AI1110 : Probability And Random Variables Assignment 2

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***Abstract*—This document provides solution of Assignment 2(ICSE 2019 12 Q.5(a))**

Question 5(a): Show that the function $f(x) = |x-4|$,$x \in R$ is continuous, but not differentiable at $x = 4$.

Key Concept :

1) A function f is said to be continuous at $x = a$,iff the following three conditions satisfied.
   - i The limit $\lim_{x \to a} f(x)$ should exist and it is finite.
   - ii The functional value $f(a)$ should exist and it is finite.
   - iii $\lim_{x \to a} f(x) = f(a)$.
2) A function f is said to be differentiable at $x = a$ if and only if the limit,
$$lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$
exists.

Solution :

Given

$$f(x) = \begin{cases} x-4, & x \geq 4 \\ 4-x, & x < 4 \end{cases}$$

We can say f is continuous at $x = 4$,iff

$$lim_{x \to 4} f(x) = f(4) \tag{1}$$

In other words f should satisfy,

$$f\left(4^-\right) = f\left(4^+\right) = f(4) \tag{2}$$

where,
$f(4^-) = \lim_{h \to 0} f(4-h)$
$f(4^+) = \lim_{h \to 0} f(4+h)$.

$$f(4) = 0 \tag{3}$$

Now,

$$f\left(4^-\right) = \lim_{h \to 0} f(4-h) \tag{4}$$
$$= \lim_{h \to 0} 4 - (4-h) \tag{5}$$
$$= \lim_{h \to 0} h \tag{6}$$
$$\implies f\left(4^-\right) = 0 \tag{7}$$

And,

$$f\left(4^+\right) = \lim_{h \to 0} f(4+h) \tag{8}$$
$$= \lim_{h \to 0} (4+h) - 4 \tag{9}$$
$$= \lim_{h \to 0} h \tag{10}$$
$$\implies f\left(4^+\right) = 0 \tag{11}$$

Using (3) ,(7),(11),we can say that f is continuous at $x = 4$ and this can be seen in Fig 1.
Now from the concept of differentiability,we can say f is differentiable at $x = 4$ iff the limit,

$$lim_{h \to 0} \frac{f(4+h) - f(4)}{h}$$

exists.
In that case f should satisfy,

$$\lim_{h \to 0} \frac{f(4+h) - f(4)}{h} = lim_{h \to 0} \frac{f(4) - f(4-h)}{h} \tag{12}$$

LHS:

$$\lim_{h \to 0} \frac{f(4+h) - f(4)}{h} = \lim_{h \to 0} \frac{((4+h)-4)-0}{h} \tag{13}$$
$$= \lim_{h \to 0} \frac{h}{h} \tag{14}$$
$$= 1 \tag{15}$$
$$\implies \lim_{h \to 0} \frac{f(4+h) - f(4)}{h} = 1. \tag{16}$$

RHS:

$$\lim_{h \to 0} \frac{f(4) - f(4-h)}{h} = \lim_{h \to 0} \frac{0-(4-(4-h))}{h} \tag{17}$$
$$= \lim_{h \to 0} \frac{-h}{h} \tag{18}$$
$$= -1 \tag{19}$$
$$\implies \lim_{h \to 0} \frac{f(4) - f(4-h)}{h} = -1. \tag{20}$$

$$\therefore LHS \neq RHS$$

Hence, function $f(x)$ is not differentiable at $x = 4$.This can be seen in Fig 2.
Therefore we proved that $f(x) = |x - 4|$ is continuous, but not differentiable at $x = 4$.

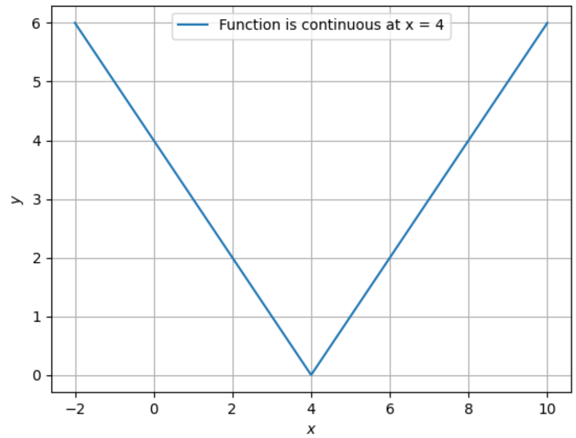

Fig. 1.

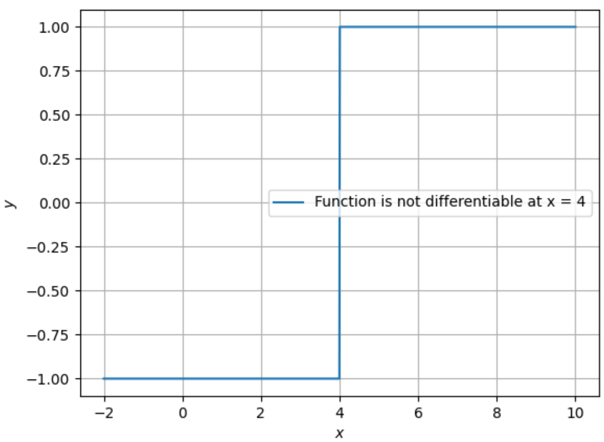

Fig. 2.