Design and Analysis of Algorithms - Quiz 1 - 28-Aug-2023 - 13.30-14.30

0. (0 marks) Expand CLRS

1. (1+2=3 marks) Given an array of size 16, what is the minimum number of comparisons required to output MAX and Second MAX. What if the array size is n. Justify your answer.

For array of size 16,

Max requires = $n-1$ Comparisons

$= 15$

Second Max $= \log_2 n - 1$

$= 4 - 1 = 3$

18 Comparisons

For n, $(n-1) + \lceil \log_2 n \rceil - 1$

2. (2 marks) What is the complexity of building a heap of size n in a bottom up fashion. Derive the time complexity.

At level i,

Cost to set right max heap is $O(i)$.

Number of nodes at level i is $\left\lceil \frac{n}{2^{i+1}} \right\rceil$

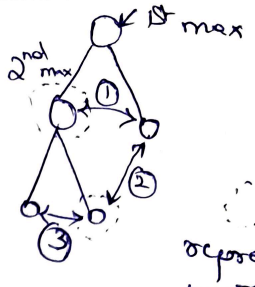
$$= \sum_{i=0}^{O(\log n)} \left\lceil \frac{n}{2^{i+1}} \right\rceil \cdot O(i)$$

$$= n \left(\sum_{i=0}^{O(\log n)} \frac{1}{2^{i+1}} \right) \cdot O(i)$$

$$= n \, O\left(\underbrace{\sum_{i=0}^{\log n} \frac{i}{2^{i+1}}}_{\leq 2} \right)$$

$$= O(n).$$

3. (1.5 marks) Given a MAX-HEAP, the number of comparisons required to output the THIRD Maximum is..... Justify.



represents maximum among two elements

1.5 Number of Comparisons required is 3

0.75 5 Comp.

1. (1.5 marks) Is $2n^3 + n \log n - 100 = o(2^n)$. Justify.

$2n^3 + n\log n - 100 \leq 1 \cdot 2^n \quad \forall c \geq 1, \forall n \geq 10$

5. (1+2=3 marks) Given a sorted array A and an element x to be searched, what is the worst case time complexity of Ternary Search to search x on A. Write the recurrence and solve using the substitution method.

$$T(n) = T\left(\frac{n}{3}\right) + 2$$

$$T\left(\frac{n}{3}\right) = T\left(\frac{n}{9}\right) + 2+2 = T\left(\frac{n}{27}\right) + 2+2+2 = T\left(\frac{n}{3^4}\right) + 2+2+2+2$$

$$\ldots = T\left(\frac{n}{3^k}\right) + \underbrace{2+2+\ldots+2}_{k \text{ times}}$$

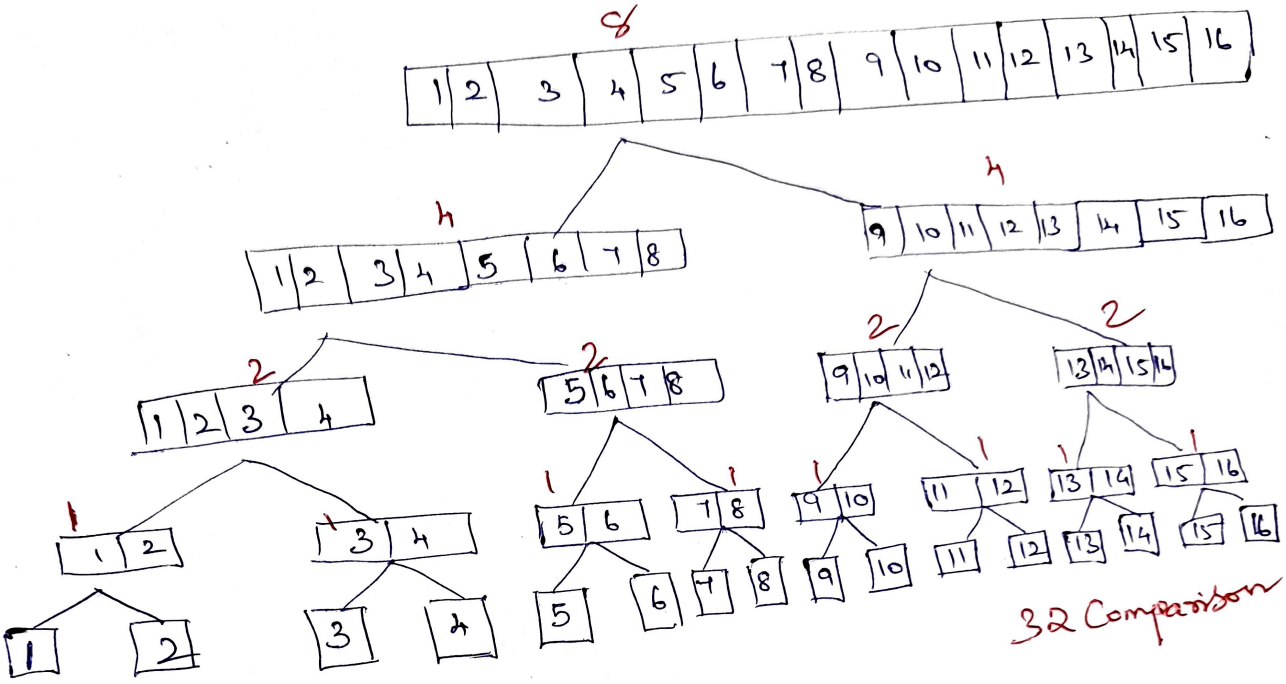
when $n = 3^k$,

$$= T(1) + \underbrace{2+\ldots+2}_{\log_3 n \text{ times}} = 1 + 2\log_3 n.$$

$$= \Theta(\log_3 n)$$

6. (2 marks) Construct an input sequence of size 16 (any example of your choice) such that the merge sort algorithm takes the least number of comparisons at each iteration of the algorithm

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16



7. (2 marks) Arrange the following functions in increasing order of growth $4^{\log n}, \sqrt{n}, n!, 3^n, n^n, \Theta(n^k)$. fixed $k \geq 2$.

$\sqrt{n}, 4^{\log n}, \Theta(n^k), 3^n, n!, n^n$