AI1110 : Probability and Random Variables

Assignment 9

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Outline

Question

Question exercise 10.27: Given an SSS process $\mathbf{x}(t)$ with zero mean, power spectrum $S(w)$, and bispectrum $S(u, v)$, we form the process $\mathbf{y}(t) = \mathbf{x}(t) + c$. Show that

$$S_{yyy}(u, v) = S(u, v) + 2\pi c\left[S(u)\delta(v) + S(v)\delta(u) + S(u)\delta(u+v)\right] + 4\pi^2 \tag{1}$$

Solution

First we will find auto correlation of y process,

$$R_{yyy}(u,v) = E\{\underline{\mathbf{x}}(t+u) + c\,[\underline{\mathbf{x}}(t+v) + c]\;[\underline{\mathbf{x}}(t) + c]\} \tag{2}$$

$$\implies = R(\mathbf{u},\mathbf{v}) + cR(\mathbf{u}) + cR(\mathbf{v}) + cR(\mathbf{u}-\mathbf{v}) + c^3 \tag{3}$$

We can write above expression since $E\{\mathbf{x}(t)\} = 0$ as $\mathbf{x}(t)$ is strict sense stationary process. And moreover , we know that

$$R(\mathbf{u}) \iff 2\pi S(u)\,\delta(v) \tag{4}$$

$$R(\mathbf{v}) \iff 2\pi\delta(v)\,S(u) \tag{5}$$

$$c^3 \iff 4\pi^3\delta(u)\,\delta(v) \tag{6}$$

where $\iff$ here represents the fourier transform.And also

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R(u-v)\,e^{-j(u\mathbf{u}+v\mathbf{v})}d\mathbf{u}d\mathbf{v} = \int_{-\infty}^{\infty} R(\tau)\,e^{-ju\tau}d\tau\int_{-\infty}^{\infty} e^{-j(u+v)\mathbf{v}}d\mathbf{v} \tag{7}$$

$$= 2\pi S(u)\,\delta(u+v) \tag{8}$$

Using (4),(5),(6),(8) we can get that,

$$S_{yyy}\left(u,v\right) = S\left(u,v\right) + 2\pi c\left[S\left(u\right)\delta\left(v\right) + S\left(v\right)\delta\left(u\right) + S\left(u\right)\delta\left(u+v\right)\right] + 4\pi^2 c^3 \delta\left(u\right)\delta\left(u\right) \tag{9}$$

Hence proved