Lorentz Transformations and The Lorentz Group

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Overview

1. Rotations
2. Special Relativity
3. Lorentz Transformations
4. Lorentz Group
5. Poincare Group

Rotations

Rotations

- Keep the length of a vector unchanged
- Can be parameterised with 3 variables

$$R_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

Rotation Matrices

In 3 Dimensions

Groups

A group is a set of elements with the following properties:

1. Identity element exists in group
2. Inverse element exists in group
3. The set is closed under some operation

Special Relativity - Recap

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$$t' = \gamma t$$

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$$x' = \frac{x}{\gamma}$$

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$$x' = \frac{x}{\gamma}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Lorentz Transformations

Lorentz Transformations

Keeps the space-time interval invariant

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$$

Satisfies the following property:

$$\Lambda \eta \Lambda^T = \eta$$

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$$\eta = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

The Lorentz Group

Proper Lorentz Group

$$det(\Lambda) = +1$$

Proper Orthochronous Lorentz Group

$$det(\Lambda) = +1$$

$$\Lambda_{00} = +1$$

Infinitesimal Generators

Infinitesimal Generators

An infinitesimal Lorentz boost can be written as:

$$\Lambda = I_4 + \xi_i K_i$$

K is called the boost generator

Poincare Group

Poincare Transformations

A Poincare transformation is of the following form:

$$x' = \Lambda x + a$$

It is an extension of the Lorentz transformation to include translations

Poincare transformation generators

The Poincare group has 10 generators, which can be written as:

$$
\begin{aligned}
P &= \{P^1, P^2, P^3\} \\
J &= \{M^{23}, M^{31}, M^{12}\} \\
K &= \{M^{01}, M^{02}, M^{03}\}
\end{aligned}
$$

These vectors correspond to momentum, and angular momentum, and boost respectively. P0 corresponds to energy

Recap

- Rotations are linear transformations that preserve the magnitude and the dot product of vectors
- Rotations form a group
- Lorentz transformations are used to move between inertial frames in relative motion
- Lorentz transformations form a group, with some interesting subgroups like proper orthochronous
- The Poincare group is a natural extension of the Lorentz group, and is used in quantum mechanics

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References

Listed below are some of the key resources used in making this presentation :

- https://gdenittis.files.wordpress.com/2016/04/ayudantiavi.pdf
- https://www2.ph.ed.ac.uk/~s0948358/mysite/Poincare%20Chapters%201&2.pdf
- http://gamelab.mit.edu/research/openrelativity/

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