$Q_{1,3}$: Sudhakar
Q_2: Jashwanth
Q_4: Shravan
Q_5: MP, MK
Q_6: Jyothish.

Design and Analysis of Algorithms - Quiz 2 - 04-Oct-2023 - 13.30-14.30

0. (0 marks) Name the Scientist whose name appears as a substring in our Institute name.

1. (1 mark) Name the two properties that any optimization problem must satisfy to become a candidate problem for the dynamic programming paradigm.

0.5 1) Optimal substructure property

0.5 2) Overlapping subproblems.

2. Consider the coin change problem; Input: Interger n, Denominations: $1, 3, 5, 7$, the objective is to find the minimum number of coins required to give change for n using the given denominations. Answer the following;

 - (i) (2 marks) Write the recursive subproblem for this problem along with base cases. Let $C[n]$ denote the minimum number of coins required to give change for n.

Soln

$C[n] = \mathrm{MIN}\{C[n-1], C[n-3], C[n-5], C[n-7]\} + 1$ (1 mark)

Base $C[1]=1$ $C[3]=1$ $C[5]=1$ $C[7]=1$; $C[i] = \mathrm{MIN}\{C[i-1], C[i-3], C[i-5], C[i-7]\}+1$
$C[2]=2$ $C[4]=2$ $C[6]=2$ $i \geq 8$ (0.25)

(0.75)

 - (ii) (2 marks) Suppose, we wish to solve using brute force approach, what would be the lower bound and upper bound on the running time of your approach.

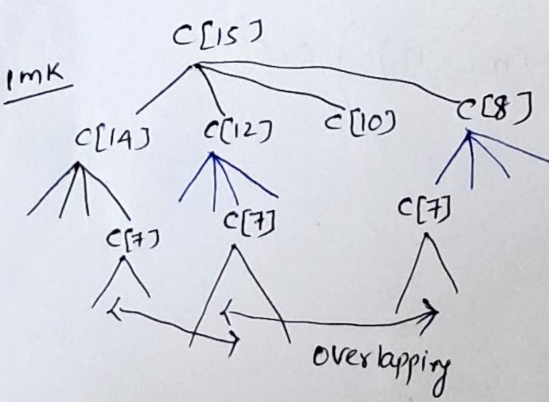
$C[n] = C[n-1] + C[n-3] + C[n-5] + C[n-7]$

$\leq 4\,C[n-1]$ $C[n] \geq 4\,C[n-7]$

(1 mk) $C[n] = O(4^{n-1})$ (1mk) $C[n] = \Omega(4^{n/7})$

 - (iii) (1 mark) Justify the overlapping subproblem property for the case $C[15]$

1mK



C[7] is Computed once and used at two other places.

3. (1.5 marks) Consider the container loading problem with two containers. Suppose, the weights of the consignments are **distinct**, and the weights of containers are $W_1 = W_2 = 5$. Will greedy strategy with respect to 'min weight' yield optimum (maximum) number of consignments that can be placed onto the containers. Justify with a proof of correctness or a counter example.

$W_1 = 5$ $\quad W_2 = 5$

w_1	w_2	w_3	w_4	w_5
1	2	3	4	5

As per Greedy; $\left.\begin{array}{l} S_1 = \{w_1, w_2\} \\ S_2 = \{w_3\} \end{array}\right\}$ 3 Consign.

↑ NOT OPT

OPT : $\left.\begin{array}{l} S_1 : \{w_1, w_4\} \\ S_2 : \{w_2, w_3\} \end{array}\right\}$ 4 Consignments

NO Marks

If wts are NOT Distinct.

4. (3 marks) Solve using Master Theorem with proper justification.

- $T(n) = 15T(\frac{n}{3}) + n^2 \log n$

$n^{\log_3 15} = n^{2+\epsilon}$, $\epsilon > 0$

$f(n) = n^2 \log n$ $\quad$ $n^{\log_b a}$ is poly larger than $f(n)$ $\quad$ 0.5

CASE 1 $\quad$ Soln : $T(n) = \Theta(n^{\log_3 15})$ $\quad$ 0.5

- $T(n) = 5T(\frac{n}{5}) + 2^n$

0.25 $\quad$ $n^{\log_b a} = n^{\log_5 5} = n$; $f(n) = 2^n$; $f(n)$ is poly larger $2^n = \Omega(n)$

Regularity Chk

0.75 $\quad$ $5 \times 2^{n/5} \leq c \cdot 2^n$; $\frac{5\, 2^{n/5}}{2^n} \leq c$; $\frac{5}{2^{\frac{4n}{5}}} \leq c$

$n \geq 5$; $\frac{5}{2^{\frac{4n}{5}}} < 1$; Choose $c = 0.9$ $\quad$ Soln $T(n) = \Theta(2^n)$

- $T(n) = 5T(\frac{n}{5}) + n$

0.5 $\quad$ $T(n) = n^{\log_5 5} = n = f(n)$ $\quad$ $n^{\log_b a}$, $f(n)$ Poly Growth is same.

CASE 2 $\quad$ Soln $\quad$ $T(n) = \Theta(n \log_5 n)$ $\quad$ 0.5

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5. (2 marks) Is the following claim true for the recurrence $T(n) = aT(\frac{n}{b}) + f(n), a \geq 1, b > 1$; **CLAIM** If $af(\frac{n}{b}) \leq cf(n)), c < 1$ then $f(n) = \Omega(n^{\log_b a+\epsilon})$, for some $\epsilon > 0$. If not true, present a counter example.

$a f(n/b) \leq c f(n)$; $f(n/b) \geq a/c \, f(n/b^2)$

$a/c \, f(n/b) \leq f(n)$

$f(n) \geq a/c \, f(n/b) \geq a/c \cdot a/c \cdot f(n/b^2) = \frac{a^2}{c^2} f(n/b^2)$

$f(n) \geq a^2/c^2 \, f(n/b^2)$

$f(n) \geq \frac{a^{\log_b n}}{c^{\log_b n}} f(1)$ ↑ $\underline{\text{Const}}$

$f(n) \geq \frac{n^{\log_b a}}{n^{\log_b E}}$

$f(n) \geq \frac{n^{\log_b a}}{n^{-\epsilon}} = n^{\log_b a+\epsilon}$

$c<1$

$\log_b c = -ve$
< 0
$= -\epsilon$
$\epsilon > 0$

$f(n) = \Omega(n^{\log_b a+\epsilon})$.

6. (2.5 marks) Given an integer m, the objective is to find x and y such that $m = 4x + 7y$.

- (i) (1 mark) What is the value of m_0 such that for all $m \geq m_0$, $m = 4x + 7y$. Justify.

$\underline{m_0 \geq 18}$

$18 = 1 \times 4 + 2 \times 7$
$19 = 3 \times 4 + 1 \times 7$
$20 = 5 \times 4 + 0 \times 7$
$21 = 0 \times 4 + 3 \times 7$

20
$22 = 2 \times 4 + 2 \times 7$
$23 = 4 \times 4 + 1 \times 7$
$24 = 6 \times 4 + 0 \times 7$
$25 = 1 \times 4 + 3 \times 7$

The pattern <u>2103</u> ....

- (ii) (1.5 marks) What is your strategy to identify x and y so that $x + y$ is minimum. Justify.

The above scheme follows

(1) If $\exists R_7$, Replace R_7 with $2 R_4$
(2) If $\exists 5R_4$, " $5R_4$ with $3R_7$
} Update Iteratively

0.5

<u>Pf by Indn</u> <u>Base</u> 18, 19, ..., 25 ; Linear Combination as above.

<u>Indn step</u>: Consider $m \geq 26$;

1

<u>By Hypo</u>; $m-1 = 4x + 7y$ for some x, y

3

<u>Obs</u> ~~CASE 1~~ Since $m \geq 26$, we observe that $m-1$ has atleast one R_7 or atleast five R_4.

<u>Case 1</u>: $\exists R_7$ $m-1+1 = 4x + 7y + 1$
$= 4x + 7(y-1) + 1 = 4x + 7(y-1) + 7 + 1$
$= 4x + 8 + 7(y-1)$
$= 4(x+2) + 7(y-1)$

<u>By Hypo</u> $x+y$ is MIN
$\therefore x+2+y-1$ is MIN

<u>Case 2</u>: $\exists 5 R_4$ $m-1+1 = 4x + 7y + 1$
$= 4(x-5) + 7y + 1 + 20 = 4(x-5) + 7(y+3)$