Exercise #1 - Merge

2. The array is recursively split into two halves until subarrays of size 1 are left. Since each time we perform the split, we divide the array size by 2. Hence, the depth of the recursion tree is log n. The merging process requires comparing elements from both subarrays, which can take up to n comparisons in the worst case. Since there are log n levels (depth of the recursion) and at each level we perform O(n) work (to merge all elements at that level), the total work done is O(n log n).

3.
Initial array: [8, 42, 25, 3, 3, 2, 27, 3]

Split into halves:

Left half: [8, 42, 25, 3]
Right half: [3, 2, 27, 3]

Continue splitting:

Left half: [8, 42]
Right half: [25, 3]

Right half further split:

Left half: [25]
Right half: [3]

Merge the rightmost halves:

Merged: [3, 25]

The array after the first merge: [8, 42, 3, 25, 3, 2, 27, 3]

Continue splitting the right part:

Left half: [3, 2]
Right half: [27, 3]

Continue splitting:

Left half: [3]
Right half: [2]
Merged: [2, 3]

Continue splitting:

Left half: [27]
Right half: [3]

Merge the rightmost halves:

Merged: [3, 27]

The array after the second merge: [8, 42, 3, 25, 2, 3, 3, 27]

Merge the remaining halves:

Merged: [2, 3, 3, 25, 3, 8, 27, 42]

The final sorted array is: [2, 3, 3, 3, 8, 25, 27, 42]

In each step, we split the array into halves, recursively sort each half, and then merge them back together in sorted order. This process continues until the entire array is sorted. The time complexity of merge sort is O(n log n), where n is the number of elements in the array.

4. The observed number of steps in the manual application of the merge sort algorithm aligns with the expected time complexity analysis of O(n log n). The algorithm recursively divides the array into halves, with each level of recursion involving a linear-time merging process. The height of the recursion tree is log(n), reflecting the logarithmic nature of the splits. Consequently, the overall time complexity remains O(n log n), as demonstrated by the consistent pattern of steps in the manual sorting process.