Fourier Series

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***Abstract*—This manual provides a simple introduction to Fourier Series**

1 Periodic Function

Let

$$x(t) = A_0 \left|\sin\left(2\pi f_0 t\right)\right| \tag{1.1}$$

1.1 Plot $x(t)$.

1.2 Show that $x(t)$ is periodic and find its period.

2 Fourier Series

Consider $A_0 = 12$ and $f_0 = 50$ for all numerical calculations.

2.1 If

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{\mathrm{J}2\pi k f_0 t} \tag{2.1}$$

show that

$$c_k = f_0 \int_{-\frac{1}{2f_0}}^{\frac{1}{2f_0}} x(t) e^{-\mathrm{J}2\pi k f_0 t}\, dt \tag{2.2}$$

2.2 Find c_k for (1.1)

2.3 Verify (1.1) using python.

2.4 Show that

$$x(t) = \sum_{k=0}^{\infty} \left(a_k \cos \mathrm{J}2\pi k f_0 t + b_k \sin \mathrm{J}2\pi k f_0 t\right) \tag{2.3}$$

and obtain the formulae for a_k and b_k.

2.5 Find a_k and b_k for (1.1)

2.6 Verify (2.3) using python.

3 Fourier Transform

3.1

$$\delta(t) = 0, \quad t \neq 0 \tag{3.1}$$

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1 \tag{3.2}$$

3.2 The Fourier Transform of $g(t)$ is

$$G(f) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi f t}\, dt \tag{3.3}$$

3.3 Show that

$$g(t - t_0) \overset{\mathcal{F}}{\longleftrightarrow} G(f) e^{-j2\pi f t_0} \tag{3.4}$$

$$\tag{3.5}$$

3.4 Show that

$$G(t) \overset{\mathcal{F}}{\longleftrightarrow} g(-f) \tag{3.6}$$

3.5 $\delta(t) \overset{\mathcal{F}}{\longleftrightarrow} ?$

3.6 $e^{-j2\pi f_0 t} \overset{\mathcal{F}}{\longleftrightarrow} ?$

3.7 $\cos(2\pi f_0 t) \overset{\mathcal{F}}{\longleftrightarrow} ?$

3.8 Find the Fourier Transform of $x(t)$ and plot it. Verify using python.

3.9 Show that

$$\mathrm{rect}\,(t) \overset{\mathcal{F}}{\longleftrightarrow} \mathrm{sinc}\,(t) \tag{3.7}$$

Verify using python.

3.10 $\mathrm{sinc}\,(t) \overset{\mathcal{F}}{\longleftrightarrow} ?$. Verify using python.

4 Filter

4.1 Find $H(f)$ which transforms $x(t)$ to DC 5V.

4.2 Find $h(t)$.

4.3 Verify your result using through convolution.

5 Filter Design

5.1 Design a Butterworth filter for $H(f)$.

5.2 Design a Chebyschev filter for $H(f)$.

5.3 Design a circuit for your Butterworth filter.

5.4 Design a circuit for your Chebyschev filter.