AI1110 : Probability and Random Variables

Assignment 6

Mannem Charan(AI21BTECH11019)

May 27, 2022

Outline

1. Question
2. Solution
   - Calculation
   - Plot

Question

The random variable x is uniform in the interval $(-2c, 2c)$. Find and sketch $f_y(y)$ and $F_y(y)$ if $y = g(x)$ and $g(x)$ is the function in Fig. 5-3.

Solution:Calculation

Given that, random variable x is uniformly distributed in interval $(-2c, 2c)$. So we can write that, the probability density function

$$f_x(x) = \frac{1}{2c - (-2c)} \tag{1}$$
$$= \frac{1}{4c} \tag{2}$$

So for $x \in (-2c, 2c)$, the probability distribution function $F_x(x)$ can be calculated as,

$$F_x(x) = \int_{-2c}^{x} f_x(x)\, dx \tag{3}$$
$$= \int_{-2c}^{x} \frac{1}{4c} dx \tag{4}$$
$$\tag{5}$$

$$= \frac{1}{4c}\int_{-2c}^{x} dx \tag{6}$$

$$= \frac{1}{4c}(x+2c) \tag{7}$$

$$= \frac{x+2c}{4c} \tag{8}$$

Since x is distributed from -2c to 2c, we can write

$$F_x(x) = \begin{cases} 0 & , x \leq -2c \\ \frac{x+2c}{4c} & , |x| < 2c \\ 1 & , x \geq 2c \end{cases}$$

Now given that random variable y is

$$y = g(x) = x^2 \tag{9}$$

When $y \geq 0$ and $y < 4c^2$ $(becausex \in (-2c, 2c))$ we can write probability distribution function in y as,

$$\begin{aligned}
F_y(y) &= \Pr(Y \leq y) && (10)\\
&= \Pr(g(x) \leq y) && (11)\\
&= \Pr\left(x^2 \leq y\right) && (12)\\
&= \Pr\left(-\sqrt{y} \leq x \leq \sqrt{y}\right) && (13)\\
&= \Pr\left(x \leq \sqrt{y}\right) - \Pr\left(x \leq -\sqrt{y}\right) && (14)\\
&= F_x\left(\sqrt{y}\right) - F_x\left(-\sqrt{y}\right) && (15)\\
&= \frac{\sqrt{y} + 2c}{4c} - \frac{-\sqrt{y} + 2c}{4c} && (16)\\
&= \frac{\sqrt{y}}{2c} && (17)
\end{aligned}$$

Now for $y < 0$ there are no values of x such that $x^2 < y$. So,

$$F_y\left(y\right) = \Pr\left(\emptyset\right) = 0\left(y < 0\right) \tag{18}$$

Overall we can write that,

$$F_y\left(y\right) = \begin{cases} 0 & , y < 0 \\ \frac{\sqrt{y}}{2c} & , 0 \leq y \leq 4c^2 \\ 1 & , y > 4c^2 \end{cases}$$

The probability density function $f_y(y)$ for $0 \leq y \leq 4c^2$ can be calculated as ,

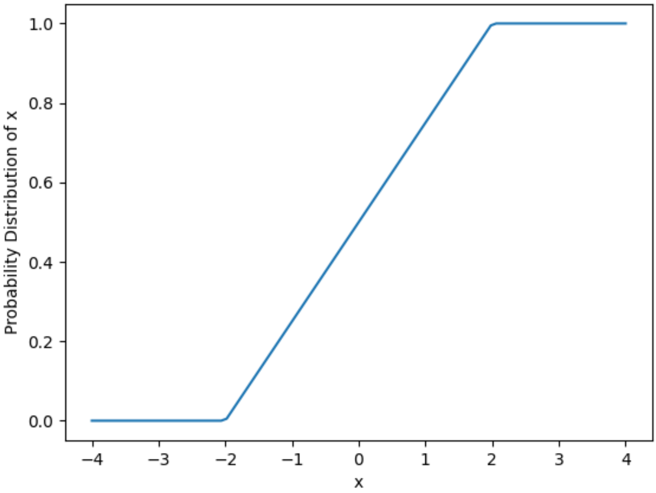
$$f_y(y) = \frac{dF_y(y)}{dy} \tag{19}$$

$$= \frac{d}{dy}\left(\frac{\sqrt{y}}{2c}\right) \tag{20}$$

$$= \frac{1}{4\sqrt{y}c} \tag{21}$$

The plot of $F_x(x), F_y(y), f_y(y)$ for $c = 1$ is shown below,

Solution:Plot



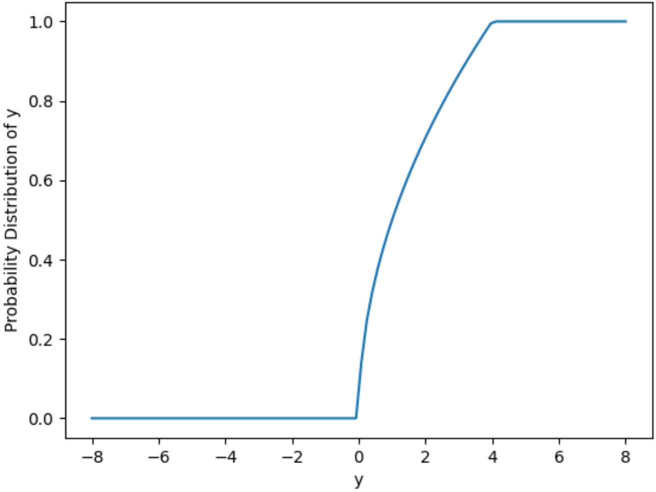


Figure 2

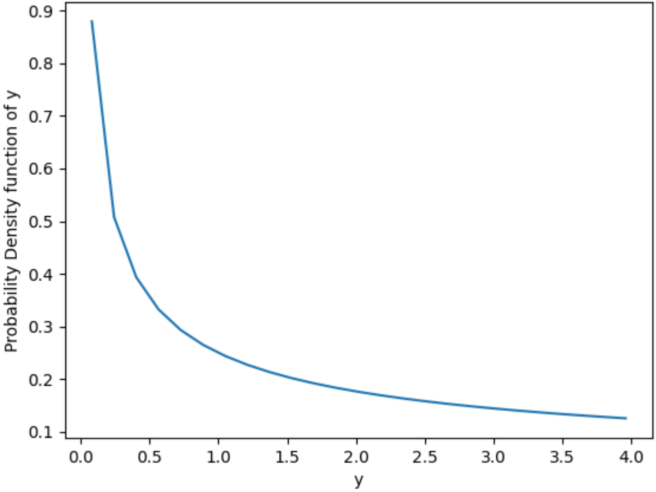

0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
Probability Density function of y
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
y

Figure 3