Softmax(xi+c) = $e^{xi+c}/\Sigma_j e^{xi+c} = e^{xi} e^c/\Sigma_j e^{xi} e^c = e^{xi} e^c/e^c\Sigma_j e^{xi} = e^{xi}/\Sigma_j e^{xi}$ =Softmax(xi)

So softmax is invariant to translation.

When c=0, the value range is 0 to infinity, when $c=-\max xi$, the value range is (0,1].

So we always set c=-max xi to simplify calculation and avoid overflow.

- The range of each element is (0, 1], the sum over all elements is 1.
- And turns it into a probability distribution between 0 and 1.
- The first step is to get the exponential result of xi, so they can have the same base.

The second step is to get the sum of all si.

The third step is to compute the probability of each si so that they can be distributed corresponding to their magnitude between 0 and 1.

If the multi-layer neural network has no activation function, the output of each neuron would be:

$$y_i = b + \sum wx_i$$

This is a linear function and the final output of the whole network would also be computed by all these linear functions.

Thus, this is equivalent to linear regression.

Gradient of
$$\sigma(x) = e^{-x}(1+e^{-x})^{-2} = e^{-x}/(1+e^{-x}) * 1/(1+e^{-x})$$

= $(1-1/(1+e^{-x})) * 1/(1+e^{-x}) = (1-\sigma(x)) * \sigma(x)$

$$y = W x + b$$

So
$$\partial y/\partial W = x$$
, $\partial y/\partial x = W$, $\partial y/\partial b = 1$

Then
$$\partial J/\partial W = \Sigma \partial J/\partial y * \partial y/\partial W = x \delta^T \in R^{dxk}$$

$$\partial J/\partial x = \Sigma \partial J/\partial y * \partial y/\partial x = W\delta \in R^{dx1}$$

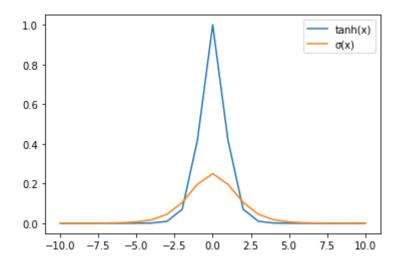
$$\partial J/\partial b = \Sigma \partial J/\partial y * \partial y/\partial b = \delta \in R^{kx1}$$

1. The final dy/dx = $\sigma'(h(x))h'(x)$

And $\sigma'(x) = (1-\sigma(x)) * \sigma(x)$, the final gradient will be close to zero if we multiply $\sigma'(x)$ which is between 0 and 1 for many times.

This phenomenon is called vanishing gradients.

- 2. The output range of tanh(x) is (-1,1), output range of sigmoid(x) is (0,1). So tanh(x) has symmetric output and this can make the average value around zero, which will increase the speed to converge.
- 3. As we can see, $\tanh'(x) = 1 \tanh(x)^2$ and $\sigma'(x) = (1 \sigma(x)) * \sigma(x)$ For the gradient value around 0, $\tanh'(x)$ is larger which means it will decrease more slowly. So $\tanh(x)$ has less of a vanishing gradient problem.



4.
$$\sigma(x) = 1/(1+e^{-x})$$

$$2\sigma(2x) = 2/(1+e^{-2x})$$

$$tanh(x) = 2\sigma(2x)-1$$

Q2.1.1

If we initialize the network with all zeros, there would be no difference for output between different input. Thus, the output would be the same after forward propagation and the errors would be the same in backpropagation. This means the same output would produce the same gradient and the corresponding weights would be updated in the same way. However, we want to update weights differently.

```
def initialize_weights(in_size,out_size,params,name=''):
    W, b = None, None
    upper = (6/(in_size+out_size))**0.5
    lower = -upper
    W = np.random.uniform(lower, upper, (in_size, out_size))
    b = np.zeros(out_size)

params['W' + name] = W
    params['b' + name] = b
```

Initializing with random numbers can improve the variance of parameters and increase the process to find optimal parameters with less loss value. Thus it can make convergence faster.

By scaling the initialization depending on layer size, the variance of weights gradient will become the same through every layer. And this can make gradient change within a limited range during training.

```
def sigmoid(x):
```

```
def softmax(x):
    res = None

    c = -np.max(x, axis=1)
    c.reshape(-1, 1)
    tmp = np.exp(x+c)
    res = tmp / np.sum(tmp, axis=1).reshape(-1, 1)

return res
```

```
def compute_loss_and_acc(y, probs):
    loss, acc = None, None

loss = -np.sum(y * np.log(probs))
    y_index = np.argmax(y, axis=1)  # get the class as label=1
    probs_index = np.argmax(probs, axis=1)  # get the most

possible class
    acc = np.sum(np.equal(y_index, probs_index)) / y.shape[0]

return loss, acc
```

```
def get_random_batches(x, y, batch_size):
  batches = []
  indexes = range(x.shape[0])
  while len(indexes) > 0:
    rand_i = np.random.choice(len(indexes), batch_size)
    select = [indexes[i] for i in rand_i]
    batch_x = [x[i] for i in select]
    batch_y = [y[i] for i in select]
    batches.append((np.array(batch_x), np.array(batch_y)))
    indexes = list(set(indexes) - set(select))
  return batches
```

```
learning rate = 1e-3
```

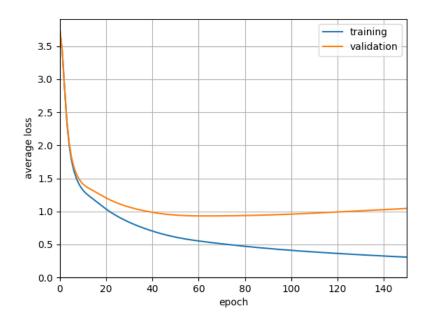
```
h1 = forward(x,params,'layer1')
probs = forward(h1,params,'output',softmax)
loss, acc = compute_loss_and_acc(y, probs)
delta1 = probs - y
delta2 = backwards(delta1,params,'output',linear deriv)
backwards(delta2,params,'layer1',sigmoid deriv)
      for i in range(v.shape[0]):
          for j in range(v.shape[1]):
```

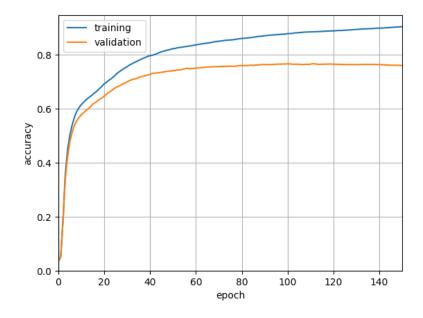
```
for i in range(v.shape[0]):
          loss2, acc2 = compute loss and acc(y, probs)
params orig[k])/np.maximum(np.abs(params[k]),np.abs(params orig[k]))
```

```
max_iters = 150
# pick a batch size, learning rate
batch_size = 15
learning_rate = 0.003
hidden_size = 64
```

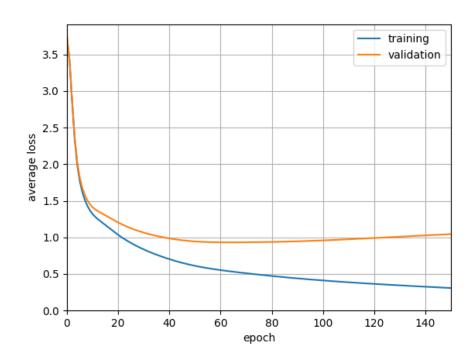
Validation accuracy: 0.76

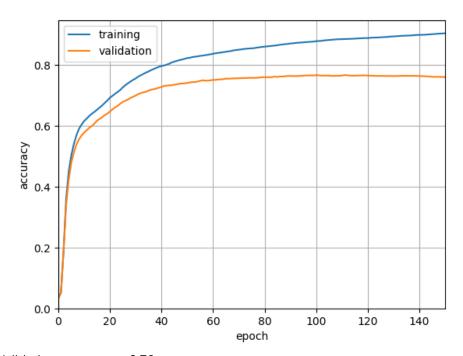
Test accuracy: 0.77722222222223





Q3.2 Learning rate with tuned learning rate=0.003:

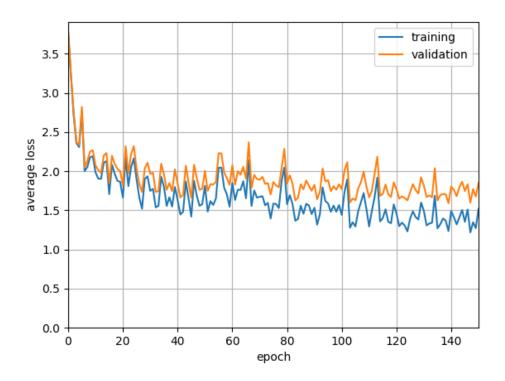


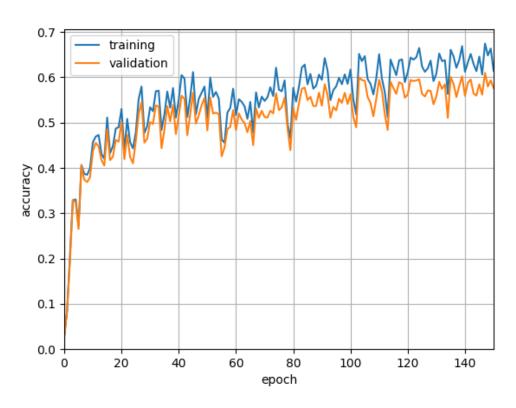


Validation accuracy: 0.76

Test accuracy: 0.77722222222223

Learning rate with 10 times that learning rate=0.03:

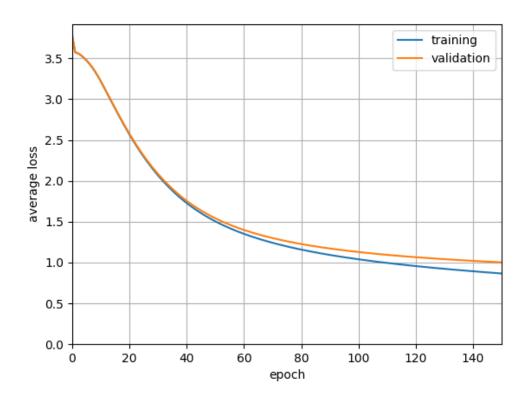


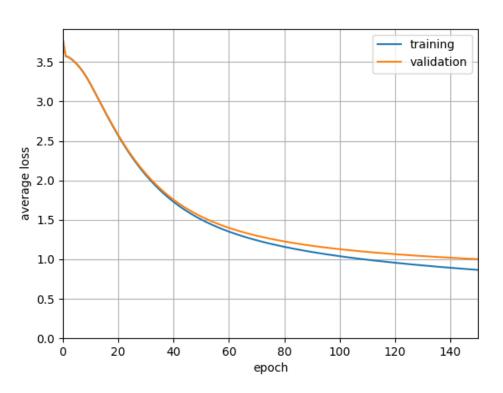


Validation accuracy: 0.5761111111111111

Test accuracy: 0.57

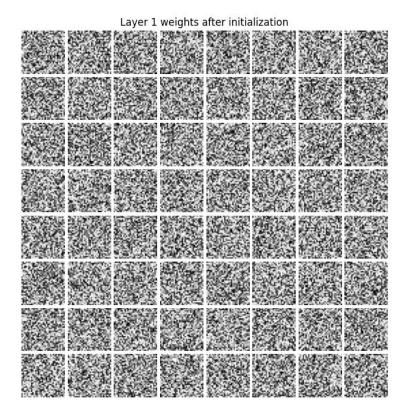
Learning rate with one tenth that learning rate=0.0003:

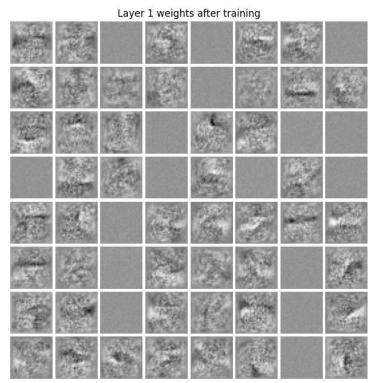




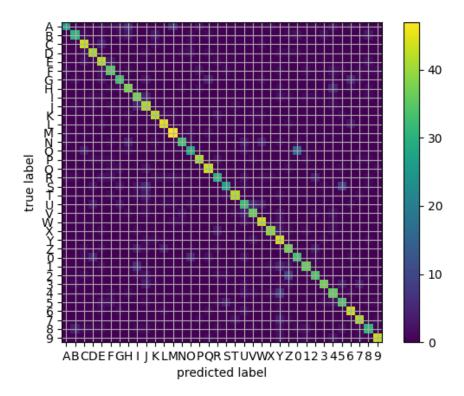
The value learning rate affects the training by the spped to change weights. Too big learning rate will make the convergence process unstable and fluctuated, which leads to unstable loss and accuracy. On the contrary, too small learning rate will make the convergence too slow and need more epoches to make the model converge.

The final accuracy of the best network on the test set is the network with learning rate=0.003, and test accuracy is 0.777222222222223.





After first intialization, the weights turn out to be random patterns. After training, weights are updated by propagation and seem more ordered according to different inputs.



The top few pairs of classes that are most commonly confused include <0, O>, <5, S> and <Z, 2>. One possible reason why these pairs are confused may be they all seem similar, especially 0 and O. So it's also hard for model to identify these similar patterns.

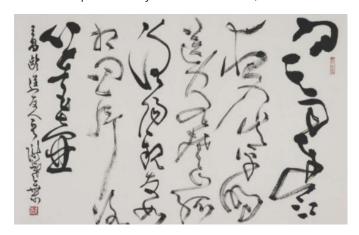
Q4.1

- (1) The whole image only contains letters and numbers, no other non-letters appear.
- (2) The size and gap between different letters are appropriate.

E.a.

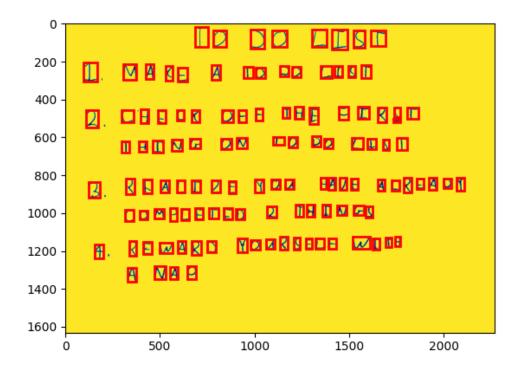


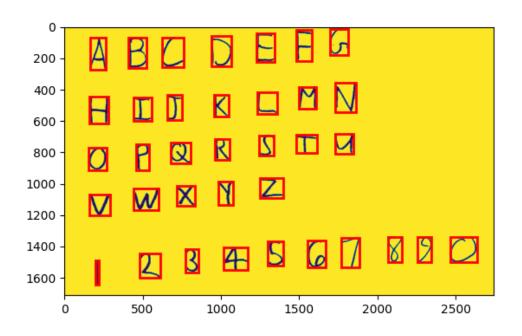
The above picture only contains one car, no other letters.

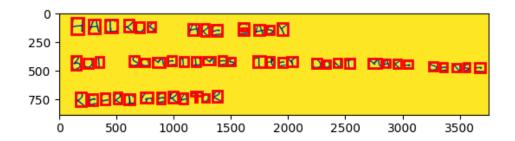


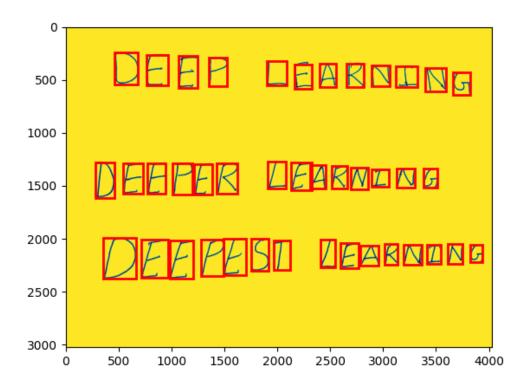
The above hand writing work doesn't include English letters and number, and the gap between characters are very casual.

```
def findLetters(image):
properties = skimage.measure.regionprops(label img)
      bboxes.append(x.bbox)
```









Q4.4

01_list:

TQ DQ LIIT
I HDX6 A TQ BQ LIIT
LH6CK JFF 3HE FIRFW
THING QW TQ DQ LI8T
RFALIZE YQU NAUE RLR6ADT
LQMPLFTCD J THING3
RFWDRD YQWRSFLF WITB
A NAP

02_letters:

2BLDFFG HIJKLMN QPQRITW VWXTZ 1Z3GBG789D

03_haiku:

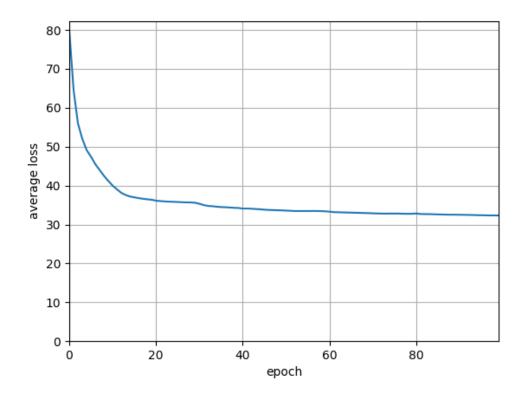
HAIWUK ARH HMAGY BWT SDMETIMBS TREY DQWT MAKG BHMGE RBGRIGBRA1MQR

04_deep:

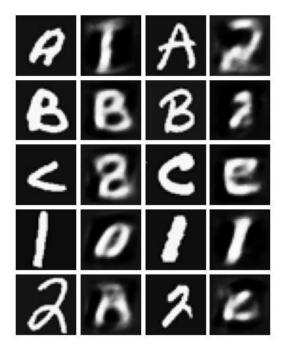
DEFP LLARHING DEDPER LEARHIMG DEEPE3T LEARNIMG

```
initialize weights(train x.shape[1], hidden size, params, 'layer1')
initialize weights(hidden size, hidden size, params, 'hidden1')
initialize weights(hidden size, train x.shape[1], params, 'output')
   losses.append(total loss/train x.shape[0])
```

```
initialize weights(train x.shape[1], hidden size , params, 'layer1')
initialize weights(hidden size , hidden size ,params, 'hidden1')
initialize weights(hidden size , hidden size ,params, 'hidden2')
initialize weights(hidden size , train x.shape[1] ,params,'output')
   params['m ' + key] = np.zeros(params[key].shape)
      loss = np.sum(np.square(probs - xb))
   losses.append(total loss/train x.shape[0])
```

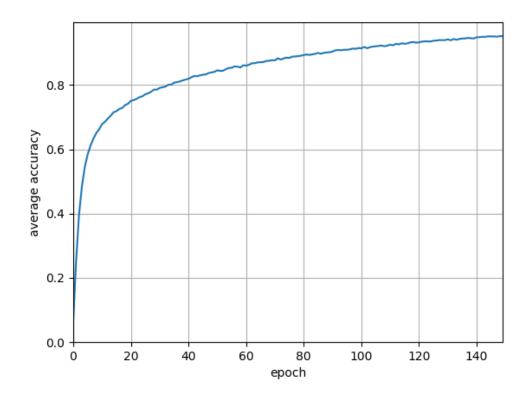


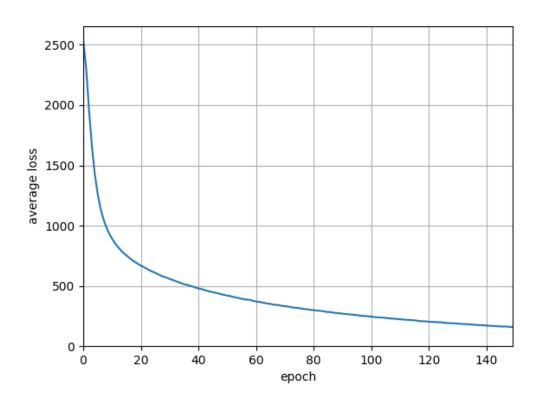
The training loss decrease fast in the first 20 epoches, then it starts decreasing more slowly and reaches a stable state after about 80 epoch.



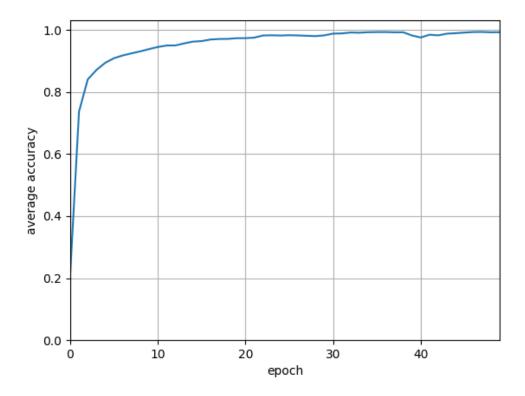
The reconstructed validation images seem more vague and blurred than the original images. And the shape of some reconstructed images are distorted or twisted.

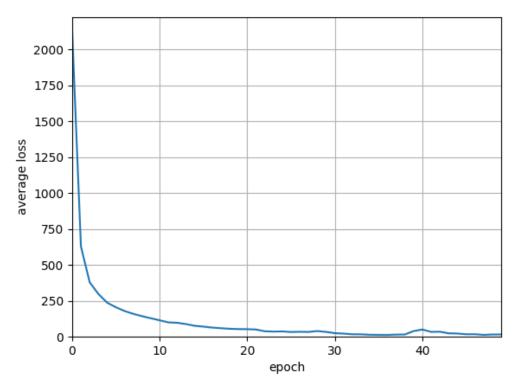
Q5.3.2 PSNR=15.81791007430785





```
x = torch.sigmoid(self.fcl(x))
      x = self.fc2(x)
simple net = orig Net(train x.shape[1], hidden size,train y.shape[1])
train loss = []
      loss = nn.functional.cross entropy(pred, targets)
   train_acc.append(avg_acc)
```

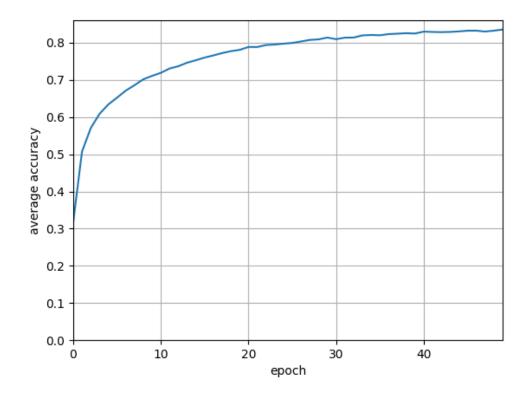


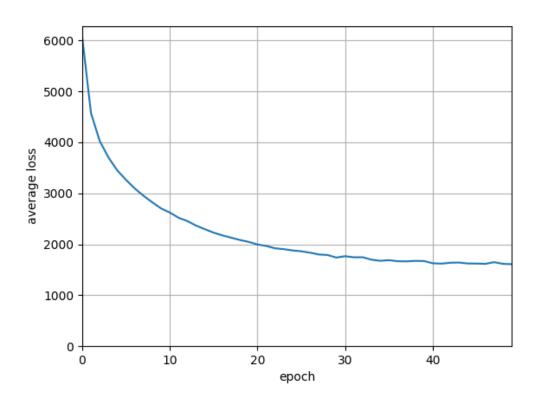


We can see this convolution model has much higher accuracy and lower loss within less epoches compared to the fully-connected network.

```
batch size = 15
learning rate = 0.003
train x = np.array([train x[i, :].reshape((32, 32)) for i in
range(test x.shape[0])])
trainset x, trainset y =
torch.from_numpy(train_x).type(torch.float32).unsqueeze(1),
torch.from numpy(train y).type(torch.long)
torch.from numpy(test x).type(torch.float32).unsqueeze(1),
torch.from numpy(test y).type(torch.long)
train loader = DataLoader(TensorDataset(trainset x, trainset y),
test loader = DataLoader(TensorDataset(test x, test y),
          nn.ReLU(),
         nn.MaxPool2d(2, 2))
          nn.MaxPool2d(2, 2))
      x = self.conv1(x)
```

```
net = ConvNet1(train x.shape[1], train y.shape[1])
train_acc = []
      loss = F.cross entropy(pred, targets)
   train loss.append(total loss)
   train acc.append(avg acc)
```





```
def forward(self, x):
   x = self.pool(F.relu(self.conv2(x)))
```

```
labels.shape[0]
      train loss.append(total loss)
      train_acc.append(avg_acc)
```