Machine Learning INF2008

Lecture 02: The Single Layer Perceptron

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Singapore Institute of Technology

SIT Open House 2023 – The Winners (Judges' Choice)

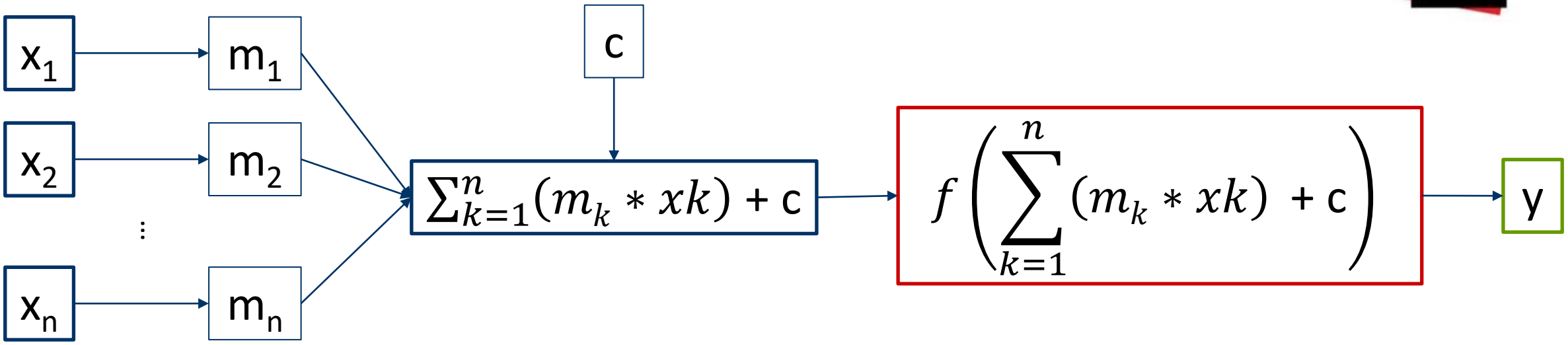
Most Attractive Booth (Judges' Choice)

Champion: Naval Architecture and Marine Engineering

1st Runner-up: Applied Artificial Intelligence

2nd Runner-up: Physiotherapy

Linear Models 3: Neural Network (Single Layer Perceptron)



$$y = f\left(\sum_{k=1}^{n} m_k * xk + c\right)$$

We “learn” the variables m_i and c

What is a non-linear function?

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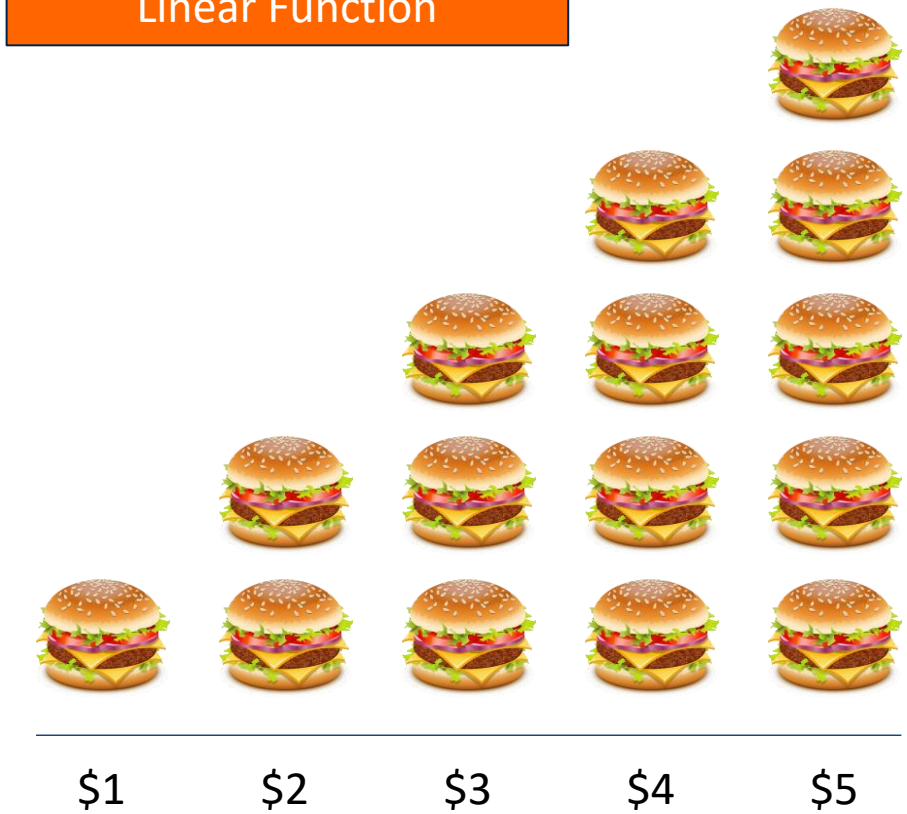
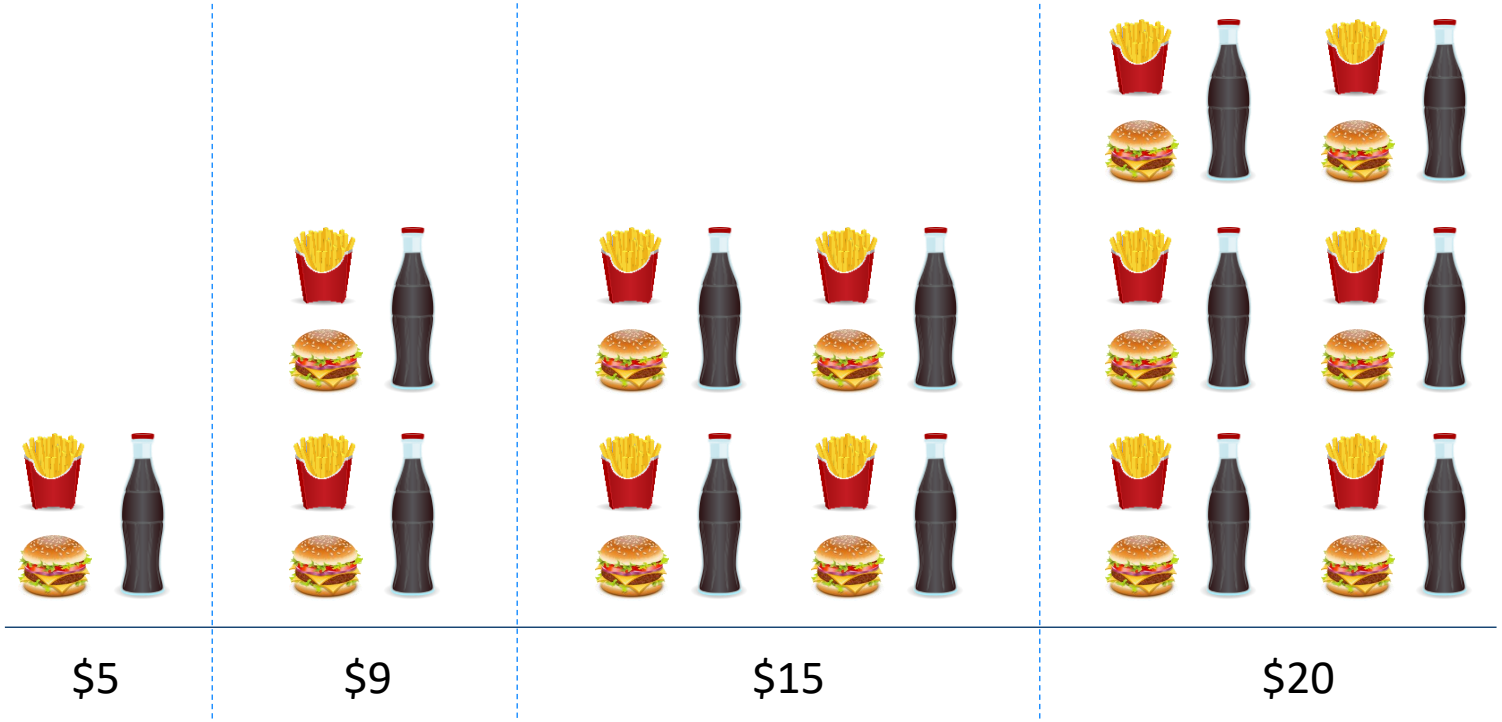
Linear Function

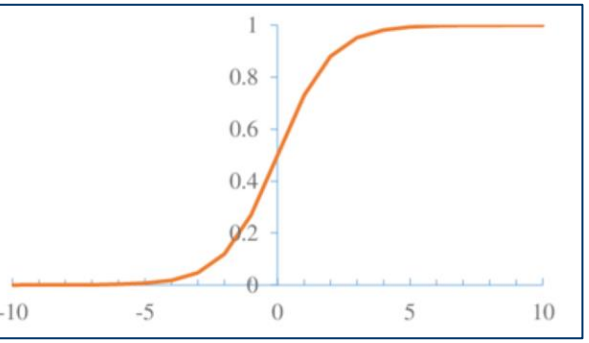


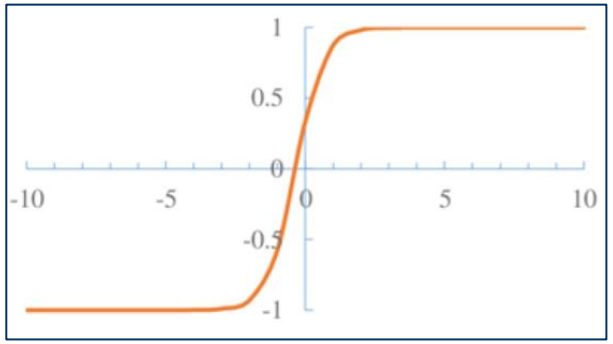
$1 $2 $3 $4 $5

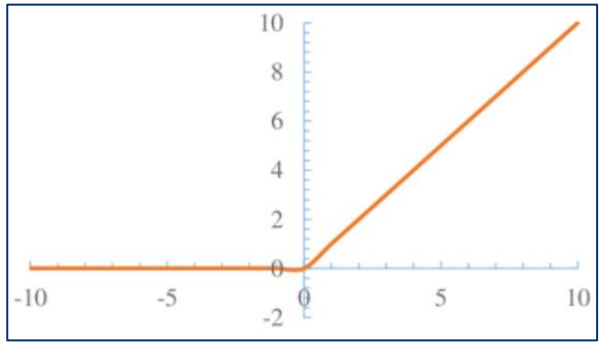
Non-Linear Function

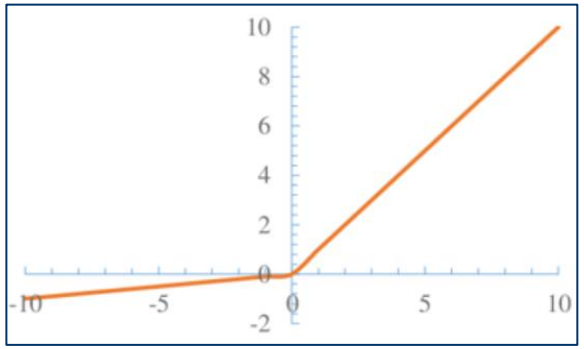


$5 $9 $15 $20

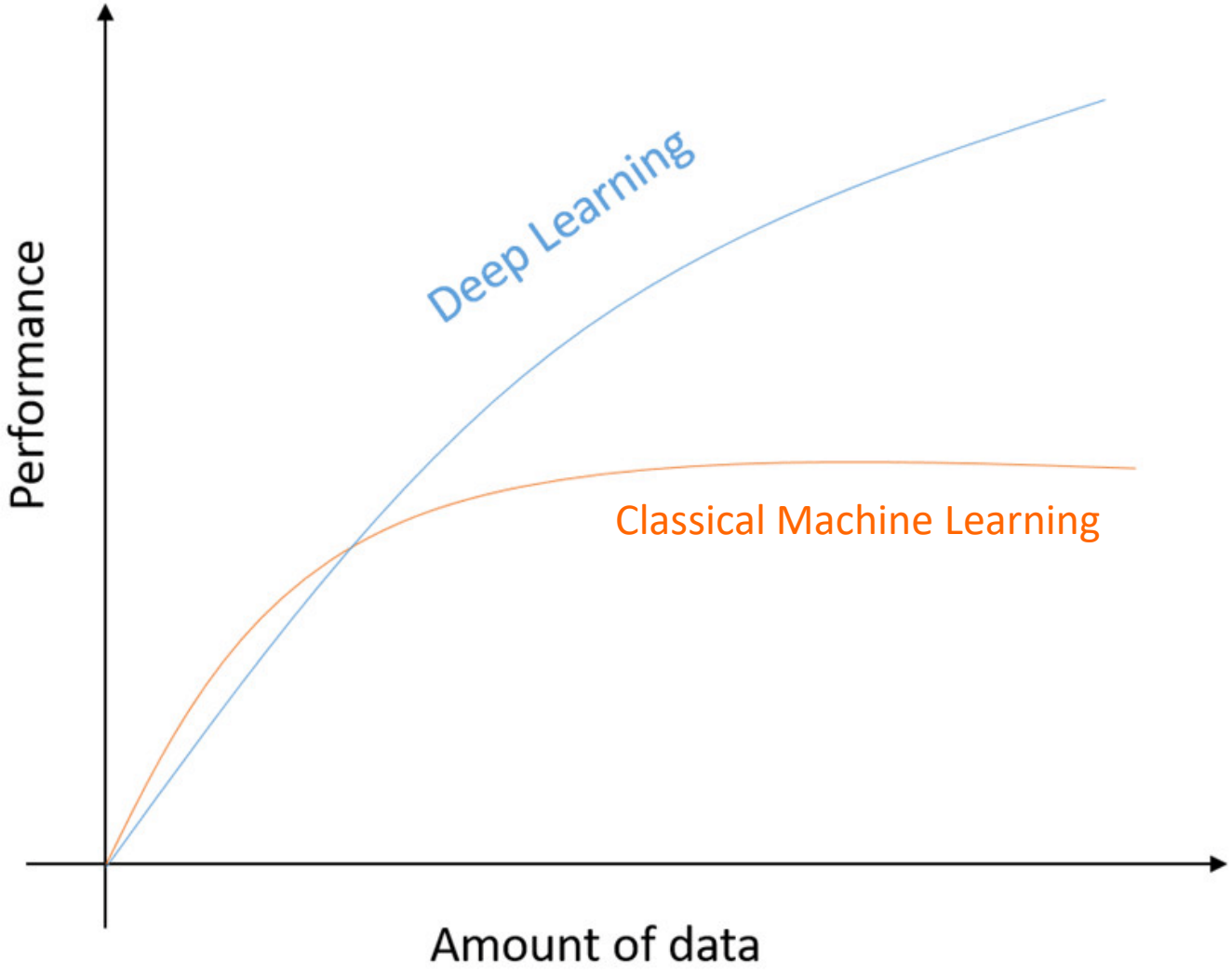




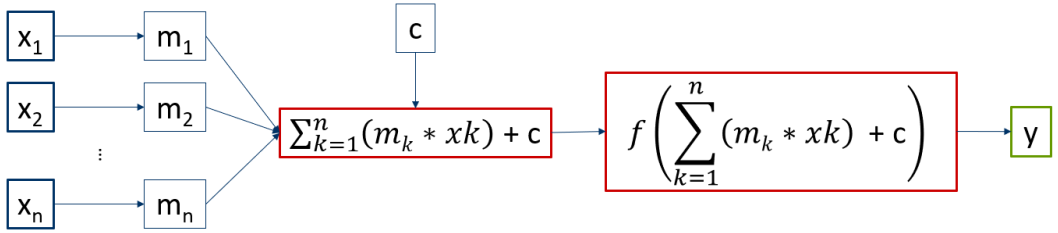




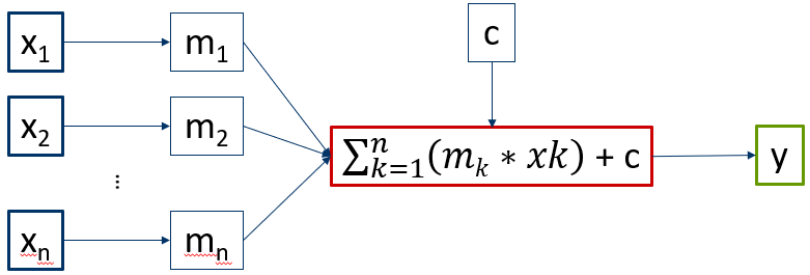
Classical Machine Learning



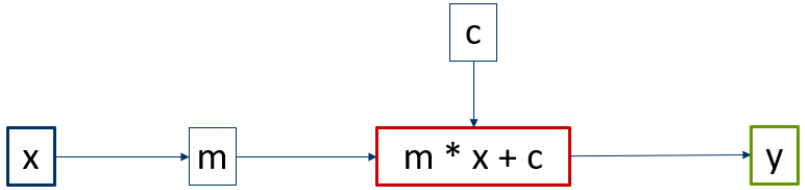
Neural Networks (Single Layer Perceptron)



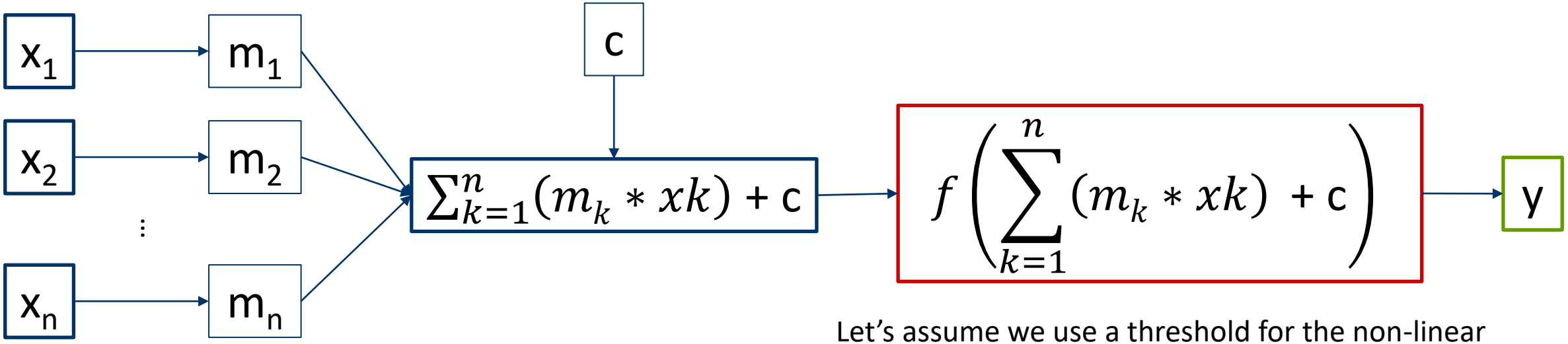
Multiple Linear Regression



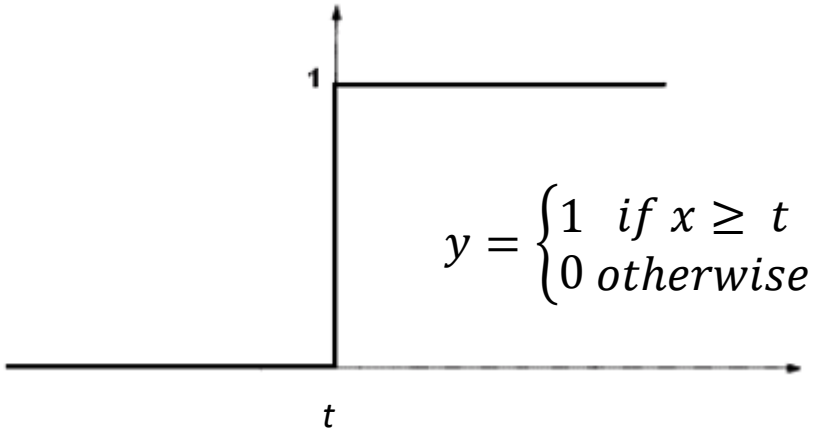
Linear Regression



Single Layer Perceptron



Let's assume we use a threshold for the non-linear function.

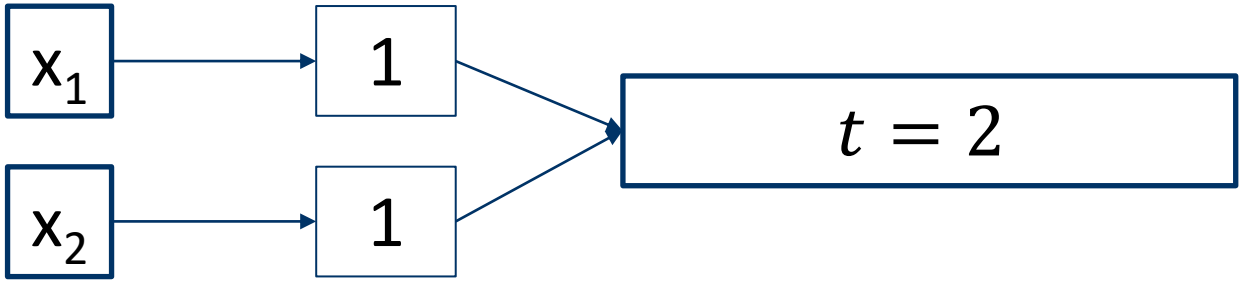


$$y = \begin{cases} 1 & if\ x \geq t \\ 0 & otherwise \end{cases}$$

Essentially, this means that for some threshold value of *t*, f(x) is 0 for x < *t* and f(x) is 1 for x >= *t*

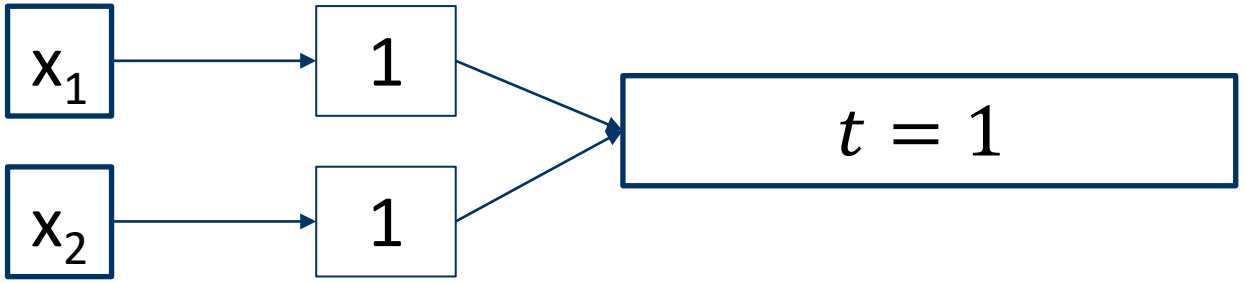
Let's draw the perceptron to mimic Boolean gates

AND Gate



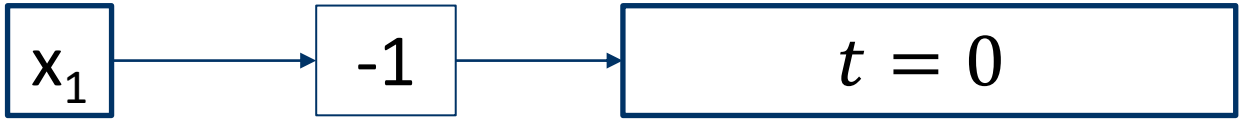
x1	x2	output	value
0	0	0	False
1	0	1	False
0	1	1	False
1	1	2	True

OR Gate



x1	x2	output	value
0	0	0	False
1	0	1	True
0	1	1	True
1	1	2	True

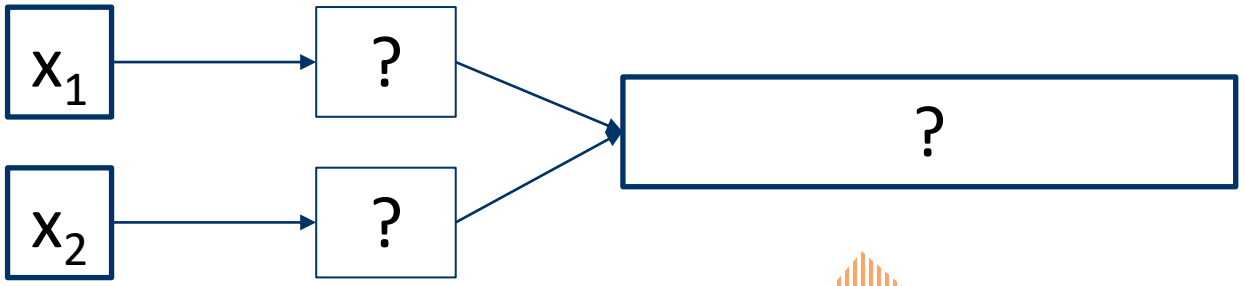
NOT Gate



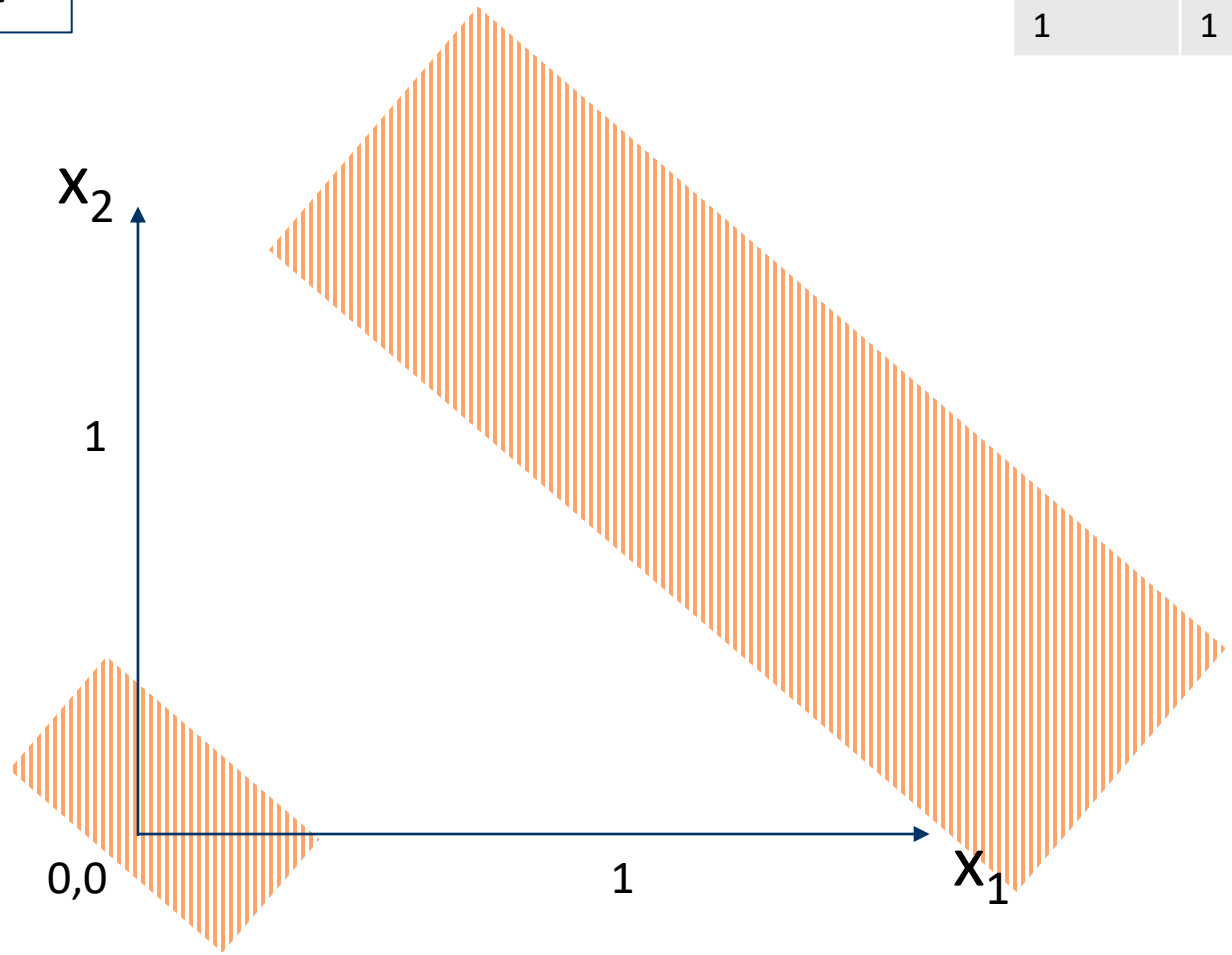
x1	output	value
0	0	True
1	-1	False

How about the XOR gate?

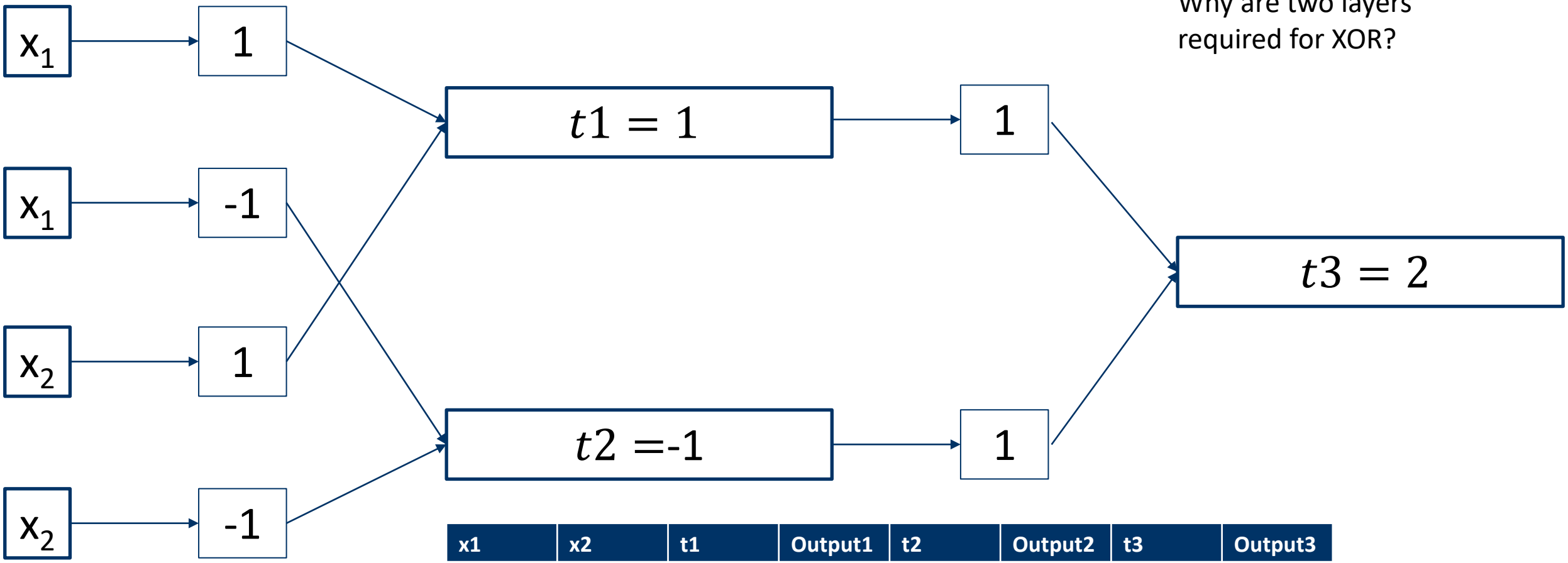
XOR Gate



x1	x2	output	value
0	0	0	False
1	0	1	True
0	1	1	True
1	1	2	False



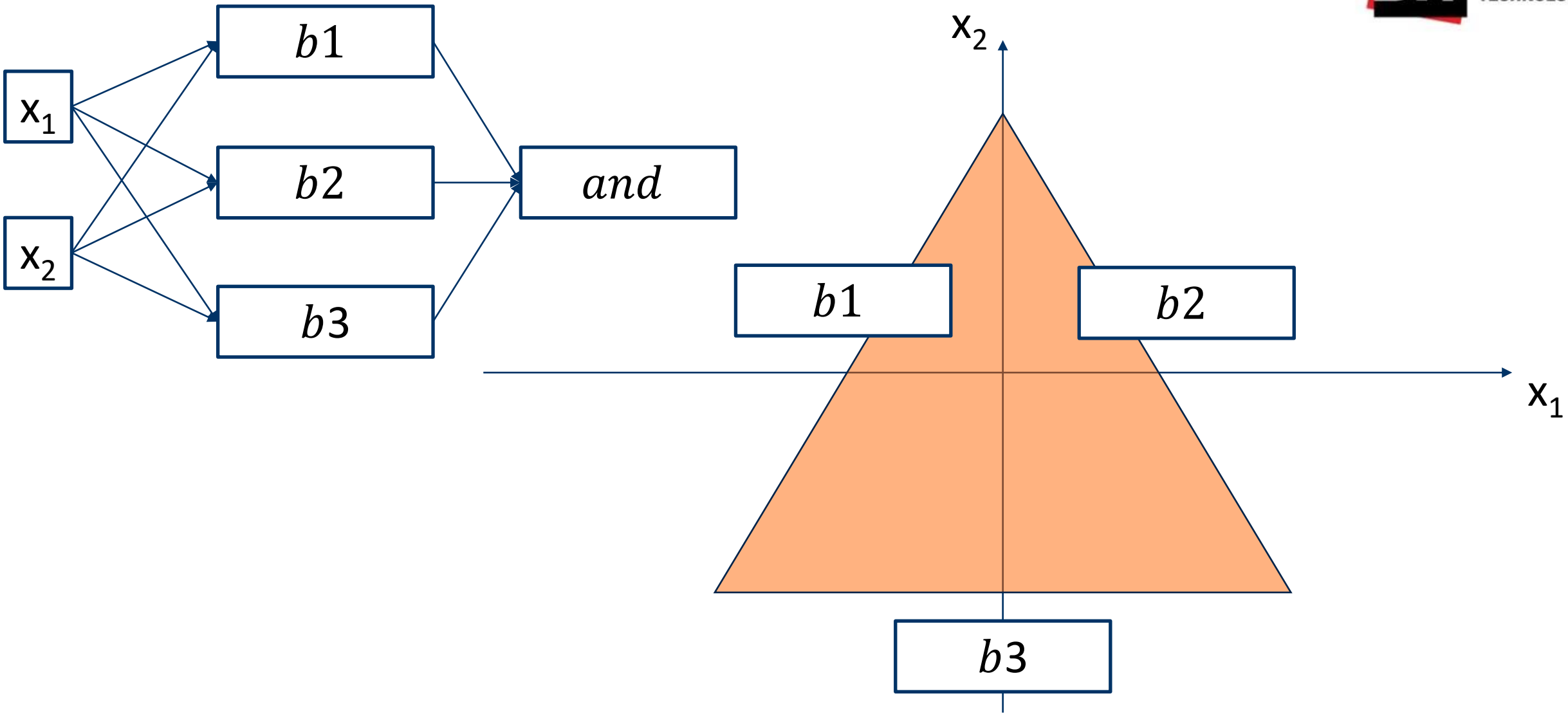
Multi Layer Perceptron is required



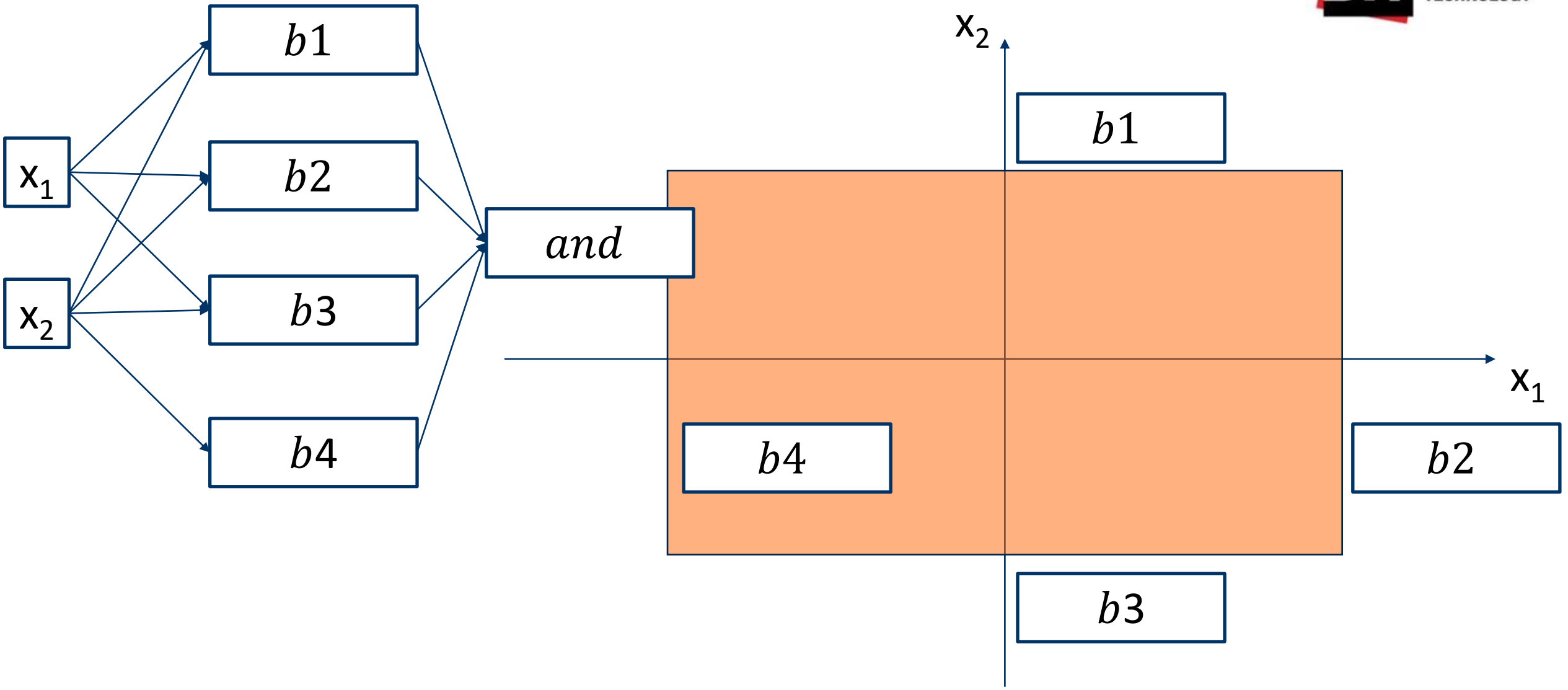
Why are two layers required for XOR?

x1	x2	t1	Output1	t2	Output2	t3	Output3
0	0	0	0 (False)	0	1 (True)	1	False
1	0	1	1 (True)	-1	1 (True)	2	True
0	1	1	1 (True)	-1	1 (True)	2	True
1	1	2	1 (True)	-2	0 (False)	1	False

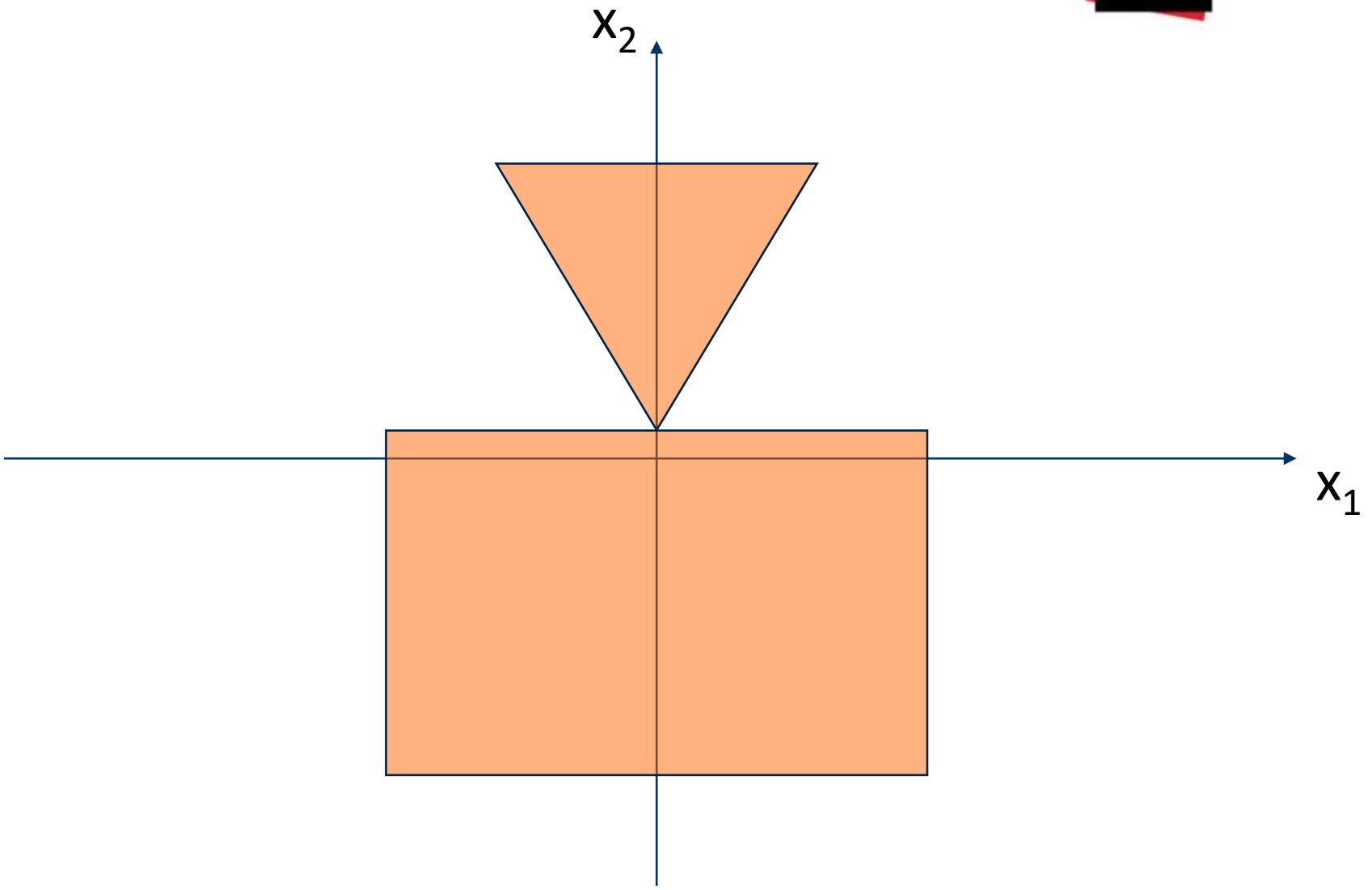
Complex Decision Boundaries: How would the representation look like?



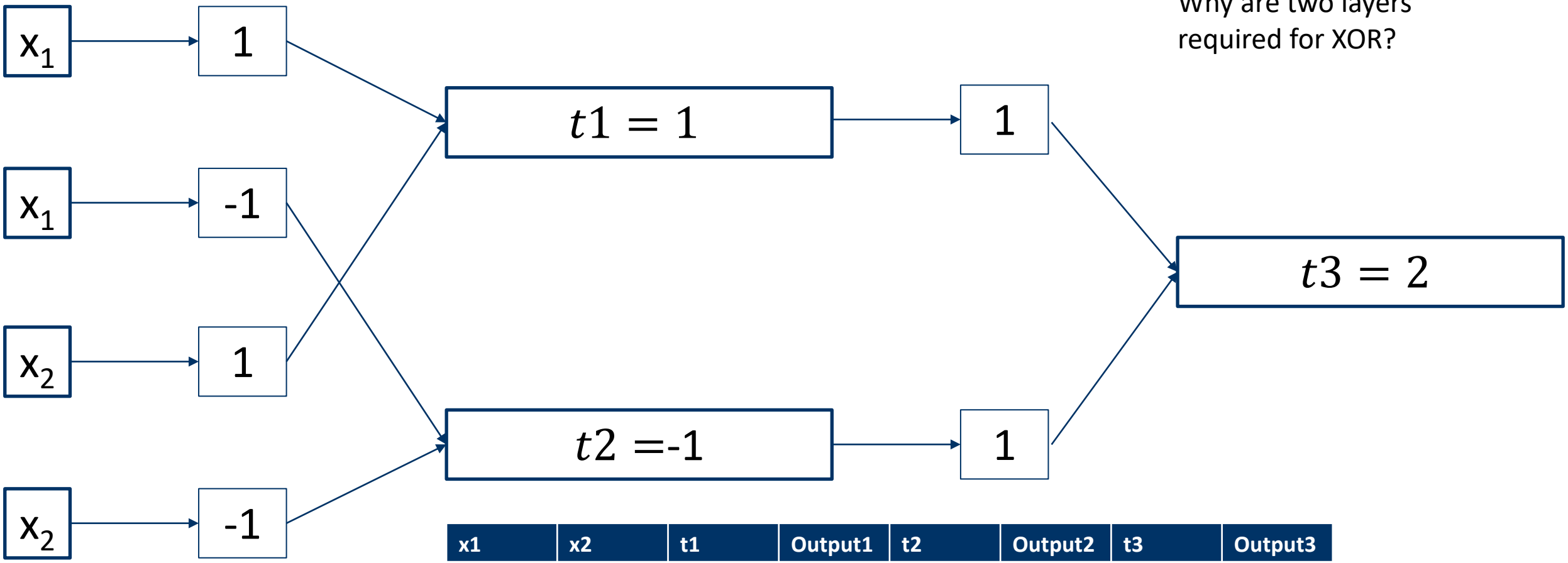
Complex Decision Boundaries: How would the representation look like?



Complex Decision Boundaries: How would the representation look like with two hidden layers?



Multi Layer Perceptron is required



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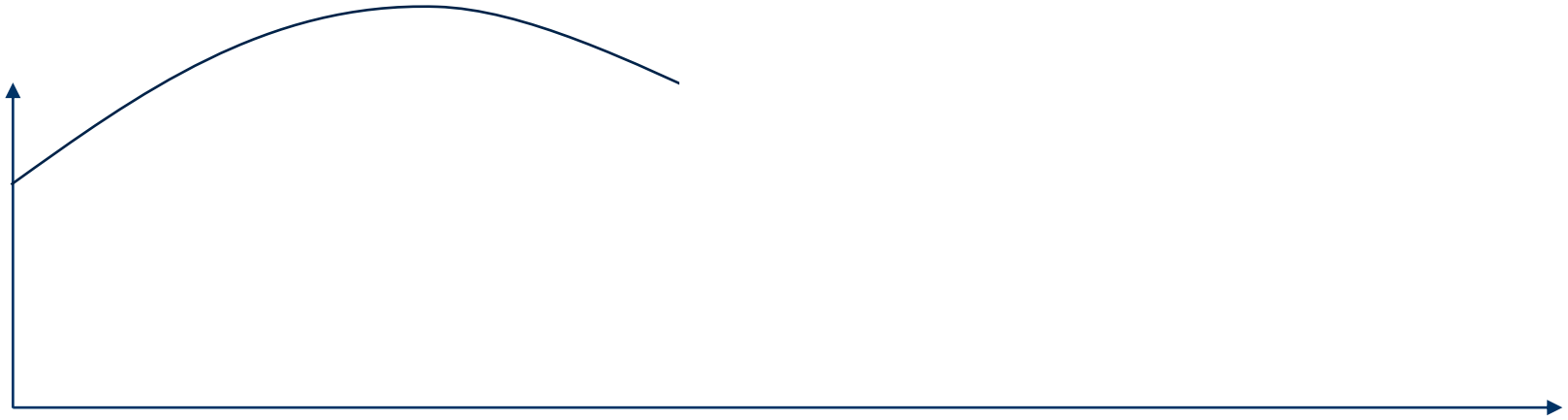
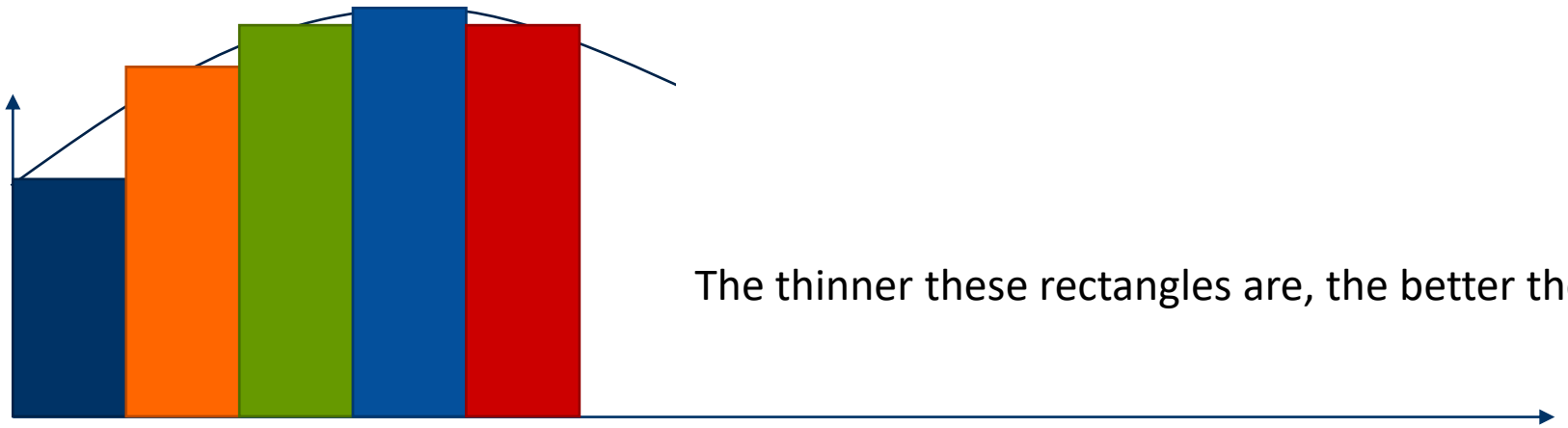
0	0	0	0 (False)	0	1 (True)	1	False
1	0	1	1 (True)	-1	1 (True)	2	True
0	1	1	1 (True)	-1	1 (True)	2	True
1	1	2	1 (True)	-2	0 (False)	1	False

What if the output value is a continuous real value?

Intuition: Suppose we have a continuous 2-d real value we wish to model with a neural network.



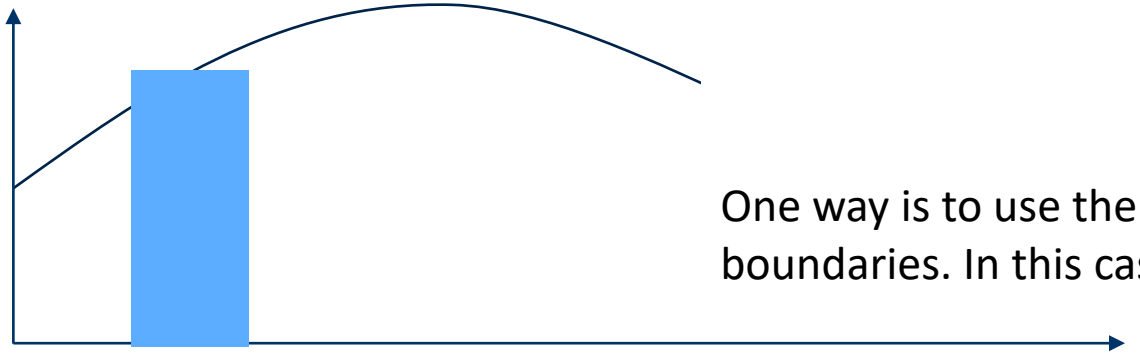
Intuition: Any continuous real value can be broken up into approximate rectangles.



The thinner these rectangles are, the better the approximation.

What if the output value is a continuous real value?

Challenge: Can we model one rectangle using a MLP?



One way is to use the technique we did earlier, having four boundaries. In this case, we can do a little bit better.

x → $t = t_1$

t_1

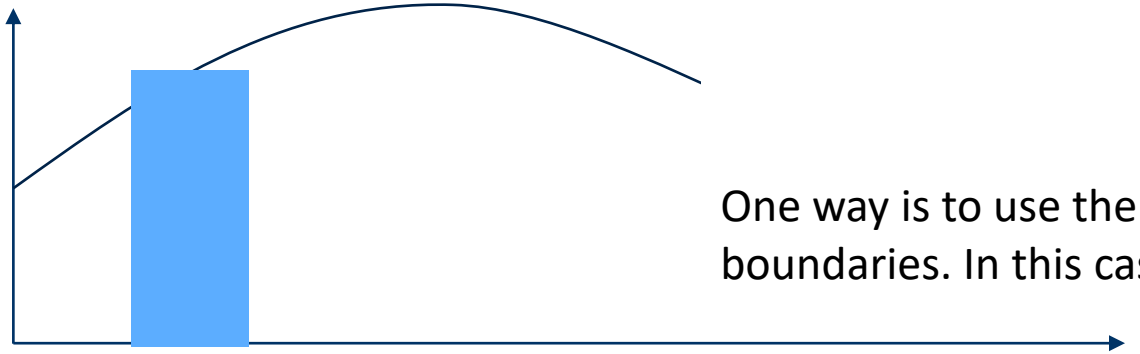
subtract

x → $t = t_2$

t_2

What if the output value is a continuous real value?

Challenge: Can we model one rectangle using a MLP?



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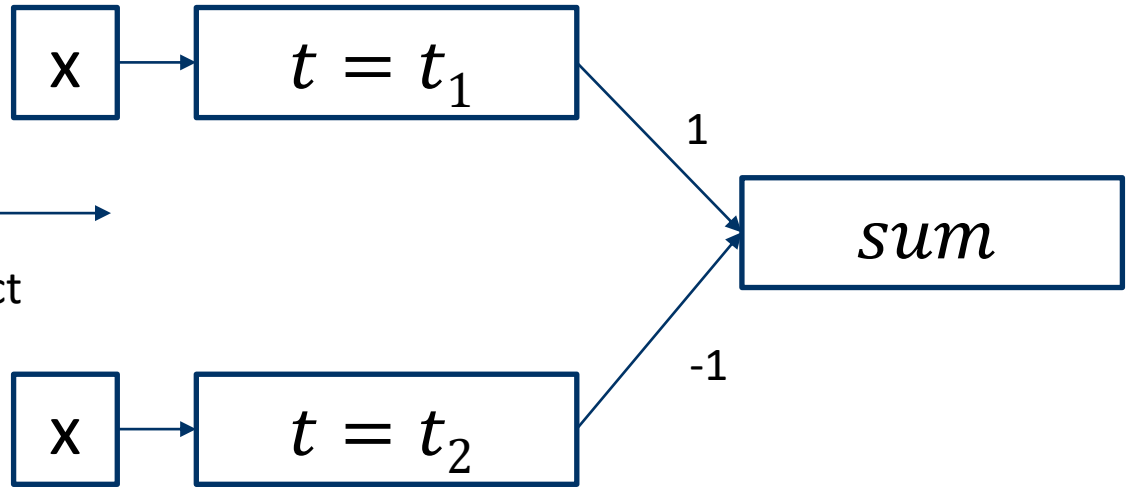
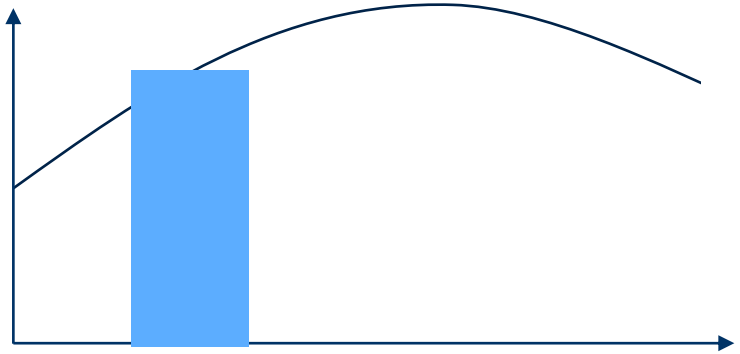
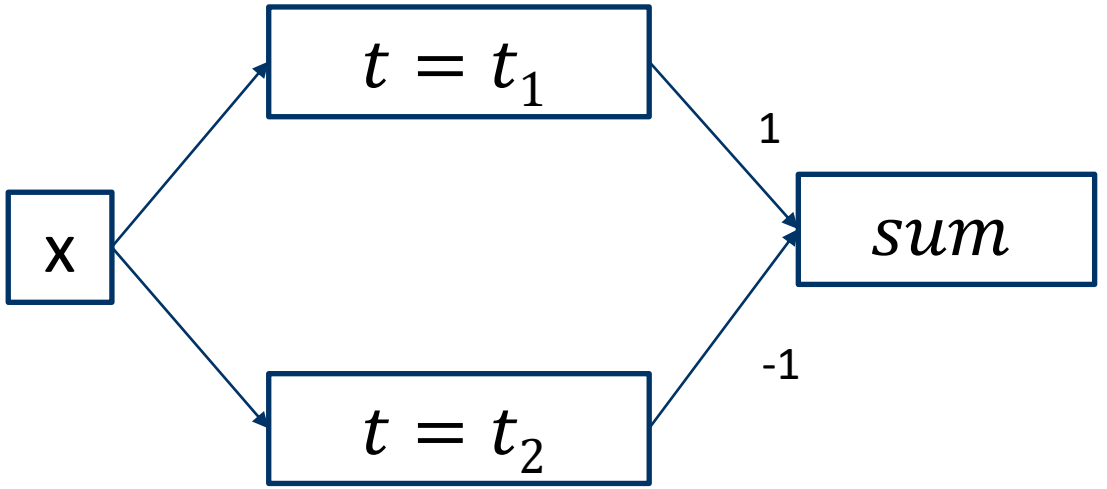
subtract

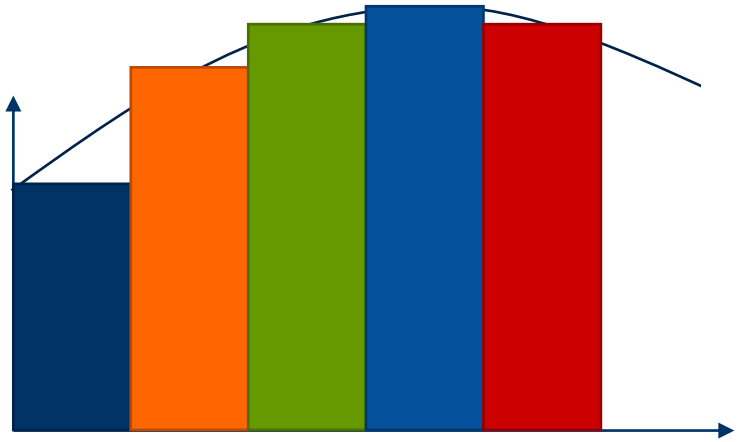


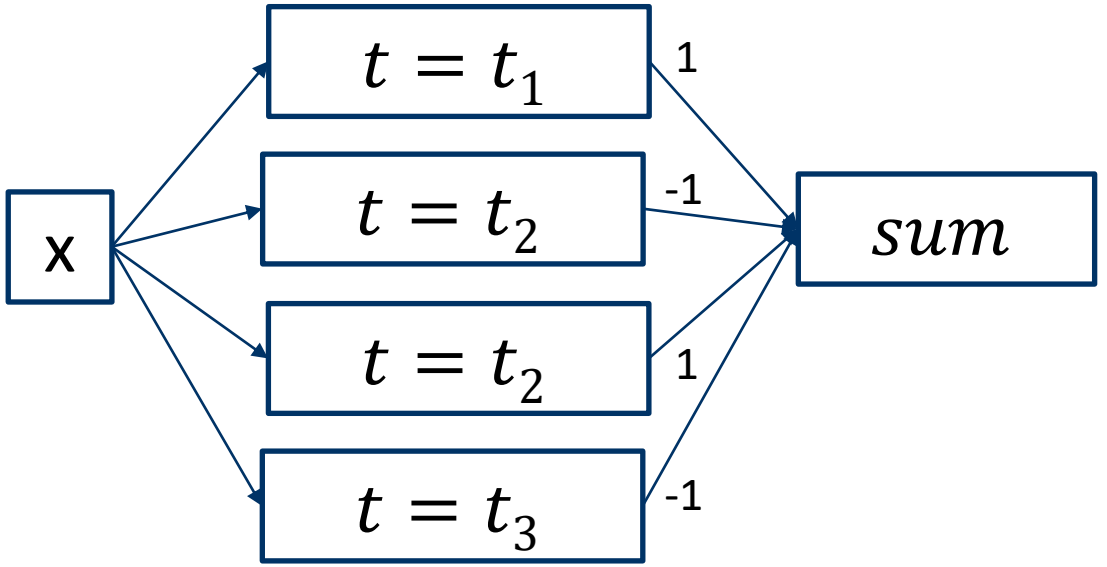
What if the output value is a continuous real value?

Challenge: Can we model one rectangle using a MLP?

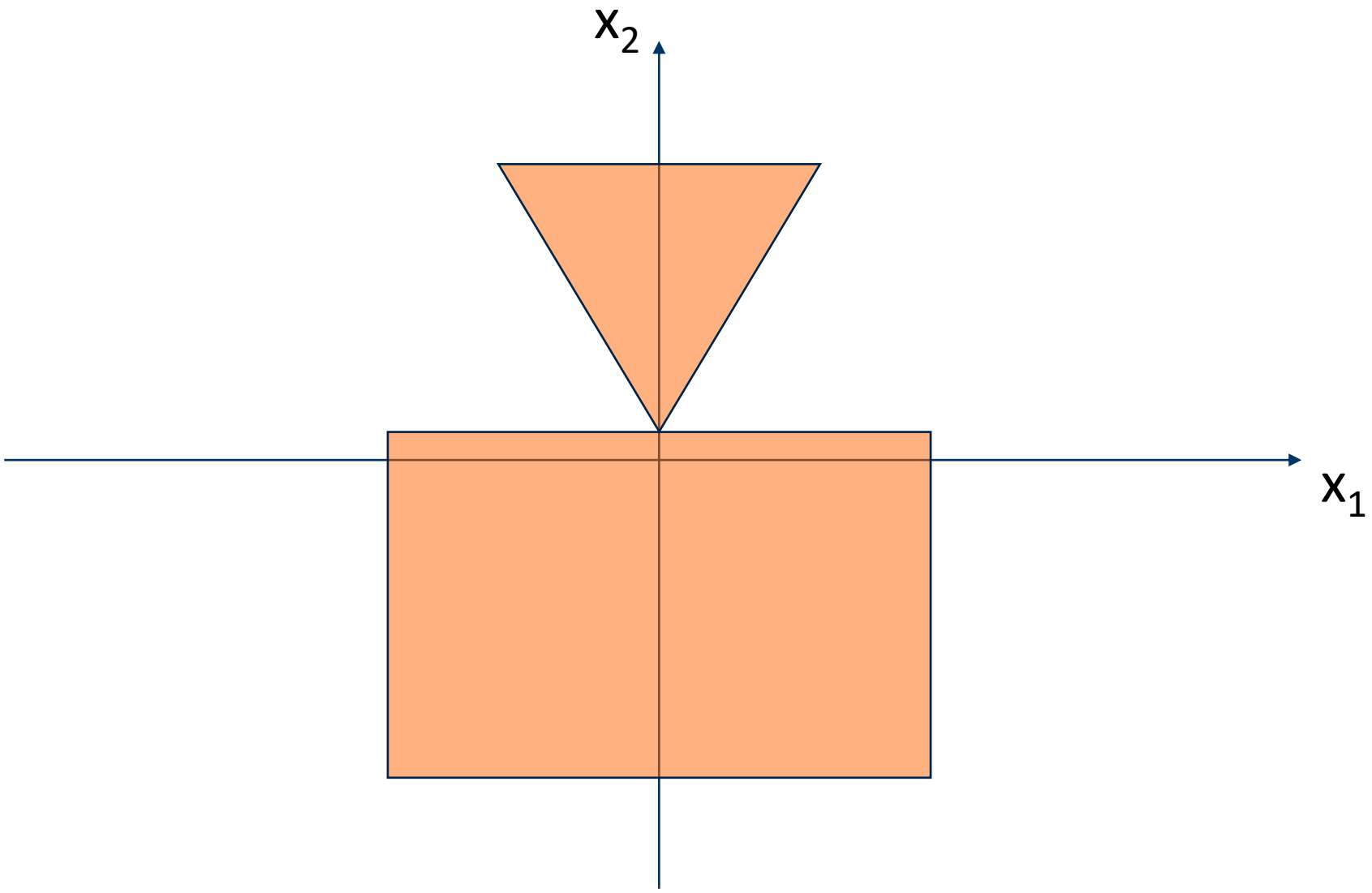




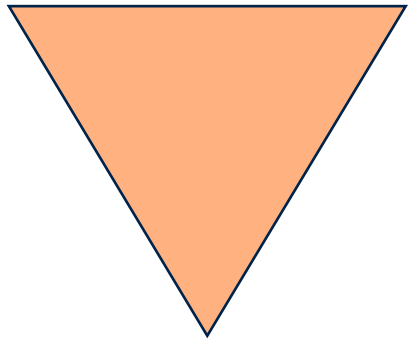
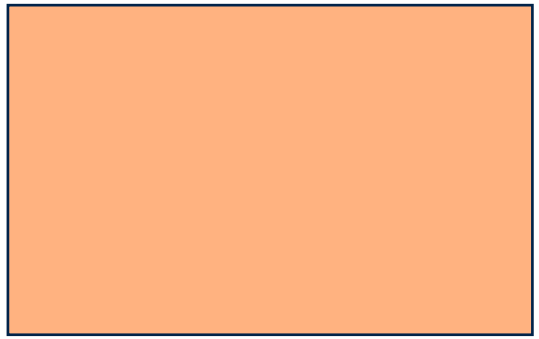
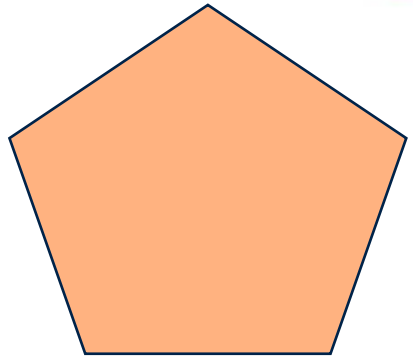
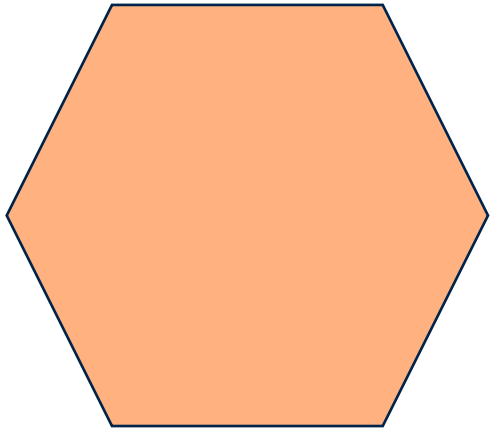
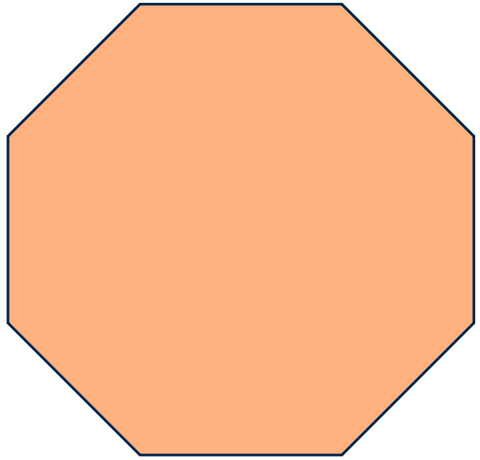
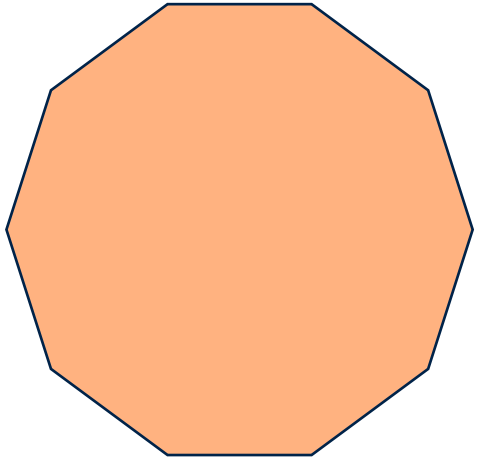
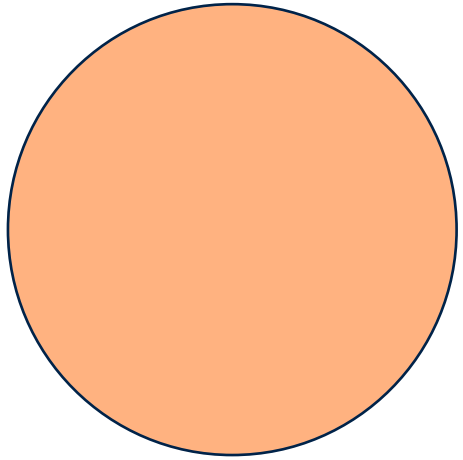




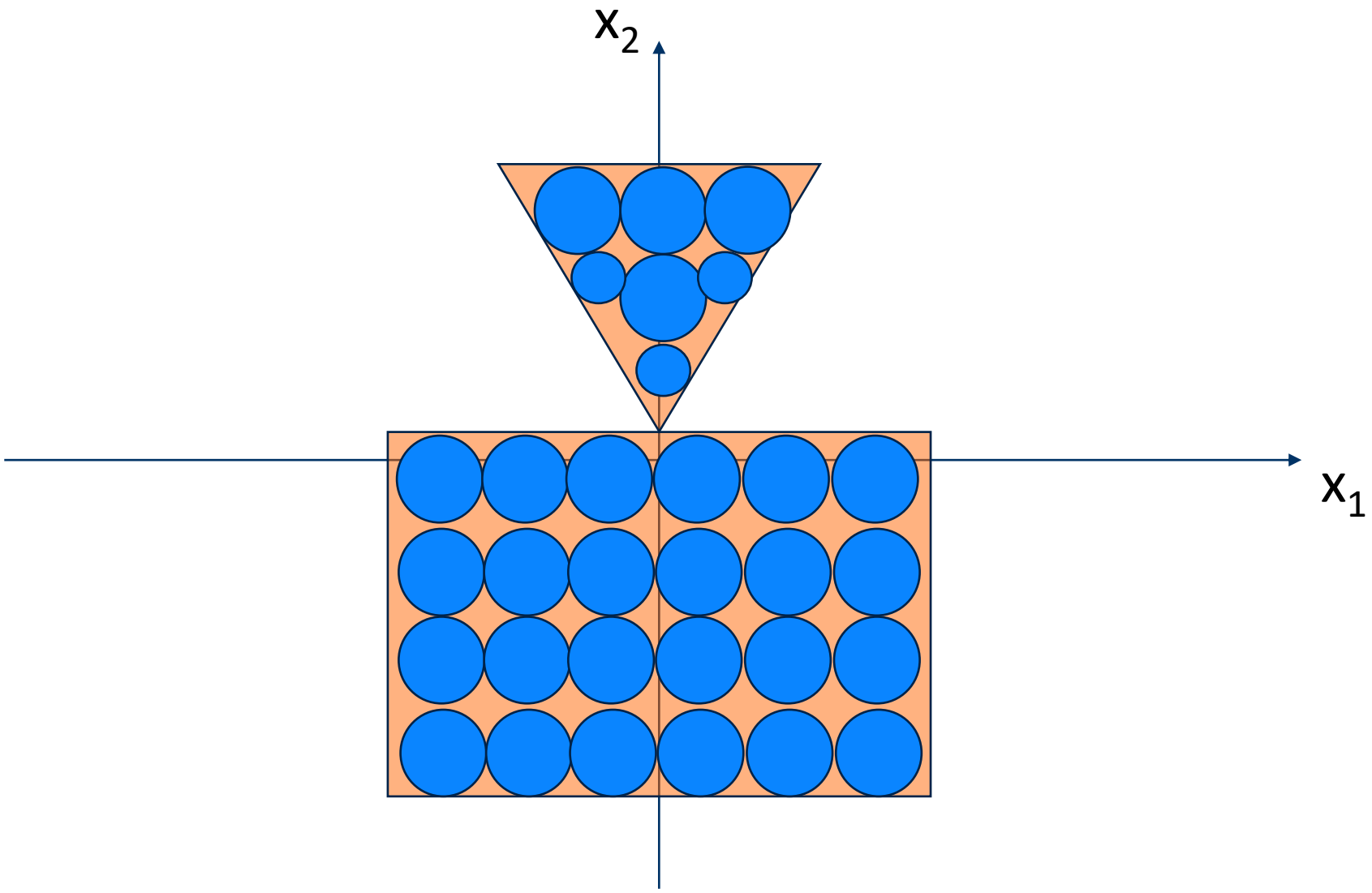
Complex Decision Boundaries: How would the representation look like with just one hidden layer?



Complex Decision Boundaries: How many neurons would we need if we have only one hidden layer?

Complex Decision Boundaries: How would the representation look like?



What are the steps in training a single layer perceptron?