JEE Main 2020
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I. Math

1) If $x = 1$ is a critical point of the function $f(x) = \left(3x^2 + ax - 2 - a\right)e^x$, then:
   a) $x = 1$ is a local minima and $x = -\frac{2}{3}$ is a local maxima of f.
   b) $x = 14$ is a local maxima and $x = -\frac{2}{3}$ is a local minima of f.
   c) $x = 1$ and $x = -\frac{2}{3}$ are local minima of f.
   d) $x = 1$ and $x = -\frac{2}{3}$ are local maxima of f.

2)
$$\lim_{x\to 0} x\frac{\left(e^{\frac{\sqrt{1+x^2+x^4}-1}{x}} - 1\right)}{\sqrt{1+x^2+x^4}-1} \tag{1}$$
   a) is equal to $\sqrt{e}$ b) is equal to 1 c) is equal to 0 d) does not exist

3) The statement $(p \to (q \to p)) \to (p \to (p \cup q))$ is:
   a) equivalent to $(p \cup q) \cap (\sim p)$
   b) equivalent to $(p \cap q) \cup (\sim p)$
   c) a contradiction
   d) a tautology

4) If $L = \sin^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$ and $M = \cos^2\left(\frac{\pi}{8}\right) - \sin^2\left(\frac{\pi}{8}\right)$, then:
   a) $M = \frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$
   b) $M = \frac{1}{4\sqrt{2}} + \frac{1}{4}\cos\frac{\pi}{8}$
   c) $L = -\frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$
   d) $L = \frac{1}{4\sqrt{2}}\frac{1}{4}\cos\frac{\pi}{8}$

5) If the sum of the first 20 terms of the series $\log_{7^{\frac{1}{2}}} x + \log_{7^{\frac{1}{3}}} x + \log_{7^{\frac{1}{4}}} x + \ldots$ is 460, then x is equal to:

   a) $7^{\frac{1}{2}}$ b) 7^2 c) e^2 d) $7^{\frac{46}{21}}$

6) There are 3 sections in a question paper and each section contains 5 questions candidate has to answer a total of 5 questions, choosing at least one question from each section. Then the number of ways, in which the candidate can choose the questions, is:

   a) 2250 b) 2255 c) 1500 d) 3000

7) If the mean and the standard deviation of the data $3, 5, 7, a, b$ are 5 and 2 respectively, then a and b are the roots of the equation:

a) $x^2 - 20x + 18 = 0$ b) $x^2 - 10x + 19 = 0$ c) $2x^2 - 20 + 19 = 0$ d) $x^2 - 10x + 18 = 0$

8) The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at $x = \frac{1}{2}$ is:

a) $\frac{2\sqrt{3}}{3}$ b) $\frac{2\sqrt{3}}{5}$ c) $\frac{\sqrt{3}}{12}$ d) $\frac{\sqrt{3}}{10}$

9) If

$$\int \frac{\cos\theta}{5 + 7\sin\theta - 2\cos^2\theta} d\theta = A \log_e |B(\theta)| + C \tag{2}$$

where C is a constant of integration, then $\frac{B(\theta)}{A}$ can be:

a) $\frac{5(2\sin\theta+1)}{\sin\theta+3}$ b) $\frac{5(\sin\theta+3)}{2\sin\theta+1}$ c) $\frac{2\sin\theta+1}{\sin\theta+3}$ d) $\frac{2\sin\theta+1}{5(\sin\theta+3)}$

10) If the length of the cord of the circle, $x^2 + y^2 = r^2\,(r > 0)$ along the line, $y - 2x = 3$ is r, then r^2 is equal to:

a) 12 b) $\frac{24}{5}$ c) $\frac{9}{5}$ d) $\frac{12}{5}$

11) If α and β are the roots of the equation, $7x^2 - 3x - 2 = 0$, then the value of $\frac{\alpha}{1-\alpha^2} + \frac{\beta}{1-\beta^2}$

a) $\frac{27}{32}$ b) $\frac{1}{24}$ c) $\frac{27}{16}$ d) $\frac{3}{8}$

12) If the sum of the second, third and fourth terms of a positive term G.P. is 3 and the sum of its sixth, seventh and eighth terms is 243, then the sum of the first 50 terms of the G.P. is:

a) $\frac{2}{13}\left(3^{50} - 1\right)$ b) $\frac{1}{26}\left(3^{49} - 1\right)$ c) $\frac{1}{13}\left(3^{50} - 1\right)$ d) $\frac{1}{26}\left(3^{50} - 1\right)$

13) If the line $y = mx + c$ is a common tangent to the hyperbola $\frac{x^2}{100} - \frac{y^2}{64} = 1$ and the circle $x^2 + y^2 = 36$, then which one of the following is true?

a) $4c^2 = 369$ b) $c^2 = 369$ c) $8m + 5 = 0$ d) $5m = 4$

14) The area (in sq.units) of the region

$$A = \left\{(x, y) : (x - 1)\,[x] \le y \le 2\sqrt{x}, 0 \le x \le 2\right\} \tag{3}$$

where $[t]$ denotes the greatest integer funtion, is:

a) $\frac{4}{3}\sqrt{2} - \frac{1}{2}$ b) $\frac{8}{3}\sqrt{2} - \frac{1}{2}$ c) $\frac{8}{3}\sqrt{2} - 1$ d) $\frac{4}{3}\sqrt{2} + 1$

15) If $a+x = b+y = c+z+1$, where a, b, c, x, y, z are non-zero distinct real numbers, then $\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix}$ is equal to:

a) $y\,(a - b)$ b) 0 c) $y\,(b - a)$ d) $y\,(a - c)$