AI1110 : Probability and Random Variables Assignment 6

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***Abstract*—This document provides the solution of Assignment 5 (Papoulis Pillai Chap 5 Ex 5.4)**

Question Exercise 5.4: The random variable X is uniform in the interval $(-2c, 2c)$. Find and sketch $f_Y(y)$ and $F_Y(y)$ if $y = g(x)$ and $g(x)$ is the function in Fig. 5-3.

Solution : Given that, random variable x is uniformly distributed in interval $(-2c, 2c)$. So we can write that, the probability density function

$$f_x(x) = \frac{1}{2c-(-2c)} \tag{1}$$

$$= \frac{1}{4c} \tag{2}$$

So for $x \in (-2c, 2c)$, the probability distribution function $F_x(x)$ can be calculated as,

$$F_x(x) = \int_{-2c}^{x} f_x(x)\,dx \tag{3}$$

$$= \int_{-2c}^{x} \frac{1}{4c}dx \tag{4}$$

$$= \frac{1}{4c}\int_{-2c}^{x} dx \tag{5}$$

$$= \frac{1}{4c}(x+2c) \tag{6}$$

$$= \frac{x+2c}{4c} \tag{7}$$

Since x is distributed from -2c to 2c, we can write

$$F_x(x) = \begin{cases} 0 & , x \leq -2c \\ \frac{x+2c}{4c} & , |x| < 2c \\ 1 & , x \geq 2c \end{cases}$$

Now given that random variable y is

$$y = g(x) = x^2 \tag{8}$$

If $y \geq 0$ and $y < 4c^2\,(because x \in (-2c, 2c))$ we can write probability distribution function in y as,

$$F_y(y) = \Pr(Y \leq y) \tag{9}$$

$$= \Pr(g(x) \leq y) \tag{10}$$

$$= \Pr\left(x^2 \leq y\right) \tag{11}$$

$$= \Pr(-\sqrt{y} \leq x \leq \sqrt{y}) \tag{12}$$

$$= \Pr(x \leq \sqrt{y}) - \Pr(x \leq -\sqrt{y}) \tag{13}$$

$$= F_x(\sqrt{y}) - F_x(-\sqrt{y}) \tag{14}$$

$$= \frac{\sqrt{y}+2c}{4c} - \frac{-\sqrt{y}+2c}{4c} \tag{15}$$

$$= \frac{\sqrt{y}}{2c} \tag{16}$$

Now for $y < 0$ there are no values of x such that $x^2 < y$. So,

$$F_y(y) = \Pr(\emptyset) = 0\,(y < 0) \tag{17}$$

Overall we can write that,

$$F_y(y) = \begin{cases} 0 & , y < 0 \\ \frac{\sqrt{y}}{2c} & , 0 \leq y \leq 4c^2 \\ 1 & , y > 4c^2 \end{cases}$$

The probability density function $f_y(y)$ for $0 \leq y \leq 4c^2$ can be calculated as ,

$$f_y(y) = \frac{dF_y(y)}{dy} \tag{18}$$

$$= \frac{d}{dy}\left(\frac{\sqrt{y}}{2c}\right) \tag{19}$$

$$= \frac{1}{4\sqrt{y}c} \tag{20}$$

The plot of $F_x(x)\,, F_y(y)\,, f_y(y)$ for $c = 1$ is shown below,

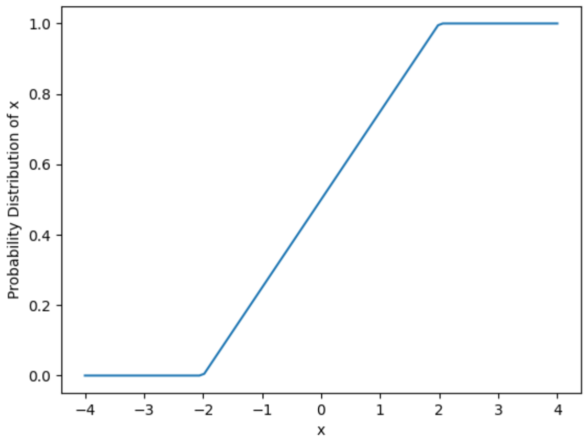

Fig. 1.

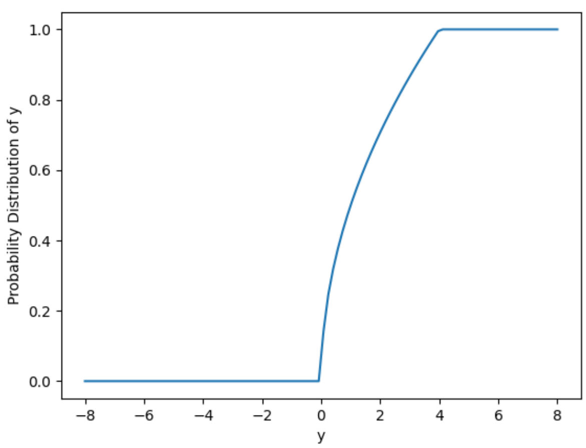

Fig. 2.

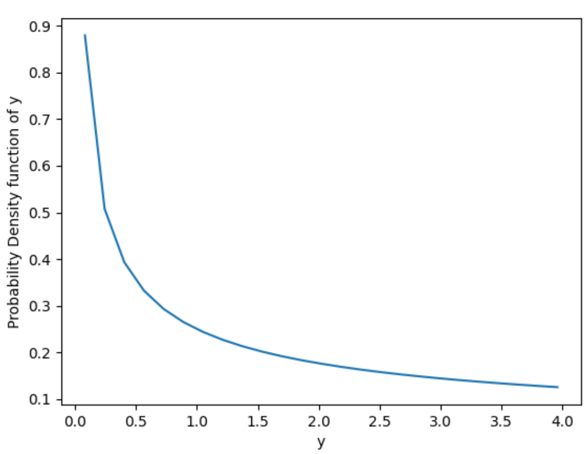

Fig. 3.