

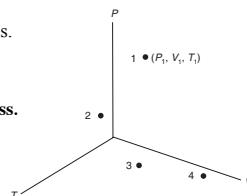
Applications    Heat engines    Refrigerators and heat pumps    Biological systems    Materials Science    Energy Systems Design  
Power Generation

Massachusetts Institute of Technology/Physical and Theoretical Chemistry, Semester 1, Massachusetts Institute of Technology, Chemistry, Thermodynamics/Chemical, Energy/CHEMICAL, Heat/PCP, and Work

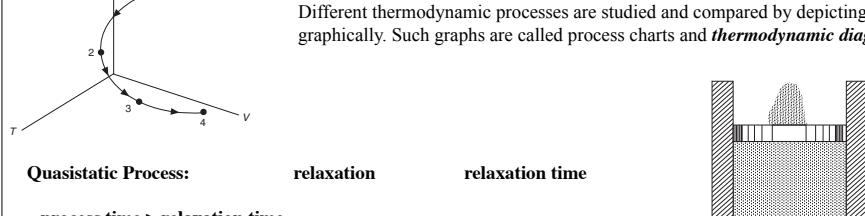
Different points on the graph correspond to different equilibrium states of the gas.

**Process:** Any change in the state of a system is called a **thermodynamic process**.

The path of the succession of states through which the system passes is called a **process**



Different thermodynamic processes are studied and compared by depicting them graphically. Such graphs are called process charts and **thermodynamic diagrams**



**Quasistatic Process:**

relaxation

relaxation time

**process time > relaxation time**

a thermodynamic process, a continuous succession of equilibrium states of the system, termed to be an equilibrium process or a **quasistatic process**.

#### Reversible and Irreversible Processes

- i. The process can be carried out with equal facility in two opposite directions.
- ii. In each direction the system passes through the same intermediate states; and
- iii. After the completion of the process, the system and its environment return to their initial state.

Any process which does not satisfy even one of these conditions is **irreversible**.

## Thermodynamics

Thermodynamics is the science that deals with work and heat, and those properties of substances related to heat and work

Two point of views of study a substance (or its properties) - a microscopic or macroscopic

microscopic quantities can be calculated by theoretical analyses and but not measured directly like macroscopic.

Temperature, pressure and volume are examples of macroscopic quantities

can tell about whole gas not about individual atom or molecule

#### TERMINOLOGY:

**System:** Open, Close and Isolated

**State:** or state function can be determined by thermodynamical variables, P,V,T.

**Equation of state:**  $PV = nRT$  Ideal gas equation

$$(P + \frac{a}{V^2})(V - b) = RT \quad \text{Van der Wall's equation}$$

a, b - Van der Wall's constants are zero for idea gas; P, V, T - if two are known, called as independent, third is dependent

**Thermodynamic Equilibrium:** A system is said to be in thermodynamic equilibrium if none of the thermodynamic variables determining its state changes with time.

a system will be in a state of thermodynamic equilibrium if it satisfies the conditions for mechanical, thermal and chemical equilibrium.

#### PV-diagram

A Cartesian coordinate system is used to plot sets of values of P, V, and T to indicate equilibrium states of a system (Fig)

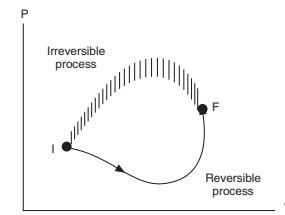
Since a reversible process is defined by a succession of equilibrium states, it can be represented by a line (**Solid line**) on a PV diagram

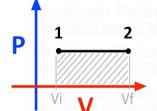
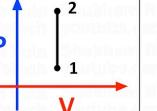
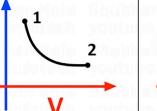
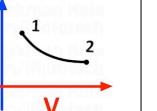
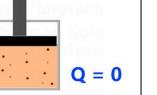
Each point on the curve represents one of the intermediate equilibrium states

an irreversible process passes from the initial state to the final state through a series of non-equilibrium states. (path by individual lines)

an irreversible process **cannot** be represented by a **line** on PV diagram

a thermodynamic process is **reversible** if the system passes from the initial state to final state through a succession of equilibrium states.



Type of Thermodynamical Process:	Isobaric Process	Isochoric Process	Isothermal Process	Adiabatic Process
	 <b>Constant Pressure</b> $P = P(V_i - V_f)$	 <b>Constant Volume</b> $\Delta U = W$	  <b>Constant Temperature</b> $Q = 0$	  <b>No Heat Transfer</b> $Q = 0$

# Work

The mechanical work performed by a force is the most familiar form of work.

$$W = \mathbf{F} \cdot \mathbf{x}$$

it serves as the measure of transfer of mechanical energy from one body to another.

Work is not stored in the body

**Work is a form of transferring energy and is a measure of the transferred energy.**

## The work done in a thermodynamic process :

Consider a thermodynamic system such as a gas contained in a cylinder fitted with a movable piston

In equilibrium the gas occupies a volume  $V$  and exerts a uniform pressure  $P$  on the piston and the walls of the cylinder.

If the cross-sectional area of the piston is  $A$ , the force exerted by the gas on the piston is

$$F = PA$$

Let gas expands quasi-statically, as the piston moves up a distance (Fig b), the work done by the gas on the piston is

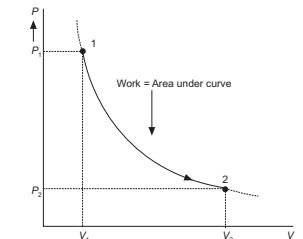
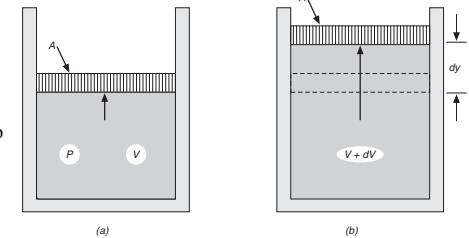
$$dW = F dy = PA dy$$

But  $A dy = dV$ , the increase in the volume of the gas

$$dW = P dV$$

work done by a system is regarded as positive

work done on a system is negative



The total work done by the gas as its volume changes from  $V_1$  to  $V_2$  can be found by integrating

$$W_{12} = \int_1^2 P dV = \int_{V_1}^{V_2} P dV$$

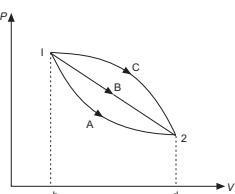
The above integration can be performed only if we know the relationship between  $P$  and  $V$  during the process

Therefore, A the total work done during the expansion of the gas from the initial state to final state is the area under the curve in a PV diagram.

It is possible to go from the initial state 1 to the final state 2 along many different paths such as A, B or C, as depicted in Fig

it is noticeable that the amount of work performed in each case is a function of the end states of the process and

also it is dependent on the path that is followed in going from one state to another



## INTERNAL ENERGY:

As per kinetic theory of gasses, ideal gas molecules/atom do not interact. But in real gas they interact.

The internal energy  $U$  of the system depends on the nature of the motion and interaction of the particles in the system

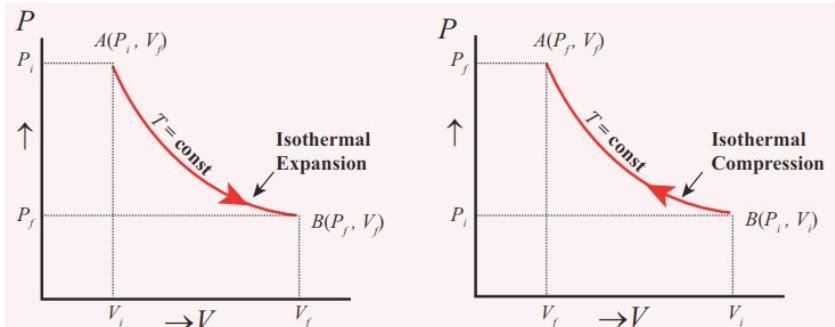
$$E_T = K.E. + P.E. + U$$

The internal energy is determined by the thermodynamic state of the system and

does not depend on how the system acquired the given state.

The internal energy is not related to the process of a change in the state of the system

Internal energy - stored in the system = (KE + intermolecular PE) of molecules; for ideal gas int. PE = 0



Importance of PV diagram:

area under the curve = work done by or on the system

during Expansion - work done by the system - curve traced in PV diagram is clockwise, "positive"

during Compression - work done on the system - curve traced in PV diagram is anti-clockwise, "negative"

# Heat

In the eighteenth century, heat was considered to be a massless and invisible substance called caloric every body contains an amount of caloric that depends on the temperature of the body.

Thus, a hot body contains more caloric than a cold body.

As per the observation of Count Rumford (Benjamin Thompson, American – British Physicist, 1753-1814) in 1798:

the generation of heat was due to the mechanical work done

Julius Robert Mayer (1814-1878), in 1842 got the idea of the equivalence of heat and work

He hypothesised that *heat is a form of energy*.

However, the relationship between work and heat was established in 1850 experimentally by James Joule (1818-1899), the British Physicist.

According to the modern view, heat is the energy that is transferred from a hot body to a cold body

**the flow of energy from one body to another solely as a result of temperature difference between them.**

Thus, heat is not an intrinsic property of a body means idea of “caloric” is incorrect.

# Zeroth Law of Thermodynamics

concept of temperature

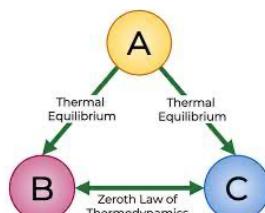
sense (subjective) of hot and cold

the quantity that characterises the degree of heating is termed as Temperature

So, temperature is a fundamental and measurable property of matter, which allows us to define temperature scales (e.g., Celsius, Fahrenheit).

The law was established by [Ralph H. Fowler](#) in the 1930s, long after the first, second, and third laws had been widely recognised.

Significance of this law:  
the thermal equilibrium principle



If two bodies are in thermal equilibrium with a third body, they are in thermal equilibrium with each other as well

# Heat

the energy in transit/transfer (between hot to cold body)

like work, heat is a path function too.

The heat transferred to or from a system depends on the process.

Heat transferred to a system is considered positive and heat transferred from a system is negative

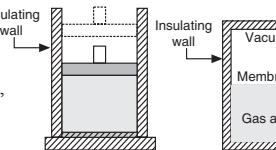
Example

gas ( $V_1$ ) is in thermal contact  
with a heat reservoir

If  $P(\text{infinitesimally}) > \text{atm } P$ .

Heat required to maintain  $T$  const of gas,  
during the process by expanding gas  $V_2$ .

Work is done by the gas in this case



gas ( $V_1$ ) is thermally insulated.

membrane breaks, the gas expands to a  $V_2$ .

No work is done in this case.

no heat transfer through the walls

initial and final states of both processes are identical but the paths followed are different

**Three modes :** Conduction, Convection and Radiation

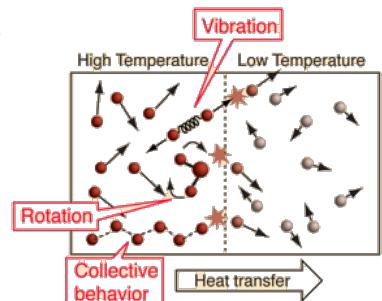
## Concept of temperature :

Zeroth law — the thermal equilibrium — means the objects in contact are no longer transferring the heat (or net energy) to one another.

Here we can say that these objects are at same temperature or temperature is a property of these objects, which implies that they will no longer transfer the net energy to one another.

An another idea of temperature is to relate the average kinetic energy of the molecules: one collection of molecules with higher average kinetic energy will have higher temperature and other collection of same type molecules with lower average kinetic energy will have lower temperature.

$$T_{\text{kinetic}} = \frac{2}{3k} \left[ \frac{1}{2} mv^2 \right] = \frac{2}{3k} KE_{\text{avg}}$$



# First Law of Thermodynamics

it is the law of conservation of energy, so thermodynamics is concerned with Heat, Work and internal Energy.

Heat is the energy transferred by virtue of temperature difference, thermodynamic work is energy (exclusive of heat) transferred between a system and surroundings, and internal energy is energy stored within a system

supplying a quantity of heat  $Q$  to system, system does the work

$$dQ = dU + dW \quad 1$$

As per, the law of conservation of energy

the quantity of heat supplied to the system will be equal to the sum of the work performed by the system and the change in the internal energy of the system.

$$dW = \text{force on piston} \cdot dx$$

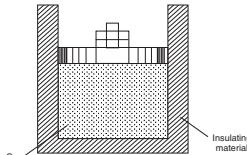
$$\text{force on piston} = P A$$

$$dW = P \cdot dx$$

$$dW = P \cdot dV$$

or eqn 1 can be written as:

$$dQ = dU + P \cdot dV$$



## Specific Heat

the amount of [heat](#) per unit mass required to raise the [temperature](#) by one degree Celsius.

### APPLICATIONS OF THE FIRST LAW :

**1. Isolated system:** does not interact with its surroundings  $\Delta U = 0$  internal energy = const

**2. Cyclic process:** the initial and final states of the system are the same  $\Delta U = 0$   $Q = W$

**3. Adiabatic process:** no heat is absorbed or ejected by the system  $Q = 0$   $\Delta U = -W$

**4. Isochoric process:** the volume remains unchanged  $W = 0$   $\Delta U = Q$   
heat that entered the system is stored as internal energy

**5. Isothermal process:** constant temperature first law does not assume any special form

**6. Isobaric process:** constant pressure  $W = \int_1^2 P dV = P(V_2 - V_1)$

**7. Isothermal expansion of an ideal gas:**  $W = nRT \ln\left(\frac{V_2}{V_1}\right)$   
ideal gas expands quasi-statically at constant T by placing the gas in good thermal contact with a heat reservoir at T

**8. Adiabatic expansion of an ideal gas:**  $PV^\gamma = \text{Constant}$

The specific heats of gases are generally expressed as [molar specific heats](#).

For a monoatomic [ideal gas](#) the [internal energy](#) is all in the form of kinetic energy,

and [kinetic theory](#) provides the expression for that energy, related to the [kinetic temperature](#)

$$U = nN_A KE_{avg} = nN_A \frac{3}{2}kT = \frac{3}{2}nRT$$

Two specific heats are defined for gases, one for [constant volume](#) ( $C_V$ ) and one for [constant pressure](#) ( $C_P$ ).

For a constant volume process with a monoatomic ideal gas the [first law of thermodynamics](#) gives:

$$Q = C_V n \Delta T \quad Q = \Delta U + P \Delta V = \Delta U$$

$$C_V = \frac{1}{n} \frac{\Delta U}{\Delta T} = \frac{3}{2}R$$

$$C_P = C_V + R$$

The ratio of the specific heats  $\gamma = C_P/C_V$  is a factor in [adiabatic](#) engine processes and in determining the [speed of sound](#) in a gas. This ratio  $\gamma = 1.66$  for an ideal monoatomic gas and  $\gamma = 1.4$  for [air](#), which is predominantly a diatomic gas.

## Isothermal expansion of an ideal gas

Let an ideal gas be allowed to expand quasi-statically at constant temperature by placing the gas in good thermal contact with a heat reservoir at the same temperature

Ideal gas equation:  $PV = nRT$

$$\text{So, } W = \int_{V_1}^{V_2} P dV = \int_{V_1}^{V_2} \frac{nRT}{V} dV$$

$$\text{For constant } T, \text{ we can write; } W = nRT \int_{V_1}^{V_2} \frac{dV}{V}$$

$$\text{or, } W = nRT \ln\left(\frac{V_2}{V_1}\right)$$

# Adiabatic expansion of an ideal gas

According to equation for adiabatic process:

$$\Delta U = -W$$

For ideal gas (constant volume)

$$\Delta U = -n C_v dT$$

Since work done during process  $W = PdV$ , so we can write:  $PdV = -n C_v dT$

from equation of state:  $d(PV) = d(nRT)$  or  $P dV + VdP = n R dT$

using above equations;  $-n C_v dT + VdP = n R dT$  or  $VdP = n dT(R + C_v)$

or the ratio between the above equations:  $PdV = -n C_v dT$  &  $VdP = n dT(R + C_v)$

$$\frac{V dP}{P dV} = -\frac{C_p}{C_v} = -\gamma$$

$$\frac{dP}{P} = -\gamma \frac{dV}{V}$$

on integrating:

$$\int_{P_1}^{P_2} \frac{dP}{P} = -\gamma \int_{V_1}^{V_2} \frac{dV}{V}$$

$$\ln \frac{P_2}{P_1} = -\gamma \ln \frac{V_2}{V_1}$$

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$PV^\gamma = \text{Constant}$$

**Thermal Efficiency:** a measure of how economical an engine is.

$$\text{Mechanical efficiency} = \frac{\text{Work output}}{\text{Work input}}$$

In heat engines, the work input is the heat input.

The work output equals the heat in minus the heat out.

in a cycle operation: heat absorbed  $Q = Q_H - Q_L$

Since the working substance returns to its initial state after the cycle is completed, its internal energy remains unchanged.

$$\Delta U = 0$$

as first law, work done is

$$W = Q = Q_H - Q_L$$

So, Thermal efficiency

$$\eta = \frac{W}{Q_H} = \frac{Q_H - Q_L}{Q_H}$$

$$\eta = 1 - \frac{Q_L}{Q_H} \quad (\text{ideal engine})$$

In case of Real engine:

$$\eta \leq 1 - \frac{Q_L}{Q_H}$$

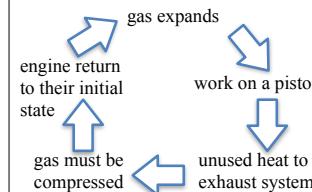
A good automobile engine's efficiency of about 20% and diesel engines have efficiencies ranging from 35% to 40%.

# Heat Engine

a device that converts heat energy to work.

three main parts: the working medium, the high-temperature reservoir and the low-temperature reservoir.

e.g., in an actual engine the energy produced in the combustion of fuel is transmitted by heat exchange to some gas

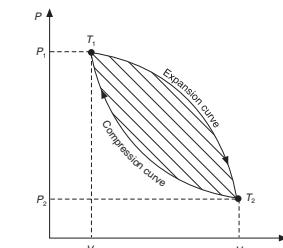
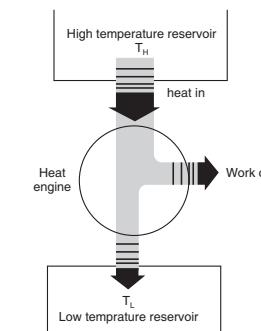


This implies that a heat engine must operate in cycles

in real engine, gas is discharged at the exhaust & a new portion admitted instead

It is called an **open cycle** operation

However, for simplicity, a **closed cycle** in which the same portion of gas expands and is compressed assumed as it does not influence the thermodynamics of the process.



# THE CARNOT CYCLE

If the efficiency of all heat engines is less than 100%, what is the most efficient cycle we can have?

In 1824 a French engineer, Sadi Carnot (1796-1832) showed that a heat engine operating in a reversible cycle between the two heat reservoirs would be the most efficient engine possible.

Every process is reversible in this ideal cycle.

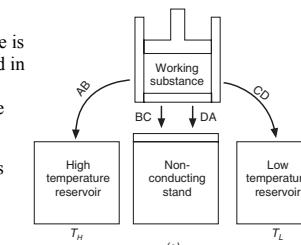
Such a cycle is now called a **Carnot cycle**.

**assumptions:**

the working substance is an ideal gas contained in a cylinder with a movable piston at one end.

base of the cylinder is **diathermic** (i.e. perfectly conducting)

walls and the piston are ideally nonconducting

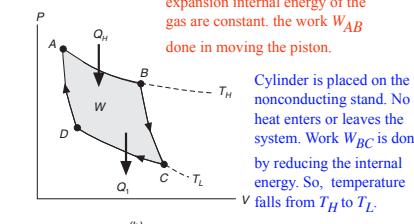


$$W = Q_H - Q_L$$

$$\eta = \frac{W}{Q_H} = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H}$$

a slow quasi-static isothermal expansion internal energy of the gas are constant, the work  $W_{AB}$  done in moving the piston.

Cylinder is placed on the nonconducting stand. No heat enters or leaves the system. Work  $W_{BC}$  is done by reducing the internal energy. So, temperature falls from  $T_H$  to  $T_L$ .



cylinder is removed the low-temperature reservoir and again placed on the non-conducting stand. gas is compressed adiabatically to initial state A. Work  $W_{DA}$  done on the gas, the system temperature increases from  $T_L$  to  $T_H$ . During this, T and U of the working substance are constant.  $W_{CD}$  done on gas

**Efficiency:**

During the isothermal expansion  $A \rightarrow B$ , the heat absorbed:

$$Q_H = W_{AB} = nRT_L \ln \frac{V_B}{V_A}$$

during compression  $C \rightarrow D$ ,

$$Q_L = W_{CD} = nRT_L \ln \frac{V_C}{V_D}$$

$$\frac{Q_L}{Q_H} = \frac{T_L \ln(V_C / V_D)}{T_H \ln(V_B / V_A)}$$

For an adiabatic process, the pressure and volume are related by :  $PV^\gamma = \text{Constant}$

During reversible process, the ideal gas obey the equation of state  $PV = nRT$

by eliminating the P from above two;  $T V^{\gamma-1} = \text{Constant}$

$$\begin{aligned} \text{So during } B \rightarrow C, D \rightarrow A; \quad T_H V_B^{\gamma-1} &= T_L V_C^{\gamma-1} \\ T_H V_A^{\gamma-1} &= T_L V_D^{\gamma-1} \end{aligned}$$

$$\begin{aligned} \text{or, } \left(\frac{V_B}{V_A}\right)^{\gamma-1} &= \left(\frac{V_C}{V_D}\right)^{\gamma-1} & \text{Substituting in above eq of Q}_L/\text{Q}_H & \frac{Q_L}{Q_H} = \frac{T_L}{T_H} \\ \left(\frac{V_B}{V_A}\right) &= \left(\frac{V_C}{V_D}\right) & \eta = 1 - \frac{T_L}{T_H} & = \frac{T_H - T_L}{T_H} \end{aligned}$$

**Carnot's Theorem:**

No heat engine operating between two heat reservoirs can be more efficient than a Carnot engine operating between the same two reservoirs.

first law of thermodynamics is concerned with conservation of energy

so as per first law, given amount heat is converted to equal amount of work in a cycle

where as second law, emphasises the fact that the process only in one direction (not reversible)

e.g., a hot cup of coffee cools by virtue of heat transfer to the surroundings but heat will not flow from the cooler surroundings to the hot cup of coffee.

**Note :** a cycle will occur only if both the laws are satisfied otherwise not

**Second law asserts** that any type of energy can be converted to thermal energy but reverse is not true.

Impossibility of converting the heat completely into mechanical work is the base of Kelvin-Planck's law

**Entropy:**

The thermal energy per unit temperature that is not available for doing useful work is called [entropy](#)

The ordered motion of molecules will result in useful work.

So, the amount of molecular disorder or randomness of the system is called entropy.

The differential form of the first law of thermodynamics is written as  $dQ = dU + dW$

Work-done by the gas  $dW = P dV$       Similarly, Heat  $dQ$  can be expressed  $dQ = T dS$

where  $dS$  — is called the change in entropy

# Second Law of Thermodynamics

## Kelvin-Planck & Clausius Statements

**Kelvin-Planck Statement:**

*It is impossible to construct a heat engine that will operate in a cycle and which will receive a given amount of heat from a high-temperature reservoir and does an equal amount of work. The only alternative is that some heat must be transferred from the working fluid to a low temperature reservoir. Therefore, work can be done by the transfer of heat only if there are two temperature levels involved*

**Rudolf Clausius Statement:**

*It is impossible to construct a device that operates in a cycle and produces no effect other than the transfer of heat from a cold body to a hot body. In effect, it is impossible to construct a refrigerator that operates without an input of work.*

**ENTROPY:**

entropy which is regarded as a measure of disorder in a system as per Clausius.

processes, in general, are irreversible and the degree of disorder increases as a result of these processes

$$\Delta S_{\text{isolated system}} \geq 0$$

$$\text{the change in entropy as } dS = \frac{dQ}{T}$$

The change in entropy  $dS$  in the course of an infinitesimal change is equal to the quantity of heat  $dQ$  divided by the absolute temperature  $T$ , where  $dQ$  is the heat absorbed (or rejected) when the change is carried out in a reversible manner.

**The net change in entropy in any reversible cycle is zero.**

during adiabatic expansion or compression,  $Q = 0$     Therefore,  $S = 0$ .

However, there is an increase in entropy during isothermal expansion, as heat  $Q_H$  is added at a constant temperature  $T_H$ .

$$\Delta S_1 = \frac{Q_H}{T_H}$$

There is a decrease in entropy during the isothermal compression in which heat  $Q_L$  is rejected at a temperature  $T_L$ .

$$\Delta S_2 = -\frac{Q_L}{T_L}$$

$$\text{The net change in entropy is given by } \Delta S = \Delta S_1 + \Delta S_2 = \frac{Q_H}{T_H} - \frac{Q_L}{T_L}$$

$$\begin{aligned} \text{but, } \frac{Q_H}{T_H} &= \frac{Q_L}{T_L} \\ \Delta S &= 0 \end{aligned}$$

**Entropy increases in all irreversible processes.**

**Entropy indicates the direction in which processes proceed in nature.**

**Entropy represents the unavailability of energy.**

$$\text{Energy wasted} = T_L \Delta S$$

If  $T_L$  is constant, the amount of energy wasted is proportional to the increase in entropy.

### THIRD LAW OF THERMODYNAMICS

With a decrease in temperature, a greater degree of order prevails in any system

W.Nernst concluded that "at 0K, any change in the state of a system takes place without a change in the entropy".

This is called **Nernst's theorem**. (also called as third law of thermodynamics)

**It is impossible to attain a temperature of 0K.**

Figure shows the Maxwell-Boltzmann distribution is affected by temperature

At lower temperatures, the molecules have less energy

Therefore, the speeds of the molecules are lower and the distribution has a smaller range

As the temperature of the molecules increases, the distribution flattens out.

Because the molecules have greater energy at higher temperature, the molecules are moving faster.

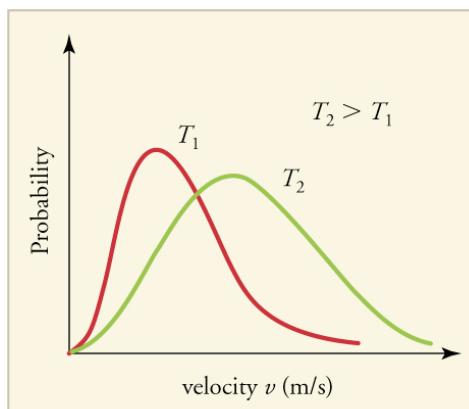
Three speed expressions can be derived from the Maxwell-Boltzmann distribution: the most probable speed, the average speed, and the root-mean-square speed.

$$- \text{RMS velocity: } v_{rms} = \sqrt{\frac{3kT}{m}}$$

$$- \text{Average velocity: } v_{avg} = \sqrt{\frac{8kT}{\pi m}}$$

$$- \text{Most probable velocity: } v_{mp} = \sqrt{\frac{2kT}{m}}$$

$$v_{mp} < v_{avg} < v_{rms}$$



Applications: diffusion, effusion, stellar atmospheres

The Maxwell-Boltzmann equation, which forms the basis of the kinetic theory of gases, defines the distribution of speeds for a gas at a certain temperature.

From this distribution function, the most probable speed, the average speed, and the root-mean-square speed can be derived.

The kinetic molecular theory is used to determine the motion of a molecule of an ideal gas under a certain set of conditions.

However, it is impossible to measure the velocity of each molecule at every instant of time

Therefore, the Maxwell-Boltzmann distribution is used to determine how many molecules are moving between velocities  $v$  and  $v+dv$ .

$$\frac{dN}{N} = \left( \frac{m}{2\pi k_B T} \right)^{1/2} e^{-\frac{mv^2}{2k_B T}} dv$$

$dN/N$  is the fraction of molecules moving at velocity  $v$  to  $v+dv$ ,  
 $m$  is the mass of the molecule,  
 $k_B$  is the Boltzmann constant, and  
 $T$  is the absolute temperature.

$$f(v) = 4\pi c^2 \left( \frac{m}{2\pi k_B T} \right)^{3/2} e^{-\frac{mv^2}{2k_B T}}$$

### Example Problems

1. Find the RMS velocity of oxygen molecules at 300 K.
2. Calculate Carnot engine efficiency between 500 K and 300 K.