AI1110 : Probability and Random Variables
Assignment 6

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Outline

Question

The random variable x is uniform in the interval $(-2c, 2c)$. Find and sketch $f_y(y)$ and $F_y(y)$ if $y = g(x)$ and $g(x)$ is the function in Fig. 5-3.

Solution:Calculation

Given that, random variable x is uniformly distributed in interval $(-2c, 2c)$. So we can write that, the probability density function

$$f_x(x) = \frac{1}{2c - (-2c)} \tag{1}$$

$$= \frac{1}{4c} \tag{2}$$

So for $x \in (-2c, 2c)$, the probability distribution function $F_x(x)$ can be calculated as,

$$F_x(x) = \int_{-2c}^{x} f_x(x) \, dx \tag{3}$$

$$= \int_{-2c}^{x} \frac{1}{4c} dx \tag{4}$$

$$\tag{5}$$

$$= \frac{1}{4c} \int_{-2c}^{x} dx \tag{6}$$

$$= \frac{1}{4c} \left(x + 2c \right) \tag{7}$$

$$= \frac{x + 2c}{4c} \tag{8}$$

Since x is distributed from -2c to 2c, we can write

$$F_x \left(x \right) = \begin{cases} 0 & , x \leq -2c \\ \frac{x+2c}{4c} & , |x| < 2c \\ 1 & , x \geq 2c \end{cases}$$

Now given that random variable *y* is

$$y = g(x) = x^2 \tag{9}$$

When $y \geq 0$ and $y < 4c^2$ (*because* $x \in (-2c, 2c)$) we can write probability distribution function in *y* as,

$$\begin{align}
F_y(y) &= \Pr(Y \leq y) \tag{10} \\
&= \Pr(g(x) \leq y) \tag{11} \\
&= \Pr(x^2 \leq y) \tag{12} \\
&= \Pr(-\sqrt{y} \leq x \leq \sqrt{y}) \tag{13} \\
&= \Pr(x \leq \sqrt{y}) - \Pr(x \leq -\sqrt{y}) \tag{14} \\
&= F_x(\sqrt{y}) - F_x(-\sqrt{y}) \tag{15} \\
&= \frac{\sqrt{y} + 2c}{4c} - \frac{-\sqrt{y} + 2c}{4c} \tag{16} \\
&= \frac{\sqrt{y}}{2c} \tag{17}
\end{align}$$

Now for $y < 0$ there are no values of x such that $x^2 < y$. So,

$$F_y (y) = \Pr(\emptyset) = 0 \, (y < 0) \qquad (18)$$

Overall we can write that,

$$F_y (y) = \begin{cases} 0 & , y < 0 \\ \frac{\sqrt{y}}{2c} & , 0 \leq y \leq 4c^2 \\ 1 & , y > 4c^2 \end{cases}$$

The probability density function $f_y(y)$ for $0 \leq y \leq 4c^2$ can be calculated as
,

$$f_y(y) = \frac{dF_y(y)}{dy} \tag{19}$$

$$= \frac{d}{dy}\left(\frac{\sqrt{y}}{2c}\right) \tag{20}$$

$$= \frac{1}{4\sqrt{y}c} \tag{21}$$
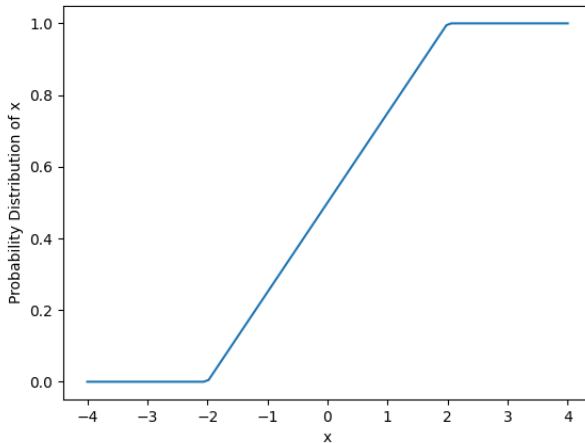
The plot of $F_x(x)$, $F_y(y)$, $f_y(y)$ for $c = 1$ is shown below,
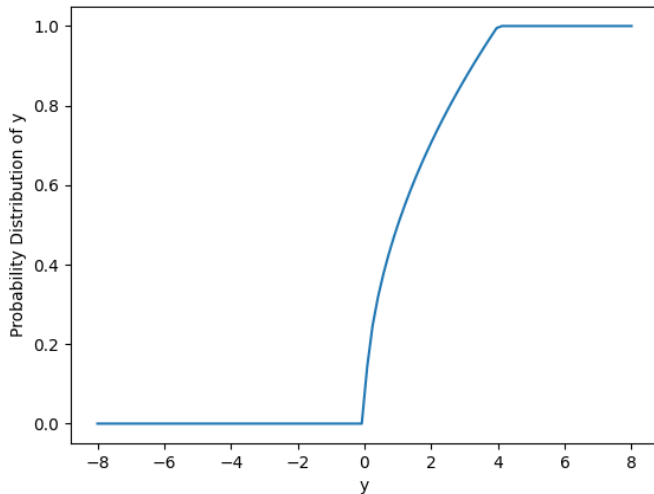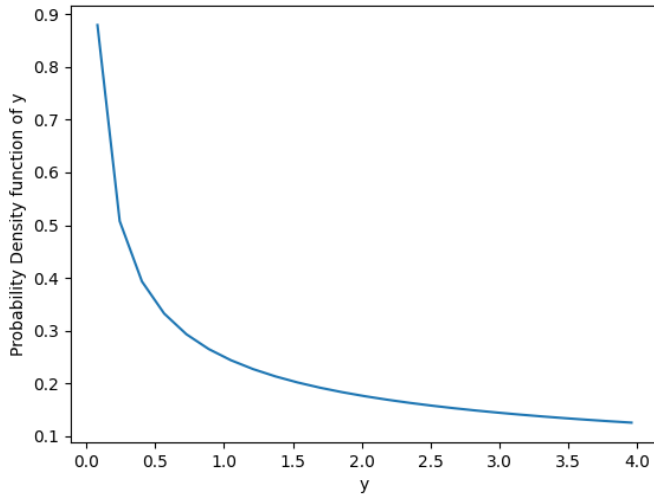
Solution:Plot

Figure 1

Figure 2

Figure 3