

AI1110 : Probability and Random Variables

Assignment 5

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May 23, 2022

Outline

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Question

Prove and generalise the following identity

$$\begin{aligned}\Pr(A + B + C) &= \Pr(A) + \Pr(B) + \Pr(C) \\ &\quad - \Pr(AB) - \Pr(BC) \\ &\quad - \Pr(CA) + \Pr(ABC)\end{aligned}\tag{1}$$

Solution:Proof

We will use the following identity,

$$\Pr(A + B) = \Pr(A) + \Pr(B) - \Pr(AB) \quad (2)$$

Now from (2)

$$\begin{aligned} \Pr(A + (B + C)) &= \Pr(A) + \Pr(B + C) \\ &\quad - \Pr(A(B + C)) \end{aligned} \quad (3)$$

$$\Pr(B + C) = \Pr(B) + \Pr(C) - \Pr(BC) \quad (4)$$

$$\Pr(A(B + C)) = \Pr(AB + AC) \quad (5)$$

$$= \Pr(AB) + \Pr(AC) \quad (6)$$

$$- \Pr(ABAC)$$

$$= \Pr(AB) + \Pr(AC) \quad (7)$$

$$- \Pr(ABC)$$

Substitute (4) and (7) in (3), we get

$$\begin{aligned} \Pr(A + B + C) &= \Pr(A) + \Pr(B) + \Pr(C) \\ &\quad - \Pr(AB) - \Pr(BC) - \Pr(CA) + \Pr(ABC) \end{aligned} \quad (8)$$

Now using induction, we can show similarly that,

$$\begin{aligned} \Pr(A_1 + \dots + A_n) &= \Pr(A_1) + \dots + \Pr(A_n) \\ &\quad - \Pr(A_1 A_2) - \dots - \Pr(A_{n-1} A_n) \\ &\quad + \dots (-1)^{n-1} \Pr(A_1 A_2 \dots A_n) \end{aligned} \quad (9)$$

Solution:Derivation

We will derive (2) using Boolean Algebra. For any two events A,B

$$A.1 = A (B + B') \quad (10)$$

$$= AB + AB' \quad (11)$$

$$\Pr(A) = \Pr(AB + AB') \quad (12)$$

$$= \Pr(AB) + \Pr(AB') \quad (13)$$

Since AB and AB' are mutually disjoint sets.

Now,

$$A + B = A (B + B') + B (A + A') \quad (14)$$

$$= (AB + BA) + BA' + B'A \quad (15)$$

$$= AB + A'B + B'A \quad (16)$$

$$= A + A'B \quad (17)$$

Now,

$$\Pr(A + B) = \Pr(A + A'B) \quad (18)$$

Since both events are mutually disjoint.

$$\Pr(A + B) = \Pr(A) + \Pr(A'B) \quad (19)$$

$$= \Pr(A) + \Pr(B) - \Pr(AB) \quad (20)$$

Verification

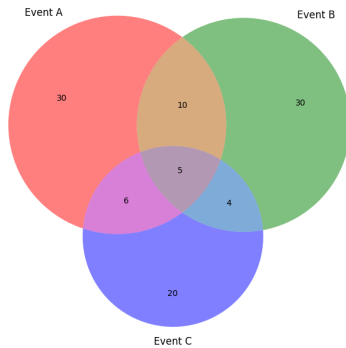


Figure 0: Figure generated by python

The python code `./codes/verify.py` is used to verify the equation (8) using above python generated figure. The output of code is shown below,

The identity is verified successfully with LHS=RHS= 1.0

Figure 0