

# Circuits and Transforms

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**Abstract**—This manual provides a simple introduction to Transforms

## 1 DEFINITIONS

1. The unit step function is

$$u(t) = \begin{cases} 1 & t > 0 \\ \frac{1}{2} & t = 0 \\ 0 & t < 0 \end{cases} \quad (1.1)$$

2. The Laplace transform of  $g(t)$  is defined as

$$G(s) = \int_{-\infty}^{\infty} g(t)e^{-st} dt \quad (1.2)$$

## 2 LAPLACE TRANSFORM

1. In the circuit, the switch S is connected to position P for a long time so that the charge on the capacitor becomes  $q_1 \mu C$ . Then S is switched to position Q. After a long time, the charge on the capacitor is  $q_2 \mu C$ .

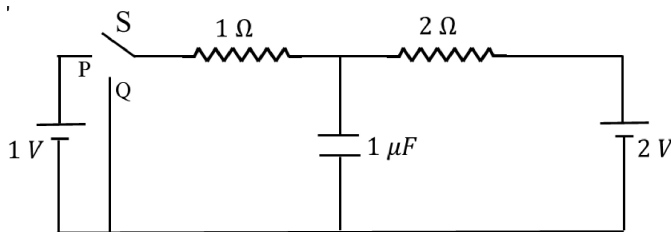


Fig. 2.1

2. Draw the circuit using latex-tikz. **Solution:**

3. Find  $q_1$ .

**Solution:** Since switch

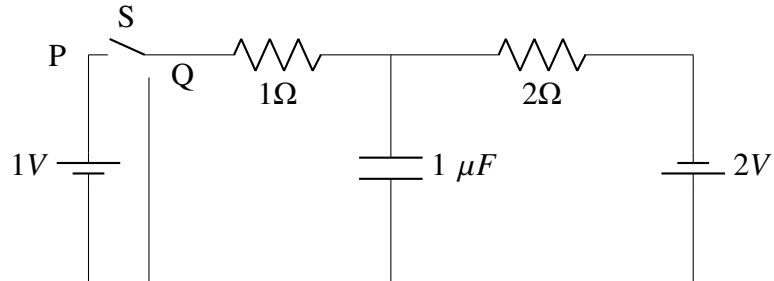


Fig. 2.2: Circuit diagram of the question

4. Show that the Laplace transform of  $u(t)$  is  $\frac{1}{s}$  and find the ROC.
5. Show that

$$e^{-at}u(t) \xleftrightarrow{\mathcal{H}} L \frac{1}{s+a}, \quad a > 0 \quad (2.1)$$

and find the ROC.

6. Now consider the following resistive circuit transformed from Fig. 2.1 where

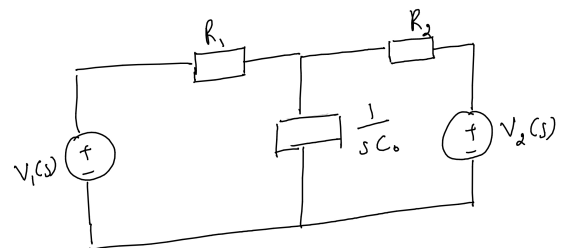


Fig. 2.3

$$u(t) \xleftrightarrow{\mathcal{H}} LV_1(s) \quad (2.2)$$

$$2u(t) \xleftrightarrow{\mathcal{H}} LV_2(s) \quad (2.3)$$

Find the voltage across the capacitor  $V_{C_0}(s)$ .

7. Find  $v_{C_0}(t)$ . Plot using python.
8. Verify your result using ngspice.
9. Obtain Fig. 2.3 using the equivalent differential equation.

## 3 INITIAL CONDITIONS

1. Find  $q_2$  in Fig. 2.1.

2. Draw the equivalent  $s$ -domain resistive circuit when  $S$  is switched to position  $Q$ . Use variables  $R_1, R_2, C_0$  for the passive elements. Use latex-tikz.
3.  $V_{C_0}(s) = ?$
4.  $v_{C_0}(t) = ?$  Plot using python.
5. Verify your result using ngspice.
6. Find  $v_{C_0}(0-), v_{C_0}(0+)$  and  $v_{C_0}(\infty)$ .
7. Obtain the Fig. in problem 3.2 using the equivalent differential equation.

#### 4 BILINEAR TRANSFORM

1. In Fig. 2.1, consider the case when  $S$  is switched to  $Q$  right in the beginning. Formulate the differential equation.
2. Find  $H(s)$  considering the output voltage at the capacitor.
3. Plot  $H(s)$ . What kind of filter is it?
4. Using trapezoidal rule for integration, formulate the difference equation by considering

$$y(n) = y(t)|_{t=n} \quad (4.1)$$

5. Find  $H(z)$ .
6. How can you obtain  $H(z)$  from  $H(s)$ ?