18.Definite Integrals

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I. G:Passage 6

1) Let $F : R \to R$ be a thrice differentiable function. Suppose that $F(1) = 0, F(3) = -4$ and $F(x) < 0$ for all x $\in \left(\frac{1}{2}, 3\right)$. Let $f(x) = xF(x)$ for all x$\in$ **R** (JEE Adv. 2015)
2) The correct statement(s) is(are)
   a) $f'(1) < 0$
   b) $f(2) < 0$
   c) $f'(x) \neq 0$ for any x $\in (1, 3)$
   d) $f'(x) = 0$ for some x $\in (1, 3)$
3) If $\int_1^3 x^2 F'(x)\, dx = -12$ and $\int_1^3 x^3 F"(x)\, dx = 40$, then the correct expression(s) is(are)
   a) $9f'(3) + f'(1) - 32 = 0$
   b) $\int_1^3 f(x)\, dx = 12$
   c) $9f'(3) - f'(1) + 32 = 0$
   d) $\int_1^3 f(x)\, dx = -12$

II. Integer

1) Let f:R$\to$ R be a continuous function which satisfies $f(x) = \int_0^x f(t)\, dt$. Then the value of $f(ln5)$ is (2009)
2) For any real number x, let $[x]$ denote the largest integer less than or equal to x. Let f be a real valued function defined on the interval $[-10, 10]$ by $f(x)$=

$$\begin{cases} \text{x- }[x] \text{ if } [x] isodd \\ \text{1+ }[x]\text{-x if } [x] \text{ is even} \end{cases}$$

   Then the value of $\frac{\pi^2}{10} \int\limits_{-10}^{10} f(x) \cos \pi$ xdx is (2011)
3) The value of $\int_0^1 4x^3 \left\{ \frac{d^2}{dx^2} \left(1 - x^2\right)^5 \right\} dx$ is (JEE Adv. 2014)
4) Let f:R $\to$ R be a function defined by $f(x) = \{ [x], x \leq 2, 0 if x > 2\}$ where $[x]$ is the greatest integer less than or equal to x, if

$$I = \int_{-1}^{2} \frac{xf\left(x^2\right)}{2 + f(x+1)} dx$$

   , then the value of (4I-1) is (JEE Adv. 2015)
5) Let $F(x) = \int_x^{x^2+\frac{\pi}{6}} 2\cos^2 t(dt)$ for all x $\in$ R and $f : \left[0, \frac{1}{2}\right] \to [0, \infty)$ be a continuous function. For a$\in \left[0, \frac{1}{2}\right]$, if $F'(a) + 2$ is the area of the region bounded by x=0, y=0, $y = f(x)$ and x=a, then $f(0)$ is (JEE Adv. 2015)
6) If $\alpha = \int_0^1 \left(e^{9x+3\tan^{-1} x}\right) \left(\frac{12+9x^2}{1+x^2}\right) dx$ where $tan^{-1} x$ takes only principal values, then the value of $\left(\log_e |1 + \alpha| - \frac{3\pi}{4}\right)$ is (JEE Adv. 2015)
7) Let f:R$\to$R be a continuous odd function which vanishes exactly at one point and $f(1) = \frac{1}{2}$. Suppose that $F(x) = \int_{-1}^x f(t)\, dt$ for all $x \in [-1, 2]$ and $G(x) = \int_{-1}^x t|f(f(t))|dt$ for all $x \in [-1, 2]$. If $lim_{x\to 1} \frac{F(x)}{G(x)} = \frac{1}{14}$, then the value of $f\left(\frac{1}{2}\right)$ is (JEE Adv. 2015)
8) The total number of distinct $x \in [0, 1]$ for which $\int_0^x \frac{t^2}{1+t^4} dt = 2x - 1$ is (JEE Adv. 2016)

9) Let $f : R \rightarrow \mathrm{R}$ be a differentiable function such that $f(0) = 0$, $f\left(\frac{\pi}{2}\right) = 3$ and $f'(0) = 1$. If $g(x) = \int_x^{\frac{\pi}{2}} [f'(t)\ \operatorname{cosec} t - \cot t \operatorname{cosec} t f(t)]\, dt$ for $x \in \left(0, \frac{\pi}{2}\right]$, then $\lim_{x\to 0} g(x) =$ (JEE Adv. 2018)

10) For each positive integer n, let $y_n = \frac{1}{n}(n+1)(n+2)\ldots(n+n)^{\frac{1}{n}}$ For $x \in R$, let $[x]$ be the greatest integer less than or equal to x. If $\lim_{n\to\infty} y_n$=L, then the value of [L] is (JEE Adv. 2018)

11) A farmer F_1 has a land in the shape of a triangle with vertices at $\mathbf{P} = (0,0)$,$\mathbf{Q} = (1,1)$ and $\mathbf{R} = (2,0)$. From this land, a neighbouring farmer F_2 takes away the region which lies between the side PQ and a curve of the form $y = x^n\ (n > 1)$. If the area of the region taken away by the farmer F_2 is exactly 30% of the area of Δ PQR, then the value of n is (JEE Adv. 2018)

12) The value of the integral $\int_0^{\frac{1}{2}} \frac{1+\sqrt{3}}{\left((x+1)^2(1-x)^6\right)^{\frac{1}{4}}} dx$ is (JEE Adv. 2018)

13) If $I = \frac{2}{\pi}\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{\left(1+e^{\sin x}\right)(2-\cos 2x)}$, then 27 I^2 equals (JEE Adv. 2019)

14) The value of the integral $\int_0^{\frac{\pi}{2}} \frac{3\sqrt{\cos\theta}}{\left(\sqrt{\cos\theta}+\sqrt{\sin\theta}\right)^5} d\theta$ equals (JEE Adv. 2019)