UNIVERSITY OF ST ANDREWS

SCHOOL OF MATHEMATICS AND STATISTICS

MMath HONOURS PROJECTS
IN MATHEMATICS AND STATISTICS
2019/2020

School of Mathematics and Statistics

Senior Honours Projects MT5599 for MMath/MPhys Students

Points of contact:

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Timeline

- *March/April*: This project booklet, containing a list of projects, important information about undertaking your project including key dates, is issued for the following academic year.
- ***18th April 2019*: Deadline for returning the form stating your project preferences** after consultation with possible supervisors as appropriate. *Complete and return the form enclosed with the booklet to the Secretary in Room 204 of the Mathematical Institute.*
- *Early May*: Your project supervisor will be confirmed.
- *May*: You are advised to have a preliminary discussion with your supervisor about your project.
- *September - April*: **Work on the project, meeting with your supervisor at least once a month** to discuss progress.
- ***31st January 2020*: Deadline for uploading an interim report** to MMS.
- *Late March/Early April*: MMS sign-up for oral presentations opens up. An email will be sent to you announcing the opening of the sign-up.
- *Semester 2, weeks 9 or 10*: Oral presentation of your project.
- ***24th April 2020*: Deadline for uploading the final electronic version of your project** on MMS.
- *May-June*: Project marked by examiners with the **final grade released at the same time as the Semester 2 examination diet results**.

Courses

- *Orientation Week, September 2019*: Honours Projects: Introductory Session — 1 hr
- *September/October*: Introduction to LaTeX— 2×1.5 hr session

- *September/October*: Project Management — 1.5 hr session
- *February/March*: Presentation Skills — 1 hr session
- *March*: Dissertation Writing — 1 hr session

The above courses are optional, but should be useful both for your project and in terms of general transferable skills.

Section 1: Guidance and Useful Information

Contents

Section 2: Project Descriptions

1 Introduction

All students studying for an MMath in Mathematics, Pure Mathematics, Applied Mathematics or Statistics must, during their final year of study, carry out a supervised project MT5599 leading to a substantial written report and an oral presentation to a small audience. Students reading for the MPhys degree in Mathematics and Theoretical Physics must either take MT5599 or the equivalent advanced project offered by the School of Physics.

There is a great deal of benefit from such a project. It allows you to work independently and in depth on a topic of your own choice and gives practice in seeking out and presenting information. Project work is well aligned with activities you are likely to encounter in your future professional life, and will enable you to develop transferable skills involved in such activities. Your project work is a common topic for discussion at job interviews.

Examiners will be looking at two broad things in assessing your project: how well you assimilate and understand a mathematical or statistical topic from your independent work, and how well you present your ideas to your intended readers.

2 Choosing a project

- Carefully examine projects offered in this booklet.
- If you would find this helpful, arrange appointments with members of staff whose projects you are interested in taking, to find out more details (this is optional).
- Select 5 projects with 5 different supervisors.
- Submit your choices by **18th April 2019** to the Secretary in Room 204.

This booklet contains a list of topics offered for 5000 level projects for the coming academic year. If a topic appeals, but you would like further information or you are not familiar with the member of staff who has proposed the project, you may contact them to discuss what might be involved. Many project titles are very broad and may be allocated to more than one student provided that the proposed treatments have minimal overlap.

You are welcome to suggest your own topic in which case you should find a staff member from inside the School who is willing to be a supervisor for your proposed topic. In this case, please indicate the title of the topic and name of the potential supervisor as one of your 5 options. In addition, two emails to the project coordinator (`nik.ruskuc@st-andrews.ac.uk`) are needed to validate this option:

- one from you, stating the academic reasons why you would like to do the proposed project;
- one from the intended supervisor (who must be a member of staff from the School of Mathematics and Statistics) supporting your case, and briefly outlining the content of the project.

You should list five choices of project with different supervisors on the form enclosed with this booklet and return it to the Secretary in Room 204 of the Mathematical Institute by **18th April 2019**. We hope that most students will be allocated one of their first three choices. However, additional considerations, e.g. the balance of workload for staff, will be also taken into consideration.

You should be aware that many projects depend on appropriate prior knowledge or background. Such prerequisites may be obvious from the project description or may be specifically stated in the description. If you are in doubt, you are advised to contact the named supervisor to make sure you do have the appropriate background for the particular project.

If you are an MPhys student in Mathematics and Theoretical Physics, you should decide whether you want to undertake a project in Mathematics or Physics. In the former case you should follow the above instructions; in the latter you need to refer to the School of Physics and Astronomy for guidance.

3 Working on the project

- The project module is based on your independent work. You are responsible for both planning and executing the necessary work.
- Your supervisor is an expert in your chosen topic, and they will be able to advise and help you with mathematical content and general advice. Make sure to use this resource by arranging regular meetings, once a month at the very least.
- You should aim to work on your project throughout the year.

The MMath project MT5599 is worth 30 credits, and therefore the time you spend on the project should be comparable with that taken on any other Level 5000 module (including private study and revision time). Notionally, 30 credits corresponds to a total of 300 hours work spread over the year. This includes time in the library, reading, working on a computer, typing drafts and the final version of the project, and the preparation and delivery of the talk. However, be aware that the actual times that an individual student may need to achieve the result they are aiming for will vary considerably. This project is intended to comprise a substantial amount of work, equivalent to two modules. Even though it might be possible to undertake all this work in a single semester, this is often a high risk strategy, especially leaving it until the 2nd Semester. It is much more prudent to spread the work across the academic year, and to make a wise use of the January break.

You should ensure that you meet regularly with your supervisor, certainly at least once a month. Ideally the first meeting should be before the summer vacation, so that you can start to turn ideas over in the back of your mind.

At the first meeting, the supervisor will discuss possibilities for the project in broad terms, and suggest background reading to help you decide on the scope and approach of your project. Your supervisor will indicate what would be expected for a 'good' project on the topic. Your initial reading and visits to web sites will lead to other references which you can consult if you think they will be useful.

Remember it is your responsibility to contact the supervisor to arrange meetings – this is best done by email. Do not expect them to chase you!

3.1 Approach and style

You should decide early on your approach and style. The range of possibilities include, but are not limited to:

- A detailed account of a specific topic, perhaps building up to a significant highlight. This could be a topic outside those met in other undergraduate modules, or could take a topic that you have already encountered somewhat further.
- A survey style project which discusses and compares a range of major ideas, with details presented briefly or intuitively.
- A mathematical or statistical investigation of a physical, biological or economic topic, or of a mathematical game, etc, perhaps using numerical, symbolic or statistical computation.
- A historical account, e.g. the work of a mathematician or development of a topic, which would include both mathematical and historical analysis.

3.2 Readership

Decide early on whom your readership will be. For instance, is your project intended to be read by any Honours student, by someone who has taken some specific Honours course or by an interested non-mathematician, etc? This may or may not be obvious from the topic. Keeping in mind who you are writing for will determine how much background you need to include and how much detailed calculation, illustration, etc. you need.

3.3 General work plan

- By about **October/November**, you should have an idea of the broad shape and content of your project. Clear organisation is important, and you should produce a skeleton outline of your project, with section and subsection headings and brief notes on what you will include under each heading; this should be discussed with your supervisor. You should also assess whether this will lead to a project of suitable length; be wary of trying to include too much material.
- During Semester 1, there should be an **opportunity to attend a project management course** which should not only help with your honours project, but should also provide you with useful transferable skills applicable to many jobs.
- The **next stage** is to flesh out this skeleton to a fuller draft of the project. At each meeting with your supervisor discuss how the project is coming together and raise any difficulties you have in finding or understanding material. Your supervisor will provide general advice, but remember that it is your project, and the overall form and detail is up to you.
- You will be asked to complete an interim report on your project by **31st January 2020** which should be submitted on MMS as a pdf or word document. Your supervisor will give you general feedback on this report indicating such things as to whether your style of writing is appropriate, whether you are giving too much or too little detail and whether you need more diagrams, explanations, etc.
- You should aim to have a complete written draft by the **middle of March** the bulk of which should, by this stage, have been typed up using a suitable word-processing package. Your supervisor will normally read through one complete draft of your project, and again you can expect to get comments of a general nature (e.g. include

more examples, there seems to be a problem with the calculation on page 11, improve the grammar), but not specific comments (e.g. spelling mistake on page 4 line 5, algebraic slip in equation (3.8)).

- Read the final version through carefully to eliminate trivial errors, to check cross-referencing, etc. It is standard practice with any major document to get someone else to read though the 'final' version, perhaps a friend or a member of your intended audience; they will almost certainly notice typos that you have missed.

- The final version of the project must be uploaded to MMS by **24th April 2020**, but it is a good idea to aim to complete it at least a week earlier in case of last minute delays, formatting problems, etc.

4 Writing up your project

4.1 Length of project

- *Font size*: 11 pt or 12 pt

- *Length*: typically 25-50 pages.

The length of the write up depends on the nature of the project. For a 'mathematical essay' style project 25-50 typed pages (about 250 words or equivalent per page) is typical. However, a shorter report (20-30 pages) could represent the result of a good deal of thought, but a project containing many figures or extended calculations might be longer (50-60 pages). You should remember that length is not an assessment criterion, but the quality of material and your engagement with it is.

4.2 Level of project

The level of mathematics or statistics in your project is expected to be comparable to that of 5000 level modules. Suitable sources and reference material will be found in advanced undergraduate level books, postgraduate level books, and in research and survey articles. Your project should not overlap significantly with topics taught in our undergraduate lecture modules or with work that you have undertaken for other purposes (such as summer vacation projects or projects that are a component of other modules). Of course, you can use ideas that you have met elsewhere as a starting point to investigate new problems. There is no expectation that the project will include original research, although this is not uncommon. You should be able to get an indication from potential supervisors as to whether the project you are considering has such scope.

4.3 Project requirements

Important notes about how your project should be laid out and what it should include.

- Your project should formatted at A4 size, using a clear, easily readable font of size 11 pt or 12 pt. Margins should be between 1.5-2.5 cm. Note, an appropriate LaTeX class file will be made available on MMS, but you are free to make your own changes within the guidelines.

- The pages should be numbered.

- Your project should have a title page, bearing the title and your name.
- You should include an introductory page with the following signed declaration:

  I certify that this project report has been written by me, is a record of work carried out by me, and is essentially different from work undertaken for any other purpose or assessment.
- You should include an abstract (i.e. a summary of the project in 100-200 words) and a list of contents in the preliminary pages. Some students even include a dedication.
- You should begin your account with a clear introduction, making clear your aims and objectives and stating your intended readership. Your project will, in part, be judged by how well you achieve these aims. Your introduction should give a brief overview of the whole project. It may also include a summary of pre-requisites for the understanding of the project and any notation assumed.
- Structure and organisation of the project is important, and you should make good use of sections and subsections. Begin each section with an outline of what is to come to provide a sense of direction and to relate the section to the whole report. Try to present a continuous chain of thought rather than jumping between topics in an arbitrary manner.
- The ending of the project should give a sense of satisfaction to the reader - don't give the impression of grinding to a halt because you can't think of any more to say. For example, you might end with the 'highlight' you have been striving towards, or with a balanced summary of your conclusions, or by considering ways in which you might develop your ideas in a sequel.
- Aim for a style of writing that is consistent, clear and precise, and follow basic standards of grammar and spelling. By all means use a style that is more lively than the scientific 'the planet was observed' or the mathematicians' 'we see that ...'!
- Remember that examples can be very effective in putting across general ideas.
- By all means include hand or computer produced graphs and diagrams, which may be scanned in or incorporated electronically. Don't spend too long producing computer drawings: if you can do a neat hand diagram in 5 minutes, it's not worth spending 3 hours doing the same thing with an electronic drawing package.
- It is essential that you give full references (see Reference section below) to the books, articles and web sites where you have found the ideas, quotations, proofs, software etc. that you have used in compiling your report. Providing full references is standard scientific practice and is a sign that you have made proper use of the literature available. It aids readers who may want to find out more about an aspect of the work. Direct quotation from other authors should be in quotation marks or italics with the author acknowledged. It is particularly important that you include references where you have quoted or paraphrased other work. Failure to give appropriate acknowledgement to material that you use is considered bad academic practice (see good academic practice section below).

- Give a list of references at the end of the project (see Appendix A). When you refer to a book or article in the text, either name the author and give the year of publication in brackets, or give a number corresponding to the item on the reference list. For references to books it is usually appropriate to include the pages, chapter or section that you have used.

- Figure and table captions should, as a rule, contain enough information that a qualified reader can understand the figure or table having read only the caption. Everything in the figure or table should be self-evident, or explained in the caption. Any figures or tables should be relevant and you should give a clean explanation of what their relevance is and/or what findings they demonstrate.

- Projects should be typed using a word-processor. This can be done in several ways depending on your skills and the software you have available. Packages such as Word may be used with equations included using an 'Equation Editor'. If you are likely to be going on to more advanced mathematical work it may be worth learning LaTeX which is the standard mathematical typesetting system. **A short course introducing LaTeX to help you get started** is offered in Semester 1.

- You are strongly advised to make regular backups for your files and keep them in a separate place, and also to make copies of any crucial notes or calculations.

4.4 Referencing and citing

In the main text when you mention ideas, equations or proofs that come from someone else's work you need to indicate this in the text using a unique marker. This is known as *citing* others work and is good academic practice. The unique marker, usually the authors name and a date or just a number, can then be used to identify the full details of the material where you found the idea, equations, etc., in a list of *references* which is placed near the back of your project, before the references.

If you look at mathematical books and articles you will see various possible ways of citing and referencing. The School does not have a specific style that you have to use in your project. In Appendix A, we have given a few examples of different styles and any one of these styles is acceptable. Just choose one that you like, then be consistent and use this style throughout your project.

4.4.1 Suggestions

Acknowledging (citing) appropriate references is an important part of any academic writing. Sometime, however, entire sections come from the same source.

- If this arises in the introduction, or another section where you are just reviewing material you will go on to use or build upon, then it may be worth stating at the start of the section that all the material in the following section is based on Author (Year). Or in a section where you want to present, for instance, the magnetohydrodynamic (MHD) equations along with the assumptions behind these equations you may instead refer to the appropriate chapter of the lecture notes either MT4510.

- If you are explaining work from a particular paper as the main part of your project, it is worth making this clear when you start to write about their calculations and work. Note, though it is useful to indicate in places if you are re-deriving a particular equation or a particular theorem that this is your work and you are showing, for example, how to get from point a in the paper to point b which the authors said could be done with "some simple algebra" (which invariably translates to many pages of work)!

- From time to time papers can involve typos or genuine errors in the calculations. If you come across one, then it is worth pointing it out and deriving the correct answer. Remember, to say "the paper contained a typo in equation .." or, if it seems to be a genuine error, point that out too, but also comment on the effect of the correction. Even if its trivial, i.e., "... the missing half from equation (3) simply reduces the velocities in the outflow jet by a half". All these small additions demonstrate that you have a good understanding of the work you are analysing.

4.5 Good academic practice

It is a basic requirement that the project you submit be your own work. This, in particular, means that the overall organisation, as well as all the text be your own. Of course, in producing your project you will (and should) use sources authored by others. All such sources should be adequately acknowledged and referenced. Failure to do so is considered plagiarism and is regarded by the School and University as a serious form of misconduct which may lead to a range of consequences, including loss of degree; see the University Good Academic Practice Policy:

http://www.st-andrews.ac.uk/students/rules/academicpractice/

Ascertaining that the project is a result of your own work is a major part of assessment for this module. As part of this, your project may be scanned by plagiarism detection software.

It may be helpful, as you write your project, instead of asking yourself the question *'How do I avoid plagiarising?'* to ask *'How can I best express what I understand of the material that I have absorbed and am trying to present to my readers?'* In particular, before resorting to any copying and pasting, you may wish to pause an assess whether the text in the original source really fits into your project: will the notation and terminology remain consistent, are all the concepts in the statement defined in the project, etc. If you do, almost invariably you will find yourself needing to modify what the source says, and, in fact, *having to* write it out in your own words.

The questions concerning proper use of sources should also be part of discussions with your supervisor, who will be able to give you opinion and guidance about any concrete issue you may be facing.

4.6 Does your project require ethical approval?

Any projects that involve data collection from questionnaires, interviews with or investigation of human subjects, or involve living tissues and/or other samples, etc, require formal approval from the University Teaching and Research Ethics Committee (UTREC). Moreover, if such work involves children under 18 it must be reviewed by the UTREC and you will need an 'Enhanced Disclosure Scotland' certificate. The letter of approval from UTREC must be included as an appendix to your project.

If you want to undertake a project of this nature you must discuss these requirements with your supervisor at the outset. See:
http://www.st-andrews.ac.uk/utrec

4.7 How to write a scientific report

There are many resources and webpages with guidance on how to write a scientific report. A list of some that you may find useful can be found in Appendix C.

5 Oral presentation

- *Duration*: 15 min plus 5 min for questions.
- *Form*: Slides (e.g. powerpoint or LaTeX slides) and/or material presented on a white board. You can use your own laptop (if you have an HDMI or VGA connector) or you can log into your University account from the computer in the room or bring your talk on a USB stick.
- *Course*: 1.5 hr Presentation Skills course offered towards the end of Semester 2

Part of the assessment involves an oral presentation on your project which you will present after returning from the spring break, in Week 9 or 10 of semester 2, and before the submission date for your written project. The audience will consist of at least two staff members and a small number of other students, including those who will be giving talks in the same session. The presentation will take place in a tutorial room or small lecture theatre. You will be notified about the arrangements for the oral presentation in due course.

You will be offered an **opportunity to attend a presentation skills course** towards the end of semester 2. Again presentation skills are important in many jobs and so this course can teach you another useful transferable skill.

Your presentation should last about (and no more than) 15 minutes followed by a few minutes for questions. Most students use pre-prepared slides or pages on a data projector or visualizer, but you may use the white/blackboard if you prefer. You should check with the staff attending about the facilities of the room; in particular if you use the data projector you should check beforehand that the system works with your software and your laptop can connect. If you use a whiteboard make sure that you have non-dried up pens available.

You should include just a few main features of your project in your talk. You will find that you can cover very little in 15 minutes. It is much better for your audience to understand fully one or two basic ideas than for them to be confused by a mass of information and formulae.

The key to the success of any talk is preparation. Your talk should have a well-defined structure: a beginning (outline of what the talk will achieve) a middle (development leading to the main highlights) and an end (conclusion, possibilities for further work). Do discuss your plans for the talk with your project supervisor.

You will wish to make notes for yourself or even write out your talk in full. However, notes are largely a confidence aid and a reminder so that you don't forget a major point. Avoid reading directly from a script - a degree of spontaneity and interaction with your audience helps retain their interest. Use pre-prepared data-projector or visualizer pages to summarise the main facts and equations you need, and expand on these in your talk.

Diagrams, pictures, etc. enliven any talk - you can scan pictures as pdf or jpg files to include in beamer or powerpoint presentations, or print pages or make photocopies (which can be in colour) for use on the visualizer. Practical demonstrations and models are also sometimes appropriate.

You may be asked questions at the end of the talk, and you should keep your answers fairly brief. Don't worry if you cannot give an immediate answer: 'I haven't explored that aspect' or 'I'll look that up for you afterwards' are perfectly acceptable responses.

Almost everyone is nervous when having to speak in public (including most of your lecturers!) and you will have a sympathetic audience. Careful preparation minimises the worry and gives you confidence that you will perform well.

6 Assessment of the project

- *Overall mark*: 80% written project + 20% oral presentation

Your project will be read and graded independently by your supervisor and another internal examiner and will also be reviewed by one of the external examiners. As has been mentioned, mathematical content and exposition are the two broad areas that are examined. Two assessors will attend and agree on a mark for the oral presentation. The Honours Grading Criteria table in the School Honours Handbook gives criteria appropriate for each grade, and these general criteria apply to projects, remembering that expository, as well as mathematical skills are taken into account.

The examiners will use the form found in Appendix D in marking the project; this indicates the factors that will be taken into consideration and their normal relative weighting. Note that the weightings may sometimes be varied so that if, for example, there is a high experimental, computational or data collection component, then appropriate credit will be given for this.

The written project carries 80% of the credit and the presentation 20%. Examiners will check that the intellectual level is appropriate for a Level 5000 project and that the amount of work evidenced reflects that expected for a 30 credit module. Projects where the level or work input is significantly lower will be marked down.

Appendices

A Referencing and citing styles

In the examples below I have written a short passage of text in which a number of articles are cited. Following this there is the list of references which give the full details of where to find the cited information.

The example in Sections A.1 is known as Harvard reference styles and is often used in applied maths or stats projects. Here articles are cited via either (Author, Year) or Author (Year) which makes it clear to the reader who the authors are within the main text. The particular citation form that you use depends on whether in your sentence you actually refer to the Author by name (in which case you would use the second citation type) or not. Both citation types are illustrated below so you can get a feel for how you should cite things. The details of each reference are then listed at the end of the article in alphabetical order rather than the order they are cited. This makes it easier to look up a particular citation.

The examples in Section A.2 and A.3 use a number reference style. Here each reference is citied via a unique number that has been assigned to either (i) due to the order in which the reference if first cited in the text (Vancouver style, used in applied maths and stats projects) or (ii) due to the order in which the reference appears when the references are alphabetically cited (used in pure and stats projects). In both cases the details of the references appear in numerical order in the reference list.

The number reference styles are preferred by some because they are very concise, but they can make reading the text harder as you need to look back at the reference list all the time to find out what work the author is referring to.

If you wish for advice on what is the most appropriate referencing style to use in your project please discuss this with your supervisor.

Note that if there are 3 or more authors then when you cite the paper you can use (FirstAuthor et al. Year) rather than listing all the authors. However, in the reference list all authors should be listed (unless there are, for example, 101 authors in which case just name the first 20 then say "and 81 others").

For further information on "referencing and citing" please look at the following link

`https://www.st-andrews.ac.uk/media/`
`capod/students/academicskills/D-Citing%20and%20Referencing.pdf`

A.1 Example using the Harvard style

This sort of style is used in both applied mathematics and statistical projects.

> A good review of the Sun and the different solar phenomena that arise in its atmosphere can may be found in Lang (2006): one such phenomena is a solar prominence (Wikipedia: Solar Prominence). During an eclipse solar prominences can been seen with the naked eye as small red flame-like features. Originally these features, which have been observed for centuries, were thought to be associated with the earth's atmosphere, however, following the analysis of the first eclipse photographs taken in 1860 it became apparent that these red features were actually solar (Foukal, 2004). Lockyer (1968) discovered that you did not need to wait for a real eclipse to observe a prominence, but they could be observed using a 'coronagraph', a telescope with a disc that blocks out much of the bright light from the Sun's surface creating an artificial eclipse (SpaceWeather: Coronagraph) combined with spectroscopic methods. Nowadays prominences are routinely observed using both ground based and space based telescopes (e.g. Lang, 2006) with considerable dynamic fine-scale structure seen from Earth, during good seeing conditions (e.g., Lin et al., 2003), as well as from space (e.g. Berger et al., 2008).
>
> *References*
>
> Berger, T. E., Shine, R. A., Slater, G. L., Tarbell, T. D., Title, A. M., Okamoto, T. J., Ichimoto, K., Katsukawa, Y., Suematsu, Y., Tsuneta, S., Lites, B. W. and Shimizu, T., 2008. *Hinode SOT Observations of Solar Quiescent Prominence Dynamics*, The Astrophysical Journal Letters 676, L89-L92.
>
> Foukal, P.V., 2004, *Solar Astrophysics*, Wiley-VCH, Weinheim, 2nd rev. edn.
>
> Lang, K., 2006. *Sun, Earth and Sky*, Springer-Verlag New York.
>
> Lin, Y., Engvold, O. R., and Wiik, J. E., 2003. *Counterstreaming in a Large Polar Crown Filament.* Solar Physics 216, 109-120.
>
> Lockyer, J.N., 1868. *Notice of an Observation of the Spectrum of a Solar Prominence, by J. N. Lockyer, Esq*, Proc. R. Soc. London, 17, 91-92.
>
> SpaceWeather, *Coronagraph.* Available at <http://spaceweather.com/glossary/coronagraph.html> [Accessed 20 July 2017].
>
> Wikipedia, *Solar Prominence.* Available at <https://en.wikipedia.org/wiki/Solar_prominence> [Accessed 01 June 2017].

A.2 Example using the Vancouver style

This sort of style is also used in applied mathematics projects.

> A good review of the Sun and the different solar phenomena that arise in its atmosphere can may be found in (1): one such phenomena is a solar prominence (2). During an eclipse solar prominences can been seen with the naked eye as small red flame-like features. Originally these features, which have been observed for centuries, were thought to be associated with the earth's atmosphere, however, following the analysis of the first eclipse photographs taken in 1860 it became apparent that these red features were actually solar (3). Lockyer (4) discovered that you did not need to wait for a real eclipse to observe a prominence, but they could be observed using a 'coronagraph', a telescope with a disc that blocks out much of the bright light from the Sun's surface creating an artificial eclipse (5) combined with spectroscopic methods. Nowadays prominences are routinely observed using both ground based and space based telescopes (1) with considerable dynamic fine-scale structure seen from Earth, during good seeing conditions (e.g. 6), as well as from space (e.g. 7).

References

1. Lang, K. Sun, Earth and Sky. New York: Springer-Verlag; 2006.
2. Wikipedia. Solar Prominence [internet].; [cited 1st June 2017]. Available from: `<https://en.wikipedia.org/wiki/Solar_prominence>`.
3. Foukal, P.V. Solar Astrophysics, Wiley-VCH: Weinheim, 2nd rev. edn; 2004
4. Lockyer, J.N. Notice of an Observation of the Spectrum of a Solar Prominence, by J. N. Lockyer, Esq. Proc. R. Soc. London. 1868; 17:91-92.
5. SpaceWeather. Coronagraph [internet].; [cited 20th July 2017]. Available from: `<http://spaceweather.com/glossary/coronagraph.html>`.
6. Lin, Y., Engvold, O. R., and Wiik, J. E. Counterstreaming in a Large Polar Crown Filament. Solar Physics. 2003; 216:109-120.
7. Berger, T. E., Shine, R. A., Slater, G. L., Tarbell, T. D., Title, A. M., Okamoto, T. J., Ichimoto, K., Katsukawa, Y., Suematsu, Y., Tsuneta, S., Lites, B. W. and Shimizu, T. Hinode SOT Observations of Solar Quiescent Prominence Dynamics. The Astrophysical Journal Letters. 2008; 676:L89-L92.

A.3 Example using numbered references

This style is often used in pure mathematics and statistics projects.

> A good review of the Sun and the different solar phenomena that arise in its atmosphere can may be found in [3]: one such phenomena is a solar prominence [7]. During an eclipse solar prominences can been seen with the naked eye as small red flame-like features. Originally these features, which have been observed for centuries, were thought to be associated with the earth's atmosphere, however, following the analysis of the first eclipse photographs taken in 1860 it became apparent that these red features were actually solar [2]. Lockyer [5] discovered that you did not need to wait for a real eclipse to observe a prominence, but they could be observed using a 'coronagraph', a telescope with a disc that blocks out much of the bright light from the Sun's surface creating an artificial eclipse [6] combined with spectroscopic methods. Nowadays prominences are routinely observed using both ground based and space based telescopes [3] with considerable dynamic fine-scale structure seen from Earth, during good seeing conditions (e.g. [4]), as well as from space (e.g. [1]).
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> *References*
>
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> [6] SpaceWeather: Coronagraph, `http://spaceweather.com/glossary/coronagraph.html` (accessed July 20, 2017).
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> [7] Wikipedia: Solar Prominence, `https://en.wikipedia.org/wiki/Solar_prominence` (accessed June 17, 2017).

B Useful resources

Below are some resources that you might find useful on how to write scientific reports:

- Alcock, L. 2014 *How to Study for a Mathematics Degree*, Oxford University Press [Contains chapters on mathematical writing and speaking]
- Alley, M. 1996 *The craft of scientific writing*, Springer.
- Day, R.A. 1994 *How to Write and Publish a Scientific Paper*, Oryx Press.
- Higham, N.J. 1998 *Handbook of Writing for the Mathematical Sciences*, SIAM
- Krantz, S.G. 1996 *A Primer of Mathematical Writing*, American Mathematical Society
- LeBrun, J. 2009 *Scientific writing: reader and writer?s guide*, World Scientific Publishing Co. Pte. Ltd.
- Olsen, R. 2015 *Houston, we have a narrative: why science needs a story.* University of Chicago Press, Chicago,
- Porush, D.A. 1995 *A Short Guide to Writing About Science* Harper Collins.
- Steenrod, N.E., Halmos, P.R., Schiffer, M.M. and Dieudonne, J. 1973 *How to Write Mathematics*, American Mathematical Society [This is a classic, but now out of print.]
- Vivaldi, F. 2014, *Mathematical Writing*, Springer Undergraduate Mathematics Series
- Williams, J. M. 2007 *Style: Lessons in clarity and grace* (9th edition). Pearson Longman, New York.

Here are also a couple of webpages which may also be useful:

- Kevin Houston, *How to Write Mathematics*, which can be found here:

  `http://www.kevinhouston.net/pdf/htwm.pdf`

- Donald E. Knuth, Tracy Larrabee, and Paul M. Roberts, *Mathematical Writing* [Notes from Stanford University - see especially pp. 1-12] which can be found here:

  `http://tex.loria.fr/typographie/mathwriting.pdf`

C Assessment forms

On the following pages there are the marking schemes for:

- written project (out of 50 and worth 80% of the overall mark)
- oral presentation (out of 10 and worth 20% of the overall mark)

MMath Honours Project Report MT5999

Student's Name:

Title of Project:

Name of Supervisor/ Examiner*: *(* = delete as appropriate)*

A. Work input and mathematical level:

The amount of work evidenced by the project is at least that expected for a 40 credit project. Yes/No*

If 'No' comment briefly:

The mathematical/statistical level is at least that expected for a 5000 level project. Yes/No*

If 'No' comment briefly:

If the amount of work or the level are below that expected, this should be taken into account when allocating marks under each heading below.

B. Comments and marks: *The distribution of marks available under each heading should be appropriate for most 5000 level projects. However they may be varied if this can be justified by the nature of the project. The prompts indicate factors that may be relevant under each heading.*

Written project:

	Mark	Comments	Prompts
Initiative, Originality and Effort	___ / 10		Degree of independence of the student; Originality of approach; Original calculation/computation; Independent thinking; Independent use of literature; Amount of work undertaken; Difficulty/ambition of the project.
Presentation and Exposition	___ / 10		Statement of aims, objectives, readership; Organisation: Structure & arrangement of material, Abstract, Introduction, Methods, Results, Discussion, References, Conclusion; Clarity and readability; Literacy and grammar; Use of examples, diagrams, tables, etc.
Understanding and Accuracy	___ / 10		Appreciation of context and significance of work, Literature review; Understanding of concepts (at 5000 level); Understanding of detail (at 5000 level); Correctness/precision of reasoning; Factual accuracy.
Mathematical, Statistical or Historical Analysis	___ / 10		Critical writing, analysis & interpretation; Were appropriate assumptions made & appropriate conclusions/inferences drawn? Were appropriate tools or methods used?
Overall Achievement	___ / 10		Does project achieve its aims? Coherence; Substance; Flair.

Oral presentation: This will be marked using the following table, and the marks will be combined in the ratio: Written project 80% ; Oral presentation 20%.

Content of talk	___ / 5		Substance; Structure; Understanding and accuracy.
Presentation of talk	___ / 5		Comprehensibility; Enthusiasm, interest, engagement; Audibility, legibility; Use of slides, whiteboard, visual aids, etc.

VA1 HOT EXPLOSIONS IN THE SUN (SOLAR BOMBS)

A recent solar mission, IRIS, has discovered the existence of solar bombs at the low atmosphere of the Sun. These hot bombs have a temperature of about 100000 Kelvin, within the cool region of solar chromosphere (10000 Kelvin). The nature of the solar bombs has not been explained yet. Advanced 3D numerical simulations will be used to study the onset and evolution of these solar explosions.

Modules: MT1002, MT2501, MT2507, MT3504, MT3802, MT4112, MT4510

VA2 FAST BULLETS IN THE SUN (JETS)

The collimated, fast (e.g. 100 km/s) emission of magnetized plasma (jets) is a generic phenomenon, which occurs in many astrophysical environments (e.g. stars, disks etc.). In the Sun, a common mechanism that explains the triggering of various types of jets is magnetic reconnection. Recently, a new class of solar jets (the so called "blow-out" jets) has been discovered but their nature is still under debate. They emit cool and hot plasma into the outer solar atmosphere and it seems that they are associated with sudden eruptions of dense material from the low solar atmosphere. Observations and 3D numerical experiments will be used to study the onset and dynamical evolution of these jets.

Modules: MT1002, MT2501, MT2507, MT3504, MT3802, MT4112, MT4510

VA3 POWERFUL ERUPTIONS IN THE SUN

One of the most dramatic phenomena in the Sun is the eruption of dense plasma from the Sun towards the outer space. These eruptions can transfer tons of heavy material, travelling with very high speeds (i.e 1000 km/s), into the interplanetary medium. They usually originate from Active Regions and they may evolve into Coronal Mass Ejections (CMEs). Observations, theoretical models and 3D numerical experiments will be used to understand the onset and dynamical evolution of these powerful solar explosions.

Modules: MT1002, MT2501, MT2507, MT3504, MT3802, MT4112, MT4510

RAB1 BLOCK DESIGNS

Block designs can be studied from either the combinatorial perspective (MT4516) or the setting of experimental design (MT4614). The student should have taken, or be taking, at least one of MT4516 and MT4614. Depending on the setting chosen, topics might include balance (MT4516), partial balance (needs MT3501), optimality (needs MT3501), construction from an initial block by using a finite group (needs MT4003), incomplete block designs in use, etc.

RAB2 ASSOCIATION SCHEMES

Association schemes are a generalization of graphs (MT4514) that arise very naturally in statistics (MT4614), in regular solids, and in combinatorics (MT4516). A student can choose any one of these areas. Their study needs an understanding of the linear algebra of real symmetric matrices, including their eigenspaces and eigenvalues (M3501, and, preferably, MT5821).

RAB3 A TOPIC IN DESIGN OF EXPERIMENTS

For anyone who has taken MT4614, a project taking part of the material further can be chosen in discussion between the student and the supervisor. For example, this could extend the work in MT4614 on any of the following: block designs; row-column designs; factorial designs; split-plot designs; multi-stratum designs; designed experiments in a particular area of application.

RAB4 NEIGHBOUR-BALANCED DESIGNS

In some experiments, it matters which treatment immediately precedes another one; in others, it matters which treatments are on either side. The project could look at either or both sorts; could concentrate on Latin squares (MT4516) or look at linear arrays; could be entirely about the combinatorics (MT4516) or could include a discussion of practical applications (MT4614).

RAB5 THE INTERFACE BETWEEN ALGEBRA AND EXPERIMENTAL DESIGN

There are many ways in which apparently abstract algebra can be used to help in the construction of experimental designs, their randomization or their analysis. Possible topics include

(1) Theory of factors (using a partially ordered set, matrix algebra and eigenspaces) (MT3501, MT4614);

(2) Randomization (properties of the relevant permutation groups) (MT4614; MT4003);

(3) Hasse diagrams to display (a) families of model subspaces (b) families of factors (MT4614; MT3501);

(4) Using graph theory to understand properties of block designs (MT4514, MT4516);

(5) Constructing factorial designs by using the characters of an elementary Abelian group (MT4614, MT4003).

CPB1 SHIFT SPACES: ALGEBRA AND DYNAMICS

If we take a non-empty, directed, connected, finite graph G, where each vertex admits at least one incoming and one outgoing edge, then the collection S of bi-infinite directed paths on G is a fundamental object in symbolic dynamical systems: it admits a topology turning it into a topological space, and also a continuous homeomorphism, the shift function $\sigma : S \to S$, which simply re-indexes any path put into it (i.e, the edge traversed at index n in an path p would be the edge appearing at index $n-1$ in the resulting path $\sigma(p)$, for each integer n). The system (S, σ) is called a *shift space*.

The shift space (S, σ) admits a canonical group $\mathrm{Aut}(S, \sigma)$, the *automorphism group of the shift* S, which is all homeomorphisms of S which commute with the shift map σ. This project is, produce any detailed study of some aspect of the theory of such automorphism groups. A canonical paper to look at is the freely available paper of Boyle, Lind, and Rudolph The automorphism group of a shift of finite type, Trans AMS, vol 306, no 1, pp 71–114, 1988.

CPB2 THE ROAD COLOURING PROBLEM AND SYNCHRONISED AUTOMATA

In 2007 Trahtman showed that Adler and Weiss's Road Colouring conjecture (1970) is true: Every finite strongly connected aperiodic directed graph of uniform out-degree has a synchronising colouring. In essence, the theorem means that given a nice enough directed graph with constant out-degree k, there is a k-colouring of the edges (each edge leaving a vertex gets its own colour) so that given any target vertex there is a uniform set of instructions (walk the red road, then the blue, then the red ...) which will take you to the target vertex no matter which vertex you start from in your graph. Any project to do with this topic, or the related Černý Conjecture (which is still open).

DLB1 HOW DO SNOW LEOPARDS USE THEIR HABITAT?

Camera traps are used to study elusive species that are difficult to observe in the wild, and snow leopards are among the most elusive of species. Little is known about how snow leopards habitat affects the way they move about their home ranges, and camera trap data provide an opportunity to investigate this. They give us time-stamped locations of individuals (identifiable from their unique pattern of spots) were whenever they were caught on camera. When combined with geographic information system data on habitat, temperature data, and/or other environmental data, the camera trap data can be used to build up a picture of how animals use their habitat. This project will involve modelling the habitat use of snow leopards in the Himalayas (or leopards in Africa, or jaguars in Central America if you prefer). The project will involve use of the R statistical programming language for analysis.

Prerequisites: MT3507 or MT3508

DLB2 HIERARCHICAL STATISTICAL MODELS IN ECOLOGY

A hierarchical statistical model is one in which there are multiple statistical models, arranged in layers such that each layer depends on a random outcome from layer below it. Such models are widely used to increase the flexibility of statistical models and/or to reflect known layers of random processes that generated observations. There are many instance is ecology where hierarchical models are useful. In this project, you will construct a hierarchical statistical model for an ecological problem of your choosing (in consultation with your supervisor), and either develop it theoretically, or implement it in R, depending on your preference.

Prerequisites: MT3507 or MT3508

DLB3 TEACH STATISTICS BETTER THAN YOU WERE TAUGHT

Frustrated with the way you have been taught statistics? Think you can do better? If so, this project is for you. Its aim is to develop material for teaching advanced statistics concepts better than you were taught. This will be done by developing material that you think will help students learn to formulate real-world problems as statistical problems better, and to illustrate the utility of statistical inference methods to address the problems. There is a great deal of flexibility in how this can be done, and you will get to decide (in consultation with your supervisor) on the scope of the project, what statistical inference problems to focus on, and (with the help of your supervisor) find real-world problems and datasets to use in the project. There is literature on how statistics can be better taught, some of which you should read in the course of the project. Teaching Statistics: A bag of tricks by Gelman and Nolan (2017) is not a bad place to start.

Prerequisites: MT3507 or MT3508

DLB4 DEVELOP A NEW STATISTICAL ESTIMATOR

Over the past two years, researchers in South Africa have been surveying a frog species on a mountain range by recording their calls on microphone arrays at selected points. The survey design involved a first stage in which they listened for frogs by ear at the selected points, and if they did not hear any, did a limited but systematic search of the area before either recording a zero for the point if no frogs were heard, or moving to the second stage, which involved setting up the array away from the selected point, where frogs were heard. Standard animal density estimators do not accommodate this two-stage process and biased estimates will result if the process is ignored.

This project will involve developing an estimation procedure that takes account of the two-stage nature of the survey process and yields unbiased estimates of frog density. This is quite a challenging project and would be suitable for someone keen to experience something like research-level statistical method development.

Prerequisites: MT3507 or MT3508
Useful but not essential: MT4608

STB1 ANALYSING PRESENCE-ONLY DATA

The governments of many nations monitor biodiversity, for example to assess whether they meet international commitments such as Convention for Biological Diversity targets, and to inform environmental policy. Until recently, most monitoring was restricted to taxa that are easy to survey. For example, birds are very visible, and many volunteers are sufficiently knowledgeable to participate in bird surveys. Schemes to monitor more difficult taxa are proliferating, as a result of digital photography combined with the development of websites where photos can be uploaded, and species identified or confirmed by experts. The resulting data are termed 'presence-only' data, because if a species is not recorded, it is not known whether it was absent or whether it was present but not detected. The data have many biases, for example geographic bias (e.g. towards southern England within the UK), and bias towards more easily identified and more easily detected species. The project will be to implement proposed models on a dataset (possibly the UK hoverfly database), and potentially to develop improved models, to allow estimation of biodiversity trends.

Pre-requisite/co-requisite: MT5753 or MT4607.

HB1 SCALING PROPERTIES OF VORTEX POPULATIONS IN SURFACE QUASIGEOSTROPHIC TURBULENCE

Proposed by: Helen Burgess & Chuong Tran

Fluids in two spatial dimensions are simple models for large-scale atmospheric and oceanic flows and other flows in thin layers, such as soap films. Many two-dimensional turbulent flows generically form coherent vortices, rapidly rotating long-lived patches of fluid. In this project the student will compare the scaling properties of vortex populations in surface quasigeostrophic (SQG) turbulence and two-dimensional vorticity (TDV) turbulence. These systems are both two-dimensional fluid models, but they have important differences: for example, coherent vortices interact with each other over longer distances in TDV flow than in SQG flow.

Students taking this project will have the opportunity to improve their coding skills. They will use pre-written software to identify vortices and study their size distributions, as well as how they group together in space. The goal is to compare vortex clusters and their statistics in SQG and TDV turbulence. Since the interaction length can be expected to affect how vortices cluster, it will be interesting to see if there are any differences with TDV flow. A more detailed project description is available upon request.

Modules that would be helpful: MT3504, MT4112, MT4509

HB2 NON-HYDROSTATIC WAVE MODELLING OF FLOWS WITH ANALOGUE HORIZONS

Proposed by: Helen Burgess & Thomas Neukirch

The equation of motion for shallow water waves on a background flow can be cast into a wave equation on a curved spacetime background, so that surface gravity waves in a moving fluid are mathematically analogous to fields propagating around black holes. In particular, analogue event horizons arise when waves propagate in a stationary counterflowing medium: at points where the flow speed equals the wave speed, the wave is unable to propagate, much as light cannot escape the event horizon of a black hole.

In this project the student will use open source wave modelling software to simulate shallow water flows with analogue horizons. The general goal is to compare the results of the simulations with analytical predictions and with experiments, and to control in the simulations for effects that are difficult to control in the experiments. We will study how the number of vertical layers used in the simulation, the amount of viscosity, and the generation of vorticity affect the results. A more detailed project description is available upon request.

Modules that would be helpful: MT3504, MT4112, MT4509

PJC1 A TOPIC IN GROUP THEORY, COMBINATORICS OR LOGIC

There is no detailed project proposed: I would like you to think about the area in which you want to do a project, and then see me to discuss your ideas and formulate a specific topic. Your project is unlikely to be approved if you haven't done this.

Projects I have supervised in the past include sum-free sets; codes over Z_4 and the Gray map; the AES cryptosystem; graphs defined by countable models of ZFC; Laplacian eigenvalues and expanders; and reverse mathematics.

MAJC1 TURING PRE-PATTERN MECHANISMS: HOW THE LEOPARD GOT ITS SPOTS AND THE TIGER GOT ITS STRIPES

Coat markings and other forms of camouflage in the animal world, from butterflies to zebras, give rise to a wide range of beautiful patterns, generally consisting of stripes, spots and other geometric shapes on the outer coats of the animals. All of these beautiful patterns can be explained by a theory initially proposed by Alan Turing in the early 1950s using reaction-diffusion equations - systems of partial differential equations governing the interaction of chemicals which control the differentiation of cells in the developing embryo. This project will explore the mechanisms of animal coat marking and attempt to answer the question of how the leopard got its spots.

Prerequisite: MT3504 Differential Equations.

MAJC2 MATHEMATICAL MODELLING OF TUMOUR-INDUCED ANGIOGENESIS

Solid tumours cannot grow beyond a few millimetres in diameter due to limitations of nutrient, mainly oxygen. Angiogenesis is the formation of blood vessels from a pre-existing vasculature and tumour cells have the ability to generate the growth of new blood vessels through the secretion of enzymes. Endothelial cells which line all blood vessels are stimulated into a sequence of events including proliferation and migration into the host tissue. This generates a new vascular network which eventually connects with the solid tumour and facilitates its subsequent growth and expansion. This project will explore the mechanisms of tumour-induced angiogenesis using partial differential equation models of cell-tissue interaction and investigate what controls successful blood vessel formation.

Prerequisite: MT3504 Differential Equations.

MAJC3 MATHEMATICAL MODELLING OF CANCER GROWTH AND INVASION

Cancers grow by excessive cell proliferation but generally stop at a size of a few millimetres in diameter due to limitations of nutrient supply. In order to overcome this deficiency, cancer cells possess the ability to secrete enzymes unto the surrounding tissue which degrade the tissue. This then enables the cancer cells to actively migrate into the transformed tissue and to spread more widely, making subsequent treatment more difficult. This project will explore the mechanisms of cancer invasion using partial differential equation models of cell-tissue interaction and investigate what controls cancer invasion.

Prerequisite: MT3504 Differential Equations.

MAJC4 MATHEMATICAL MODELLING OF THE IMMUNE-RESPONSE TO CANCER

At early stages of its growth, a solid tumour can be attacked by cells from the hosts immune system. Such cells e.g. lymphocytes, macrophages, are stimulated to attack the cancer cells through the secretion of enzymes and respond chemotactically. Once they detect the cancer cells the immune cells have the ability to kill them, but there is also a (small) chance that they themselves are killed. In some successful cases, the immune cells can remove the tumour completely. In other cases, the cancer wins out. This project will explore these different scenarios using ordinary and partial differential equation models of cell-cell interaction.

Prerequisite: MT3504 Differential Equations.

TC1 HOMOGENEOUS STRUCTURES

Automorphisms of infinite mathematical structures (such as an order, a graph or a group) are widely studied due to their deep connections with infinite permutation group theory. A structure $\mathcal{M}$ is *homogeneous* if every isomorphism between substructures of $\mathcal{M}$ extends to an automorphism of $\mathcal{M}$; it follows that $\mathcal{M}$ has many automorphisms. Examples of infinite homogeneous structures include the random graph R and the countable dense linear order without endpoints $(\mathbb{Q}, <)$.

This project will survey some topics involving homogeneous structures, including the famous Fraïssé theorem and some classification theorems. There is then potential to explore homogeneous structures in a range of settings; including infinite permutation group theory, infinite combinatorics, model theory, topology and constraint satisfaction problems.

Prerequisites: MT2504, MT3505. MT4003 would be useful.

TC2 HOMOTOPY THEORY

If you have two continuous functions f, g from one topological space to another, you may be able to 'continuously deform' f into g. This deformation is called a *homotopy* between f and g; an example of a homotopy is the famous deformation from a donut to a coffee cup. This concept forms the basis for important invariants in algebraic topology; these are known as *homotopy groups*.

This project will introduce the student to homotopy theory. The main focus will be the *fundamental group* of a topological space, and the student will describe some examples of fundamental groups as well as some applications of homotopy theory to prove some important mathematical results. Further topics could include higher-order homotopy groups or fibrations (which generalise product of spaces).

Prerequisites: MT2505 and MT3503. It would help if the student had some prior experience with topology; MT4526 should ideally be taken concurrently.

IDM1 MHD WAVES

The constant buffeting of the solar magnetic field by the subsurface convection motions generates an abundance of waves and oscillations propagating into the solar atmosphere. In this project, we will look at some of the properties of these waves and oscillations and investigate how the basic properties are modified by the presence of, for example, gravity or inhomogeneous structures. MMath students should have taken MT4510 before the start of the project.

IDM2 CORONAL SEISMOLOGY

Recent observations have shown an abundance of waves and oscillations in the solar atmosphere. These waves and oscillations carry information about the medium in which they propagate and hence we can use them to extract information about the environment. In this project, we will look at some of the most promising models proposed for coronal seismology. MMath students should have taken MT4510 before the start of the project.

IDM3 CORONAL HEATING WITH MHD WAVES

The solar atmosphere is at least two orders of magnitude hotter than the solar surface. Although it has long been believed that the Sun's magnetic field is responsible for the supply of energy to the atmosphere, the details of how this energy is converted into heat are still not fully understood. In this project, we will look at how the energy carried by waves and oscillations in the solar atmosphere could be converted into heat. MMath students should have taken MT4510 before the start of the project.

CRD1 PREDICTING TRENDS IN P2P BETTING MARKETS

There are a number of online trading markets available. One such area, covered here, is the peer-to-peer sports-betting market. In this project youll look at predicting market shifts some seconds into the future, on the basis of streams of correlated market data. Even a small ability to do this presents a huge advantage in high-frequency trading. The principles apply to share and foreign-exchange markets also.

Youll need to do substantial coding in R (although Python would be another option) to put historic data into a useable form, generate strategies, test strategies and then apply to live streams. In totality this is a large task, but small bite-sized bits can be tackled. The project can focus on any of: efficient coding/data treatment, machine learning/predictive models or some more theoretical aspects of correlated stochastic processes however their actual application is key.

Search on "pairs-trading" or "correlated random walks" (e.g. ARMA models and OrnsteinUhlenbeck processes) for indications of some methodological directions. This project would suit someone with an affinity for programming and keen to work on tough real-world problems.

CRD2 FUNDAMENTALS OF BETTING MARKETS

There are a number of online trading markets available. One such area, covered here, is the peer-to-peer sports-betting market. In this project youll look at the fundamental properties of betting markets. This might cover power to detect edges, trading volume-price relationships, queuing effects, useful hypothesis tests and simulations to inform betting strategies.

This project would suit someone with an affinity for programming and keen to work on tough real-world problems.

CRD3 PREDICTIVE MODELLING ON REMOTE-SENSED DATA

There is growing satellite-generated data, which lends itself to new predictive modelling possibilities. One particular area of interest is the monitoring of fishing activities on the basis of GPS/VMS (Vessel Monitoring System), AIS (Automatic Identification System) or satellite imagery data (such as freely available from ESA satellite systems). Global Fishing Watch (https://globalfishingwatch.org/) is an example of real-time monitoring that is freely explored.

This project will mainly involve building/improving predictive models for the prediction of vessel behaviours, primarily on location-time, vessel characteristics and movements. There is the possibility of moving this to related image-processing using satellite feeds.

The project will be heavily computational, so suits students with an interest in programming. Various machine learning skills will be developed throughout.

DGD1 THE MOTION OF VORTICES ON CURVED SURFACES

Fluid dynamics is a well developed subject stemming back three centuries. The equations of motion are well established but in practice difficult to solve in most cases. Some special cases have afforded great insight, such as focusing on the motion of point vortices, singular structures spinning infinitely fast but inducing a regular swirling flow field around them. This regular field causes other vortices to move, and in general the complex fluid dynamical system reduces to a finite dynamical system, governed by ordinary differential equations, for N point vortices - each interacting with all others. Much is known about this system, at least for small N and for vortex motion in planar geometry. Much less is known for large N or for vortex motion in curved geometry, such as an ellipsoid. This project will examine the motion of a small number of vortices on such curved surfaces, and explore when the motion may be regular or chaotic. This will involve using existing computer codes written in Python, though some additional programming may be required.

Pre-requisites: MT2503, MT2506 and MT3504 (MT2507 and MT3506 would be advantageous)

DGD2 THE MOTION OF MASSES IN CURVED SPACES

Newtonian gravity is a familiar concept in physics involving an inverse-square law of attraction between any pair of masses. The concept is less familiar however in curved spaces, such as the two-dimensional surface of a sphere, or the three-dimensional hyper-surface of a four-dimensional sphere. This project will look at recent work in this rapidly evolving research area and examine the potentially chaotic behaviour of two or more masses. This will involve using existing computer codes written in Python, though some additional programming may be required.

Pre-requisites: MT2503, MT2506 and MT3504 (MT2507 and MT3506 would be advantageous)

IJF1 A TOPIC IN THE HISTORY OF 18th or 19th CENTURY MATHEMATICS

This is a demanding option that requires both good mathematical understanding, and competence in historical approaches. It would be highly desirable to have taken the history of mathematics module MT4501.

The period from the death of Newton in 1727 to the early 20th century saw the rise of many areas of mathematics, and techniques, that are being actively pursued today: the ongoing development of calculus; the introduction of rigour; groups and fields; complex analysis, vector analysis; non-Euclidean and projective geometry; probability and statistics, to name but a few. It also saw the introduction of widespread mathematical education, and an increasingly technological society that relied on mathematical practice. The choice of a particular topic will depend on a students background and other interests, and students should contact me to discuss options.

KJF1 A TOPIC IN ADVANCED ANALYSIS

The project will study and develop a topic in advanced analysis to be agreed between student and supervisor. There are many possibilities, for example topics following on from the functional analysis module such as operator algebras or applications of the category theorems, also Fourier analysis, harmonic analysis, maximal operators, fixed point theorems, etc, etc.

Prerequisite: At least one 4000 or 5000 level analysis module.

KJF2 A TOPIC IN FRACTAL GEOMETRY

Fractal geometry is a subject that impinges on many areas of mathematics and science. This project will investigate an area of fractal geometry beyond that covered in the Honours course. Possibilities include Julia sets and the Mandelbrot set, random fractals, fractals in dynamical systems, multifractals, fractal groups, geometric properties of dimensions, differential equations on fractals, etc, etc. The project will involve reading advanced books and papers, and perhaps an original investigation of a class of fractal.

Advisable prerequisite: MT4513 Fractal Geometry

KJF3 A TOPIC IN PROBABILITY

Probability theory is a wide-ranging subject of which only a small part is covered in our taught courses. It can be approached from a theoretical, application-focused or computational viewpoint. The many topics suitable for projects include random processes (of many sorts), random walks and other Markov chains, renewal processes, queuing models, Brownian motion, etc., with applications such as share prices, gambling, actuarial problems, random networks, etc. or to other areas of maths such as group theory or graph theory.

Pre- or co-requisite: Some statistics or analysis at level 3000 or above

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JF1 LIMIT SETS OF KLEINIAN GROUPS AND HYPERBOLIC GEOMETRY

Kleinian groups are discrete subgroups of the group of isometries of hyperbolic space. In the 2-dimensional case, the group of orientation preserving isometries is isomorphic to PSL(2,R) and Kleinian groups are more commonly referred to as Fuchsian groups. These groups act properly discontinuously on hyperbolic space, but may act ‘continuously’ on the boundary of the space. The subset of the boundary where the action is continuous is often a complicated fractal set, known as the limit set. This project will focus on geometrical properties of these limit sets and how they relate to the action of the group.

This project may be of interest to students who enjoy the interactions between analysis, algebra and geometry. Especially, those who have taken the module MT5830 Topics in Geometry and Analysis, which focused on hyperbolic geometry (although this is certainly not a prerequisite).

Prerequisites: MT2502, MT2505, MT3502, MT3503.

JF2 THE KAKEYA PROBLEM

The Kakeya problem can refer to several (related) problems in geometry and analysis concerning the 'size' of sets which contain a lot of 'lines'. For example, Besicovitch proved that one may construct subsets of the plane with zero area but which contain a unit line segment pointing in every possible direction. A major open problem in analysis is whether or not this is sharp: if a set in d-dimensional Euclidean space contains a unit line segment in every direction, then does it have full dimension? This project will investigate various forms of the Kakeya problem, including some partial results concerning the above question. It will also look at Dvir's proof of the 'finite field Kakeya problem', which was a major breakthrough in combinatorial geometry published in 2008 but is surprisingly easy to read.

This project may be of interest to students who are familiar with fractal geometry and/or finite fields. A good grasp of analysis at the 3000 level is essential.

Prerequisites: MT2502, MT3502, MT3505.

JF3 FRACTAL GEOMETRY AND THE ASSOUAD DIMENSION

Roughly speaking, a fractal is an object which displays interesting features at arbitrarily small scales. Such objects appear naturally in numerous areas of pure and applied science and fractal geometry is the subject which aims to develop a rigorous mathematical framework for studying them. The 'dimension' of a fractal describes how the fractal fills up space on small scales. This project will focus on a particular notion of dimension, known as the Assouad dimension.

This project may be of interest to students fond of analysis, who may even have taken the fractal geometry module MT4513 (although this is not a prerequisite). Indeed, the Assouad dimension is not covered in this module and this will be the focus of the project.

Prerequisites: MT2502, MT3502.

AWH1 SPECIAL FUNCTIONS IN MATHEMATICAL MODELLING

Many classes of differential equations regularly appear in the modelling of physical systems. These equations are so important that the linearly independent solutions are given special names. The most well known functions are the trigonometric functions that are solutions to the equation of simple harmonic motion. This project will investigate the properties of some special functions, e.g. Bessel functions, Airy functions, Parabolic Cylinder functions that are solutions to well known differential equations.

AWH2 AVALANCHE MODELS OF SOLAR FLARES

Solar Flares are probably the most violent events in the solar system. The plasma in the solar corona is heated rapidly from about one million degrees to over 10 million degrees in the order of a minute or so. This requires the release of a vast amount of energy and the only available source of energy is the coronal magnetic field. Recently the idea that a solar flare occurs as the result of an avalanche process have been proposed. The coronal field is modelled as a lattice and simple cellular automata rules are prescribed. This project will investigate how CA have been used to describe solar flares in terms of avalanches and sequences of avalanches.

SH1 LATIN SQUARES

A latin square is an $n \times n$ matrix L, whose entries are taken from a set S of n symbols, and which has the property that each symbol from S occurs exactly once in each row and each column of L. Interest in these squares dates from at least the time of Euler in the eighteenth century (a famous conjecture which he made about them was not resolved until the twentieth century). These squares have connections with algebra (we can consider a latin square as the multiplication table of an algebraic structure called a quasigroup), and there are interesting questions about the number and structures of such squares. Latin squares have numerous and varied applications, for example in coding theory, geometry and experimental design.

No specific module prerequisites. MT2505 may be helpful.

SH2 APPLICATIONS OF COMBINATORICS TO INFORMATION SECURITY

There are numerous situations in information security, generally involving the safe and private transmission of information, which may be expressed as problems in combinatorics. Some classical combinatorial structures such as designs, difference sets and difference families, and generalisations of these, can be applied to solve these problems. Moreover, new combinatorial objects can be defined which possess the necessary properties to address a given problem. In this project, we will explore this interplay between combinatorics and problems in information security.

Necessary: MT2501 and MT2505; MT3505.

SH3 NOETHERIAN AND ARTINIAN RINGS

Noetherian and Artinian rings, characterized by satisfying the ascending or descending chain condition (respectively) on their ideals, play a key role in ring theory. In MT3505, we investigate their basic properties in the commutative case; this project offers the chance to further investigate their properties and significance. Topics which could be investigated include Hilbert's Basis Theorem, the role of prime ideals, and primary decompositions of ideals. You could also investigate the non-commutative case, in which matrix rings play a key role. There is also the possibility to explore the more general concept of modules: a Noetherian/Artinian module is one which satisfies ACC/DCC on its submodules.

Prerequisites: MT2501 and MT3505.

TL1 MATHEMATICAL MODELS FOR THE DYNAMICS OF LEUKAEMIA

Although technological progress in molecular cell biology has resulted in large amounts of data documenting the progression of leukaemia, our understanding of the principles that underpin the dynamics of leukaemic cells is filled with gaps and unresolved questions. This project is presented with the perspective that mathematical modelling can help to address some of these gaps in our knowledge. The project deals with the analysis and numerical simulation of mathematical models for the dynamics of leukaemia formulated in terms of integrodifferential equations and nonlocal partial differential equations.

TL2 MATHEMATICAL MODELS OF TUMOUR GROWTH

A thorough understanding of the mechanisms that drive tumour growth is a timely and key challenge for the cancer research community. In this respect, mathematical modelling can complement experimental cancer research by offering an alternative means of understanding the results of in vitro and in vivo experiments. This project deals with the analysis and numerical simulation of nonlinear partial differential equations modelling tumour growth.

TL3 NONLOCAL PARTIAL DIFFERENTIAL EQUATIONS MODELLING EVOLUTIONARY DYNAMICS

Evolution can be thought of as a complex and dynamical interplay between hereditary phenotypic modifications and natural selection. This project deals with the analysis and numerical simulation of nonlocal partial differential equations modelling evolutionary dynamics in asexual and sexual species.

AL1 NON-NEGATIVE MATRIX FACTORIZATION IN CANCER RESEARCH

Non-negative matrix factorization (NNMF) is a an approach to matrix decomposition that ensures all entires are non-negative. It has proven popular in a number of areas of cancer research including gene-expression, methylation and (most notably) DNA mutation signature analyses. This project would review methods for performing NNMF and, via simulation, examine the uniqueness of results and stability of results under measurement error. The implications for results in cancer research will be discussed.

This would be suitable for somebody interested in statistics who had covered topics such as principle components analysis and was confident in R programming.

AL2 ESTABLISHING AGE FROM METHYLATION DATA

In 2013 Steve Horvath demonstrated that it is possible to estimate a subjects biological age from DNA methylation patterns in various tissues. This project involves examining the models used in this and subsequent papers on the topic to determine the limitations of the approach. The adventurous may wish to apply the approach to publicly available data.

This would be suitable for somebody interested in statistics who had covered topics such as principle components analysis and was confident in R programming.

DHM1 SOLAR AND STELLAR MAGNETIC FIELDS

The evolution of magnetic fields on the surface of the Sun and other stars may be mathematically modelled through combined advection and diffusion equations called flux transport equations. The project will consider aspects of flux transport and how the same processes with different transport parameters lead to very different magnetic configurations on the Sun compared to other stars. The project is computationally based so knowledge of a programming language such as Fortran is essential.

Required Modules: MT3504, MT4510, MT4112

DHM2 SOLAR PROMINENCES

Solar prominences are cool dense regions of plasma that are suspended in a much hotter and rarer en vironment of the solar corona. They exist in the solar corona due to the presence of coronal magnetic fields which contain magnetic dips. The project will consider under what conditions dipped magnetic fields may be produced with linear force-free field solutions. While the project is mostly analytically based some computing knowledge is required.

Required Modules: MT3504, MT4510

GM1 STOCHASTIC SIMULATION OF REACTION NETWORKS IN MOLECULAR BIOLOGY

The human cell consists of various different molecules, such as genes, micro and messenger RNAs, proteins, and metabolites, that interact with each other for example to respond to changes in their environment, to communicate and control each other, give birth to new cells or lead old cells to death. These reaction networks have long been thought to be stochastic, and the recent advances in biotechnology that allow us to observe molecules in single cells over time, confirm the stochasticity of these interactions. This project will explore the stochastic processes that describe those reaction networks. Various directions can be taken by the student upon discussion including focusing on simulation of a specific reaction network of interest and comparing different stochastic models for a class of networks with similar behaviour.

GM2 STATISTICAL INFERENCE FOR REACTION NETWORKS IIN MOLECULAR BIOLOGY

This project will apply computational statistical methods, particularly Markov Chain Monte Carlo (MCMC) methods, for inferring the parameters of reaction networks in molecular biology. You will spend some time on developing your understanding of the stochastic processes that describe the time-evolution of biochemical reaction networks. These networks describe the interactions between molecules, such as genes, RNA, and proteins, that enable communication of signals, regulate the expression of genes and ultimately control almost all functions of life in a cell. Then you will use theory and/or simulation to compare some of the MCMC methods for parameter estimation in this setting and explore a potential improvement to those methods.

GM3 MATHEMATICAL ANALYSIS OF BIOCHEMICAL SIGNALLING PATHWAYS

Biochemical signalling pathways are molecular networks that are responsible for the communication of signals arising either from inside or outside of cells. They respond to signals appropriately in a sense assessing the nature and strength of the signal. Based on this evaluation signalling pathways decide whether to initiate or not appropriate actions. The robustness, and sensitivity of those pathways is very important, because signalling errors are linked to various diseases, including cancer. This project will use various information theory tools to study signalling in a pathway related to inflammation. The aim is to use analytical methods and simulation to compare various measures used to quantify the amount of information that a signalling pathway can process and therefore identify their strengths and weaknesses.

JDM1 COMPUTATIONAL (SEMI)GROUP THEORY

This project is about using a computer to answer fundamental questions about a (semi)group defined by a set of generators. For example, how do you compute the number of elements, test membership, factorise the elements over a generating set, or answer any other question about the structure of the (semi)group? In this project you will study the theoretical and practical aspects of the answers to questions of this type. This might include studying the complexity of these problems, issues of decidability, and/or implementation of algorithms.

Required prerequisites: MT2505 or MT3505 or MT3852
Desirable prerequisites: MT4003 or MT5823

JDM2 GRAPH ALGORITHMS

This project is about using a computer to answer fundamental questions about a graphs and digraphs. For example, how do you compute the strongly connected components, the automorphism group, the complete subgraphs, the shortest distance from one vertex to another, the transitive closure of the graph? In this project you will study the theoretical and practical aspects of the answers to questions of this type. This might include studying the complexity of these problems and/or implementation of algorithms.

Required prerequisites: MT2504 or MT3852
Desirable prerequisites: MT4514 or MT5821

JDM3 GRAPH HOMOMORPHISMS

This project is about graph homomorphisms. A homomorphism is a map from a graph Γ to another graph Γ that preserves the edges of Γ. The study of graph homomorphisms is a relatively new area with connections to many other areas of mathematics as well as computer science and statistical physics. In this project you will study a classical result in the field: that every finite group is isomorphic to the group of automorphisms of a finite graph. Related results will also be investigated, for example, every finite group is isomorphic to the group of automorphisms of a 3-colourable graph.

Required prerequisites: MT2504 or MT3852
Desirable prerequisites: MT4514 or MT5821

JDM4 COMPACT SEMIGROUPS

A *topological semigroup* is a semigroup S with a topology where the multiplication is a continuous function from $S \times S$ to S. A *compact semigroup* is a topological semigroup whose topology is compact. In this project, you will start from the basic properties of topological spaces and semigroups and build up to the famous theorems stating that every compact semigroup S contains an idempotent and that Green's $\mathcal{J}$-and $\mathcal{D}$-relations coincide on S.

Prerequisites: MT4526 or MT5823.

JDM5 INFINITE SYMMETRIC GROUPS

In this project we will explore some of the fascinating properties of the infinite symmetric group. Some of these properties are: it cannot be given as a union of a countable chain of subgroups; given any generating set there exists a natural number n such that every element is a product of length at most n in the generators; it has as many maximal subgroups as subsets; every Hausdorff group topology must extend the usual topology; every element is a commutator; every permutation of the real plane can be obtained by composing permutations of the form $(x, y) \to (x, y + f(x))$ and $(x, y) \to (x + g(y), y)$ where f and g are arbitrary functions from the reals to itself.

Required prerequisites: MT2505 or MT3505
Desirable prerequisites: MT4003 or MT5823

APN1 NUMERICAL METHODS FOR DIFFERENTIAL EQUATIONS

Analytical solutions to differential equations are rare and typically limited to idealised problems. Numerical methods such as Eulers method have been used to give approximate solutions for years and the recent explosion in computing power has led to enormous developments in this field. This project looks at programming some of these methods, the estimation of errors and the numerical order of convergence. Knowledge of some programming language is essential.

APN2 MAXIMUM PRINCIPLES IN DIFFERENTIAL EQUATIONS

Maximum principles are among the most powerful tools used in the study of partial differential equations. Although based on elementary calculus, maximum principles are used extensively in the study of partial differential equations. They provide information about the nature of the solution to a boundary-value problem without actually knowing what the solution is. Interested students should consult “Maximum principles in differential equations” [Protter/Weinberger], (it would be useful to have taken MT3504 Differential Equations)

TN1 NONLINEAR EVOLUTION EQUATIONS AND THE INVERSE SCATTERING TRANSFORMATION

Integrable nonlinear evolution equations are a cornerstone of modern applied mathematics and theoretical physics. This project will explore the connection between the possibilities of finding exact solution to certain nonlinear evolution equations by using transformation techniques, in particular the so-called inverse scattering transformation.

Students who consider choosing this project would benefit from having taken MT4005 Linear and Nonlinear Waves.

TN2 3D MHD EQUILIBRIA

Equilibrium solutions of the magnetohydrodynamic equations can be used as a starting point for modelling many plasma systems such as nuclear fusion devices, planetary magnetospheres, or stellar coronae (including the solar corona). The theory of MHD equilibria with spatial symmetries (2D) is quite well understood and a number of analytical solutions are known, the same cannot be said about MHD equilibria without spatial symmetries (3D). The aim of this project is to look into special classes of 3D equilibria and to calculate example solutions that can be used to model some of the plasma systems mentioned above. (Lit: Neukirch, Introduction to the Theory of MHD Equilibria, Lecture Notes, 1998)

Students who consider choosing this project should have taken MT4510 Solar Theory.

LO1 COMPLEX DIMENSIONS

The box dimension $\dim_{\mathrm{B}}(C)$ of the middle third Cantor set C equals $\frac{\log 2}{\log 3}$, i.e.

$$\dim_{\mathrm{B}}(C) = \frac{\log 2}{\log 3}\,.$$

However, surprisingly, the box dimension is just one amongst an infinite set of *complex* dimensions of C. Indeed, the infinitely many *complex* dimensions of C are given by

$$\frac{\log 2}{\log 3} + i\frac{2\pi}{\log 3}n \quad \text{for } n \in \mathbb{Z}.$$

Note that the box dimension is obtained by putting n equal to 0. Where do the other complex dimensions come from? What is the significance of these dimensions? What is going on here?

In this project you will learn complex dimensions of fractals. Since this topic is related to geometric measure theory, which is one of the active research areas at this school, this project is well suited for a student aspiring to do post graduate work. For an introduction to the topic see M Lapidus & M van Frankenhuysen, Fractal Geometry and Number Theory, Birkhäuser, 2000. Prerequisites: Knowledge of complex analysis corresponding to MT3503 is essential and knowledge of fractal geometry corresponding to MT4513 is useful.

LO2 THE PRIME NUMBER THEOREM

For each positive number x, let $\pi(x)$ denote the number of prime numbers less than x. The prime number theorem (proved independently by Hadamard and de la Vallée Poussin in 1896) states that

$$\frac{\pi(x)\log x}{x} \to 1 \text{ as } x \to \infty,$$

i.e. there are approximately $\dfrac{x}{\log x}$ prime numbers less than x.

The purpose of this project is to prove the prime number theorem. Several proofs are available. For a simple proof involving complex analysis, see D. J. Newman, Simple analytic proof of the prime number theorem, *American Mathematical Monthly* **87** (1980), 693-696. For an elementary proof (i.e. a proof that does not involve complex analysis), see E. Wright, The elementary proof of the prime number theorem, *Proc. Royal Soc. Edinburgh* **A63** (1954), 257-267. A further excellent reference is Jameson, The Prime Number Theorem, Springer Verlag. Prerequisites: Knowledge of complex analysis corresponding to MT3503 is essential and knowledge of analysis corresponding to MT3502 is useful.

LO3 NON-COMMUTATIVE DIMENSIONS

A very fashionable trend in the 1980's and 1990's has been to construct and study so-called noncommutative versions of various classical mathematical theories, for example: non-commutative analysis, non-commutative geometry, and non-commutative probability theory.

Typically a non-commutative version of a classical mathematical theory is obtained by interpreting the original theory in a functional analytic setting and subsequently "extend the domain of the theory to a space of non-commutative operators." For an introduction to the subject see E. Effros, *Why the circle is connected: An introduction to quantized topology*, The Mathematical Intelligencer **11** (1989), 27-37, or A. Connes, *Non-commutative geometry and physics, Gravitation et quantications*, 805-950, North Holland, 1995.

In this project, you will study non-commutative box dimension formulated in the setting of C^*-algebras. You will compute the non-commutative box-dimensons of various classical C^*-algebras, and you will prove a "quantum mechanical correspondence principle" saying that the non-commutative box dimensions reduce to the usual box-dimensions in the "classical commutative case". For a good introduction, see D. Kerr, *Dimension and Dynamical Entropy for Metrized C^*-Algebras*, Commun. Math. Phys. **232** (2003), 501-534.

Prerequisites: knowledge of functional analysis (corresponding to the module MT4515) is essential, and MT4004 (Real and Abstract Analysis), MT4513 (Fractal Geometry) and MT4003 (Groups) are desirable.

PP1 NUMERICAL MODELLING OF MHD WAVES

MHD waves are ubiquitous in astrophysics with many systems, from the solar corona to galactic arms. Due to their complexity, techniques to numerically solve the MHD equations are an essential research tool. However, it does come at a cost, as numerical techniques have other disadvantages, such as the inherent presence of numerical diffusivity. In this project, we will numerically solve the MHD equations with different numerical schemes in 1D for the case of the propagation of MHD waves and will investigate weakly non-linear effect as well as the role of numerical diffusion in the energy loss of the waves. We will focus also on physical effects, such as the reflection of waves on a boundary where the plasma and magnetic field are tied and how this changes when a buffer is placed before the fixed boundary. This is a configuration highly relevant for the solar atmosphere. MMath students should have taken MT4510 before the start of the project.

MP1 INVESTIGATING FREQUENTIST AND BAYESIAN VARIABLE SELECTION METHODS FOR LOGISTIC REGRESSION

When fitting a multiple logistic regression model with a large number of covariates or parameters, it is of interest to remove the terms that are not significant in explaining the binary outcome. In frequentist statistics, this is usually done with the forward or backward selection approach, or an Akaike type criterion. In Bayesian statistics, it can be done with the use of binary selection indicators within the Bayesian software OpenBugs, or by considering specific priors that induce sparseness. The aim of the project is to implement these approaches using real or simulated data sets, report similarities and differences, and examine the effect of the choice of prior distribution on the Bayesian inferences.

Prerequisites: MT2508, MT4531/MT5831 (knowledge of some MT4607 material would be beneficial)

MP2 COMPARING CLASSICAL CLUSTERING APPROACHES WITH BAYESIAN PARTITIONING

In many studies it is of interest to apply clustering and create homogenous groups of subjects in order to reduce the dimensionality of the data set. Classical clustering approaches often use various distance criteria for constructing the groups of subjects, where the number of groups is predetermined. Bayesian approaches usually assume a random number of clusters and evaluate the uncertainty regarding the number of groups and the subjects allocation to groups. The aim of the project is to apply both approaches to real or simulated data, and report on the differences between the investigated methodologies. A good knowledge of the statistical software R is required.

Prerequisites: MT2504, MT2508 (knowledge of MT4609, MT5758, MT4531/MT5831 material would be beneficial).

CEP1 CORONAL HEATING

It has been discovered that the temperature of some star's atmospheres are hotter than their thermonuclear cores. This feature is true for the Sun's outer atmosphere, the corona, which is some 150 times hotter than its surface, the photosphere, and has a temperature in excess of a million degrees Kelvin. Understanding how the solar corona is heated has been a puzzle for solar physicists for more than 60 years. Possible heating mechanisms include various reconnection or waves dissipation processes. In his project, you will look at what is known from the observations to determine what features affect the temperature and density of the solar atmosphere. Then you will consider some of the main reconnection mechanisms and discuss where these might be applicable in the solar atmosphere.

Pre-requisites: MT2501, MT2503, MT2506, MT2507, MT3504, MT3506
Desirable: MT4510

CEP2 SOLAR FLARES

Solar flares are the biggest explosions in the solar system and can cause major disruptions to the Sun's magnetic field resulting in intense heating and accelerated particles. They affect all layers of the solar atmosphere, and even sometimes affect us here on earth, and are observed in a wide range of solar emission lines. In this project, you will consider what is known from the observations to determine the main properties of these amazing solar events. Then you will look at some of the main theories for solar flares.

Pre-requisites: MT2501, MT2503, MT2506, MT2507, MT3504, MT3506
Desirable: MT4510

VP1 INTRADAY RANGE-BASED ESTIMATION OF VOLATILITY

In finance, the range, defined as the distance between the highest and the lowest return (or price) over a certain time interval, has over 30 years of history as a measure of volatility. In the last decade, its application for high-frequency data has received a great popularity. The purpose of this project is to discuss the properties of range-based estimators in the framework of semimartingales. In particular, it should include a comparison with the realized volatility (considered a classical estimator for the quadratic variation) in terms of efficiency and robustness.

Relevant module: MT4527 Time Series Analysis or MT4539 QRM

VP2 A SHRINKAGE APPROACH TO LARGE-SCALE COVARIANCE MATRIX WITH AN APPLICATION TO PORTFOLIO SELECTION

When the number of stocks in a portfolio exceeds the sample size, the sample covariance matrix is rank-deficient. The purpose of this project is to analyse estimators obtained from the sample covariance matrix through a transformation called shrinkage. This tends to pull the most extreme coefficients towards more central values, thereby systematically reducing estimation error. In terms of statistical analysis, the challenge is to determine the optimal shrinkage intensity. An empirical application of the shrinkage estimators should be provided.

Relevant module: MT2508

MRQ1 STRUCTURAL ASPECTS OF INFINITE GROUP THEORY

Two of the principal tools in the study of finite groups are the application of mathematical induction to the order of the group and the use of Sylow's Theorem. When examining infinite groups, these are no longer available and indeed there are examples (e.g., Ol'shanskii's construction of the Tarski monsters) that illustrate that infinite groups can have very surprising properties. Accordingly different methods are required in this context and frequently one needs to assume what are known as *finiteness properties* or to make use of topological or geometric methods. In this project you will examine some of the methods used and some of the results that can be proved. The exact choice of methods and material covered will depend on your interests and, perhaps, modules previously taken. You might consider the study of infinite soluble and nilpotent groups, situations where an analogue of Sylow's Theorem for an infinite setting do apply, or results concerning groups that have prescribed quotient groups or have particular restrictions on the subgroups occurring.

Prerequisite: MT4003 Groups

MRQ2 THE STRUCTURE OF GROUPS OF PRIME-POWER ORDER

Sylow's Theorem tells us that every finite group G possesses a subgroup of order p^n for every prime-power p^n dividing the order of G. As a consequence the groups of prime-power order (called p-groups for the relevant prime p) are of considerable importance in the study of finite groups. It is also the case that Sylow's Theorem gives little information about such groups. Instead other techniques are needed to investigate their structure. In this project, you will investigate some of the methods used, such as the lower central series, the lower p-central series, the Frattini subgroup, and related commutator calculus. Some important classes of p-groups could be studied in detail, such as regular p-groups and p-groups of maximal class.

Prerequisite: MT4003 Groups

MRQ3 STUDY OF A RESEARCH PAPER ON GROUP THEORY

Research papers in mathematics vary from short to very long and in the modern era often rely on the many years of development of the subject. In this project you will study a relatively substantial paper, but one not necessarily depending on a large amount of background material, and understand its content. In the process, you will need to learn some mathematics not covered in modules taught at St Andrews. Your dissertation will present this mathematics, expanding the material in the original paper so as to make it understandable for an undergraduate student.

Prerequisite: MT4003 Groups

JNR1 INTERACTIONS BETWEEN TWO OCEANIC SURFACE VORTICES

Vortices are ubiquitous on the ocean's surface. They are also key dynamical features responsible for a large part of the transport of mass and tracers in the oceans. The project would consider the nonlinear interaction between two vortices at the surface of an idealised ocean. Whether the interaction between two large vortices may lead to their breaking and to the generation of smaller scale vortices are of particular interest.

JNR2 UNSTABLE OCEANIC JETS

Jets are frequently observed in the atmosphere and the oceans, as well as in industrial applications. They are often unstable and may break into sets of coherent structures or vortices. The project would focus on some fundamental properties of simple jet configurations.

CMRD1 THE FINITE SIMPLE GROUPS

The finite simple groups are the basic building blocks of all finite groups. The Classification of Finite Simple Groups was one of the landmark results of twentieth century mathematics, and the proof runs to thousands of published pages by hundreds of authors. The main families of finite simple groups have been known for considerably longer than that, with the first infinite families (the cyclic groups of prime order and the alternating groups) being known even to Galois in the 1820s. This project would involve looking at various classes of finite simple groups, as chosen by the student in conversation with me.

Prequisites: MT4003. It would also be helpful to have taken MT3505.

CMRD2 HYPERBOLIC GROUPS

This project would involve the study of an important class of infinite groups called hyperbolic groups. These groups can be characterised in several different ways and the main task in this project would be to understand these descriptions, starting from whichever approach best suits you. For example, these are the groups for which it is easy to solve the word problem, so if you enjoy thinking algorithmically about mathematics then we would look briefly at the history of the word problem and its importance. As another example, these are the groups that act on hyperbolic metric spaces, so if your interests are more geometric or topological then we could start by considering them from this angle.

Prerequisites: MT4003.

NR1 A TOPIC IN UNIVERSAL ALGEBRA

Universal Algebra can be viewed as an overarching algebraic theory: it introduces a common language in which hitherto separate algebraic theories - such as groups, rings, semigroups, Lie algebras, etc. - can be formulated, and their properties can be compared and contrasted. In this project you will introduce the foundations of this theory, and then you will concentrate on one particular topic. This may include the theory of varieties (classes of algebras defined by identities), congruence permutable varieties and Malcev conditions, or the structure of finite algebras.

NR2 A TOPIC IN COMBINATORIAL GROUP THEORY

Combinatorial Group Theory is a study of groups defined by generators and defining relations. For instance, $\langle a, b \mid a^n = 1,\ b^2 = 1,\ ba = a^{-1}b\}$ defines the dihedral group, while $\langle a, b \mid ab = ba\rangle$ defines the free abelian group $\mathbb{Z} \times \mathbb{Z}$, and $\langle a, b \mid \ \rangle$ defines the free group on $\{a, b\}$. In this project you will investigate an advanced topic in this area. Possibilities include: undecidability of the word problem (which includes a foray into Turing machines), other algorithmic problems (conjugacy, membership, etc.), one-relator groups, or subgroups and properties of free groups.

NR3 A TOPIC IN SEMIGROUP THEORY

Semigroups are a very general type of algebraic structure – just a set with an associative binary operation – and they model a number of fundamental mathematical objects, most notably mappings on a set under composition, and words over an alphabet under concatenation. Research in Semigroup Theory is one of the major areas of expertise in our School. With my assistance you will select an article of interest form the recent literature in the field, and present its background, context and findings on a level appropriate for an advanced undergraduate/beginning postgraduate student. Likely possible areas are the theory of semigroup presentations, algorithmic problems, connections with automata and regular languages, or infinite transformation semigroups.

RKS1 CHAOTIC ADVECTION AND FLUID MIXING

Chaotic advection in fluid flows refers to the mixing of fluid by a large scale, smooth, and persistent velocity field. Despite the relatively simple nature of the flows, repeated stretching and folding of material fluid elements leads to the rapid formation of small scale structures in such a way that efficient fluid mixing may be achieved. The project will review the underlying principles of chaotic advection and examine in greater detail a particular area of application, such as mixing in planetary atmospheres, mixing in the earth's mantle, industrial mixing, or biological mixing. It will develop a numerical scheme to generate Poincaré sections for a chosen family of flows and to obtain an explicit characterization of chaotic trajectories and mixing. (Initial reading: Ottino, 1989, "The kinematics of mixing".)

Prerequisites: MT4508 or MT4509.

RKS2 SOLITONS

One of the earliest observations of a soliton, or solitary wave, was made by J. Scott Russell on the Edinburgh–Glasgow canal in 1834; it is characterised by a localised disturbance that propagates as a single entity without change of form. Mathematically, solitons are exact solutions to certain nonlinear evolution equations in which nonlinearity is typically balanced by dissipation or dispersion. This project will investigate methods for obtaining exact analytic solutions to one or more types of evolution equation and develop a simple numerical scheme to obtain examples of different types of soliton interactions. (Initial reading: Drazin and Johnson, 1989, "Solitons: an introduction".)

Pre or co-requisite: MT4005.

RKS3 HURRICANES

Why do hurricanes form? What sets their intensity? Why do they form where they do? This project will investigate these and other important questions relating to the development and structure of tropical cyclones, using reduced fluid dynamical models of the tropical atmosphere and simple numerical experimentation. Students should have taken or be taking MT5809, Advanced Fluid Dynamics.

LST1 CLASSICAL TESSELATIONS

There are 17 “wallpaper patterns”, a finite number of spherical and frieze patterns. This project is about the enumeration of the classical tesselations of Euclidean, spherical, and hyperbolic spaces in 2d. The techniques connect group theory to work of Thurston and Conway on the geometry of surfaces and orbifolds.

Required modules: MT2501, MT2505, MT3501, MT3502, MT3505

Helpful modules: MT4003, MT4526

LST2 POLYNOMIALS IN COMBINATORICS/DISCRETE GEOMETRY

Imagine a set of n integer points in R^m lies on on the zero set of a non-trivial degree d polynomial. It is not too hard to see that n can’t be too big. This surprisingly simple idea (and elaborations) has lead to breakthrough progress in combinatorics and discrete geometry in recent years. Students will learn about the “polynomial method” and explore one or two of its applications (e.g., unique distances or cap set) in depth.

Required modules: MT2501, MT2504, MT2505, MT3501, MT3505

Helpful modules: MT4514

LT1 ESTIMATING MARINE MAMMAL BYCATCH IN UK FISHERIES (with Simon Northridge, Biology)

Commercial fishing vessels sometimes inadvertently catch marine mammals such as porpoises and dolphins in their nets. This could potentially be a major conservation problem, and so it's important to quantify how many animals are caught, and where. The UK government pays to have observers on a sample of fishing trips. This project will involve spatio-temporal modelling of the resulting data. Modelling needs to account for the "zero-inflated" nature of the data (many trips have zero bycatch, a few have much bycatch), as well as the hierarchical nature of the survey design, with multiple hauls observed on the same trip, and multiple trips on the same vessel.

Required modules: MT2508, (MT4607 or MT5753); MT5802 desirable

LT2 INFERENTIAL APPROACHES TO FITTING STATE-SPACE MODELS OF WILDLIFE POPULATION DYNAMICS

State-space models are doubly stochastic models of discrete time series. The first stochastic component describes the true, but unobserved process of interest, which evolves according to a Markovian process. The second stochastic component links these true states to observations made on them. Such models are being increasingly used to describe how wildlife population numbers change through time, and allow inferences on population parameters based on incomplete survey data. Various approaches are available to fit the models to data, and in this project the student will investigate one or more of these. Of particular interest is a potentially efficient method of fitting a Bayesian version of the models, using a piece of software called AD Model Builder (http://admb-project.org/). The models will be applied to British Grey Seals, building on lots of previous work in this area.

Required modules: MT2508, MT3507; (MT4531 or MT5831) desirable

LT3 ADVANCED SEQUENTIAL MONTE CARLO METHODS

Sequential Monte Carlo (SMC) is a set of computer-intensive methods for generating samples from some target distribution, where you sequentially feed in more of the data as the algorithm proceeds. It was originally used for time series data, where there is a natural ordering, but more recently has seen wider use. It is particularly useful in Bayesian analyses of so-called "online" problems, where data arrives in real time and updated answers (posterior distributions) are required as fast as possible. This project involves the student writing a review of SMC methods, particularly the more advanced methods suited to parameter estimation, programming one or more of these more advanced approaches on a computer (in R, C or FORTRAN), and applying the program to both simulated and real-world data. This project would suit a student interested in computing and finance, since the approach is often used for financial time series. However, I know almost nothing about financial models, so the student would be responsible for providing the financial dataset and appropriate time series model.

Required modules: MT2508, MT3507

LT4 ASSESSING BEHAVIOURAL RESPONSE IN MIGRATING GRAY WHALES
(With Prof Peter Tyack, Biology)

Each year, gray whales migrate along the west coast of North America, between feeding and breeding grounds. They may be disturbed by unfamiliar or loud noises, and deflect from their path. This project will develop and implement methods for detecting deflection of migrating whales; the methods will be applied to experimental data collected by Prof. Tyack during 1997-1998 in coastal California, where a sound source that simulated Navy low-frequency active sonar was played to a sample of whales while their behaviour was being observed from shore. The data therefore comprise a time series of locations, taken on multiple individuals; our initial thoughts are that an appropriate analysis approach will be a state-space model, possibly fit within the Bayesian framework.

Required modules: MT2508, MT3507; (MT4531 or MT5831) desirable

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LT5 ESTIMATING SURVIVAL POPULATION SIZE OF BOTTLENOSE DOLPHINS IN A GULF OF MEXICO STOCK FOLLOWING THE DEEPWATER HORIZON OIL SPILL
(With Dr Lori Schwacke, NOAA, USA)

The Deepwater Horizon oil spill was by far the largest offshore spill in American history, with millions of barrels of oil discharged into the Gulf of Mexico. The impact on marine mammals was assessed by a research team, co-ordinated by Dr. Lori Schwacke of the US NOAA (DWH MMIQT 2015). One part of this assessment was a cutting-edge analysis, where photographic identification data was used to perform a spatially-explicit analysis of survival and population size of one stock of bottlenose dolphins, in Mississippi Sound (see Section 2.1 of DWH MMIQT 2015). The purpose of this project will be to repeat this analysis on a different stock where similar data are available. The project will suit a student with an interest in statistics and conservation; knowledge of Bayesian statistics will be very useful and of methods of estimating animal abundance will be helpful (although not essential).

Reference:

McDonald, T.L, F.E. Hornsby, T.R. Speakman, E.S. Zolman, K.D. Mullin, C. Sinclair, P.E. Rosel, L. Thomas and L.H. Schwacke. 2017 Survival, density, and abundance of common bottlenose dolphins in Barataria Bay following the Deepwater Horizon oil spill. Endangered Species Research 33: 193-209. Available from http://dx.doi.org/10.3354/esr00806

Required modules: MT2508, MT3507; (MT4531 or MT5831) highly desirable

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LT6 MONTSERRAT FOREST BIRDS (with emeritus Prof. Jeremy Greenwood)

Standardized bird surveys have taken place in the Caribbean island of Montserrat, using a variety of methods, from 1997 to present. Some of the data have been analysed to look at population size and trend for some species, but most has not. We have been granted access to the data by the RSPB and Montserrat forest rangers. Multiple projects are possible, including analysis of individual datasets, trend estimation, comparison of monitoring protocols, spatial modelling and biodiversity analysis. The exact project would be agreed between student and supervisor, depending on their preferences.

Required modules: MT2508, MT3507; MT5751 would be helpful but is not required

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MT1 PATH SPACES

Cadlag functions are ‘continuous on the right and have limits on the left’: paths which are allowed to jump. The set of cadlag functions is called Skorokhod space. There are various metrics on, and extensions of, this space: which you use depends on what application you’re interested in - standard applications are to convergence of point processes in probability theory. This project is concerned with finding out about these different metrics and assessing their good points and bad points. This can go in a more topological direction, focussing on the spaces themselves, or towards specific applications in probability theory. Prerequisite: MT3502.

MT2 INTERMITTENCY

Intermittent systems are characterised as having long periods of fairly predictable behaviour interspersed with short bursts of ‘chaotic’ behaviour. Intermittency is observed in many real-world systems, such as financial markets and turbulent fluids.

This project will focus on intermittent behaviour in simplified models: either a Markov chain or an interval map. For the Markov chain model to see intermittency, it needs a countable (rather than finite) state space. For the interval map (the Manneville-Pomeau map) there needs to be some non-expanding behaviour. Depending on which direction the project takes, some prior knowledge of Markov chains (eg MT4528) could be useful, but this is certainly not essential.

CVT1 FINITE-TIME BLOWUPS IN NONLINEAR PDEs

The development of finite-time blowups (singularities or shocks) from smooth initial conditions is common in nonlinear PDEs. A popular example is the inviscid Burgers equation, for which the position and time of a shock can be readily determined for a given initial wave profile. Such a blowup can be delayed or completely suppressed by natural dissipation mechanisms. This project is concerned with such suppression. Some elements of the classical energy method, one of the main research tools in mathematical fluid dynamics, can be explored.

Prerequisite: MT4005 Linear and Nonlinear Waves

CVT2 THE NAVIER-STOKES EQUATIONS

A long-standing issue in mathematical fluid dynamics is concerned with whether solutions of the three-dimensional Navier–Stokes equations evolving from smooth (but otherwise arbitrary) initial velocity fields remain globally smooth (regular) in time. Decades of active research since Leray’s seminal studies in the 1930s have resulted in a rich literature. Yet, the prospect of a definitive answer to the problem remains remote, leading to its designation as one of the millennium prize problems by the Clay Mathematics Institute (see http://www.claymath.org/millennium). This project introduces the student to this fascinating problem. Either the long history or a piece of contemporary knowledge of the problem (or both) can be explored.

Prerequisite: MT4509 Fluid Dynamics

AWS1 MAGNETIC HELICITY

Helicity is a measure of the pairwise linkage of field lines of a divergence-free vector field. It is now widely used to understand the dynamics of magnetic fields in plasmas, including the magnetic field of the Solar Corona. The project aims to explore magnetic helicity and this relationship. Pre-requisite module: MT2506.

AWS2 BRAIDED MAGNETIC FIELDS

In Mathematics, braid theory is a branch of topology which studies the possible types of braids that one can form with a number of strands, n. Analogously, one can study the structure of magnetic braids, which now have an infinite number of strands, the magnetic field lines. Magnetic fields can take on a braided structure in a wide variety of physical situations, from magnetic fusion devices on Earth to the atmosphere of the Sun and of other stars. The magnetic field plays an critical role in governing the dynamics of these environments, and by developing tools to measure and quantify the degree of braiding in such fields we hope to understand more about the permitted physical evolutions and why they arise. For example, an unanswered question is whether non-trivially braided force-free magnetic fields with smooth electric currents even exist, the answer to which has important implications for solar coronal heating. In this project we will construct models for braided magnetic fields and look at ways to quantify their complexity. There are also applications in other fields such as the physics of mixing, and in turbulence. Pre-requisite module: MT2506.

HW1 INCORPORATING COVARIATES IN STOPOVER MODELS

Wildlife populations are often of interest in order to understand the ecosystem and for conservation and management purposes. Whilst interest is often focussed on survival probabilities, for some populations the time of arrival at a breeding site is also of interest. This project will investigate how explanatory variables may be incorporated into the analysis of capture-recapture data to explain temporal and/or individual heterogeneity in demographic parameters of interest along with associated model-fitting issues.

Module Requirements: MT3507 or MT3508, (MT4113 and MT4528 preferable)

HW2 INDIVIDUAL HETEROGENEITY FOR INCOMPLETE CONTINGENCY TABLES

Contingency tables summarise information from several sources of information, however, they are typically incomplete since a proportion of the population will be missed by all sources. This project will investigate methods for estimating population size using incomplete contingency tables and the effect of ignoring individual heterogeneity. Methods will be developed to include individual heterogeneity in the model structure and demonstrated through a simulation study.

Module Requirements: MT3507 or MT3508, (MT4113 preferable)

ANW1 THE INDIAN ROPE-TRICK

Eye-witness accounts of the celebrated Indian rope-trick date back to the 14th century. Have these accounts been exaggerated with the passage of time, or has one of India's magical secrets been forgotten?

The mechanics of standing a rope on end can be investigated via the simpler system of an inverted rigid pendulum that can be made to stand upright if the pivot point is moved appropriately. By linking several pendulums together the system begins to mimic a rope. So far, a practical demonstration of this effect has been achieved for a "rope" with four sections and also a stiff cable. Investigate the limitations, practicalities and behaviour of these systems. (See http://www.jesus.ox.ac.uk/ dacheson/res11.html and its links for further information.) Competence in programming with Maple or Fortran is desirable.

ANW2 BATTING THE BALL

From sporting heroes to those who putt on the Himalayas, they all know the feel of hitting a good shot - and a bad shot. For many the learning process is intuitive and based on experience, but for the scientist there is another way. Whether your recreation be tennis, golf, cricket, baseball, squash, or hockey, you hit a ball with a bat, club or racket. Hitting the ball from the "sweet spot" (or center of percussion) feels great, but does this produce the maximum ball speed? Investigate these issues, find out what the "dead spot" is and, of course, try to improve your game. (See http://www.physics.usyd.edu.au/ cross/ and http://www.npl.uiuc.edu/ a-nathan/pob/ for further details.) Competence in programming with Maple or Fortran is desirable.

PROJECT SUPERVISORS FOR 2019/2020

The following table matches initials to names of supervisors, in the order in which they appear in the booklet.

Initials	Name	Initials	Name
VA	Dr V Archontis	NR	Prof N Ruskuc
RAB	Prof R A Bailey	RKS	Dr R K Scott
CPB	Dr C P Bleak	LST	Dr L Theran
DLB	Dr D L Borchers	LT	Dr L Thomas
STB	Prof S T Buckland	MT	Dr M Todd
HB	Dr Helen Burgess	CVT	Dr C V Tran
PJC	Prof P J Cameron	AWS	Dr A Wilmot-Smith
MAJC	Prof M A J Chaplain	ANW	Dr A Wright
TC	Dr T Coleman		
IDM	Prof I De Moortel		
CRD	Dr C R Donovan		*****
DGD	Prof D G Dritschel		
IJF	Dr I J Falconer		
KJF	Prof K J Falconer		
JF	Dr J Fraser		
AWH	Prof A W Hood		
SH	Dr S Huczynska		
TL	Dr T Lorenzi		
AL	Prof A Lynch		
DHM	Dr D H Mackay		
GM	Dr G Minas		
JDM	Dr J D Mitchell		
APN	Dr A P Naughton		
TN	Prof T Neukirch		
LO	Prof L Olsen		
PP	Dr P Pagano		
MP	Dr M Papathomas		
CEP	Prof C E Parnell		
VP	Dr V Popov		
MRQ	Dr M R Quick		
JNR	Dr J N Reinaud		
CMRD	Dr C M Roney-Dougal		