Using Matrices to Find the External Division Point of a Line Segment

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Problem Statement

Given points $\mathbf{X}$ and $\mathbf{Y}$ with position vectors:

$$\mathbf{X} = 3\mathbf{a} + \mathbf{b}, \quad \mathbf{Y} = \mathbf{a} - 3\mathbf{b}$$

find the position vector of point $\mathbf{V}$ which divides the line segment XY in the ratio $2 : 1$ externally.

Section Formula for External Division

If a point $\mathbf{V}$ divides the line segment joining $\mathbf{X}$ and $\mathbf{Y}$ externally in the ratio $m : n$, then:

$$\mathbf{V} = \frac{m\mathbf{Y} - n\mathbf{X}}{m - n}$$

Here, $m = 2$ and $n = 1$. This approach is based on the section formula, which is commonly used in analytic geometry.

Why Use Matrices?

Efficiency in Computation: Using matrices allows us to represent and manipulate vectors efficiently, especially when working with multiple points or vector transformations.

Code Simplification: Matrix representations make coding vector calculations easier, reducing the risk of errors when handling individual coordinates.

Scalability: The matrix approach can easily be extended to higher dimensions or more complex transformations, which is useful in fields such as computer graphics and machine learning.

Calculating $\mathbf{V}$ Using the Section Formula

Substitute $\mathbf{X} = 3\mathbf{a} + \mathbf{b}$ and $\mathbf{Y} = \mathbf{a} - 3\mathbf{b}$ into the formula:

$$\mathbf{V} = \frac{2\begin{pmatrix}1 & -3\end{pmatrix}\begin{pmatrix}\mathbf{a}\\ \mathbf{b}\end{pmatrix} - 1\begin{pmatrix}3 & 1\end{pmatrix}\begin{pmatrix}\mathbf{a}\\ \mathbf{b}\end{pmatrix}}{2-1}$$

Simplifying,

$$\mathbf{V} = -\mathbf{a} - 7\mathbf{b} \tag{0.1}$$

Results and Visualization

`https://github.com/CharithaAI11014/Matgeo/blob/master/codes/plot.py` plots the position vector (**??**)
The code in `https://github.com/CharithaAI11014/Matgeo/blob/master/codes/section.c` uses matrices and Matsecext function and calculates the position vector.