

CSCI 551

Numerical and Parallel

Programming

*Lecture 5-2 – Introduction to MPI and
Exercise #3 Help*



MPI Simple Demonstrations

■ MPI Hello world

- mpicc -g -Wall -o greetings greetings.c
- mpirun -n 4 ./greetings

■ Use starter Makefile in CSCI 551 code directories!

■ MPI book examples

- Always works mpiexec
- Using mpirun on multiple nodes can hang (program error)
OR due to cluster node issue

```
sbsiewert@o244-01:~/code/hello_custer$ ls
cl_machine_file c2_machine_file greetings greetings.c Makefile
sbsiewert@o244-01:~/code/hello_custer$ mpirun -n 4 ./greetings
Hello from process 0 of 4 on o244-01
Hello from process 1 of 4 on o244-01
Hello from process 2 of 4 on o244-01
Hello from process 3 of 4 on o244-01

sbsiewert@o244-01:~/code/hello_custer$ █
4.ecc-linux (sbsiewert) ×
sbsiewert@o251-01:~/public_html/csci551/code/MPI_Examples$ make
gcc -g -Wall -I/opt/intel/compilers_and_libraries_2020.0.166/linux/mpi/intel64/include/
debug -L/opt/intel/compilers_and_libraries_2020.0.166/linux/mpi/intel64/lib
gcc -g -Wall -I/opt/intel/compilers_and_libraries_2020.0.166/linux/mpi/intel64/include/
lib/debug -L/opt/intel/compilers_and_libraries_2020.0.166/linux/mpi/intel64/lib
gcc -g -Wall -I/opt/intel/compilers_and_libraries_2020.0.166/linux/mpi/intel64/include/
l64/lib/debug -L/opt/intel/compilers_and_libraries_2020.0.166/linux/mpi/intel64/lib
gcc -g -Wall -I/opt/intel/compilers_and_libraries_2020.0.166/linux/mpi/intel64/include/
g -L/opt/intel/compilers_and_libraries_2020.0.166/linux/mpi/intel64/lib
mpicc -g -Wall -O2 -I/opt/intel/compilers_and_libraries_2020.0.166/linux/mpi/intel64/include/
nux/mpi/intel64/lib/debug -L/opt/intel/compilers_and_libraries_2020.0.166/linux/mpi/intel64
mpicc -g -Wall -O2 -I/opt/intel/compilers_and_libraries_2020.0.166/linux/mpi/intel64/include/
intel64/lib/debug -L/opt/intel/compilers_and_libraries_2020.0.166/linux/mpi/intel64/lib
mpicc -g -Wall -O2 -I/opt/intel/compilers_and_libraries_2020.0.166/linux/mpi/intel64/include/
0.166/linux/mpi/intel64/lib/debug -L/opt/intel/compilers_and_libraries_2020.0.166/linux/mpi/
mpicc -g -Wall -O2 -I/opt/intel/compilers_and_libraries_2020.0.166/linux/mpi/intel64/include/
0.166/linux/mpi/intel64/lib/debug -L/opt/intel/compilers_and_libraries_2020.0.166/linux/mpi/intel64
mpicc -g -Wall -O2 -I/opt/intel/compilers_and_libraries_2020.0.166/linux/mpi/intel64/include/
0.166/linux/mpi/intel64/lib/debug -L/opt/intel/compilers_and_libraries_2020.0.166/linux/mpi/intel64
mpicc -g -Wall -O2 -I/opt/intel/compilers_and_libraries_2020.0.166/linux/mpi/intel64/include/
intel64/lib/debug -L/opt/intel/compilers_and_libraries_2020.0.166/linux/mpi/intel64/lib
mpicc -g -Wall -O2 -I/opt/intel/compilers_and_libraries_2020.0.166/linux/mpi/intel64/include/
intel64/lib/debug -L/opt/intel/compilers_and_libraries_2020.0.166/linux/mpi/intel64/lib -lm
mpicc -g -Wall -O2 -I/opt/intel/compilers_and_libraries_2020.0.166/linux/mpi/intel64/include/
intel64/lib/debug -L/opt/intel/compilers_and_libraries_2020.0.166/linux/mpi/intel64/lib
mpicc -g -Wall -O2 -I/opt/intel/compilers_and_libraries_2020.0.166/linux/mpi/intel64/include/
intel64/lib/debug -L/opt/intel/compilers_and_libraries_2020.0.166/linux/mpi/intel64/lib -lm
mpicc -g -Wall -O2 -I/opt/intel/compilers_and_libraries_2020.0.166/linux/mpi/intel64/include/
intel64/lib/debug -L/opt/intel/compilers_and_libraries_2020.0.166/linux/mpi/intel64/lib -lm
gcc -g -Wall -I/opt/intel/compilers_and_libraries_2020.0.166/linux/mpi/intel64/include/
b/debug -L/opt/intel/compilers_and_libraries_2020.0.166/linux/mpi/intel64/lib -lm
sbsiewert@o251-01:~/public_html/csci551/code/MPI_Examples$ ls
bubble ex4vel.h mpi_hello.c mpi_mat_vect_mult mpi_mat_vect_time.c mpi_outpu
bubble.c Makefile mpi_many_msgs mpi_mat_vect_mult.c mpi_odd_even mpi_outpu
ex4accel.h mpi_hello mpi_many_msgs.c mpi_mat_vect_time mpi_odd_even.c mpi_trap
sbsiewert@o251-01:~/public_html/csci551/code/MPI_Examples$ mpiexec ./mpi_hello
Greetings from process 0 of 4!
Greetings from process 1 of 4!
Greetings from process 2 of 4!
Greetings from process 3 of 4!
sbsiewert@o251-01:~/public_html/csci551/code/MPI_Examples$ █
```

Exercise #3

- **Read chapter 3 through p. 113 – Intro to MPI**
- **Pacheco Code - [csci551/code/MPI_Examples/](#)**
 - Integrate any function – code walkthrough of math function and interpolation methods
 - [csci551/code/MPI_Examples/mpi_trap3.c](#)
 - [csci551/code/MPI_Examples/mpi_trap4.c](#)
-  **Start here!**
- Review [LLNL tutorials](#) (for more on Pthreads as you are interested)
- Review Intel [MPI benchmark user's guide](#) ([csci551/code/mpi-benchmarks-master-csu/](#)) – **Broken now ... ignore**
- Simple MPI program – based on book to explore MPI and cluster features
- Top Down - MPI benchmarks and examples
 - Build, run, observe
 - Review code and explain
- Bottom Up – Methods of integration
 - Simple forward integration, large, medium, small step size
 - Trapezoidal integration, large, medium, small step size
- Application – Train Acceleration and Braking
 - Create a profile of position and velocity over time
 - Given acceleration timeline
 - Compute arrival time
 - Use MPI to speed up the analysis (6 runs, “n” segments to integrate)

Notes on Simulation

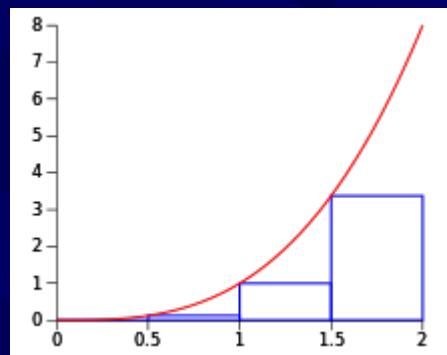
- Functions with known anti-derivative simpler (Ex #3)
 - We know acceleration, velocity, and even position functions
 - Integration here can be verified with symbolic math
- Function Look-up Table with Interpolation (Ex #4)
 - Review of interpolation - models any function
 - Harder loop-carried dependency - [csci551/code/functiongen/](https://csci551.com/functiongen/)
- Discussion of Simulation and Parallel Divide in Conquer
 - OpenMP version with speed-up
 - Phases of integration
- Checking correctness for integration
 - Ex-3 model - [Ex-3-Smooth-Function-Train-profile.xlsx](https://csci551.com/Ex-3-Smooth-Function-Train-profile.xlsx)
 - Ex-4 model - [Ex-4-Segmented-Function-Train-profile.xlsx](https://csci551.com/Ex-4-Segmented-Function-Train-profile.xlsx)

Exercise #3 & #4 – MPI for Simulation

- Continuous physics of 1-DOF translation
 - Position = $P_0 + \text{Velocity} \times \text{time}$, if $V(t)$ is constant
 - Velocity = $V_0 + \text{Acceleration} \times \text{time}$, if $A(t)$ is constant
 - Acceleration can be constant, or could be a function of time
- If Acceleration is $f(t)$, we need to integrate
 - Simple Riemann Sum Integration
 - $\text{Position}(t) = P_0 + \sum V(t)dt$
 - $\text{Velocity}(t) = V_0 + \sum A(t)dt$
 - $\text{Acceleration}(t) = f(t)$, provided as acceleration profile (or from sensor)
- Approximate the area under curve $A(t)$ for a dt (step size)
- Break $A(t)$ into segments or make it a look-up-table
- Interpolate for $A(t)$ at times between profile given
- Consider accuracy (method of integration & step size) and precision (double or float)

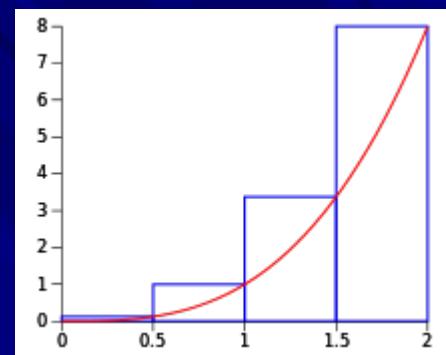
Recall Integration Methods (Riemann Sum)

Left Riemann Sum (Forward)
 $A_n = f(a) dx$
 $dx = [b-a]/n$



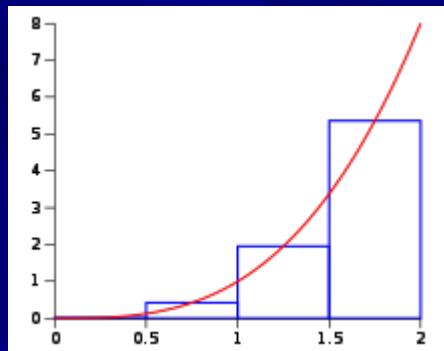
$$A_{b-a} = dx [(f(a) + f(a+dx) + f(a+2dx) + \dots + f(b-dx))]$$

Right Riemann Sum (Backward)
 $A_n = f(a+dx) dx$
 $dx = [b-a]/n$



$$A_{b-a} = dx [f(a+dx) + f(a+2dx) + \dots + f(b)]$$

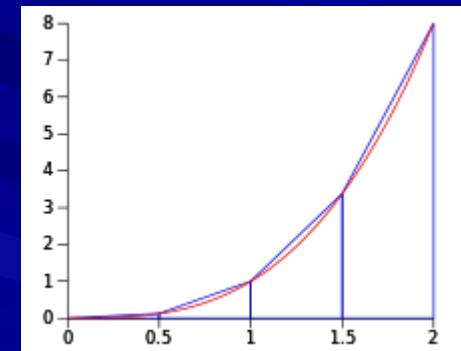
Midpoint Rule
 $A_n = f(a+dx/2) dx$
 $dx = [b-a]/n$



$$A_{b-a} = dx [f(a+dx/2) + f(a+3dx/2) + \dots + f(b-dx/2)]$$

Trapezoidal Rule

$$A_{\text{trap}} = \frac{1}{2} h (f(a)+f(b))$$
$$dx = [b-a]/n$$
$$A_n = \frac{1}{2} dx (f(a)+f(a+dx))$$



$$\frac{1}{2} dx [(f(a) + f(a+dx)) + (f(a+dx) + f(a+2dx)) \dots]$$
$$\frac{1}{2} dx [f(a) + 2f(a+dx) + 2f(a+2dx) + \dots + f(b)]$$

Numerical integration works for any function – solved by symbolic calculus or not
Can also work on well-defined functions that have known symbolic solutions

Verifying Numerical Integration

- Test with definite integral from Calculus – exact solution
- E.g., area under $\sin(x)$ over well-defined interval (2.0)

Known anti-derivative of sine and evaluation

- Definite integral solution
- We know from calculus
- Can be proven

Use this fact to verify your numerical integration

- Riemann Sum (function)
- Trapezoidal (function)
- Simpson's Rule (function)
- Runge-Kutta (differential equations)
- Other methods of quadrature (math topic)
- Estimation of area under a curve (function)
- Integration in general of a differential equation
- In applications, the functions generally represent physical quantities, the derivatives represent their rates of change, and the differential equation defines a relationship between the two.

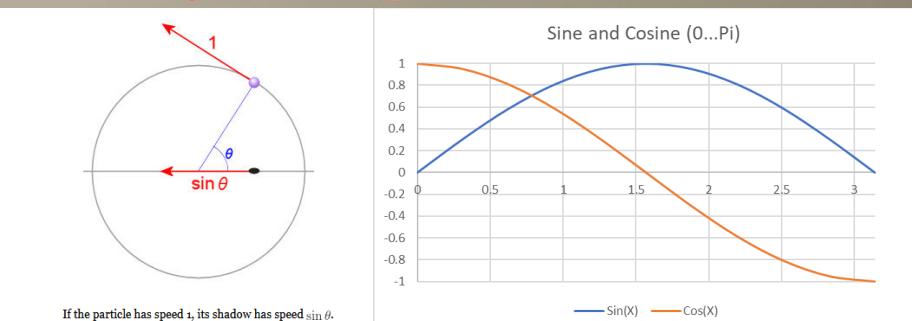
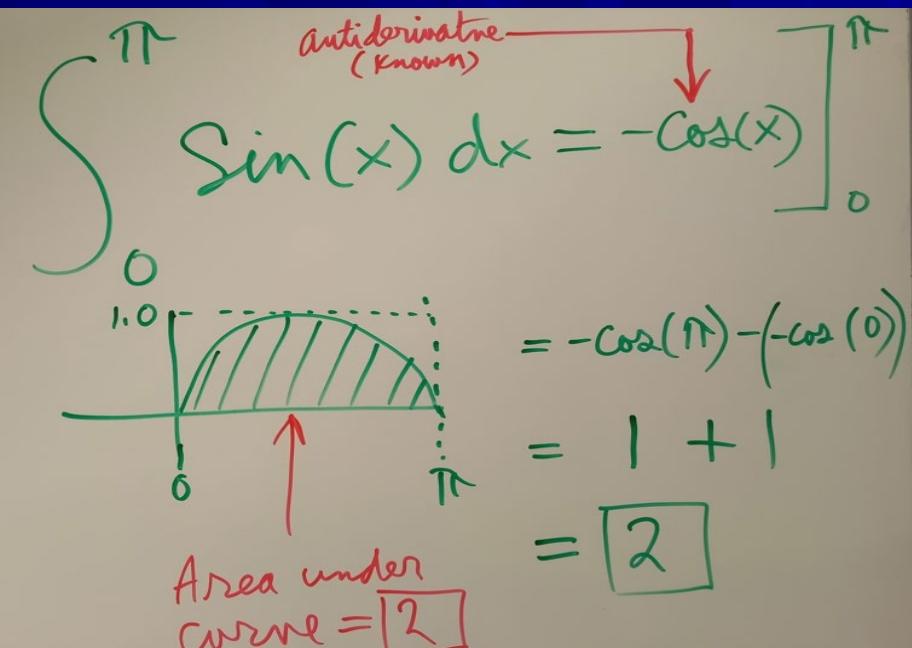
Unit Circle integration of velocity of shadow

- Assume light source on vertical axis
- Velocity of the shadow is $\sin(\theta)$
- Position is the Integral of velocity
- Distance is 2.0 for unit circle

Sine and Cosine Unit Circle Calculus Videos (diameter=2.0):

[Unit Circle – Derivative](#), [Integral of Sine\(x\) Video](#)

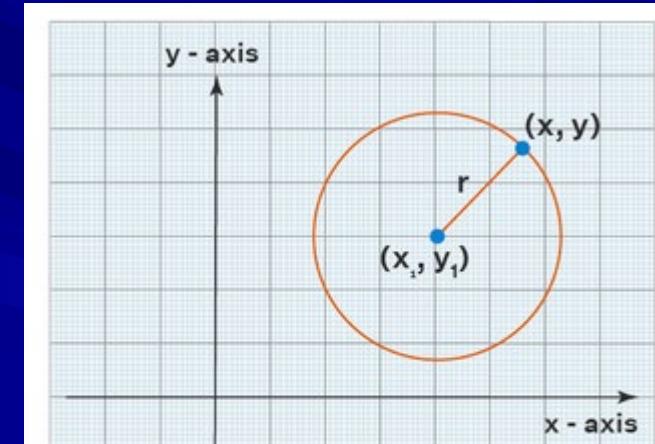
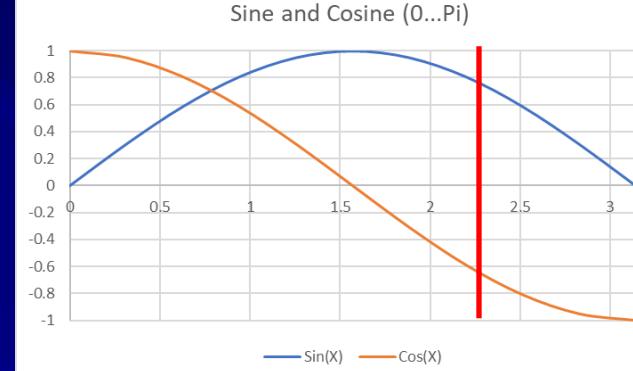
<https://betterexplained.com/articles/integral-sinx/>



Integration Tips – Stick to Functions

- Keep it Simple! If possible.
- Remember, we are just estimating the area under a curve (function) for most problems
- Recall the definition of a function
 - A vertical line intersects only ONE value of a function $y=f(x)$
 - Therefore, a circle is not a function, but it is a shape that can be described by an equation
- For the train problems, stick to integration of simple functions if possible
- For Ex #3, a function to model Acceleration(t) will have to be piecewise
- Initial conditions for divide and conquer are a challenge

Function – one vertical intersection

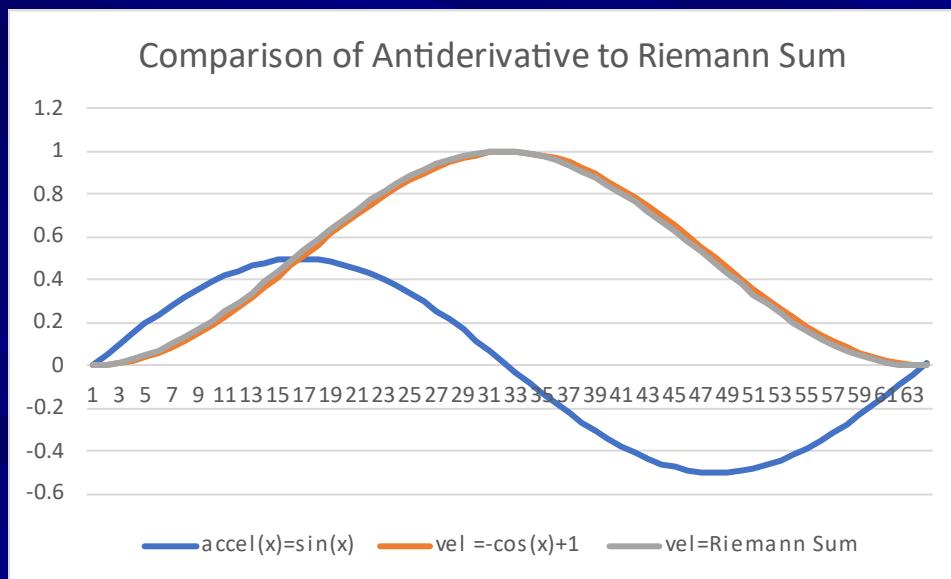


The standard equation of a circle gives precise information about the center of the circle and its radius and therefore, it is much easier to read the center and the radius of the circle at a glance. The standard equation of a circle with center at (x_1, y_1) and radius r is $(x - x_1)^2 + (y - y_1)^2 = r^2$, where (x, y) is an arbitrary point on the circumference of the circle.

<https://www.cuemath.com/geometry/equation-of-circle/>

Ex #3 Help

- Train Problem is Non-Linear, but Simple $\sin(x)$ acceleration function (known from calculus)
- Velocity has a definite integral solution of $-\cos(x)+C$
- Table Look-Up or interpolation (not required)
- Re-Try MPI implementation for $n=1 \dots m$ processes
- Focus on Accuracy and Precision rather than speed-up



Antiderivative is Exact Solution ([Spreadsheet](#))

Riemann or Trapezoidal is Numerical Approximation

Train acceleration profile is simple sine curve, but scaled for time and amplitude

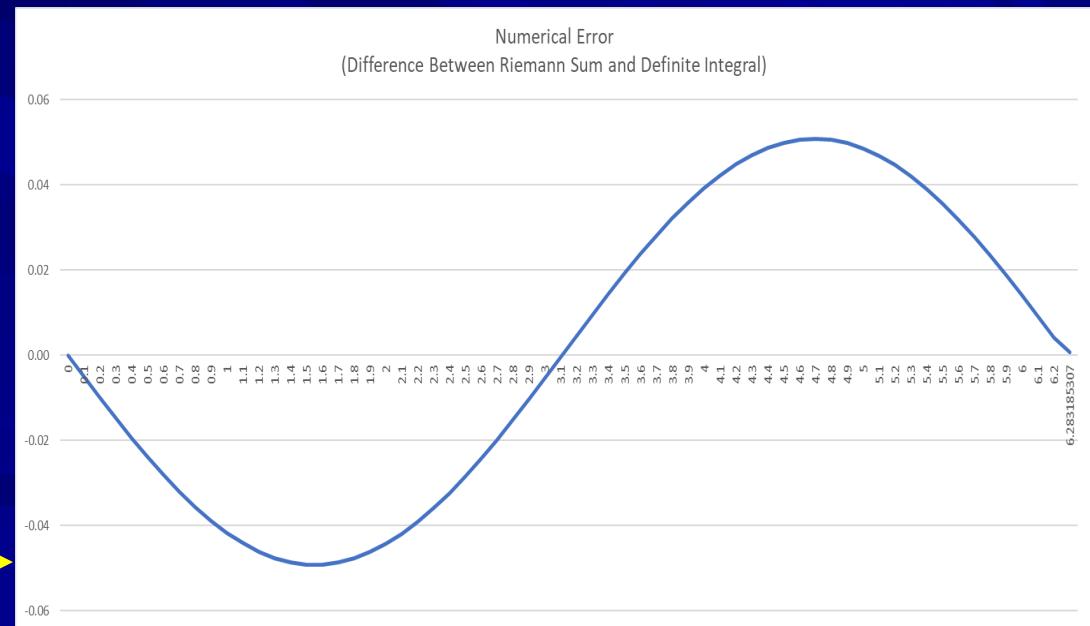
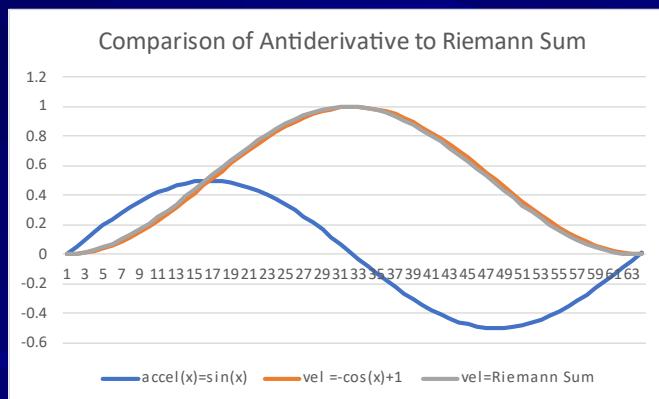
Non-Linear Functions

- Polynomials of degree 2 or higher
- Transcendental functions (trigonometric)
- Others (e.g., logarithmic), but these 2 are our primary interest
- Non-linear can be treated as “piece-wise linear”, but often can be directly implemented and solved with mathematics – e.g., antiderivative of $\sin(x)$

- Linear systems include linear functions as well as linear systems of equations
- Non-linear functions and non-linear systems of equations (interesting, but we will keep it simple)
- Ex #3 and Ex #6 are the main practice we have

Most of the world is non-linear

- The world is mostly non-linear, engineers and scientists like to simplify with linear models (**do you agree?**)
- Potential accuracy issues – e.g., linear interpolation with non-linear functions (piecewise linear, small steps)
- In Ex #4, use direct function generator rather than linear interpolation



Complex Functions

- Most measured functions for real-world applications are hard to model with a simple function and math library
 - Measured acceleration with accelerometer
 - Output is Acceleration(t) as data values in a table
 - Create look-up table and interpolate to find values between measurements

- Look-up tables preclude use of symbolic calculus integrals
 - Look-up combined with numerical integration
 - Allows for integration of any function
 - Recall a function is simply one-to-one and onto
 - Means that a vertical line crosses the function only once!