AI1110 : Probability and Random Variables

Assignment 4

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Outline

Question

In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these students is selected at random, find the probability that

(a) The student opted for NCC or NSS.

(b) The student has opted neither NCC nor NSS.

(c) The student has opted NSS but not NCC.

Solution : Classifying the data

The given data can be represented as,

Description	Count
opted for NCC	30
opted for NSS	32
opted for NCC and NSS	24
total students	60

Table 1

Solution : Defining the Random Variables

Let us define random variables X, Y, $whereX$, $Y \in (0, 1)$,such that

Random variable	Value	Description
X	0	**not opted for NCC**
X	1	**opted for NCC**
Y	0	**not opted for NSS**
Y	1	**opted for NSS**

Table 2

From Table 1 we can write,

$$\Pr(X=1) = \frac{30}{60} \tag{1}$$

$$= \frac{1}{2} \tag{2}$$

$$\Pr(Y=1) = \frac{32}{60} \tag{3}$$

$$= \frac{8}{15} \tag{4}$$

$$\Pr(X+Y=2) = \frac{24}{60} \tag{5}$$

$$= \frac{2}{5} \tag{6}$$

Solution : Part 1

The probability of the event "the student opted for NCC or NSS" can be described as $\Pr(X + Y > 0)$. In other words, either of random variables X, Y should be equal to 1.

$$\Pr(X + Y > 0) = \Pr(X = 1) + \Pr(Y = 1) - \Pr(X + Y = 2) \tag{7}$$

$$\implies = \frac{1}{2} + \frac{8}{15} - \frac{2}{5} \tag{8}$$

$$\implies = \frac{19}{30}. \tag{9}$$

$$\therefore \Pr(X + Y > 0) = \frac{19}{30}. \tag{10}$$

Solution : Part 2

The probability of the event ”the student has opted neither NCC nor NSS” can be described as ”$\Pr(X + Y = 0)$ ”.In other words,both the random variables X, Y should be 0. Now we can write,

$$\Pr(X + Y = 0) = 1 - \Pr(X + Y > 0) \tag{11}$$

Using (10)

$$\Pr(X + Y = 0) = 1 - \frac{19}{30} \tag{12}$$

$$= \frac{11}{30} \tag{13}$$

Solution : Part 3

The probability of the event "the student has opted NSS but not NCC" can be described as "$\Pr(Y = 1 and X = 0)$". We can write

$$\Pr(Y = 1 and X = 0) = \Pr(Y = 1) - \Pr(Y = 1 and X = 1) \tag{14}$$

$$= \Pr(Y = 1) - \Pr(X + Y = 2) \tag{15}$$

Using (4) and (6),

$$\Pr(Y = 1 and X = 0) = \frac{8}{15} - \frac{2}{5} \tag{16}$$

$$= \frac{2}{15} \tag{17}$$

$$\therefore \Pr(Y = 1 and X = 0) = \frac{2}{15}. \tag{18}$$