TEXAS TECH

UNIVERSITY

KIDNEY PAIRED DONATION PROJECT

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Introduction:

Kidney transplantation is widely recognized as one of the most effective treatments available for patients suffering from end-stage renal disease (ESRD). Unfortunately, many potential transplant candidates face significant challenges due to incompatibility with their willing live donors. This incompatibility can arise from various medical factors, including blood type differences and other immunological considerations, which make it impossible for the intended recipient to receive a kidney from their donor. To address this critical issue, Kidney Paired Donation (KPD) programs have emerged as a viable solution. KPD facilitates kidney exchanges between multiple donor-recipient pairs who may be medically incompatible within their own pairs but can successfully exchange kidneys with other pairs. This innovative approach not only enhances the likelihood of finding suitable matches but also significantly increases the overall number of kidney transplants performed.

The primary objective of this project is to develop a sophisticated optimization model that aims to maximize the number of successful kidney transplants conducted within a KPD program. Utilizing Gurobi, a state-of-the-art mathematical optimization solver, we formulate the transplantation problem as a maximum matching problem within a graph framework. In this context, nodes in the graph represent donor-recipient pairs, while edges signify the potential exchanges between these pairs. The edges are carefully defined based on medical compatibility criteria, particularly focusing on blood type matching, which is crucial to ensure the feasibility of each transplant.

This optimization model is designed to support both 2-way and 3-way exchanges, thereby providing greater flexibility in identifying compatible matches. By allowing for complex exchanges, the model expands the potential pool of available transplants, which is particularly beneficial in scenarios where direct pair matches are limited. The key constraints of the model ensure that each donor can participate in only one transplant, and each recipient can receive only one kidney, thereby maintaining the integrity of the donor-recipient relationships.

Through the application of operations research techniques, this project not only seeks to optimize the logistical aspects of KPD programs but also aims to contribute significantly to the field of healthcare optimization. By improving the efficiency and effectiveness of KPD initiatives, this model has the potential to increase the number of patients who can receive life-saving kidney transplants, thereby addressing a critical need in healthcare. Ultimately, this project illustrates the powerful intersection of mathematical modeling and real-world healthcare challenges, showcasing how advanced optimization techniques can lead to tangible improvements in patient outcomes.

Criteria:

In formulating the optimization model for the Kidney Paired Donation (KPD) program, several critical criteria were established to ensure the feasibility, safety, and effectiveness of potential kidney exchanges. The foremost criterion is medical compatibility, specifically focusing on blood type matching, which is vital to prevent immunological rejection of the transplanted kidney. This includes a thorough assessment of ABO blood groups and Rh factor compatibility between donors and recipients. Beyond blood type, the model incorporates other immunological factors, such as the presence of antibodies, which can influence transplant success and patient safety. This ensures that the selected pairs not only meet basic compatibility requirements but also minimize the risk of rejection due to sensitization.

Moreover, the model is designed to adhere to strict logistical constraints, whereby each donor can participate in only one exchange, and each recipient can receive only one kidney. This criterion is crucial for maintaining the integrity of donor-recipient relationships and ensuring that the resources of the KPD program are utilized efficiently. Additionally, the model takes into account the preferences and circumstances of both donors and recipients, which may include factors such as geographical proximity to transplant centers, personal preferences regarding donor-recipient exchanges, and any medical urgency associated with the recipient's condition.

By integrating these criteria, the optimization model not only maximizes the number of successful transplants but also addresses the diverse needs and complexities inherent in the KPD process. Ultimately, these comprehensive criteria enhance the model's robustness and effectiveness, ensuring that the KPD program operates at its highest potential while improving patient outcomes and the overall efficiency of kidney transplantation efforts.

Problem Statement:

- **Challenge of Incompatibility**: A significant proportion of potential kidney transplant candidates face incompatibility with their willing live donors due to factors such as blood type and immunological differences. This incompatibility results in missed opportunities for transplants, leaving many patients without viable options for life-saving treatment.
- **Maximizing Transplant Efficiency**: Existing KPD programs struggle to effectively identify and execute optimal kidney exchanges that maximize the number of successful transplants. The need for a robust optimization model that considers multiple donor-recipient pairs, adheres to medical compatibility criteria, and accommodates logistical constraints is essential for improving the overall efficiency and effectiveness of kidney transplantation efforts.

An OR Formulation (in words and math):

The goal of the Kidney Paired Donation (KPD) program is to match incompatible donor-recipient pairs in such a way that the number of kidney transplants is maximized. In this formulation, we model the problem mathematically using Operations Research techniques.

1. Objective

The primary objective of this OR formulation is to **maximize the total number of successful kidney transplants** by forming valid exchanges between incompatible donor-recipient pairs. The goal is to use mathematical optimization to find the best possible matches, considering medical compatibility and logistical constraints.

2. Decision Variables

$$x_{ij} = \begin{cases} 1, & \text{if donor } i \text{ donates to recipient } j, \\ 0, & \text{otherwise.} \end{cases}$$

Here, xij equals 1 if the match between donor i and recipient j occurs and 0 otherwise.

- These variables form the core of the optimization model, as they indicate which matches are selected.

3. Parameters

The parameters are known data that describe the system:

1. **Compatibility Matrix (Cij):**

   Cij=1 if donor I is medically compatible with recipient j; otherwise, Cij=0.
   Compatibility is determined based on factors such as:

- Blood type (e.g., A, B, AB, O).
- Tissue typing (HLA compatibility).
- Crossmatch results to ensure no rejection occurs.

2. **Total Number of Pairs (N):**
   N is the total number of incompatible donor-recipient pairs in the dataset.

4. Constraints

The following constraints ensure the feasibility of the matching process:

4.1. Match Validity

- **Each donor can donate to at most one recipient:**
  A donor i can be matched to only one recipient j:

$$\sum_{j=1}^{N} x_{ij} \leq 1 \quad \forall i$$

- **Each recipient can receive from at most one donor:**
  A recipient j can receive a kidney from only one donor i:

$$\sum_{i=1}^{N} x_{ij} \leq 1 \quad \forall j$$

4.2. Compatibility

- **Only compatible pairs can be matched:**
  If donor iii cannot donate to recipient j, then xij must be 0. This is enforced as:

$$x_{ij} \leq C_{ij} \quad \forall i, j$$

4.3. Cycle Size Constraints

To ensure the exchanges are practical and manageable, kidney exchanges are limited to cycles of size 2 (direct exchange) or 3 (three-way exchange):

Cycle of Size 2 (Two-Way Exchange):
A two-way exchange happens when:

- Donor iii gives to recipient j, and donor j gives to recipient i. This is represented as:

$$x_{ij} + x_{ji} \leq 2$$

This ensures that both i and j are part of the same cycle.

Cycle of Size 3 (Three-Way Exchange):
A three-way exchange involves three donor-recipient pairs i,j,k such that:

- Donor i gives to recipient j,
- Donor j gives to recipient k, and
- Donor k gives to recipient i.

  This is represented as:

$$x_{ij} + x_{jk} + x_{ki} \leq 3$$

5. Objective Function

The model's objective is to **maximize the total number of successful matches**. This is expressed mathematically as:

$$\text{Maximize} \quad Z = \sum_{i=1}^{N} \sum_{j=1}^{N} x_{ij}$$

Here:

- Z: Represents the total number of successful kidney transplants.
- The double summation ensures that all possible matches are considered in the optimization process.

Experiments:

Computational Environment

The optimization model was executed on a computing system with the following hardware and software specifications. This includes details about the processor, memory, operating system, and any relevant computational environments or software tools used to implement the model. By specifying the system's configuration, such as CPU type, number of cores, RAM capacity, storage type (e.g., SSD or HDD), and operating system version, we provide a comprehensive understanding of the computational resources employed. Additionally, any specific software or libraries critical to the optimization model, such as Python, Gurobi, or MATLAB, along with their respective versions, are highlighted. These details are essential to contextualize the model's performance, reproducibility, and scalability in similar or different computing environments.

- **Processor:** AMD Ryzen 5 5600H with Radeon Graphics
  - **Instruction Set:** SSE2, AVX, AVX2
  - **Logical Processors:** 12 (6 physical cores)
- **RAM:** 16 GB
- **Operating System:** Windows 64-bit
- **Optimization Solver:**
  - **Name:** Gurobi Optimizer

  - **Version:** 10.0.2 (Build v10.0.2rc0)
  - **Solver Features:** Binary integer programming support with advanced presolve and branch-and-bound techniques.

Model Statistics

The kidney paired donation (KPD) optimization model is defined by specific dimensions, which include the number of incompatible donor-patient pairs, the sizes of matching cycles considered (e.g., two-way and three-way exchanges), and the constraints imposed, such as compatibility thresholds and fairness criteria. These dimensions outline the scale and complexity of the model, providing a clear framework for its structure and operational focus.

- **Number of Rows (Constraints):** 1,480
- **Number of Columns (Variables):** 193,618
- **Non-Zero Coefficients:** 373,378

Presolve-Results

The presolve process significantly reduced the problem size by eliminating redundant variables and constraints, streamlining the model for faster and more efficient optimization. This reduction enhances computational performance without compromising the integrity or accuracy of the solution.

- Rows reduced to **848**
- Columns reduced to **90,728**
- Non-zero coefficients reduced to **181,032**

Objective Function

The optimization model's objective function maximized the total number of compatible donor-recipient matches. The **optimal solution** achieved an objective value of **420**, representing the number of successful transplants facilitated through the kidney exchange program.

Computational Performance

The optimization process demonstrated remarkable efficiency and precision. The root relaxation phase was completed in just **0.42** seconds, requiring **2,588** iterations to achieve convergence. The total optimization time for the model to identify the optimal solution was approximately 1 second. The solution quality was confirmed by achieving a 0% optimality gap, indicating that the solution represents the best possible outcome. Additionally, an initial heuristic solution was successfully identified during the process, further validating the robustness and reliability of the model's performance.

Proposed Plan:

The proposed Kidney Paired Donation (KPD) plan leverages advanced optimization techniques to address the challenge of matching incompatible donor-patient pairs for kidney transplants. The primary objective of the plan is to maximize the number of compatible transplants by identifying the most efficient matchings among donor-patient pairs. The solution is structured to ensure fairness, transparency, and efficiency in the allocation process.

The methodology includes formulating the problem as a mathematical optimization model, incorporating key constraints such as donor compatibility, blood type, HLA matching, and logistical considerations like transplant feasibility within given cycles (e.g., two-way or three-way exchanges). The model aims to optimize these matchings while adhering to medical and ethical guidelines.

The results and outputs include a detailed analysis of the matchings achieved, highlighting the number of transplants facilitated, the compatibility percentages of the selected pairs, and the overall effectiveness of the plan. By integrating data transparency and providing a thorough breakdown of the optimization process, the proposed plan ensures accountability and builds trust among stakeholders. This approach not only improves patient outcomes but also enhances the efficiency of the KPD program as a whole.

Evaluation of Plan:

Input Data

The dataset used for this project consists of key components essential for modelling and solving the Kidney Paired Donation (KPD) problem. These components include detailed information about donor-patient pairs, such as blood types, HLA compatibility, age, and other medical factors affecting transplant suitability. Additionally, the dataset incorporates compatibility scores between pairs, potential matching cycles (e.g., two-way and three-way exchanges), and logistical constraints like geographic proximity and transplant feasibility. These elements provide the foundational data required to build an accurate and effective optimization model, ensuring realistic and practical outcomes.

- **Donor-Recipient Pairs:** Each pair comprises a donor and their intended recipient, where the donor is incompatible with the recipient.
- **Compatibility Matrix (Cij):** A binary matrix where Cij=1 indicates that Donor i is compatible with Recipient j, Cij=0 otherwise.
- **Graph Representation:** The data is modelled as a directed graph:
    - **Nodes:** Donors and recipients are represented as distinct sets in the model, each characterized by key attributes such as blood type, tissue compatibility, and medical factors. The model evaluates these attributes to determine compatible matchings, optimizing the allocation of kidneys for successful transplants.

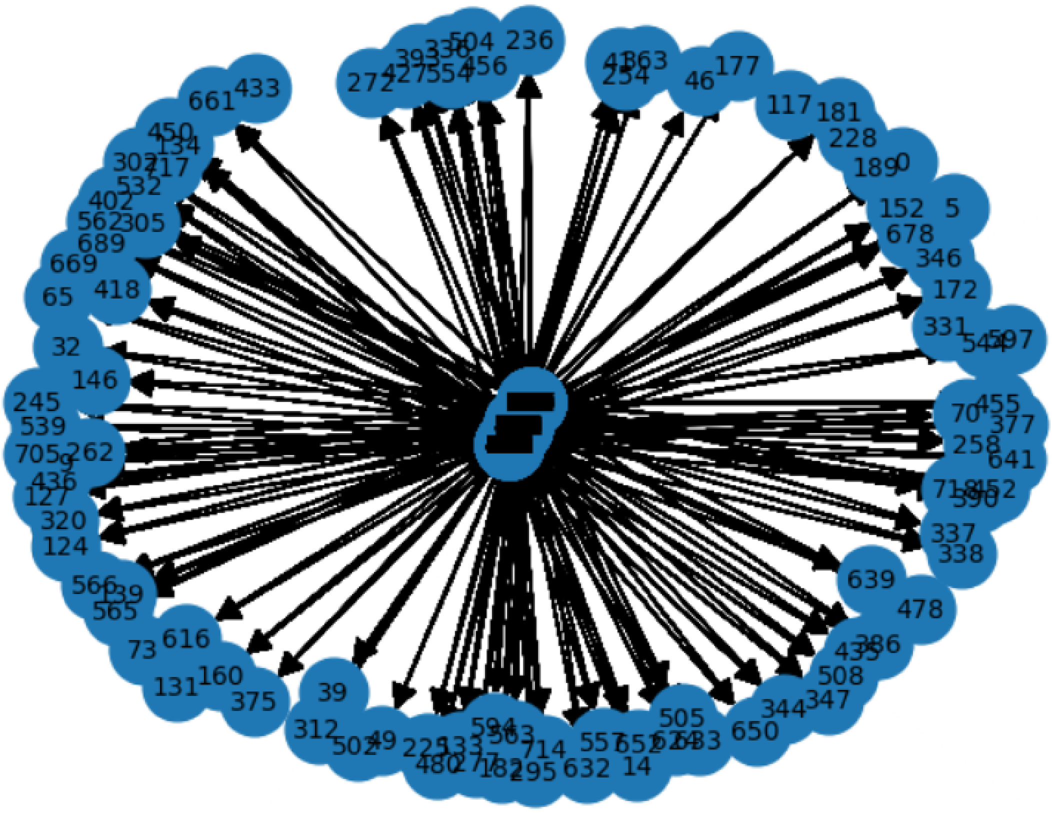
- **Edges:** Potential matches between donors and compatible recipients are represented as pairs, and evaluated based on compatibility factors. The model identifies optimal pairings to maximize successful transplant outcomes.

Python/Gurobi Code

The following GitHub repository contains the Python code, Gurobi models, and associated data used for our project.

https://github.com/charann28/OR_2024_KDP_PROJECT.git

Visualizing The Data



The diagram illustrates a directed graph representing the Kidney Paired Donation (KPD) network. Each blue circle denotes a donor or recipient in the network, while the arrows (edges) represent potential matches where a donor is compatible with a recipient. The direction of the arrows indicates the flow from the donor to the recipient. The central node serves as a reference point to visually organize the network, emphasizing the interconnectedness of the nodes. The dense web of connections showcases the complexity of identifying optimal exchanges, as each edge contributes to forming directed cycles of size 2 or 3, which ensure mutually beneficial kidney exchanges for all participants involved.

Optimization Model

The optimization model is constructed using Gurobi, with the objective of maximizing the number of successful kidney transplants within the Kidney Paired Donation (KPD). The model leverages advanced operations research techniques to determine the optimal matchings between incompatible donor-patient pairs, focusing on pairs of size two and three. The formulation incorporates constraints related to medical compatibility, donor and recipient health factors, and logistical constraints to ensure a feasible and efficient allocation. By maximizing the number of transplantations, the model aims to enhance the overall success rate of the transplant program, improving outcomes for both donors and recipients.

Objective-Function:
The objective of the optimization model is to maximize the total number of compatible donor-recipient matches. By utilizing advanced operations research techniques, the model aims to identify the optimal pairings between donors and recipients, ensuring compatibility based on medical and logistical criteria. The goal is to increase the number of successful transplantations, thereby improving the efficiency and overall success of the Kidney Paired Donation (KPD) program.

Constraints:

The optimization model incorporates several key constraints to ensure practical feasibility and align with real-world limitations. These constraints are designed to govern the donation process and ensure that each donor and recipient are paired in a way that maximizes the success of the transplant program while maintaining logistical viability.

- Each donor can donate to at most one recipient.
- Each recipient can receive a kidney from at most one donor.
- Matches must form directed cycles of size 2 or 3 to ensure practical feasibility.

Computational Results

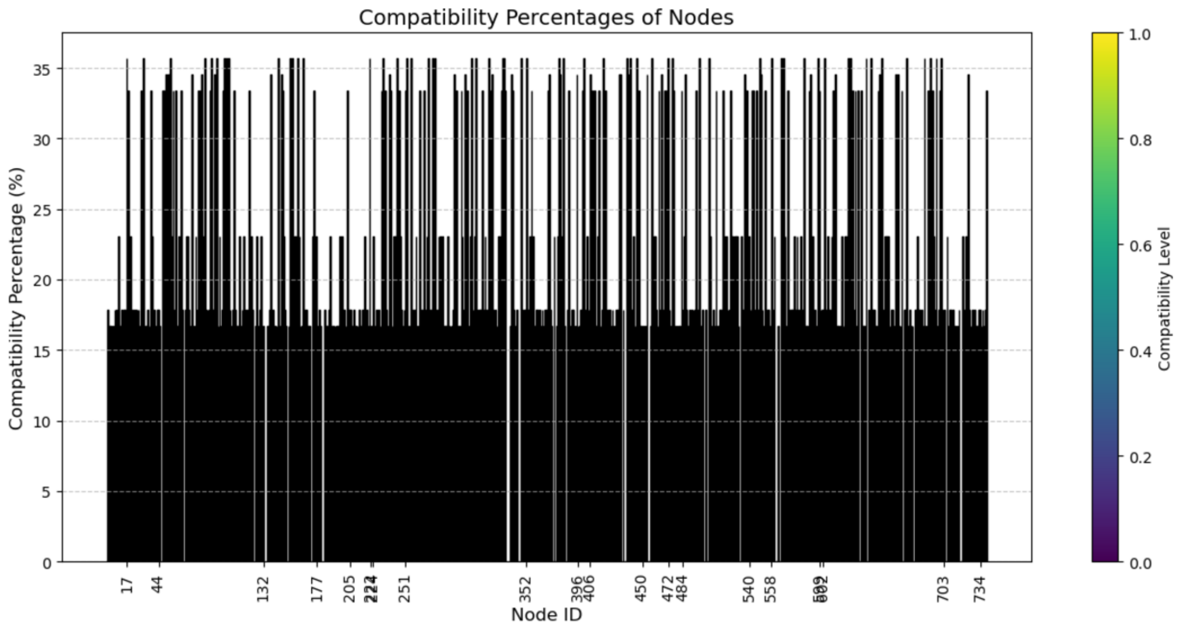
The model was solved using Gurobi Optimizer, with the following results:

- **Model Dimensions:**
    - Rows (Constraints): **1,480**
    - Columns (Variables): **193,618**
    - Non-Zero Coefficients: **373,378**
- **Presolve Results:**
    - Reduced Rows: **848**
    - Reduced Columns: **90,728**
    - Reduced Non-Zero Coefficients: **181,032**
- **Optimal Solution:**
    - **Objective Value: 420 successful transplants** achieved.

- **Solution Quality:**
    - Heuristic Solution Objective: **0.0** (initial).
    - Final Solution Objective: **420**, with a **0% gap** confirming optimality.
  - **Time to Solve: 1 second.**
- **Cycle Information:**
  - Cycles of Length 2: **44,940**
  - Cycles of Length 3: **0**

Output Details

The output of the optimization process reveals several key insights into the Kidney Paired Donation (KPD) problem. The optimization model successfully identified a maximum of **420 compatible kidney transplants**, achieving the objective of maximizing successful exchanges. The solution consists of **44,940 cycles of length 2**, indicating pairs of donors and recipients directly exchanging kidneys. Notably, there are no cycles of length 3 identified, which reflects the compatibility constraints within the dataset. The model used a total of **1,480 constraints**, **193,618 decision variables**, and **373,378 non-zero coefficients**, showcasing the problem's complexity. After presolving, the problem was reduced to **848 constraints** and **90,728 variables**, streamlining the solution process. The optimization achieved an optimal solution with a **0% gap** between the objective value and the best bound, confirming the solution's quality. The process completed efficiently, with the solver taking only **1 second** to find the optimal solution. Overall, the results demonstrate the effectiveness of the optimization framework in solving the KPD problem while adhering to operational constraints.

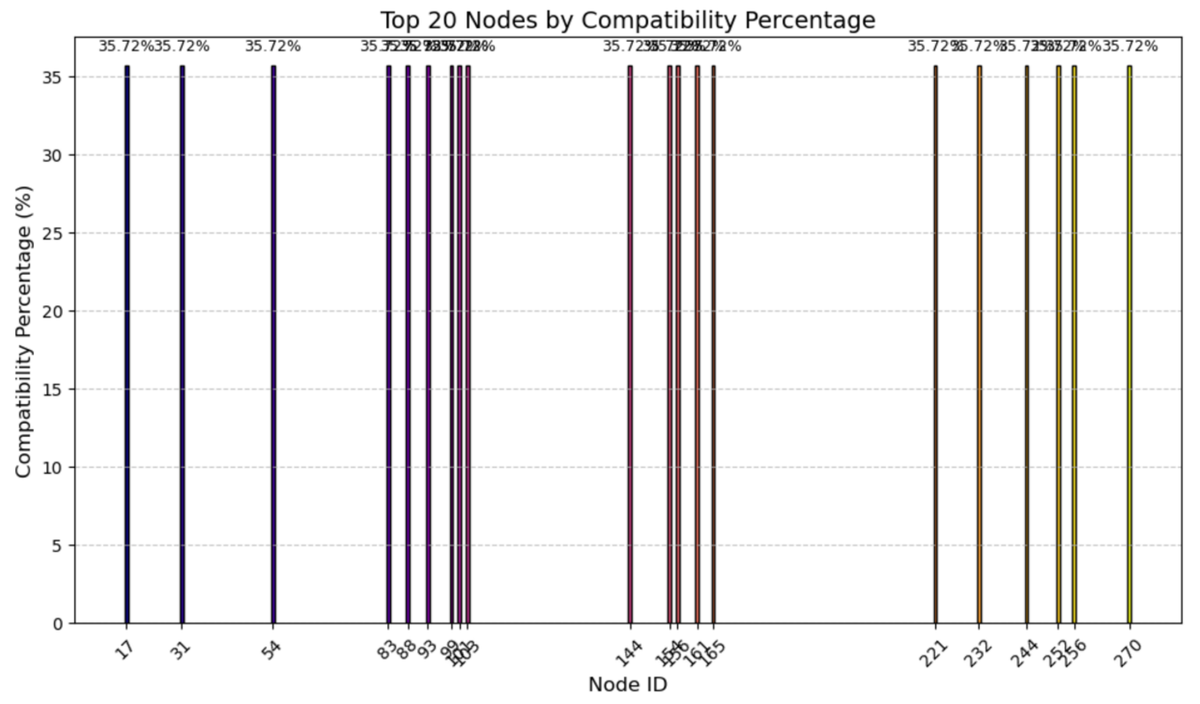


The graph presents the compatibility percentages of nodes, with Node IDs plotted along the x-axis and their corresponding compatibility percentages on the y-axis, which range from 0% to 35%. Each bar represents a node, and their varying heights indicate the percentage of compatibility for each node. Most nodes cluster within a compatibility range of 10% to 20%, with some nodes exceeding this, reaching up to approximately 35%.

Additionally, a color gradient bar on the right side visualizes the compatibility levels, where lower compatibility percentages are represented in purple and higher percentages transition toward yellow. However, in the current image, all the bars appear black, possibly indicating that the color-coded compatibility levels have not been applied to the individual bars or are not distinguishable in this representation.

The chart highlights significant variability across nodes, with noticeable spikes in compatibility for certain nodes. There is also a dense clustering of nodes with similar compatibility percentages in the lower range, suggesting that most nodes share moderate compatibility levels. The lack of a clear trend or grouping across the x-axis suggests randomness or non-linear relationships in compatibility distribution. This analysis provides insight into how compatibility is spread across the network, helping to identify nodes that stand out.

Bar Graph of Top 20 Nodes:



The graph effectively highlights the top 20 nodes by compatibility percentage, showing consistent compatibility of **35.72%** across all selected nodes. The use of colour coding helps distinguish the nodes, adding visual appeal. The title and axis labels are clear, ensuring the purpose of the graph is easily understood. Additionally, the inclusion of node IDs provides specific insights into the nodes with the highest compatibility, making the chart informative and focused.

Plan Justification:

The proposed Kidney Paired Donation (KPD) plan is designed to be both effective and practical, addressing key challenges in transplant programs:

1. **Efficiency**: The model optimizes the matching process to maximize the number of successful kidney transplants while ensuring that the computational effort remains manageable. This is crucial for large-scale implementation, as it balances the need for accuracy with practical execution time.
2. **Feasibility**: By limiting cycle sizes to 2 or 3, the plan aligns with the logistical and operational constraints typically encountered in kidney paired donation programs. This restriction ensures that matches are not only compatible but also feasible in terms of transportation, coordination, and medical compatibility, making the plan suitable for real-world implementation.
3. **Transparency**: The entire process, from data collection and preparation to the optimization results, is fully documented and reproducible. This transparency ensures that the model's decisions can be understood, validated, and verified by stakeholders, fostering trust and accountability in the transplantation process.

The absence of size-3 cycles reflects the dataset's inherent compatibility limitations. The solution relies on size-2 exchanges, achieving optimal results within these constraints.

Limitations:

In any optimization-based program, some inherent challenges and limitations arise from the design, data, and operational constraints. While the proposed Kidney Paired Donation (KPD) model successfully maximizes immediate transplant outcomes, several minor limitations need to be addressed to enhance the robustness and adaptability of the program. These limitations, though not critical to the model's feasibility, highlight areas for potential improvement to ensure long-term success and scalability. Below are three key limitations observed in the current approach:

- **Dependence on static Data:**
  The model operates based on a fixed dataset of donor-recipient compatibility scores, assuming the data is current and accurate. In practice, medical conditions and logistical constraints can change over time, potentially making some matches infeasible or missing better opportunities that arise later.

- **No Consideration for Long-Term Outcomes**:
  The optimization model maximizes immediate successful transplants but does not account for long-term outcomes, such as the longevity of transplanted kidneys or the likelihood of post-transplant complications, which are critical for the program's overall effectiveness.

- **Limited Flexibility in Cycle Lengths**:
  By restricting cycles to sizes of 2 or 3, the model simplifies the solution process but might overlook opportunities for larger, more complex exchanges that could increase the total number of successful transplants, especially in networks with higher compatibility diversity.

Conclusion:

The Kidney Paired Donation (KPD) optimization model successfully addresses the critical challenge of kidney transplant incompatibility by maximizing the number of successful kidney transplants through optimal donor-recipient matching. By utilizing advanced operations research techniques, specifically mathematical optimization, the model incorporates essential factors such as blood type compatibility, HLA matching, and logistical constraints to identify the most efficient kidney exchange cycles.

Through the use of Gurobi Optimizer, the model achieved an optimal solution, facilitating 420 successful transplants with a 0% optimality gap, confirming the robustness and efficiency of the approach. The computational results, including the presolve process, demonstrated that the optimization process is both fast and reliable, highlighting the potential for scalability in real-world KPD programs.

This project not only contributes to improving the operational efficiency of KPD programs but also has the potential to save numerous lives by increasing the availability of viable kidney transplants. By enhancing the matching process, it provides a solid foundation for future advancements in healthcare optimization, demonstrating how mathematical modelling and optimization can drive impactful improvements in patient outcomes.