Assignment-4

③

④ $Y_1 \sim N(\mu, \sigma^2)$ $\quad Y_2 \sim N(\mu+\delta, \sigma^2)$

$\delta \rightarrow 0$

with σ^2 known, Jeffrey prior $\pi(\mu, \delta) = 1$

$$\delta | Y, \sigma^2 \sim N\left(\bar{Y}_2 - \bar{Y}_1, \frac{\sigma^2}{n_1} + \frac{\sigma^2}{n_2}\right)$$

with σ^2 unknown, Jeffrey prior $\pi(\mu, \delta, \sigma^2) \propto \left(\frac{1}{\sigma^2}\right)^2$

$$\delta | Y \sim t_n\left(\bar{Y}_2 - \bar{Y}_1, \hat{\sigma}^2\left[\frac{1}{n_1} + \frac{1}{n_2}\right]\right)$$

$$\hat{\sigma}^2 = \left[\frac{n_2 \sum_{i=1}^{n_1} (Y_{1i} - \bar{Y}_1) + n_1 \sum_{i=1}^{n_2} (Y_{2i} - \bar{Y}_2)}{n_1 + n_2}\right]$$

⑤ $Y = \text{medv}$ $\quad X = \text{Boston}[, -14]$

a) $Y_i = \beta_0 + \sum_{i=1}^{P} X_i \beta_i + \varepsilon_i$ $\quad P = 13$, $i = 1, \dots 506$

$\beta_0 \sim N(0, 100^2)$ $\quad \varepsilon_i \sim N(0, \sigma^2)$

$\sigma^2 \sim IG(0.01, 0.01)$

$\beta_1 \dots \beta_P \Rightarrow$ Non informative Gaussian prior.

$\beta_j \overset{iid}{\sim} N(0, 100^2)$ over j $\quad j = 1, 2, \dots P$

beta[1] = β_0

beta[j] = β_j $\quad j = 2, \dots 14$

$\beta_1 \dots \beta_{13}$

Almost similar

b) lm()

c) Bayesian LASSO

$\beta_j \sim$ Double Exponential $(0, \sigma^2 \times \sigma_\beta^2)$

$\sigma_\beta^2 \sim IG(0.01, 0.01)$

d) Posterior predictive distribution (PPD)

$p(Y_{new} | Y) = \int f(Y_{new}, \beta, \sigma | Y) \, d\beta \, d\sigma$

$= \int f(Y_{new} | \beta, \sigma) . f(\beta, \sigma | Y) \, d\beta \, d\sigma$

using MCMC (Gibbs sampling) gives draws from Ynew's PPD.

Y - 500 observations

$\beta_0 \sim N(0, 100^2)$, $\sigma^2 \sim IG(0.01, 0.01)$

~~$\beta_1 \ldots \beta_{13} \rightarrow \beta_j \overset{iid}{\sim} N(0, 100^2)$~~

$\beta_1, \ldots \beta_{13} \Longrightarrow \beta_j \overset{iid}{\sim} N(0, \sigma^2 \times \sigma_B^2)$

$\sigma_B^2 \sim IG(1, 1)$

1

10th) $Y_t | \alpha, \beta \overset{ind}{\sim}$ Poisson (λ_t)

$\lambda_t = e^{\alpha + \beta t}$ $\alpha, \beta \overset{ind}{\sim} N(0, 10^2)$

11th) Metroplois Hasting.

3 $Y | n, p \sim$ Binomial (n, p) $Y = 10$

$n \sim$ Poisson (λ)

$p \sim$ Beta (a, b)

③ ② $Y_i|\theta \overset{iid}{\sim} f(y|\theta) = \frac{1}{2}[\phi(y-\theta) + \phi(y)]$

$$\phi(z) = \frac{\exp\left(-\frac{z^2}{2}\right)}{\sqrt{2\pi}} \qquad i=1,\dots n$$

Pr

$\theta \sim N(0, 10^2)$

$$\hat{\theta}_{MAP} = \log(p(\theta|Y))$$

$$= \log f(Y|\theta) + \log p(\theta)$$

$$\prod_{i=1}^{n} f(y_i|\theta)$$

$$= \sum_{i=1}^{n} \log f(y_i|\theta) + \log p(\theta)$$