

# 18. Definite Integrals

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## I. G:PASSAGE 6

- 1) Let  $F : R \rightarrow R$  be a thrice differentiable function. Suppose that  $F(1) = 0, F(3) = -4$  and  $F(x) < 0$  for all  $x \in (\frac{1}{2}, 3)$ . Let  $f(x) = xF(x)$  for all  $x \in R$  (JEE Adv. 2015)
- 2) The correct statement(s) is(are)
  - a)  $f'(1) < 0$
  - b)  $f(2) < 0$
  - c)  $f'(x) \neq 0$  for any  $x \in (1, 3)$
  - d)  $f'(x) = 0$  for some  $x \in (1, 3)$
- 3) If  $\int_1^3 x^2 F'(x) dx = -12$  and  $\int_1^3 x^3 F''(x) dx = 40$ , then the correct expression(s) is(are)
  - a)  $9f'(3) + f'(1) - 32 = 0$
  - b)  $\int_1^3 f(x) dx = 12$
  - c)  $9f'(3) - f'(1) + 32 = 0$
  - d)  $\int_1^3 f(x) dx = -12$

## II. INTEGER

- 1) Let  $f : R \rightarrow R$  be a continuous function which satisfies  $f(x) = \int_0^x f(t) dt$ . Then the value of  $f(\ln 5)$  is (2009)
- 2) For any real number  $x$ , let  $[x]$  denote the largest integer less than or equal to  $x$ . Let  $f$  be a real valued function defined on the interval  $[-10, 10]$  by  $f(x) =$

$$\begin{cases} x - [x] & \text{if } [x] \text{ is odd} \\ 1 + [x] - x & \text{if } [x] \text{ is even} \end{cases}$$

Then the value of  $\frac{\pi^2}{10} \int_{-10}^{10} f(x) \cos \pi x dx$  is (2011)

- 3) The value of

$$\int_0^1 4x^3 \left\{ \frac{d^2}{dx^2} (1 - x^2)^5 \right\} dx \quad (1)$$

is

(JEE Adv. 2014)

- 4) Let  $f : R \rightarrow R$  be a function defined by

$$f(x) = \{ [x], x \leq 2, 0 \text{ if } x > 2 \} \quad (2)$$

where  $[x]$  is the greatest integer less than or equal to  $x$ , if

$$I = \int_{-1}^2 \frac{xf(x^2)}{2 + f(x+1)} dx$$

, then the value of  $(4I - 1)$  is

(JEE Adv. 2015)

- 5) Let

$$F(x) = \int_x^{x^2 + \frac{\pi}{6}} 2 \cos^2 t (dt) \quad (3)$$

for all  $x \in \mathbb{R}$  and  $f : \left[0, \frac{1}{2}\right] \rightarrow [0, \infty)$  be a continuous function. For  $a \in \left[0, \frac{1}{2}\right]$ , if  $F'(a) + 2$  is the area of the region bounded by  $x = 0, y = 0, y = f(x)$  and  $x = a$ , then  $f(0)$  is (JEE Adv. 2015)

6) If

$$\alpha = \int_0^1 \left( e^{9x+3 \tan^{-1} x} \right) \left( \frac{12+9x^2}{1+x^2} \right) dx \quad (4)$$

where  $\tan^{-1} x$  takes only principal values, then the value of  $\left( \log_e |1 + \alpha| - \frac{3\pi}{4} \right)$  is (JEE Adv. 2015)

7) Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a continuous odd function which vanishes exactly at one point and  $f(1) = \frac{1}{2}$ . Suppose that  $F(x) = \int_{-1}^x f(t) dt$  for all  $x \in [-1, 2]$  and  $G(x) = \int_{-1}^x t|f(f(t))| dt$  for all  $x \in [-1, 2]$ . If

$$\lim_{x \rightarrow 1} \frac{F(x)}{G(x)} = \frac{1}{14} \quad (5)$$

, then the value of  $f\left(\frac{1}{2}\right)$  is (JEE Adv. 2015)

8) The total number of distinct  $x \in [0, 1]$  for which

$$\int_0^x \frac{t^2}{1+t^4} dt = 2x - 1 \quad (6)$$

is (JEE Adv. 2016)

9) Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a differentiable function such that  $f(0) = 0$ ,  $f\left(\frac{\pi}{2}\right) = 3$  and  $f'(0) = 1$ . If

$$g(x) = \int_x^{\frac{\pi}{2}} [f'(t) \operatorname{cosec} t - \cot t \operatorname{cosec} t f(t)] dt \quad (7)$$

for  $x \in \left(0, \frac{\pi}{2}\right]$ , then  $\lim_{x \rightarrow 0} g(x) =$  (JEE Adv. 2018)

10) For each positive integer  $n$ , let

$$y_n = \frac{1}{n} (n+1)(n+2) \dots (n+n)^{\frac{1}{n}} \quad (8)$$

For  $x \in \mathbb{R}$ , let  $[x]$  be the greatest integer less than or equal to  $x$ . If  $\lim_{n \rightarrow \infty} y_n = L$ , then the value of  $[L]$  is (JEE Adv. 2018)

11) A farmer  $F_1$  has a land in the shape of a triangle with vertices at  $\mathbf{P} = (0, 0)$ ,  $\mathbf{Q} = (1, 1)$  and  $\mathbf{R} = (2, 0)$ . From this land, a neighbouring farmer  $F_2$  takes away the region which lies between the side PQ and a curve of the form  $y = x^n$  ( $n > 1$ ). If the area of the region taken away by the farmer  $F_2$  is exactly 30% of the area of  $\Delta PQR$ , then the value of  $n$  is (JEE Adv. 2018)

12) The value of the integral

$$\int_0^{\frac{1}{2}} \frac{1 + \sqrt{3}}{\left((x+1)^2 (1-x)^6\right)^{\frac{1}{4}}} dx \quad (9)$$

(JEE Adv. 2018)

13) If

$$I = \frac{2}{\pi} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{(1 + e^{\sin x})(2 - \cos 2x)}, \quad (10)$$

then  $27I^2$  equals (JEE Adv. 2019)

14) The value of the integral

$$\int_0^{\frac{\pi}{2}} \frac{3\sqrt{\cos \theta}}{(\sqrt{\cos \theta} + \sqrt{\sin \theta})^5} d\theta \text{ equals} \quad (11)$$

(JEE Adv. 2019)