CS771: Assignment-1

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Abstract

This document contains the solutions of Assignment-1 of CS771 (Introduction of Machine Learning) by the group member mentioned above. This document submitted to the course instructor Prof. Purushottam Kar in the summer semester of 2023-24.

1 Cross Connection PUF

1.1 Solution of 1^{st} question

Given $\phi(\mathbf{c}) : \{0,1\}^{32} \to \mathbb{R}^D$ mapping 32-bit $0/1$-valued challenge vectors to D-dimensional feature vectors (for some $D > 0$) so that for any arbiter PUF, there exists a D- dimensional linear model $\mathbf{W} \in \mathbb{R}^D$ and a bias term $b \in \mathbb{R}$ such that for all CRPs $\mathbf{c} \in \{0,1\}^{32}$, we have

$$\mathbf{W}^T\phi(\mathbf{c}) + b = t^u(\mathbf{c})$$

where $t^u(\mathbf{c})$ is the time it takes for the upper signal to reach the finish line when challenge $\mathbf{c}$ is input

Let $d_i = 1 - 2c_i \ \ \forall i = 0, 1, ..., 31$

To derive $t^u(\mathbf{c})$,

$$\begin{aligned}
t_1^u &= (1 - c_1)(t_0^u + p_1) + c_1(t_0^l + s_1) \\
&= (1 - 2c_1 + 1)\frac{(t_0^u + p_1)}{2} - (1 - 2c_1 - 1)\frac{(t_0^l + s_1)}{2} \\
&= (d_1 + 1)\frac{(t_0^u + p_1)}{2} - (d_1 - 1)\frac{(t_0^l + s_1)}{2} \\
&= d_1\frac{(t_0^u - t_0^l)}{2} + d_1\frac{(p_1 + s_1)}{2} + \frac{(t_0^u + t_0^l)}{2} + \frac{(p_1 + s_1)}{2} \\
t_1^l &= (1 - c_1)(t_0^l + q_1) + c_1(t_0^u + r_1) \\
&= (1 - 2c_1 + 1)\frac{(t_0^l + q_1)}{2} - (1 - 2c_1 - 1)\frac{(t_0^u + r_1)}{2} \\
&= (d_1 + 1)\frac{(t_0^l + q_1)}{2} - (d_1 - 1)\frac{(t_0^u + r_1)}{2} \\
&= -d_1\frac{(t_0^u - t_0^l)}{2} + d_1\frac{(q_1 + r_1)}{2} + \frac{(t_0^u + t_0^l)}{2} + \frac{(q_1 + r_1)}{2}
\end{aligned}$$

Let $x_i = t_i^u - t_i^l \ \forall i = 0, 1, ..., 31$

$$
\begin{aligned}
x_1 &= t_1^u - t_1^l \\
&= d_1(t_0^u - t_0^l) + d1\frac{p_1 - q_1 + s_1 - r_1}{2} + \frac{p_1 - q_1 + s_1 - r_1}{2} \\
&= d_1 x_0 + d_1\alpha_1 + \alpha_1 \;\; ; \alpha_1 = \frac{p_1 - q_1 + s_1 - r_1}{2}
\end{aligned}
$$

Let $x_{-1} = 0$ so

$$
\begin{aligned}
x_0 &= d_0 x_{-1} + d_0\alpha_0 + \alpha_0 \\
&= d_0\alpha_0 + \alpha_0 \\
x_1 &= d_1 x_0 + d_1\alpha_1 + \alpha_1 \\
&= \alpha_0 \cdot d_1 d_0 + (\alpha_0 + \alpha_1) \cdot d_1 + \alpha_1 \\
x_2 &= d_2 x_1 + d_2\alpha_2 + \alpha_2 \\
&= d_2(\alpha_0 \cdot d_1 d_0 + (\alpha_0 + \alpha_1) \cdot d_1 + \alpha_1) + d_2\alpha_2 + \alpha_2 \\
&= \alpha_0 \cdot d_2 d_1 d_0 + (\alpha_0 + \alpha_1) \cdot d_2 d_1 + (\alpha_1 + \alpha_2) \cdot d_2 + \alpha_2 \\
&\vdots \\
x_{31} &= \mathbf{U}^T\gamma(\mathbf{c}) + b_x
\end{aligned}
$$

$x_{31} = \mathbf{U}^T\gamma(\mathbf{c}) + b_x$

$$
\begin{aligned}
\mathbf{U} &= [U_0, U_1, ..., U_{31}]_{\text{32x1}} \\
U_0 &= \alpha_0 \\
U_i &= \alpha_{i-1} + \alpha_i \;\; \forall i = 1, 2, ..., 31 \\
\gamma(\mathbf{c}) &= [\gamma_0, \gamma_1, ..., \gamma_{31}]_{\text{32x1}} \\
\gamma_i &= d_i d_{i+1} d_{i+2} ......d_{31} \;\; \forall i = 0, 1, ..., 31 \\
b_x &= \alpha_{31}
\end{aligned}
$$

Let $y_i = t_i^u + t_i^l \; \forall i = 0, 1, ..., 31$

$$
\begin{aligned}
y_1 &= t_1^u + t_1^l \\
&= d_1\frac{p_1 + s_1 + q_1 + r_1}{2} + (t_0^u + t_0^l) + \frac{p_1 + s_1 + q_1 + r_1}{2} \\
&= d_1\beta_1 + y_0 + \beta_1 \;\; ; \beta_1 = \frac{p_1 + s_1 + q_1 + r_1}{2}
\end{aligned}
$$

Let $y_{-1} = 0$

$$
\begin{aligned}
y_0 &= d_0\beta_0 + y_{-1} + \beta_0 \\
&= d_0\beta_0 + \beta_0 \\
y_1 &= y_0 + d_1\beta_1 + \beta_1 \\
&= d_0\beta_0 + d_1\beta_1 + (\beta_0 + \beta_1) \\
y_2 &= y_1 + d_2\beta_2 + \beta_2 \\
&= d_0\beta_0 + d_1\beta_1 + d_2\beta_2 + (\beta_0 + \beta_1 + \beta_2) \\
&\vdots \\
y_{31} &= \mathbf{V}^T\delta(\mathbf{c}) + b_y
\end{aligned}
$$

$y_{31} = \mathbf{V}^T \delta(\mathbf{c}) + b_y$

$$
\begin{aligned}
\mathbf{V} &= [V_0, V_1, ..., V_{31}]_{32\text{x}1} \\
V_i &= \beta_i \;\; \forall i = 0, 1, ..., 31 \\
\delta(\mathbf{c}) &= [\delta_0, \delta_1, ..., \delta_{31}]_{32\text{x}1} \\
\delta_i &= d_i \;\; \forall i = 0, 1, ..., 31 \\
b_y &= \sum_{i=0}^{31} \beta_i
\end{aligned}
$$

Thus, $t^u(\mathbf{c}) = \frac{x_{31}+y_{31}}{2}$

$$
\begin{aligned}
t^u(\mathbf{c}) &= \frac{x_{31} + y_{31}}{2} \\
&= \frac{\mathbf{U}^T \gamma(\mathbf{c}) + b_x + \mathbf{V}^T \delta(\mathbf{c}) + b_y}{2} \\
&= \frac{\mathbf{U}^T \gamma(\mathbf{c}) + \mathbf{V}^T \delta(\mathbf{c})}{2} + \frac{b_x + b_y}{2} \\
&= \mathbf{W}^T \phi(\mathbf{c}) + b \\
\phi(\mathbf{c}) &= [\phi_0, \phi_1, ..., \phi_{62}]_{63\text{x}1} \\
\phi_i &= \begin{cases} d_i d_{i+1} d_{i+2} ...... d_{31} & , 0 \leq i \leq 31 \\ d_{63-i-1} & , 32 \leq i \leq 62 \end{cases} \\
\mathbf{W} &= [W_0, W_1, ..., W_{62}]_{63\text{x}1} \\
W_i &= \begin{cases} \frac{\alpha_0}{2} & , i = 0 \\ \frac{(\alpha_{i-1} + \alpha_i)}{2} & , 1 \leq i \leq 30 \\ \frac{(\alpha_{i-1} + \alpha_i + \beta_i)}{2} & , i = 31 \\ \frac{\beta_i}{2} & , 32 \leq i \leq 62 \end{cases} \\
b &= \frac{b_x + b_y}{2} \\
&= \frac{\alpha_{31} + \sum_{i=0}^{31} \beta_i}{2}
\end{aligned}
$$

Therefore, $t^u(\mathbf{c}) = \mathbf{W}^T \phi(\mathbf{c}) + b$

Thus $t^l(\mathbf{c}) = \frac{y_{31} - x_{31}}{2}$

$$
\begin{aligned}
t^l(\mathbf{c}) &= \frac{y_{31} - x_{31}}{2} \\
&= \frac{\mathbf{V}^T\delta(\mathbf{c}) + b_y - \mathbf{U}^T\gamma(\mathbf{c}) - b_x}{2} \\
&= \frac{\mathbf{V}^T\delta(\mathbf{c}) - \mathbf{U}^T\gamma(\mathbf{c})}{2} + \frac{b_y - b_x}{2} \\
&= \mathbf{P}^T\phi(\mathbf{c}) + q \\
\phi(\mathbf{c}) &= [\phi_0, \phi_1, ..., \phi_{62}]_{\text{63x1}} \\
\phi_i &= \begin{cases} d_i d_{i+1} d_{i+2} ...... d_{31} & , 0 \le i \le 31 \\ d_{63-i-1} & , 32 \le i \le 62 \end{cases} \\
\mathbf{P} &= [P_0, P_1, ..., P_{62}]_{\text{63x1}} \\
P_i &= \begin{cases} \frac{-\alpha_0}{2} & , i = 0 \\ \frac{(-\alpha_{i-1} - \alpha_i)}{2} & , 1 \le i \le 30 \\ \frac{(\beta_i - \alpha_{i-1} - \alpha_i)}{2} & , i = 31 \\ \frac{\beta_i}{2} & , 32 \le i \le 62 \end{cases} \\
q &= \frac{b_y - b_x}{2} \\
&= \frac{\sum_{i=0}^{31} \beta_i - \alpha_{31}}{2}
\end{aligned}
$$

1.2 Solution of 2^{nd} question

The dimensionality of the linear model needs to predict the arrival time of the upper signal for an arbiter PUF.

1. The dimension of $\mathbf{W}$is 63 x 1 $(D = 63)$
2. The dimension of $\phi(\mathbf{c})$ is 63 x 1
3. $t^u(\mathbf{c}), b \in \mathbb{R}$ (a real number)

1.3 Solution of 3^{rd} question

Let the time taken

- For the upper signal of PUF0 to reach the finish line be $t^{u0} = \mathbf{G_0}^T\phi(\mathbf{c}) + h_0$
- For the lower signal of PUF0 to reach the finish line be $t^{l0} = \mathbf{W_0}^T\phi(\mathbf{c}) + b_0$
- For the upper signal of PUF1 to reach the finish line be $t^{u1} = \mathbf{G_1}^T\phi(\mathbf{c}) + h_1$
- For the lower signal of PUF1 to reach the finish line be $t^{l1} = \mathbf{W_1}^T\phi(\mathbf{c}) + b_1$

[In the 3^{rd} question, the values of $\mathbf{W}, \mathbf{G}, b, h$ are

- $\mathbf{W} = [W_0, W_1, ..., W_{62}]_{\text{63x1}}$
  W_i = $\begin{cases} \frac{-\alpha_0}{2} & , i = 0 \\ \frac{(-\alpha_{i-1} - \alpha_i)}{2} & , 1 \le i \le 30 \\ \frac{(\beta_i - \alpha_{i-1} - \alpha_i)}{2} & , i = 31 \\ \frac{\beta_i}{2} & , 32 \le i \le 62 \end{cases}$
- $\mathbf{G} = [G_0, G_1, ..., G_{62}]_{\text{63x1}}$
  G_i = $\begin{cases} \frac{\alpha_0}{2} & , i = 0 \\ \frac{(\alpha_{i-1} + \alpha_i)}{2} & , 1 \le i \le 30 \\ \frac{(\alpha_{i-1} + \alpha_i + \beta_i)}{2} & , i = 31 \\ \frac{\beta_i}{2} & , 32 \le i \le 62 \end{cases}$

- $h_i = \frac{\alpha_{31}+\sum_{j=0}^{31}\beta_j}{2} \; \forall \; i = 0, 1$
- $b_i = \frac{\sum_{j=0}^{31}\beta_j - \alpha_{31}}{2} \; \forall \; i = 0, 1$

]

Thus,

- In Arbiter1, $\Delta_1 = t^{u1} - t^{u0}$, then the Response1($r^1(\mathbf{c})$) = $\frac{1+\text{sign}(\Delta_1)}{2}$
- In Arbiter0, $\Delta_0 = t^{l1} - t^{l0}$, then the Response0($r^0(\mathbf{c})$) = $\frac{1+\text{sign}(\Delta_0)}{2}$

The Response0,

$$\begin{aligned} r^0(\mathbf{c}) &= \frac{1+\text{sign}(\Delta_0)}{2} \\ \Delta_0 &= t^{l1} - t^{l0} \\ &= \mathbf{W_1}^T\phi(\mathbf{c}) + b_1 - \mathbf{W_0}^T\phi(\mathbf{c}) - b_0 \\ &= (\mathbf{W_1} - \mathbf{W_0})^T\phi(\mathbf{c}) + (b_1 - b_0) \end{aligned}$$

Let

$$\begin{aligned} &\mathbf{W_1} - \mathbf{W_0} = \widetilde{\mathbf{W}} \\ &b_1 - b_0 = \widetilde{b} \end{aligned}$$

Thus,

$$\begin{aligned} \Delta_0 &= \widetilde{\mathbf{W}}^T\phi(\mathbf{c}) + \widetilde{b} \\ r^0(\mathbf{c}) &= \frac{1+\text{sign}(\widetilde{\mathbf{W}}^T\phi(\mathbf{c}) + \widetilde{b})}{2} \end{aligned}$$

Similarly, Response1 ($r^1(\mathbf{c})$),

$$\begin{aligned} r^1(\mathbf{c}) &= \frac{1+\text{sign}(\Delta_1)}{2} \\ \Delta_1 &= t^{u1} - t^{u0} \\ &= \mathbf{G_1}^T\phi(\mathbf{c}) + h_1 - \mathbf{G_0}^T\phi(\mathbf{c}) - h_0 \\ &= (\mathbf{G_1} - \mathbf{G_0})^T\phi(\mathbf{c}) + (h_1 - h_0) \end{aligned}$$

Let

$$\begin{aligned} &\mathbf{G_1} - \mathbf{G_0} = \widetilde{\mathbf{G}} \\ &h_1 - h_0 = \widetilde{h} \end{aligned}$$

Thus,

$$\begin{aligned} \Delta_1 &= \widetilde{\mathbf{G}}^T\phi(\mathbf{c}) + \widetilde{h} \\ r^1(\mathbf{c}) &= \frac{1+\text{sign}(\widetilde{\mathbf{G}}^T\phi(\mathbf{c}) + \widetilde{h})}{2} \end{aligned}$$

The response values $(r^1(\mathbf{c}), r^0(\mathbf{c}))$ are $r^i(\mathbf{c})$ = $\begin{cases} 0 & , \Delta_i < 0 \\ 1 & , \Delta_i > 0 \end{cases}$ for $i = 0, 1$

1.4 Solution of 4^{th} question

The dimensionality of the linear model needs to predict Response0 and Response1.

1. The dimension of $\widetilde{\mathbf{W}}, \widetilde{\mathbf{G}}$ is 63 x 1 ($\widetilde{D} = 63$)
2. The dimension of $\phi(\mathbf{c})$ is 63 x 1
3. $\widetilde{b}_i, \widetilde{h}_i \in \mathbb{R}$(a real number) $\forall \; i = 0, 1$
4. Response0($r^0(\mathbf{c})$) , Response1($r^1(\mathbf{c})$) give binary outcome [$r^0(\mathbf{c}), r^1(\mathbf{c}) \in \{0, 1\}$]

1.5 Solution of 5^{th} question

Submitted via the code submission page.

1.6 Solution of 6^{th} question

Various hyperparameters affected training time and test accuracy using tables and charts

(a) changing the loss hyperparameter in LinearSVC (hinge vs squared hinge)

Loss Function	Training time(s)	Accuracy(Response0)	Accuracy(Response1)
hinge	1.8766	98.41	99.6
squared hinge	5.4145	98	99.78

(b) setting C in LinearSVC and LogisticRegression to high/low/medium values

- C v/s Training time

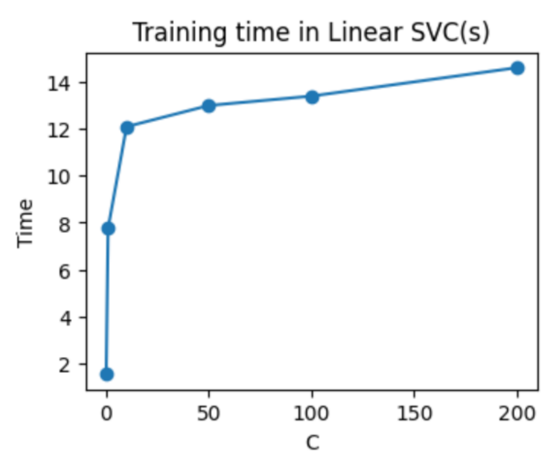

Figure 1: C v/s Training time for Linear SVC

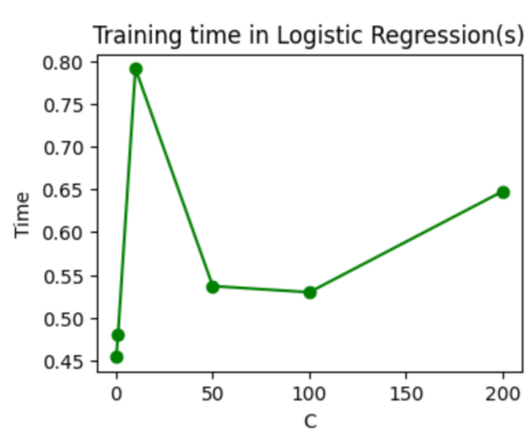

Figure 2: C v/s Training time for Logistic Regression

- C v/s Accuracy

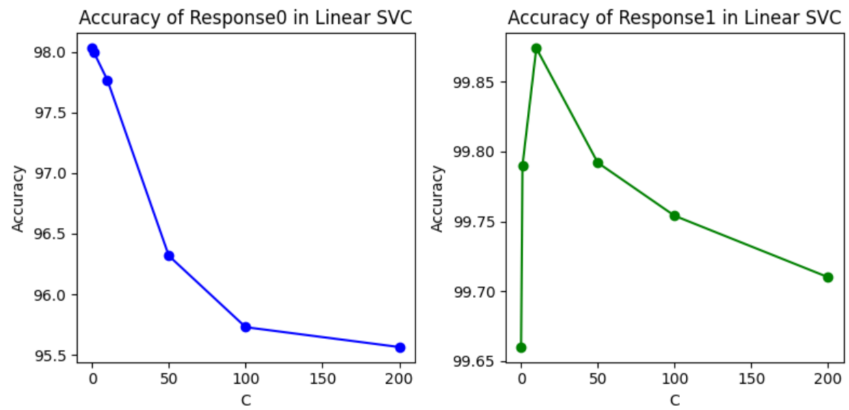

Figure 3: C v/s Accuracy for Linear SVC

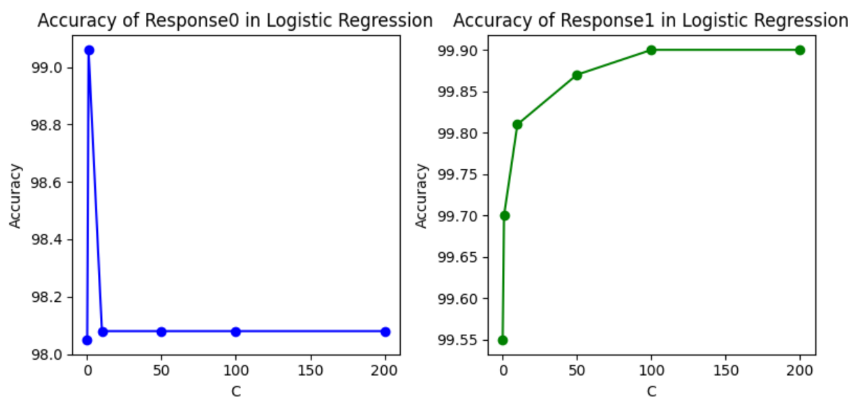

Figure 4: C v/s Accuracy for Logistic Regression

(c) changing tol in LinearSVC and LogisticRegression to high/low/medium values

– Linear SVC

tol values	Training time(s)	Accuracy(Response0)	Accuracy(Response1)
1e-3	5.6226	98	99.78
1e-6	5.353	98	99.78
1e-8	5.335	98	99.78

– Logistic Regression

tol values	Training time(s)	Accuracy(Response0)	Accuracy(Response1)
1e-3	0.5884	98.06	99.7
1e-6	0.5214	98.06	99.7
1e-8	0.4723	98.06	99.7

Note:

- For Response0, we opted to LinearSVC which have **hinge** loss, C = 0.2, tol = $1e-03$,
- For Response1, we opted to LogisticRegression which have C = 400, tol = $1e-08$