AI1110 : Probability and Random Variables Assignment 5

Mannem Charan(AI21BTECH11019)

***Abstract*—This document provides the solution of Assignment 5 (Papoulis Pillai Chap 2 Ex 2.5)**

Question Exercise 2.5: Prove and generalise the following identity

$$\Pr(A+B+C) = \Pr(A)+\Pr(B)+\Pr(C) - \Pr(AB)-\Pr(BC) - \Pr(CA)+\Pr(ABC) \tag{1}$$

Solution: We will use the following identity,

$$\Pr(A+B) = \Pr(A)+\Pr(B)-\Pr(AB) \tag{2}$$

Now from (2)

$$\Pr(A+(B+C)) = \Pr(A)+\Pr(B+C) - \Pr(A(B+C)) \tag{3}$$

$$\Pr(B+C) = \Pr(B)+\Pr(C)-\Pr(BC) \tag{4}$$

$$\begin{aligned} \Pr(A(B+C)) &= \Pr(AB+AC) &(5)\\ &= \Pr(AB)+\Pr(AC) - \Pr(ABAC) &(6)\\ &= \Pr(AB)+\Pr(AC) - \Pr(ABC) &(7) \end{aligned}$$

Substitute (4) and (7) in (3), we get

$$\Pr(A+B+C) = \Pr(A)+\Pr(B)+\Pr(C) - \Pr(AB)-\Pr(BC)-\Pr(CA)+\Pr(ABC) \tag{8}$$

Now using induction, we can show similarly that,

$$\Pr(A1+...+An) = \Pr(A1)+.....+\Pr(An) - \Pr(A1A2)-...-\Pr(An-1An) +...(-1)^{n-1}\Pr(A1A2....An) \tag{9}$$

Derivation : We will derive (2) using Boolean Algebra.For any two events A,B

$$\begin{aligned} A.1 &= A(B+B') &(10)\\ &= AB+AB' &(11)\\ \Pr(A) &= \Pr(AB+AB') &(12)\\ &= \Pr(AB)+\Pr(AB') &(13) \end{aligned}$$

Since AB and AB' are mutually disjoint sets. Now,

$$\begin{aligned} A+B &= A(B+B')+B(A+A') &(14)\\ &= (AB+BA)+BA'+B'A &(15)\\ &= AB+A'B+B'A &(16)\\ &= A+A'B &(17) \end{aligned}$$

Now,

$$\Pr(A+B) = \Pr(A+A'B) \tag{18}$$

Since both events are mutually disjoint.

$$\begin{aligned} \Pr(A+B) &= \Pr(A)+\Pr(A'B) &(19)\\ &= \Pr(A)+\Pr(B)-\Pr(AB) &(20) \end{aligned}$$

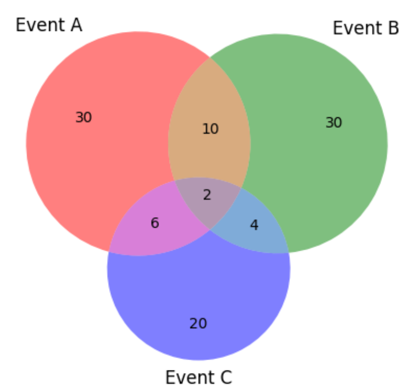

Fig. 1. Figure generated by python

The python code `./codes/verify.py` verifies the (8) using Fig 1.