

Indian Institute of Technology, Kanpur
Econometrics
MTH676
Project Report

Debarghya Jana* Shailza Sharma† Dasari Charithambika‡
Roll: 221306 Roll: 221416 Roll: 210302

Tests for Heteroscedasticity :

0.1 Dataset Description:

- * Dataset Used : We have used Real Estate price prediction dataset for testing heteroscedasticity. The source of our dataset is “Kaggle”.
- * Data Description : It contains 1 response variable and 6 predictors. The total number of data points are 414.
 - Y : House price per unit area
 - X1 : Transaction date of the property
 - X2 : Age of the house
 - X3 : Distance to the nearest MRT (Mass Rapid Transit) station
 - X4 : Number of convenience stores nearby
 - X5 : Latitude of the location
 - X6 : Longitude of the location

*final year Student, M.Sc. Statistics, IIT Kanpur

†final year Student, M.Sc. Statistics, IIT Kanpur

‡3rd year Student, B.S. Statistics and Data Science, IIT Kanpur

* Following is the summary of the dataset :

```
>
> summary(Y)
  Min. 1st Qu. Median   Mean 3rd Qu.   Max.
  7.60  27.70  38.45  37.98  46.60 117.50
>
> summary(X)
      V1          V2          V3          V4          V5
  Min. :2013  Min. : 0.000  Min. : 23.38  Min. : 0.000  Min. :24.93
  1st Qu.:2013 1st Qu.: 9.025  1st Qu.: 289.32 1st Qu.: 1.000  1st Qu.:24.96
  Median :2013 Median :16.100  Median : 492.23 Median : 4.000  Median :24.97
  Mean   :2013 Mean   :17.713  Mean   :1083.89 Mean   : 4.094  Mean   :24.97
  3rd Qu.:2013 3rd Qu.:28.150  3rd Qu.:1454.28 3rd Qu.: 6.000  3rd Qu.:24.98
  Max.   :2014 Max.   :43.800  Max.   :6488.02 Max.   :10.000  Max.   :25.01
      V6
  Min. :121.5
  1st Qu.:121.5
  Median :121.5
  Mean   :121.5
  3rd Qu.:121.5
  Max.   :121.6
>
```

Figure 1: Summary of the dataset

0.2 Tests used:

* Glejser Test:

```
> # Glejser test
> glejser(model)
# A tibble: 1 × 4
  statistic p.value parameter alternative
     <dbl>    <dbl>    <chr>    <chr>
1     28.1 0.0000892       6 greater
>
```

Figure 2: Performance of Glejser test

The test statistic is 28.1 with a p-value of 0.0000892. This indicates evidence against the null hypothesis of homoscedasticity (equal variance of residuals) in favor of the alternative hypothesis of heteroscedasticity (unequal variance of residuals). Since the p-value is less than 0.05, we reject the null hypothesis of homoscedasticity in favor of heteroscedasticity.

* Breusch Pagan Godfrey test:

```
> #Breusch-Pagan Test
> bptest(model)

studentized Breusch-Pagan test

data: model
BP = 8.4591, df = 6, p-value = 0.2064
```

Figure 3: Performance of Breusch Pagan Godfrey test

The test statistic is 8.4591 with a p-value of 0.2064. This indicates weak evidence against the null hypothesis of homoscedasticity, suggesting that the variance of the residuals may not be constant across all levels of the independent variables. Since the p-value is greater than 0.05, we fail to reject the null hypothesis of homoscedasticity.

* Harvey Test:

```
> #Harvey's test
> harvey(model)
# A tibble: 1 × 4
  statistic    p.value parameter alternative
     <dbl>      <dbl>     <chr>   <chr>
1     33.2 0.00000978       6 greater
>
```

Figure 4: Performance of Harvey Test

The test statistic is 33.2 with a p-value of 0.00000978. This provides strong evidence against the null hypothesis of homoscedasticity in favor of the alternative hypothesis of heteroscedasticity. Since the p-value is less than 0.05, we reject the null hypothesis of homoscedasticity in favor of heteroscedasticity.

* White test:

```
> #White's test
> white_test(model)
White's test results

Null hypothesis: Homoskedasticity of the residuals
Alternative hypothesis: Heteroskedasticity of the residuals
Test Statistic: 0.55
P-value: 0.758388
>
```

Figure 5: Performance of White's Test

The test statistic is 0.55 with a p-value of 0.758388. This provides no evidence against the null hypothesis of homoscedasticity. Since the p-value is greater than 0.05, we fail to reject the null hypothesis of homoscedasticity.

Conclusion: Overall, the results suggest that there is some evidence of heteroscedasticity in the residuals of our model, particularly supported by the Glejser and Harvey tests, while the Breusch-Pagan test provides weaker evidence. White's test, however, indicates homoscedasticity.

Tests for Autocorrelation:

For testing the Autocorrelation we used two tests i.e. Durbin Watson test and BreuschGodfrey test. For Durbin Watson test we used "Vehicle dataset" from <https://www.kaggle.com/datasets/nehalbirla/vehicle-dataset-from-cardekho/data> where it contains information about used cars listed on different websites. The columns in the given dataset are as follows: Car_Name, year, selling_price, Present_Price, Kms_Driven, Fuel_Type, Seller_Type, Transmission, Owner. We first made a linear regression model using selling_price as response variable and Present_Price and Kms_Driven as covariates. Then we perform Durbin Watson test and For BreuschâGodfrey test both on that model. Summary of the both tests are given below.

Durbin Watson test:

```
> model
Call:
lm(formula = Selling_Price ~ Present_Price + Kms_Driven, data = car_data)

Coefficients:
(Intercept) Present_Price     Kms_Driven
1.331e+00    5.356e-01   -2.043e-05

> durbinWatsonTest(model) ## Performing Durbin Watson test
lag Autocorrelation D-W Statistic p-value
1      0.215915      1.56752       0
Alternative hypothesis: rho != 0
```

Figure 6: Performance of Durbin Watson test

From the output we can see that the Durbin Watson test statistic is 1.56752 and the corresponding p-value is 0.034. Since this p-value is less than 0.05 in Durbin Watson test, we can reject the null hypothesis and conclude that the residuals in this regression model are autocorrelated.

BreuschGodfrey test :

```
> ## Performing Breusch-Godfrey test for serial correlation of order up to 3
> bgtest(Selling_Price ~ Present_Price + Kms_Driven, order = 3, data = car_data)

Breusch-Godfrey test for serial correlation of order up to 3

data: Selling_Price ~ Present_Price + Kms_Driven
LM test = 15.344, df = 3, p-value = 0.001545
```

Figure 7: Performance of BreuschGodfrey test

From the output we can see that the test statistic is $\chi^2 = 15.344$ with 3 degrees of freedom. The corresponding p-value is 0.001545. Since this p-value is less than 0.05, we can reject the null hypothesis and conclude that autocorrelation exists among the residuals at some order less than or equal to 3.

Seemingly Unrelated Regression Equations:

SURE estimation : For SURE estimation we have downloaded data from <https://pages.stern.nyu.edu/~wgreene/Text/Edition7/tablelist8new.htm>. We worked on Munnell Productivity Data, 48 Continental U.S. States, 17 years, 1970 to 1986, Source: Baltagi (2005), Munnell (1990). It is a panel data. So, we performed SURE estimation upon this dataset. In this data set there are 50 States of USA and there are 17 years from 1970 to 1986 and columns are STATE, ST_ABB, YR, P_CAP, HWY, WATER, UTIL, PC, GSP, EMP, UNEMP. Now, We have taken Gross state product(GSP) as the response and Private capital(PC), Water utility capital(WATER), Public capital(P_CAP), state unemployment rate(UNEMP), Highway capital(HWY), Utility capital(UTIL) are covariates.

```
systemfit results
method: SUR

Coefficients:
eq1_(Intercept)      eq1_P_CAP      eq1_HWY      eq1_WATER      eq1_UTIL
  3.25357e+03     -5.24883e+04    5.24895e+04    5.24935e+04    5.24894e+04
          eq1_PC      eq1_UNEMP
  4.13365e-01     -1.21889e+03
```

Figure 8: Performance of SURE model

First, the estimation method is reported and a few summary statistics for the entire system and for each equation are given. Then, the covariance matrix used for estimation and the covariance matrix as well as the correlation matrix of the (final) residuals are printed. Finally, the estimation results of each equation are reported: the formula of the estimated equation, the estimated coefficients, their standard errors, t values, P values and codes indicating their statistical significance, as well as some other statistics like the standard error of the residuals and the R^2 value of the equation. Adjusted R-squared is

used to determine how reliable the correlation is and how much it is determined by the addition of independent variables. The adjusted R^2 of the model is 0.981487 which is less than it's Multiple R-squared = 0.981623 which is very close to 1 that implies predictors improved the model as expected. Other summaries are given below.

```
> summary(Sure_model)

systemfit results
method: SUR

      N   DF       SSR  detRCov   OLS-R2 McElroy-R2
system 816 809 73333632909 90647259 0.981623  0.981623

      N   DF       SSR       MSE      RMSE       R2     Adj R2
eq1 816 809 73333632909 90647259 9520.89 0.981623 0.981487

The covariance matrix of the residuals used for estimation
eq1
eq1 90646526

The covariance matrix of the residuals
eq1
eq1 90647259

The correlations of the residuals
eq1
eq1 1

SUR estimates for 'eq1' (equation 1)
Model Formula: GSP ~ P_CAP + HWY + WATER + UTIL + PC + UNEMP

            Estimate  Std. Error  t value  Pr(>|t|)    
(Intercept) 3.25357e+03 1.11866e+03 2.90844 0.0037318 ** 
P_CAP       -5.24883e+04 5.65352e+04 -0.92842 0.3534669  
HWY         5.24895e+04 5.65352e+04 0.92844 0.3534567  
WATER        5.24935e+04 5.65352e+04 0.92851 0.3534198  
UTIL         5.24894e+04 5.65352e+04 0.92844 0.3534573  
PC           4.13365e-01 1.49275e-02 27.69152 < 2.22e-16 *** 
UNEMP        -1.21889e+03 1.52694e+02 -7.98261 4.885e-15 *** 
---
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 9520.885435 on 809 degrees of freedom
Number of observations: 816 Degrees of Freedom: 809
SSR: 73333632908.5861 MSE: 90647259.466732 Root MSE: 9520.885435
Multiple R-Squared: 0.981623 Adjusted R-Squared: 0.981487
```

Figure 9: Performance of SURE model

Panel Data model:

Dataset Details: For Panel data analysis, we have used the dataset **Cost Data of U.S Airlines** consisting of 90 Observations on 6 firms for 15 years, 1970 – 1984

Predictors:

- ※ I = Airline
- ※ T = Year
- ※ Q = Output, in revenue passenger miles, index number
- ※ PF = fuel price
- ※ LF = Load factor, the average capacity utilization of the fleet.

Response:

- ※ C = Total cost, in \$1000

We are making a matrix over Airline and Time.

- ※ Y = vector of Total cost, in \$1000
- ※ X = matrix of Q, PF, and LF.

```
> summary(Y)
   Min. 1st Qu. Median   Mean 3rd Qu.   Max.
 68978 292046 637001 1122524 1345968 4748320
> summary(X)
      V1                  V2                  V3
Min. :0.03768  Min. : 103795  Min. :0.4321
1st Qu.:0.14213 1st Qu.: 129848 1st Qu.:0.5288
Median :0.30503 Median : 357434 Median :0.5661
Mean   :0.54499 Mean   : 471683 Mean   :0.5605
3rd Qu.:0.94528 3rd Qu.: 849840 3rd Qu.:0.5947
Max.   :1.93646  Max.  :1015610  Max.  :0.6763
```

Figure 10: Summary of Y and X

Ways to handle a pooled model

- ※ Pooling model:

```

Pooling Model

Call:
plm(formula = Y ~ X, data = pdata, model = "pooling")

Balanced Panel: n = 6, T = 15, N = 90

Residuals:
    Min. 1st Qu. Median 3rd Qu.    Max.
-520654 -250270   37333  208690  849700

Coefficients:
            Estimate Std. Error t-value Pr(>|t|)
(Intercept) 1.1586e+06 3.6059e+05 3.2129  0.00185 **
X1          2.0261e+06 6.1807e+04 32.7813 < 2.2e-16 ***
X2          1.2253e+00 1.0372e-01 11.8138 < 2.2e-16 ***
X3          -3.0658e+06 6.9633e+05 -4.4027 3.058e-05 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Total Sum of Squares:  1.2647e+14
Residual Sum of Squares: 6.8177e+12
R-Squared: 0.94609
Adj. R-Squared: 0.94421
F-statistic: 503.118 on 3 and 86 DF, p-value: < 2.22e-16

```

Figure 11: Pooling model

※ Between model:

```

Oneway (individual) effect Between Model

Call:
plm(formula = Y ~ X, data = pdata, model = "between")

Balanced Panel: n = 6, T = 15, N = 90
Observations used in estimation: 6

Residuals:
    1      2      3      4      5      6
-38528  58079 -44440  98838  32460 -106409

Coefficients:
            Estimate Std. Error t-value Pr(>|t|)
(Intercept) 6.2014e+06 1.8381e+07 0.3374  0.76795
X1          1.8183e+06 5.4493e+05 3.3367  0.07928 .
X2          -9.4242e+00 4.2111e+01 -0.2238  0.84370
X3          -2.8987e+06 3.9612e+06 -0.7318  0.54045
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Total Sum of Squares:  5.0464e+12
Residual Sum of Squares: 2.8978e+10
R-Squared: 0.99426
Adj. R-Squared: 0.98564
F-statistic: 115.432 on 3 and 2 DF, p-value: 0.008601

```

Figure 12: Between model

※ Within model:

```

Oneway (individual) effect Within Model

Call:
plm(formula = Y ~ X, data = pdata, model = "within")

Balanced Panel: n = 6, T = 15, N = 90

Residuals:
    Min. 1st Qu. Median     Mean 3rd Qu.     Max.
-551783 -159259     1796      0 137226 499296

Coefficients:
            Estimate Std. Error t-value Pr(>|t|)
X1  3.3190e+06  1.7135e+05 19.3694 < 2.2e-16 ***
X2  7.7307e-01  9.7319e-02  7.9437 9.698e-12 ***
X3 -3.7974e+06  6.1377e+05 -6.1869 2.375e-08 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Total Sum of Squares:  5.0776e+13
Residual Sum of Squares: 3.5865e+12
R-Squared: 0.92937
Adj. R-Squared: 0.92239
F-statistic: 355.254 on 3 and 81 DF, p-value: < 2.22e-16

```

Figure 13: Within model

* Random model:

```

Oneway (individual) effect Random Effect Model
(Swamy-Arora's transformation)

Call:
plm(formula = Y ~ X, data = pdata, model = "random")

Balanced Panel: n = 6, T = 15, N = 90

Effects:
           var   std.dev share
idiosyncratic 4.428e+10 2.104e+05 0.793
individual    1.154e+10 1.074e+05 0.207
theta:        0.5486

Residuals:
    Min. 1st Qu. Median 3rd Qu.     Max.
-535726 -238494    49890 207491 722934

Coefficients:
            Estimate Std. Error z-value Pr(>|z|)
(Intercept) 1.0743e+06  3.7747e+05 2.8461  0.004427 **
X1          2.2886e+06  1.0949e+05 20.9015 < 2.2e-16 ***
X2          1.1236e+00  1.0344e-01 10.8622 < 2.2e-16 ***
X3         -3.0850e+06  7.2568e+05 -4.2512 2.126e-05 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Total Sum of Squares:  6.6198e+13
Residual Sum of Squares: 5.8721e+12
R-Squared: 0.91129
Adj. R-Squared: 0.9082
Chisq: 883.501 on 3 DF, p-value: < 2.22e-16

```

Figure 14: Random model

* First Difference Model:

```

Oneway (individual) effect First-Difference Model

Call:
plm(formula = Y ~ X, data = pdata, model = "fd")

Balanced Panel: n = 6, T = 15, N = 90
Observations used in estimation: 84

Residuals:
    Min. 1st Qu. Median 3rd Qu. Max.
-232631 -58504 -25086 31884 493212

Coefficients:
            Estimate Std. Error t-value Pr(>|t|)
(Intercept) 7.3268e+04 1.6544e+04 4.4286 2.975e-05 ***
X1          1.1493e+06 2.1346e+05 5.3842 7.099e-07 ***
X2          5.7761e-01 1.3371e-01 4.3197 4.449e-05 ***
X3         -1.7024e+06 4.7366e+05 -3.5941 0.000561 ***
---
Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Total Sum of Squares: 1.5613e+12
Residual Sum of Squares: 9.9318e+11
R-Squared: 0.36388
Adj. R-Squared: 0.34002
F-statistic: 15.2539 on 3 and 80 DF, p-value: 6.1466e-08

```

Figure 15: First Difference Model

Testing:

- * Lagrange Multiplier Test for Random effects v/s OLS:

Lagrange Multiplier Test - (Honda)

```

data: Y ~ X
normal = 0.783, p-value = 0.2168
alternative hypothesis: significant effects

```

Figure 16: LM test for random effects v/s OLS

The outcomes of the Lagrange Multiplier Test could suggest that there would be random effects.

- * F test for individual effects:

F test for individual effects

```

data: Y ~ X
F = 14.595, df1 = 5, df2 = 81, p-value = 3.467e-10
alternative hypothesis: significant effects

```

Figure 17: LM test for fixed effects v/s OLS

The outcomes of the F-Test could suggest that there would be fixed effects.

* Hausman Test:

Hausman Test

```

data: Y ~ X
chisq = 60.87, df = 3, p-value = 3.832e-13
alternative hypothesis: one model is inconsistent

```

Figure 18: Hausman test for fixed v/s random effects model

The outcome of the Hausman test is an alternative hypothesis, which is one model is inconsistent. It recommends using the Random Effect Model.

Multivariate Multiple Linear Regression Model:

For Multivariate Multiple Linear Regression we have downloaded data from <https://pages.stern.nyu.edu/~wgreene/Text/Edition7/tablelist8new.htm>. We worked on Munnell Productivity Data, 48 Continental U.S. States, 17 years, 1970 to 1986, Source: Baltagi (2005), Munnell (1990). It is a panel data. So, we performed SURE estimation upon this dataset. In this data set there are 50 States of USA and there are 17 years from 1970 to 1986 and columns are STATE, ST_ABB, YR, P_CAP, HWY, WATER, UTIL, PC, GSP, EMP, UNEMP. Now, We have taken Gross state product(GSP) as the response and Private capital(PC), Water utility capital(WATER), Public capital(P_CAP), state unemployment rate(UNEMP), Highway capital(HWY), Utility capital(UTIL) are covariates. Now, we combined the response of all years with respect to each state and made a matrix of response variable and similarly made the matrix of covariates. The data set has 816 rows and 17 columns.

Response: GSP, Covariate: PCAP We have now taken GSP as response variable and PCAP as covariate. Summary for the model is described below. There are 17 summary outputs for this session of response and covariates.

```

> summary(model_1)
Response Y1 :

Call:
lm(formula = Y1 ~ P_CAP_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-13167.7 -2678.1   224.6  4411.3 10509.2 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -3140.730   350.489 -8.961 < 2e-16 ***
P_CAP_state_1_matrix1 -17.595    2.212 -7.956 6.08e-15 ***
P_CAP_state_1_matrix2  32.317    4.324  7.474 2.05e-13 ***
P_CAP_state_1_matrix3 -27.957    3.508 -7.970 5.48e-15 ***
P_CAP_state_1_matrix4  10.729    3.904  2.748 0.006128 ** 
P_CAP_state_1_matrix5   9.375    3.424  2.738 0.006316 ** 
P_CAP_state_1_matrix6  -1.666    4.740 -0.352 0.725291  
P_CAP_state_1_matrix7 -13.573    4.292 -3.162 0.001626 ** 
P_CAP_state_1_matrix8  10.491    4.887  2.147 0.032123 *  
P_CAP_state_1_matrix9   8.773    5.038  1.741 0.081999 .  
P_CAP_state_1_matrix10 -12.242    3.760 -3.256 0.001177 ** 
P_CAP_state_1_matrix11  17.157    4.536  3.783 0.000167 *** 
P_CAP_state_1_matrix12 -49.802    5.268 -9.453 < 2e-16 *** 
P_CAP_state_1_matrix13  70.025    4.467 15.675 < 2e-16 *** 
P_CAP_state_1_matrix14 -50.049    4.221 -11.858 < 2e-16 *** 
P_CAP_state_1_matrix15  -7.953    5.450 -1.459 0.144912  
P_CAP_state_1_matrix16  39.399    4.786  8.231 7.52e-16 *** 
P_CAP_state_1_matrix17 -15.689    1.902 -8.250 6.53e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 5580 on 798 degrees of freedom
Multiple R-squared:  0.9905,    Adjusted R-squared:  0.9903 
F-statistic:  4902 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 19: Performance of Multivariate Multiple Linear Regression MODEL(GSP-PCAP)

```

Response Y2 :

Call:
lm(formula = Y2 ~ P_CAP_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-13282.6 -3013.0   343.8  4373.2 10642.9 

Coefficients:
                                         Estimate Std. Error t value Pr(>|t|)    
(Intercept)           -2929.0633   360.5059 -8.125 1.70e-15 ***  
P_CAP_state_1_matrix1 -17.5879    2.2747 -7.732 3.19e-14 ***  
P_CAP_state_1_matrix2  34.5424    4.4477  7.766 2.48e-14 ***  
P_CAP_state_1_matrix3 -32.8925    3.6081 -9.116 < 2e-16 ***  
P_CAP_state_1_matrix4  15.9792    4.0158  3.979 7.55e-05 ***  
P_CAP_state_1_matrix5  4.9929    3.5217  1.418 0.156652    
P_CAP_state_1_matrix6  0.4411    4.8759  0.090 0.927939    
P_CAP_state_1_matrix7 -13.3725    4.4151 -3.029 0.002535 **  
P_CAP_state_1_matrix8  6.4201    5.0269  1.277 0.201921    
P_CAP_state_1_matrix9  15.6944    5.1818  3.029 0.002535 **  
P_CAP_state_1_matrix10 -16.4827    3.8670 -4.262 2.26e-05 ***  
P_CAP_state_1_matrix11 16.9085    4.6655  3.624 0.000308 ***  
P_CAP_state_1_matrix12 -47.9266    5.4190 -8.844 < 2e-16 ***  
P_CAP_state_1_matrix13 68.8235    4.5951 14.978 < 2e-16 ***  
P_CAP_state_1_matrix14 -48.6767    4.3414 -11.212 < 2e-16 ***  
P_CAP_state_1_matrix15 -7.8323    5.6060 -1.397 0.162762    
P_CAP_state_1_matrix16 37.6864    4.9233  7.655 5.60e-14 ***  
P_CAP_state_1_matrix17 -14.9351    1.9561 -7.635 6.46e-14 ***  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 5740 on 798 degrees of freedom
Multiple R-squared:  0.9902,   Adjusted R-squared:  0.99
F-statistic:  4730 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 20: Performance of Multivariate Multiple Linear Regression MODEL(GSP-PCAP)

```

Response Y3 :

Call:
lm(formula = Y3 ~ P_CAP_state_1_matrix)

Residuals:
    Min      1Q Median      3Q     Max 
-13318 -3325    282   4365  11057 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) -2970.422    375.590 -7.909 8.66e-15 ***
P_CAP_state_1_matrix1 -18.793     2.370 -7.930 7.39e-15 ***
P_CAP_state_1_matrix2  37.247     4.634  8.038 3.28e-15 ***
P_CAP_state_1_matrix3 -34.488     3.759 -9.175 < 2e-16 ***
P_CAP_state_1_matrix4  18.149     4.184  4.338 1.62e-05 ***
P_CAP_state_1_matrix5  3.076     3.669  0.838  0.40211  
P_CAP_state_1_matrix6 -1.713     5.080 -0.337  0.73612  
P_CAP_state_1_matrix7 -12.987     4.600 -2.823  0.00487 ** 
P_CAP_state_1_matrix8  8.365     5.237  1.597  0.11060  
P_CAP_state_1_matrix9  17.315     5.399  3.207  0.00139 ** 
P_CAP_state_1_matrix10 -20.359     4.029 -5.053 5.39e-07 *** 
P_CAP_state_1_matrix11 20.736     4.861  4.266 2.23e-05 *** 
P_CAP_state_1_matrix12 -51.718     5.646 -9.161 < 2e-16 *** 
P_CAP_state_1_matrix13 71.774     4.787 14.992 < 2e-16 *** 
P_CAP_state_1_matrix14 -48.691     4.523 -10.765 < 2e-16 *** 
P_CAP_state_1_matrix15 -9.256     5.841 -1.585  0.11341  
P_CAP_state_1_matrix16 36.882     5.129  7.191 1.49e-12 *** 
P_CAP_state_1_matrix17 -13.659     2.038 -6.702 3.89e-11 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 5980 on 798 degrees of freedom
Multiple R-squared:  0.9902,    Adjusted R-squared:  0.99 
F-statistic:  4751 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 21: Performance of Multivariate Multiple Linear Regression MODEL(GSP-PCAP)

```

Response Y4 :

Call:
lm(formula = Y4 ~ P_CAP_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-14006.2 -3549.8   260.8  3982.9 12932.7 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) -3084.339   386.810  -7.974 5.32e-15 ***
P_CAP_state_1_matrix1 -22.385    2.441  -9.171 < 2e-16 ***
P_CAP_state_1_matrix2  45.449    4.772   9.524 < 2e-16 ***
P_CAP_state_1_matrix3 -42.281    3.871 -10.921 < 2e-16 ***
P_CAP_state_1_matrix4  23.693    4.309   5.499 5.15e-08 *** 
P_CAP_state_1_matrix5  2.320    3.779   0.614 0.539324  
P_CAP_state_1_matrix6 -6.541    5.232  -1.250 0.211556  
P_CAP_state_1_matrix7 -10.773   4.737  -2.274 0.023231 *  
P_CAP_state_1_matrix8  6.635    5.394   1.230 0.219002  
P_CAP_state_1_matrix9  26.152   5.560   4.704 3.01e-06 *** 
P_CAP_state_1_matrix10 -26.563   4.149  -6.402 2.62e-10 *** 
P_CAP_state_1_matrix11 18.574    5.006   3.710 0.000221 *** 
P_CAP_state_1_matrix12 -50.165   5.814  -8.628 < 2e-16 *** 
P_CAP_state_1_matrix13 69.977   4.930  14.193 < 2e-16 *** 
P_CAP_state_1_matrix14 -42.740   4.658  -9.175 < 2e-16 *** 
P_CAP_state_1_matrix15 -10.657   6.015  -1.772 0.076815 .  
P_CAP_state_1_matrix16 32.441    5.282   6.141 1.29e-09 *** 
P_CAP_state_1_matrix17 -11.177   2.099  -5.325 1.31e-07 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 6158 on 798 degrees of freedom
Multiple R-squared:  0.9905,   Adjusted R-squared:  0.9903
F-statistic: 4905 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 22: Performance of Multivariate Multiple Linear Regression MODEL(GSP-PCAP)

```

Response Y5 :

Call:
lm(formula = Y5 ~ P_CAP_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-13421.8 -3253.0   494.5  3831.8 12302.4 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) -3565.178   376.206 -9.477 < 2e-16 ***
P_CAP_state_1_matrix1 -21.729    2.374 -9.154 < 2e-16 ***
P_CAP_state_1_matrix2  41.156    4.641  8.867 < 2e-16 ***
P_CAP_state_1_matrix3 -31.021    3.765 -8.239 7.11e-16 ***
P_CAP_state_1_matrix4 12.724    4.191  3.036  0.00247 ** 
P_CAP_state_1_matrix5  6.045    3.675  1.645  0.10037  
P_CAP_state_1_matrix6 -7.462    5.088 -1.466  0.14292  
P_CAP_state_1_matrix7 -10.998   4.607 -2.387  0.01722 *  
P_CAP_state_1_matrix8 12.598    5.246  2.402  0.01655 *  
P_CAP_state_1_matrix9 16.900    5.407  3.125  0.00184 ** 
P_CAP_state_1_matrix10 -22.261   4.035 -5.516 4.68e-08 *** 
P_CAP_state_1_matrix11 19.074    4.869  3.918 9.71e-05 *** 
P_CAP_state_1_matrix12 -54.357   5.655 -9.612 < 2e-16 *** 
P_CAP_state_1_matrix13 75.412    4.795 15.727 < 2e-16 *** 
P_CAP_state_1_matrix14 -42.922   4.530 -9.474 < 2e-16 *** 
P_CAP_state_1_matrix15 -15.627   5.850 -2.671  0.00771 ** 
P_CAP_state_1_matrix16 34.324    5.138  6.681 4.45e-11 *** 
P_CAP_state_1_matrix17 -9.900    2.041 -4.850 1.49e-06 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 5989 on 798 degrees of freedom
Multiple R-squared:  0.9909,    Adjusted R-squared:  0.9907 
F-statistic:  5130 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 23: Performance of Multivariate Multiple Linear Regression MODEL(GSP-PCAP)

```

Response Y6 :

Call:
lm(formula = Y6 ~ P_CAP_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-14454.0 -2529.0   582.7  3654.9  9617.8 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept) -3949.616   366.692 -10.771 < 2e-16 ***
P_CAP_state_1_matrix1 -19.381    2.314  -8.376 2.44e-16 ***
P_CAP_state_1_matrix2  34.923    4.524   7.720 3.49e-14 ***
P_CAP_state_1_matrix3 -19.268    3.670  -5.250 1.95e-07 ***
P_CAP_state_1_matrix4   1.617    4.085   0.396 0.692236  
P_CAP_state_1_matrix5   7.182    3.582   2.005 0.045318 *  
P_CAP_state_1_matrix6  -3.307    4.960  -0.667 0.505089  
P_CAP_state_1_matrix7 -13.068    4.491  -2.910 0.003715 ** 
P_CAP_state_1_matrix8  18.662    5.113   3.650 0.000280 *** 
P_CAP_state_1_matrix9   5.271    5.271   1.000 0.317550  
P_CAP_state_1_matrix10 -16.234    3.933  -4.127 4.06e-05 *** 
P_CAP_state_1_matrix11 18.211    4.746   3.837 0.000134 *** 
P_CAP_state_1_matrix12 -55.075    5.512  -9.992 < 2e-16 *** 
P_CAP_state_1_matrix13 78.383    4.674  16.770 < 2e-16 *** 
P_CAP_state_1_matrix14 -43.575    4.416  -9.868 < 2e-16 *** 
P_CAP_state_1_matrix15 -20.624    5.702  -3.617 0.000317 *** 
P_CAP_state_1_matrix16  37.771    5.008   7.542 1.26e-13 *** 
P_CAP_state_1_matrix17 -9.501    1.990  -4.775 2.14e-06 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 5838 on 798 degrees of freedom
Multiple R-squared:  0.9913,    Adjusted R-squared:  0.9911 
F-statistic:  5319 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 24: Performance of Multivariate Multiple Linear Regression MODEL(GSP-PCAP)

```

Response Y7 :

Call:
lm(formula = Y7 ~ P_CAP_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-14648.8 -2621.1   511.5  3834.1 10225.4 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) -3806.128   390.381 -9.750 < 2e-16 ***
P_CAP_state_1_matrix1 -20.711    2.463 -8.408 < 2e-16 ***
P_CAP_state_1_matrix2  40.022    4.816  8.310 4.10e-16 ***
P_CAP_state_1_matrix3 -25.161    3.907 -6.440 2.07e-10 ***
P_CAP_state_1_matrix4  5.923     4.349  1.362 0.173546  
P_CAP_state_1_matrix5  4.683     3.814  1.228 0.219769  
P_CAP_state_1_matrix6 -3.234     5.280 -0.612 0.540404  
P_CAP_state_1_matrix7 -13.030    4.781 -2.725 0.006563 ** 
P_CAP_state_1_matrix8  18.710    5.444  3.437 0.000618 *** 
P_CAP_state_1_matrix9  8.305     5.611  1.480 0.139261  
P_CAP_state_1_matrix10 -19.484   4.187 -4.653 3.83e-06 ***
P_CAP_state_1_matrix11 15.031     5.052  2.975 0.003017 ** 
P_CAP_state_1_matrix12 -51.634    5.868 -8.799 < 2e-16 ***
P_CAP_state_1_matrix13 80.416    4.976 16.161 < 2e-16 ***
P_CAP_state_1_matrix14 -47.204    4.701 -10.041 < 2e-16 ***
P_CAP_state_1_matrix15 -17.813    6.071 -2.934 0.003440 ** 
P_CAP_state_1_matrix16  37.655    5.331  7.063 3.55e-12 *** 
P_CAP_state_1_matrix17 -10.347   2.118 -4.885 1.25e-06 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 6215 on 798 degrees of freedom
Multiple R-squared:  0.9909,    Adjusted R-squared:  0.9907 
F-statistic:  5084 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 25: Performance of Multivariate Multiple Linear Regression MODEL(GSP-PCAP)

```

Response Y8 :

Call:
lm(formula = Y8 ~ P_CAP_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-15920.9 -2671.8   421.1  3402.9 10919.8 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) -3889.836   410.521 -9.475 < 2e-16 ***
P_CAP_state_1_matrix1 -22.076    2.590 -8.522 < 2e-16 ***
P_CAP_state_1_matrix2  44.702    5.065  8.826 < 2e-16 ***
P_CAP_state_1_matrix3 -29.877    4.109 -7.272 8.48e-13 ***
P_CAP_state_1_matrix4  9.252     4.573  2.023  0.04339 *  
P_CAP_state_1_matrix5  2.886     4.010  0.720  0.47192  
P_CAP_state_1_matrix6 -4.569     5.552 -0.823  0.41079  
P_CAP_state_1_matrix7 -11.368    5.028 -2.261  0.02402 *  
P_CAP_state_1_matrix8  18.668    5.724  3.261  0.00116 ** 
P_CAP_state_1_matrix9  10.816    5.901  1.833  0.06719 .  
P_CAP_state_1_matrix10 -23.781    4.404 -5.400 8.78e-08 ***
P_CAP_state_1_matrix11 13.909    5.313  2.618  0.00901 ** 
P_CAP_state_1_matrix12 -48.926    6.171 -7.929 7.46e-15 ***
P_CAP_state_1_matrix13  80.706    5.233 15.424 < 2e-16 ***
P_CAP_state_1_matrix14 -47.515    4.944 -9.611 < 2e-16 *** 
P_CAP_state_1_matrix15 -17.796    6.384 -2.788  0.00543 ** 
P_CAP_state_1_matrix16  37.165    5.606  6.629 6.22e-11 *** 
P_CAP_state_1_matrix17 -9.917     2.228 -4.452 9.71e-06 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 6536 on 798 degrees of freedom
Multiple R-squared:  0.9908,    Adjusted R-squared:  0.9906 
F-statistic:  5039 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 26: Performance of Multivariate Multiple Linear Regression MODEL(GSP-PCAP)

```

Response Y9 :

Call:
lm(formula = Y9 ~ P_CAP_state_1_matrix)

Residuals:
    Min      1Q Median      3Q     Max 
-16943 -2491   1312   3477  12021 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) -3838.310    433.590 -8.852 < 2e-16 ***
P_CAP_state_1_matrix1 -22.011     2.736 -8.045 3.10e-15 ***
P_CAP_state_1_matrix2  44.311     5.349  8.283 5.03e-16 ***
P_CAP_state_1_matrix3 -27.132     4.340 -6.252 6.59e-10 ***
P_CAP_state_1_matrix4  8.901     4.830  1.843 0.065721 .  
P_CAP_state_1_matrix5  0.220     4.236  0.052 0.958590  
P_CAP_state_1_matrix6 -4.674     5.864 -0.797 0.425647  
P_CAP_state_1_matrix7 -10.716    5.310 -2.018 0.043928 *  
P_CAP_state_1_matrix8  20.983    6.046  3.471 0.000547 *** 
P_CAP_state_1_matrix9  7.937     6.232  1.274 0.203199  
P_CAP_state_1_matrix10 -25.545    4.651 -5.492 5.33e-08 *** 
P_CAP_state_1_matrix11 18.193     5.611  3.242 0.001235 ** 
P_CAP_state_1_matrix12 -53.858    6.518 -8.264 5.87e-16 *** 
P_CAP_state_1_matrix13  85.814    5.527 15.527 < 2e-16 *** 
P_CAP_state_1_matrix14 -48.858    5.221 -9.357 < 2e-16 *** 
P_CAP_state_1_matrix15 -21.989    6.742 -3.261 0.001156 ** 
P_CAP_state_1_matrix16  41.662     5.921  7.036 4.27e-12 *** 
P_CAP_state_1_matrix17 -10.788    2.353 -4.585 5.26e-06 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 6903 on 798 degrees of freedom
Multiple R-squared:  0.9906,    Adjusted R-squared:  0.9904 
F-statistic:  4935 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 27: Performance of Multivariate Multiple Linear Regression MODEL(GSP-PCAP)

```

Response Y10 :

Call:
lm(formula = Y10 ~ P_CAP_state_1_matrix)

Residuals:
    Min      1Q Median      3Q     Max 
-19066 -2880   1129  3654  11645 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) -3819.1851   440.3867 -8.672 < 2e-16 ***
P_CAP_state_1_matrix1 -22.2577    2.7788 -8.010 4.06e-15 ***
P_CAP_state_1_matrix2  42.0826    5.4332  7.745 2.89e-14 ***
P_CAP_state_1_matrix3 -19.1928   4.4076 -4.354 1.51e-05 ***
P_CAP_state_1_matrix4  3.2016   4.9056  0.653  0.5142  
P_CAP_state_1_matrix5 -0.8291   4.3021 -0.193  0.8472  
P_CAP_state_1_matrix6 -3.2920   5.9563 -0.553  0.5806  
P_CAP_state_1_matrix7 -10.8608   5.3935 -2.014  0.0444 *  
P_CAP_state_1_matrix8  25.2640   6.1408  4.114 4.29e-05 *** 
P_CAP_state_1_matrix9 -0.2232   6.3300 -0.035  0.9719  
P_CAP_state_1_matrix10 -23.2237   4.7239 -4.916 1.07e-06 *** 
P_CAP_state_1_matrix11  23.7915   5.6992  4.175 3.31e-05 *** 
P_CAP_state_1_matrix12 -61.3626   6.6197 -9.270 < 2e-16 *** 
P_CAP_state_1_matrix13  91.1015   5.6133 16.230 < 2e-16 *** 
P_CAP_state_1_matrix14 -49.4940   5.3033 -9.333 < 2e-16 *** 
P_CAP_state_1_matrix15 -27.5877   6.8482 -4.028 6.15e-05 *** 
P_CAP_state_1_matrix16  45.9214   6.0142  7.636 6.43e-14 *** 
P_CAP_state_1_matrix17 -10.5304   2.3896 -4.407 1.19e-05 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 7011 on 798 degrees of freedom
Multiple R-squared:  0.9907,    Adjusted R-squared:  0.9905 
F-statistic:  5010 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 28: Performance of Multivariate Multiple Linear Regression MODEL(GSP-PCAP)

```

Response Y11 :

Call:
lm(formula = Y11 ~ P_CAP_state_1_matrix)

Residuals:
    Min      1Q Median      3Q     Max 
-22662 -2890   1168  3941 12762 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) -4014.194    466.176 -8.611 < 2e-16 ***
P_CAP_state_1_matrix1 -21.160     2.941  -7.194 1.45e-12 ***
P_CAP_state_1_matrix2  34.457     5.751   5.991 3.15e-09 ***
P_CAP_state_1_matrix3 -1.934     4.666  -0.414 0.678658  
P_CAP_state_1_matrix4 -9.793     5.193  -1.886 0.059676 .  
P_CAP_state_1_matrix5 -1.107     4.554  -0.243 0.807974  
P_CAP_state_1_matrix6 -1.997     6.305  -0.317 0.751557  
P_CAP_state_1_matrix7 -8.824     5.709  -1.546 0.122614  
P_CAP_state_1_matrix8 32.085     6.500   4.936 9.72e-07 ***
P_CAP_state_1_matrix9 -14.952    6.701  -2.231 0.025933 *  
P_CAP_state_1_matrix10 -19.422    5.001  -3.884 0.000111 *** 
P_CAP_state_1_matrix11 34.849     6.033   5.776 1.09e-08 *** 
P_CAP_state_1_matrix12 -73.189    7.007  -10.445 < 2e-16 *** 
P_CAP_state_1_matrix13 97.807     5.942   16.460 < 2e-16 *** 
P_CAP_state_1_matrix14 -51.019    5.614  -9.088 < 2e-16 *** 
P_CAP_state_1_matrix15 -38.349    7.249  -5.290 1.58e-07 *** 
P_CAP_state_1_matrix16 56.895     6.366   8.937 < 2e-16 *** 
P_CAP_state_1_matrix17 -11.878    2.530  -4.696 3.13e-06 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 7422 on 798 degrees of freedom
Multiple R-squared:  0.9895,    Adjusted R-squared:  0.9893 
F-statistic:  4438 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 29: Performance of Multivariate Multiple Linear Regression MODEL(GSP-PCAP)

```

Response Y12 :

Call:
lm(formula = Y12 ~ P_CAP_state_1_matrix)

Residuals:
    Min      1Q Median      3Q     Max 
-24476 -3532   1294  4546 13961 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) -3929.4542   496.2083 -7.919 8.02e-15 *** 
P_CAP_state_1_matrix1 -22.1694    3.1310 -7.081 3.15e-12 *** 
P_CAP_state_1_matrix2  33.7005    6.1219  5.505 4.98e-08 *** 
P_CAP_state_1_matrix3 -0.1412    4.9663 -0.028  0.977    
P_CAP_state_1_matrix4 -8.9182    5.5274 -1.613  0.107    
P_CAP_state_1_matrix5 -3.0710    4.8474 -0.634  0.527    
P_CAP_state_1_matrix6 -2.3403    6.7113 -0.349  0.727    
P_CAP_state_1_matrix7 -5.6359    6.0771 -0.927  0.354    
P_CAP_state_1_matrix8 31.9760    6.9192  4.621 4.44e-06 *** 
P_CAP_state_1_matrix9 -16.7633   7.1324 -2.350  0.019 *  
P_CAP_state_1_matrix10 -22.5481   5.3227 -4.236 2.54e-05 *** 
P_CAP_state_1_matrix11 39.8741    6.4216  6.209 8.55e-10 *** 
P_CAP_state_1_matrix12 -77.4083   7.4588 -10.378 < 2e-16 *** 
P_CAP_state_1_matrix13 101.2382   6.3248 16.007 < 2e-16 *** 
P_CAP_state_1_matrix14 -48.9574   5.9756 -8.193 1.01e-15 *** 
P_CAP_state_1_matrix15 -47.8955   7.7162 -6.207 8.67e-10 *** 
P_CAP_state_1_matrix16 64.6359    6.7765  9.538 < 2e-16 *** 
P_CAP_state_1_matrix17 -13.1830   2.6925 -4.896 1.18e-06 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 7900 on 798 degrees of freedom
Multiple R-squared:  0.9884,   Adjusted R-squared:  0.9881
F-statistic: 3989 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 30: Performance of Multivariate Multiple Linear Regression MODEL(GSP-PCAP)

```

Response Y13 :

Call:
lm(formula = Y13 ~ P_CAP_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-26637.1 -3988.9   816.8  4599.6 14736.0 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -4196.008   521.062 -8.053 2.94e-15 ***
P_CAP_state_1_matrix1 -21.275    3.288 -6.471 1.70e-10 ***
P_CAP_state_1_matrix2  28.022    6.429  4.359 1.48e-05 ***
P_CAP_state_1_matrix3 10.513    5.215  2.016  0.04414 *  
P_CAP_state_1_matrix4 -16.043    5.804 -2.764  0.00584 ** 
P_CAP_state_1_matrix5 -7.325    5.090 -1.439  0.15052  
P_CAP_state_1_matrix6  6.088    7.047  0.864  0.38792  
P_CAP_state_1_matrix7 -5.578    6.381 -0.874  0.38230  
P_CAP_state_1_matrix8 30.096    7.266  4.142 3.81e-05 *** 
P_CAP_state_1_matrix9 -20.349    7.490 -2.717  0.00673 ** 
P_CAP_state_1_matrix10 -22.023    5.589 -3.940 8.85e-05 *** 
P_CAP_state_1_matrix11 43.562    6.743  6.460 1.82e-10 *** 
P_CAP_state_1_matrix12 -79.370    7.832 -10.134 < 2e-16 *** 
P_CAP_state_1_matrix13 100.377   6.642 15.113 < 2e-16 *** 
P_CAP_state_1_matrix14 -43.501    6.275 -6.933 8.53e-12 *** 
P_CAP_state_1_matrix15 -57.400    8.103 -7.084 3.08e-12 *** 
P_CAP_state_1_matrix16  68.643    7.116  9.646 < 2e-16 *** 
P_CAP_state_1_matrix17 -12.147    2.827 -4.296 1.95e-05 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 8296 on 798 degrees of freedom
Multiple R-squared:  0.9868,    Adjusted R-squared:  0.9865 
F-statistic: 3511 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 31: Performance of Multivariate Multiple Linear Regression MODEL(GSP-PCAP)

```

Response Y14 :

Call:
lm(formula = Y14 ~ P_CAP_state_1_matrix)

Residuals:
    Min      1Q Median      3Q     Max 
-27528 -3931   1406   4806  14266 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) -4731.052   537.610  -8.800 < 2e-16 ***
P_CAP_state_1_matrix1 -22.993    3.392  -6.778 2.37e-11 ***
P_CAP_state_1_matrix2  30.740    6.633   4.635 4.18e-06 ***
P_CAP_state_1_matrix3 11.226    5.381   2.086  0.03727 *  
P_CAP_state_1_matrix4 -14.822    5.989  -2.475  0.01353 *  
P_CAP_state_1_matrix5 -16.185    5.252  -3.082  0.00213 ** 
P_CAP_state_1_matrix6 13.993    7.271   1.924  0.05466 .  
P_CAP_state_1_matrix7 -5.198    6.584  -0.790  0.43003  
P_CAP_state_1_matrix8 24.869    7.496   3.317  0.00095 *** 
P_CAP_state_1_matrix9 -15.394    7.727  -1.992  0.04671 *  
P_CAP_state_1_matrix10 -28.448   5.767  -4.933 9.85e-07 *** 
P_CAP_state_1_matrix11 49.463    6.957   7.109 2.59e-12 *** 
P_CAP_state_1_matrix12 -77.243   8.081  -9.559 < 2e-16 *** 
P_CAP_state_1_matrix13 92.201    6.853  13.455 < 2e-16 *** 
P_CAP_state_1_matrix14 -33.045   6.474  -5.104 4.16e-07 *** 
P_CAP_state_1_matrix15 -60.792   8.360  -7.272 8.48e-13 *** 
P_CAP_state_1_matrix16 60.380    7.342   8.224 7.96e-16 *** 
P_CAP_state_1_matrix17 -6.387    2.917  -2.189  0.02886 *  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 8559 on 798 degrees of freedom
Multiple R-squared:  0.987,   Adjusted R-squared:  0.9867
F-statistic:  3566 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 32: Performance of Multivariate Multiple Linear Regression MODEL(GSP-PCAP)

```

Response Y15 :

Call:
lm(formula = Y15 ~ P_CAP_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-29829 -5200   1996   5556  15589 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) -5249.484   578.999 -9.066 < 2e-16 ***
P_CAP_state_1_matrix1 -24.758    3.653 -6.777 2.39e-11 ***
P_CAP_state_1_matrix2  34.638    7.143  4.849 1.49e-06 ***
P_CAP_state_1_matrix3   7.424    5.795  1.281 0.200498  
P_CAP_state_1_matrix4 -10.936    6.450 -1.696 0.090343 .  
P_CAP_state_1_matrix5 -20.000    5.656 -3.536 0.000430 *** 
P_CAP_state_1_matrix6  17.590    7.831  2.246 0.024963 *  
P_CAP_state_1_matrix7 -8.941    7.091 -1.261 0.207723  
P_CAP_state_1_matrix8  27.833    8.074  3.447 0.000596 *** 
P_CAP_state_1_matrix9 -17.044    8.322 -2.048 0.040889 *  
P_CAP_state_1_matrix10 -29.650    6.211 -4.774 2.15e-06 *** 
P_CAP_state_1_matrix11  52.268    7.493  6.976 6.40e-12 *** 
P_CAP_state_1_matrix12 -80.972    8.703 -9.304 < 2e-16 *** 
P_CAP_state_1_matrix13  96.562    7.380 13.084 < 2e-16 *** 
P_CAP_state_1_matrix14 -30.446    6.973 -4.367 1.43e-05 *** 
P_CAP_state_1_matrix15 -66.238    9.004 -7.357 4.68e-13 *** 
P_CAP_state_1_matrix16  57.731    7.907  7.301 6.91e-13 *** 
P_CAP_state_1_matrix17 -2.550     3.142 -0.812 0.417300  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 9218 on 798 degrees of freedom
Multiple R-squared:  0.9869,    Adjusted R-squared:  0.9866 
F-statistic:  3534 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 33: Performance of Multivariate Multiple Linear Regression MODEL(GSP-PCAP)

```

Response Y16 :

Call:
lm(formula = Y16 ~ P_CAP_state_1_matrix)

Residuals:
    Min      1Q Median      3Q     Max 
-31602 -6676  1964  6205 14783 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) -5890.704   611.495 -9.633 < 2e-16 ***
P_CAP_state_1_matrix1 -27.009    3.858 -7.000 5.43e-12 ***
P_CAP_state_1_matrix2  39.237    7.544  5.201 2.52e-07 ***
P_CAP_state_1_matrix3  6.199    6.120  1.013 0.311440  
P_CAP_state_1_matrix4 -10.291   6.812 -1.511 0.131234  
P_CAP_state_1_matrix5 -25.415   5.974 -4.255 2.34e-05 ***
P_CAP_state_1_matrix6 22.328    8.271  2.700 0.007087 ** 
P_CAP_state_1_matrix7 -9.391    7.489 -1.254 0.210239  
P_CAP_state_1_matrix8 26.707    8.527  3.132 0.001799 ** 
P_CAP_state_1_matrix9 -16.038   8.789 -1.825 0.068421 .  
P_CAP_state_1_matrix10 -31.361   6.559 -4.781 2.08e-06 *** 
P_CAP_state_1_matrix11 52.900    7.914  6.685 4.35e-11 *** 
P_CAP_state_1_matrix12 -78.430   9.192 -8.533 < 2e-16 *** 
P_CAP_state_1_matrix13 93.449    7.794 11.989 < 2e-16 *** 
P_CAP_state_1_matrix14 -26.972   7.364 -3.663 0.000266 *** 
P_CAP_state_1_matrix15 -67.098   9.509 -7.056 3.72e-12 *** 
P_CAP_state_1_matrix16 52.299    8.351  6.263 6.18e-10 *** 
P_CAP_state_1_matrix17  1.506    3.318  0.454 0.650014  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 9735 on 798 degrees of freedom
Multiple R-squared:  0.9867,    Adjusted R-squared:  0.9864 
F-statistic:  3477 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 34: Performance of Multivariate Multiple Linear Regression MODEL(GSP-PCAP)

```

Response Y17 :

Call:
lm(formula = Y17 ~ P_CAP_state_1_matrix)

Residuals:
    Min      1Q Median      3Q     Max 
-31944 -6926   2321   6533  15878 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) -6015.530   626.181 -9.607 < 2e-16 ***
P_CAP_state_1_matrix1 -29.081    3.951 -7.360 4.57e-13 ***
P_CAP_state_1_matrix2  42.759    7.725  5.535 4.23e-08 ***
P_CAP_state_1_matrix3  6.821    6.267  1.088 0.276735  
P_CAP_state_1_matrix4 -10.249    6.975 -1.469 0.142117  
P_CAP_state_1_matrix5 -33.019    6.117 -5.398 8.90e-08 *** 
P_CAP_state_1_matrix6  31.169    8.469  3.680 0.000249 *** 
P_CAP_state_1_matrix7 -12.191    7.669 -1.590 0.112294  
P_CAP_state_1_matrix8  20.697    8.732  2.370 0.018008 *  
P_CAP_state_1_matrix9 -8.488    9.001 -0.943 0.345953  
P_CAP_state_1_matrix10 -35.768   6.717 -5.325 1.31e-07 *** 
P_CAP_state_1_matrix11 59.169    8.104  7.301 6.90e-13 *** 
P_CAP_state_1_matrix12 -80.249   9.412 -8.526 < 2e-16 *** 
P_CAP_state_1_matrix13 85.198    7.981 10.674 < 2e-16 *** 
P_CAP_state_1_matrix14 -13.483   7.541 -1.788 0.074158 . 
P_CAP_state_1_matrix15 -65.817   9.737 -6.759 2.68e-11 *** 
P_CAP_state_1_matrix16 34.259    8.551  4.006 6.75e-05 *** 
P_CAP_state_1_matrix17 10.981    3.398  3.232 0.001280 ** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 9969 on 798 degrees of freedom
Multiple R-squared:  0.9869,    Adjusted R-squared:  0.9866 
F-statistic:  3541 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 35: Performance of Multivariate Multiple Linear Regression MODEL(GSP-PCAP)

Response: GSP, Covariate: HWY We have now taken GSP as response variable and HWY as covariate. Summary for the model is described below. There are 17 summary outputs for this session of response and covariates.

```

> summary(model_2)
Response Y1 :

Call:
lm(formula = Y1 ~ HWY_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-16144.1 -3892.4 - 556.2  3272.5 20021.7 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -6241.796   576.659 -10.824 < 2e-16 ***
HWY_state_1_matrix1  16.973    3.600   4.714 2.86e-06 ***
HWY_state_1_matrix2 -4.466    8.424  -0.530 0.596194  
HWY_state_1_matrix3 -27.746   7.564  -3.668 0.000261 *** 
HWY_state_1_matrix4  37.580   6.291   5.974 3.50e-09 *** 
HWY_state_1_matrix5 -68.638   9.061  -7.575 9.93e-14 *** 
HWY_state_1_matrix6  98.354   12.565   7.827 1.58e-14 *** 
HWY_state_1_matrix7 -83.265   13.881  -5.998 3.02e-09 *** 
HWY_state_1_matrix8  68.973   14.798   4.661 3.69e-06 *** 
HWY_state_1_matrix9 -22.710   11.023  -2.060 0.039694 *  
HWY_state_1_matrix10 5.529    16.997   0.325 0.745044  
HWY_state_1_matrix11 9.007    16.933   0.532 0.594922  
HWY_state_1_matrix12 -43.315   13.957  -3.104 0.001980 ** 
HWY_state_1_matrix13 21.198    9.419   2.251 0.024677 *  
HWY_state_1_matrix14 -67.904   11.594  -5.857 6.90e-09 *** 
HWY_state_1_matrix15 116.021   11.236   10.326 < 2e-16 *** 
HWY_state_1_matrix16 -47.593   8.095  -5.880 6.04e-09 *** 
HWY_state_1_matrix17 -1.930    4.736  -0.408 0.683711  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 7172 on 798 degrees of freedom
Multiple R-squared:  0.9843,    Adjusted R-squared:  0.984 
F-statistic: 2949 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 36: Performance of Multivariate Multiple Linear Regression MODEL(GSP-HWY)

```

Response Y2 :

Call:
lm(formula = Y2 ~ HWY_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-17498.5 -4244.7 -574.8  3174.3 20389.6 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) -6390.1928   604.5270 -10.571 < 2e-16 ***
HWY_state_1_matrix1  16.6464    3.7744   4.410 1.17e-05 ***
HWY_state_1_matrix2   1.7931    8.8311   0.203   0.8392    
HWY_state_1_matrix3 -36.0292    7.9299  -4.543 6.39e-06 ***
HWY_state_1_matrix4  38.9577    6.5950   5.907 5.15e-09 *** 
HWY_state_1_matrix5 -71.5827    9.4986  -7.536 1.31e-13 *** 
HWY_state_1_matrix6 103.6649   13.1728   7.870 1.16e-14 *** 
HWY_state_1_matrix7 -85.9174   14.5518  -5.904 5.24e-09 *** 
HWY_state_1_matrix8  70.2098   15.5127   4.526 6.93e-06 *** 
HWY_state_1_matrix9 -24.9184   11.5556  -2.156   0.0314 *  
HWY_state_1_matrix10 10.9692   17.8186   0.616   0.5383    
HWY_state_1_matrix11  2.5913   17.7511   0.146   0.8840    
HWY_state_1_matrix12 -36.8030   14.6311  -2.515   0.0121 *  
HWY_state_1_matrix13 16.3937   9.8738   1.660   0.0972 .  
HWY_state_1_matrix14 -64.5740   12.1544  -5.313 1.40e-07 *** 
HWY_state_1_matrix15 114.6973  11.7788   9.738 < 2e-16 *** 
HWY_state_1_matrix16 -50.4244   8.4858  -5.942 4.20e-09 *** 
HWY_state_1_matrix17  0.5271   4.9644   0.106   0.9155    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 7519 on 798 degrees of freedom
Multiple R-squared:  0.9831,    Adjusted R-squared:  0.9828 
F-statistic: 2736 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 37: Performance of Multivariate Multiple Linear Regression MODEL(GSP-HWY)

```

Response Y3 :

Call:
lm(formula = Y3 ~ HWY_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-18072.0 -4265.6 -754.3  3139.9 21401.6 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -6825.000   626.323 -10.897 < 2e-16 ***
HWY_state_1_matrix1  17.057    3.910   4.362 1.46e-05 ***
HWY_state_1_matrix2   2.984    9.150   0.326  0.74443  
HWY_state_1_matrix3 -38.647    8.216  -4.704 3.01e-06 ***
HWY_state_1_matrix4  37.375    6.833   5.470 6.03e-08 *** 
HWY_state_1_matrix5 -64.052    9.841  -6.509 1.34e-10 *** 
HWY_state_1_matrix6  91.422   13.648   6.699 3.97e-11 *** 
HWY_state_1_matrix7 -74.930   15.076  -4.970 8.20e-07 *** 
HWY_state_1_matrix8  69.879   16.072   4.348 1.55e-05 *** 
HWY_state_1_matrix9 -38.329   11.972  -3.201  0.00142 ** 
HWY_state_1_matrix10 32.487   18.461   1.760  0.07883 .  
HWY_state_1_matrix11 -18.897   18.391  -1.028  0.30449  
HWY_state_1_matrix12 -18.793   15.159  -1.240  0.21543  
HWY_state_1_matrix13   8.437   10.230   0.825  0.40976  
HWY_state_1_matrix14 -60.572   12.593  -4.810 1.80e-06 *** 
HWY_state_1_matrix15 111.239   12.204   9.115 < 2e-16 *** 
HWY_state_1_matrix16 -56.013    8.792  -6.371 3.17e-10 *** 
HWY_state_1_matrix17   5.843    5.143   1.136  0.25632  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 7790 on 798 degrees of freedom
Multiple R-squared:  0.9834,    Adjusted R-squared:  0.983 
F-statistic:  2780 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 38: Performance of Multivariate Multiple Linear Regression MODEL(GSP-HWY)

```

Response Y4 :

Call:
lm(formula = Y4 ~ HWY_state_1_matrix)

Residuals:
    Min      1Q Median      3Q     Max 
-18366 -4646   -673   3214  22665 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) -7368.2997   652.1454 -11.299 < 2e-16 ***
HWY_state_1_matrix1 18.6629    4.0717   4.584 5.30e-06 ***
HWY_state_1_matrix2  0.6012    9.5268   0.063 0.949699  
HWY_state_1_matrix3 -38.0269   8.5545  -4.445 1.00e-05 *** 
HWY_state_1_matrix4 37.7024   7.1145   5.299 1.50e-07 *** 
HWY_state_1_matrix5 -59.9369  10.2468  -5.849 7.20e-09 *** 
HWY_state_1_matrix6 76.3010   14.2104   5.369 1.04e-07 *** 
HWY_state_1_matrix7 -52.5846  15.6981  -3.350 0.000847 *** 
HWY_state_1_matrix8 53.0252   16.7346   3.169 0.001590 ** 
HWY_state_1_matrix9 -33.3177  12.4658  -2.673 0.007677 ** 
HWY_state_1_matrix10 36.3126   19.2222   1.889 0.059241 .  
HWY_state_1_matrix11 -31.5275  19.1494  -1.646 0.100076  
HWY_state_1_matrix12  0.7904   15.7836   0.050 0.960072  
HWY_state_1_matrix13 -9.9443   10.6516  -0.934 0.350795  
HWY_state_1_matrix14 -35.2298  13.1118  -2.687 0.007363 ** 
HWY_state_1_matrix15 89.0142   12.7067   7.005 5.24e-12 *** 
HWY_state_1_matrix16 -57.8496  9.1542  -6.319 4.36e-10 *** 
HWY_state_1_matrix17 12.8702   5.3555   2.403 0.016480 *  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 8111 on 798 degrees of freedom
Multiple R-squared:  0.9836,    Adjusted R-squared:  0.9832 
F-statistic:  2807 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 39: Performance of Multivariate Multiple Linear Regression MODEL(GSP-HWY)

```

Response Y5 :

Call:
lm(formula = Y5 ~ HWY_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-16371.2 -4601.1 - 931.7  2603.9 21052.7 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) -7229.323   608.933 -11.872 < 2e-16 ***
HWY_state_1_matrix1  17.057    3.802   4.486 8.31e-06 ***
HWY_state_1_matrix2 -10.267    8.896  -1.154 0.248778  
HWY_state_1_matrix3 -22.050    7.988  -2.760 0.005904 ** 
HWY_state_1_matrix4  36.277    6.643   5.461 6.33e-08 *** 
HWY_state_1_matrix5 -47.287    9.568  -4.942 9.41e-07 *** 
HWY_state_1_matrix6  46.567   13.269   3.510 0.000474 *** 
HWY_state_1_matrix7 -23.957   14.658  -1.634 0.102562  
HWY_state_1_matrix8  38.226   15.626   2.446 0.014647 *  
HWY_state_1_matrix9 -43.457   11.640  -3.733 0.000202 *** 
HWY_state_1_matrix10 62.414   17.948   3.477 0.000534 *** 
HWY_state_1_matrix11 -69.264   17.881  -3.874 0.000116 *** 
HWY_state_1_matrix12  30.768   14.738   2.088 0.037142 *  
HWY_state_1_matrix13 -13.177   9.946  -1.325 0.185576  
HWY_state_1_matrix14 -37.396   12.243  -3.054 0.002330 ** 
HWY_state_1_matrix15  84.129   11.865   7.091 2.94e-12 *** 
HWY_state_1_matrix16 -57.352    8.548  -6.710 3.70e-11 *** 
HWY_state_1_matrix17  15.425    5.001   3.085 0.002108 ** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 7574 on 798 degrees of freedom
Multiple R-squared:  0.9855,   Adjusted R-squared:  0.9852
F-statistic:  3190 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 40: Performance of Multivariate Multiple Linear Regression MODEL(GSP-HWY)

```

Response Y6 :

Call:
lm(formula = Y6 ~ HWY_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-13823.7 -4433.7 -195.1  2291.8 18117.5 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) -6709.075   569.732 -11.776 < 2e-16 ***  
HWY_state_1_matrix1 12.029    3.557   3.382 0.000755 ***  
HWY_state_1_matrix2 -11.222    8.323  -1.348 0.177936    
HWY_state_1_matrix3 -16.007    7.473  -2.142 0.032505 *  
HWY_state_1_matrix4  40.336    6.215   6.490 1.51e-10 ***  
HWY_state_1_matrix5 -42.318    8.952  -4.727 2.69e-06 ***  
HWY_state_1_matrix6  31.272   12.415   2.519 0.011964 *  
HWY_state_1_matrix7 -11.168   13.714  -0.814 0.415702    
HWY_state_1_matrix8  33.035   14.620   2.260 0.024116 *  
HWY_state_1_matrix9 -57.029   10.890  -5.237 2.09e-07 ***  
HWY_state_1_matrix10 92.144   16.793   5.487 5.49e-08 ***  
HWY_state_1_matrix11 -111.324   16.729  -6.654 5.29e-11 *** 
HWY_state_1_matrix12  59.563   13.789   4.320 1.76e-05 ***  
HWY_state_1_matrix13 -9.791    9.306  -1.052 0.293047    
HWY_state_1_matrix14 -56.754   11.455  -4.955 8.85e-07 ***  
HWY_state_1_matrix15 104.171   11.101   9.384 < 2e-16 ***  
HWY_state_1_matrix16 -65.165    7.997  -8.148 1.42e-15 ***  
HWY_state_1_matrix17  14.624    4.679   3.126 0.001838 ** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 7086 on 798 degrees of freedom
Multiple R-squared:  0.9871, Adjusted R-squared:  0.9868
F-statistic:  3595 on 17 and 798 DF, p-value: < 2.2e-16

```

Figure 41: Performance of Multivariate Multiple Linear Regression MODEL(GSP-HWY)

```

Response Y7 :

Call:
lm(formula = Y7 ~ HWY_state_1_matrix)

Residuals:
    Min      1Q Median      3Q     Max 
-15437 -4476   -419   2304  17509 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) -6775.944   597.092 -11.348 < 2e-16 ***
HWY_state_1_matrix1   5.758    3.728   1.545  0.122852  
HWY_state_1_matrix2   3.456    8.723   0.396  0.692075  
HWY_state_1_matrix3  -28.032   7.832  -3.579  0.000366 *** 
HWY_state_1_matrix4   44.168   6.514   6.781  2.33e-11 *** 
HWY_state_1_matrix5  -37.563   9.382  -4.004 6.81e-05 *** 
HWY_state_1_matrix6   21.947  13.011   1.687  0.092020 .  
HWY_state_1_matrix7  -3.374   14.373  -0.235  0.814457  
HWY_state_1_matrix8   26.010  15.322   1.698  0.089980 .  
HWY_state_1_matrix9  -55.599  11.413  -4.871 1.34e-06 *** 
HWY_state_1_matrix10  107.111  17.599   6.086  1.80e-09 *** 
HWY_state_1_matrix11 -145.190  17.533  -8.281 5.13e-16 *** 
HWY_state_1_matrix12   93.429  14.451   6.465  1.76e-10 *** 
HWY_state_1_matrix13  -25.109   9.752  -2.575  0.010213 *  
HWY_state_1_matrix14  -51.878  12.005  -4.321 1.75e-05 *** 
HWY_state_1_matrix15  104.984  11.634   9.024 < 2e-16 *** 
HWY_state_1_matrix16  -67.653   8.381  -8.072 2.54e-15 *** 
HWY_state_1_matrix17   14.210   4.903   2.898  0.003858 ** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 7427 on 798 degrees of freedom
Multiple R-squared:  0.9869,    Adjusted R-squared:  0.9867 
F-statistic:  3547 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 42: Performance of Multivariate Multiple Linear Regression MODEL(GSP-HWY)

```

Response Y8 :

Call:
lm(formula = Y8 ~ HWY_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-16236.9 -4735.8 -366.1  2242.1 17540.6 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) -7063.1802   631.1232 -11.191 < 2e-16 ***
HWY_state_1_matrix1  0.8331    3.9404   0.211 0.832608    
HWY_state_1_matrix2 14.5944    9.2197   1.583 0.113827    
HWY_state_1_matrix3 -36.7929    8.2788  -4.444 1.01e-05 *** 
HWY_state_1_matrix4 47.5688    6.8852   6.909 9.99e-12 *** 
HWY_state_1_matrix5 -33.7070    9.9165  -3.399 0.000710 *** 
HWY_state_1_matrix6 10.7241   13.7523   0.780 0.435737    
HWY_state_1_matrix7 10.9934   15.1920   0.724 0.469505    
HWY_state_1_matrix8 16.0882   16.1952   0.993 0.320818    
HWY_state_1_matrix9 -59.4358   12.0640  -4.927 1.02e-06 *** 
HWY_state_1_matrix10 126.4779   18.6025   6.799 2.06e-11 *** 
HWY_state_1_matrix11 -178.9149   18.5321  -9.654 < 2e-16 *** 
HWY_state_1_matrix12 125.2030   15.2748   8.197 9.82e-16 *** 
HWY_state_1_matrix13 -38.2844   10.3082  -3.714 0.000218 *** 
HWY_state_1_matrix14 -46.6113   12.6892  -3.673 0.000255 *** 
HWY_state_1_matrix15 104.1367   12.2971   8.468 < 2e-16 *** 
HWY_state_1_matrix16 -73.0774    8.8591  -8.249 6.57e-16 *** 
HWY_state_1_matrix17 17.1960    5.1828   3.318 0.000948 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 7850 on 798 degrees of freedom
Multiple R-squared:  0.9867,   Adjusted R-squared:  0.9864
F-statistic:  3479 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 43: Performance of Multivariate Multiple Linear Regression MODEL(GSP-HWY)

```

Response Y9 :

Call:
lm(formula = Y9 ~ HWY_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-17648.4 -5307.0 -500.1  2319.5 18279.9 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) -7094.096   664.790 -10.671 < 2e-16 ***
HWY_state_1_matrix1   -4.277    4.151  -1.031 0.303072    
HWY_state_1_matrix2   21.057    9.711   2.168 0.030431 *  
HWY_state_1_matrix3  -39.529    8.720  -4.533 6.71e-06 *** 
HWY_state_1_matrix4   46.727    7.252   6.443 2.02e-10 *** 
HWY_state_1_matrix5  -17.457   10.445  -1.671 0.095070 .  
HWY_state_1_matrix6  -19.963   14.486  -1.378 0.168553    
HWY_state_1_matrix7   43.734   16.002   2.733 0.006416 **  
HWY_state_1_matrix8  -3.390    17.059  -0.199 0.842515    
HWY_state_1_matrix9  -67.536   12.708  -5.315 1.39e-07 *** 
HWY_state_1_matrix10  157.461   19.595   8.036 3.34e-15 *** 
HWY_state_1_matrix11 -223.036   19.521 -11.426 < 2e-16 *** 
HWY_state_1_matrix12  164.576   16.090   10.229 < 2e-16 *** 
HWY_state_1_matrix13 -50.308    10.858  -4.633 4.20e-06 *** 
HWY_state_1_matrix14 -48.496    13.366  -3.628 0.000303 *** 
HWY_state_1_matrix15  100.576   12.953   7.765 2.51e-14 *** 
HWY_state_1_matrix16 -74.719    9.332  -8.007 4.15e-15 *** 
HWY_state_1_matrix17  21.776    5.459   3.989 7.25e-05 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 8269 on 798 degrees of freedom
Multiple R-squared:  0.9865,   Adjusted R-squared:  0.9862
F-statistic:  3425 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 44: Performance of Multivariate Multiple Linear Regression MODEL(GSP-HWY)

```

Response Y10 :

Call:
lm(formula = Y10 ~ HWY_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-17678.0 -5656.0 -123.1  3372.0 18405.7 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) -6980.515   685.732 -10.180 < 2e-16 ***
HWY_state_1_matrix1 -8.093     4.281 -1.890 0.059090 .  
HWY_state_1_matrix2 21.030    10.017  2.099 0.036101 *  
HWY_state_1_matrix3 -33.735    8.995 -3.750 0.000189 *** 
HWY_state_1_matrix4 42.084     7.481  5.625 2.56e-08 *** 
HWY_state_1_matrix5 -2.429    10.775 -0.225 0.821672  
HWY_state_1_matrix6 -45.650    14.942 -3.055 0.002325 ** 
HWY_state_1_matrix7 69.558    16.507  4.214 2.80e-05 *** 
HWY_state_1_matrix8 -13.040    17.596 -0.741 0.458859  
HWY_state_1_matrix9 -85.874    13.108 -6.551 1.02e-10 *** 
HWY_state_1_matrix10 192.102   20.212   9.504 < 2e-16 *** 
HWY_state_1_matrix11 -261.921   20.136 -13.008 < 2e-16 *** 
HWY_state_1_matrix12 190.333   16.596  11.468 < 2e-16 *** 
HWY_state_1_matrix13 -50.580    11.200 -4.516 7.25e-06 *** 
HWY_state_1_matrix14 -55.727    13.787 -4.042 5.81e-05 *** 
HWY_state_1_matrix15 101.515    13.361   7.598 8.44e-14 *** 
HWY_state_1_matrix16 -80.547    9.626 -8.368 2.61e-16 *** 
HWY_state_1_matrix17  28.169    5.631   5.002 6.97e-07 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 8529 on 798 degrees of freedom
Multiple R-squared:  0.9863,    Adjusted R-squared:  0.986 
F-statistic:  3370 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 45: Performance of Multivariate Multiple Linear Regression MODEL(GSP-HWY)

```

Response Y11 :

Call:
lm(formula = Y11 ~ HWY_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-17768.2 -5767.0 -103.1  4300.8 19112.0 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) -6599.436   718.580 -9.184 < 2e-16 ***  
HWY_state_1_matrix1 -11.135    4.486 -2.482 0.01327 *  
HWY_state_1_matrix2  18.031   10.497  1.718 0.08624 .  
HWY_state_1_matrix3 -25.103    9.426 -2.663 0.00790 **  
HWY_state_1_matrix4  39.252    7.839  5.007 6.81e-07 ***  
HWY_state_1_matrix5   3.257   11.291  0.288 0.77309    
HWY_state_1_matrix6 -63.656   15.658 -4.065 5.27e-05 ***  
HWY_state_1_matrix7  90.093   17.297  5.209 2.42e-07 ***  
HWY_state_1_matrix8 -9.272   18.439 -0.503 0.61522    
HWY_state_1_matrix9 -121.952   13.736 -8.878 < 2e-16 ***  
HWY_state_1_matrix10 238.065   21.180 11.240 < 2e-16 ***  
HWY_state_1_matrix11 -297.430   21.100 -14.096 < 2e-16 ***  
HWY_state_1_matrix12  202.401   17.391 11.638 < 2e-16 ***  
HWY_state_1_matrix13 -37.491   11.737 -3.194 0.00146 **  
HWY_state_1_matrix14 -80.718   14.448 -5.587 3.17e-08 ***  
HWY_state_1_matrix15 115.119   14.001  8.222 8.07e-16 ***  
HWY_state_1_matrix16 -82.758   10.087 -8.205 9.23e-16 ***  
HWY_state_1_matrix17  30.172    5.901  5.113 3.97e-07 ***  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 8938 on 798 degrees of freedom
Multiple R-squared:  0.9848,   Adjusted R-squared:  0.9845
F-statistic:  3046 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 46: Performance of Multivariate Multiple Linear Regression MODEL(GSP-HWY)

```

Response Y12 :

Call:
lm(formula = Y12 ~ HWY_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-18795 -5781 -1112   3755  21114 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -6244.477   757.982 -8.238 7.13e-16 ***
HWY_state_1_matrix1 -12.770     4.732 -2.698 0.007114 ** 
HWY_state_1_matrix2  17.893    11.073  1.616 0.106497    
HWY_state_1_matrix3 -22.831     9.943 -2.296 0.021924 *  
HWY_state_1_matrix4  45.945     8.269  5.556 3.76e-08 *** 
HWY_state_1_matrix5 -9.924    11.910 -0.833 0.404944    
HWY_state_1_matrix6 -59.649    16.517 -3.611 0.000323 *** 
HWY_state_1_matrix7  88.378    18.246  4.844 1.53e-06 *** 
HWY_state_1_matrix8  1.709    19.450  0.088 0.929988    
HWY_state_1_matrix9 -129.269   14.489 -8.922 < 2e-16 *** 
HWY_state_1_matrix10 228.851    22.342 10.243 < 2e-16 *** 
HWY_state_1_matrix11 -281.419   22.257 -12.644 < 2e-16 *** 
HWY_state_1_matrix12  189.736   18.345 10.343 < 2e-16 *** 
HWY_state_1_matrix13 -33.242    12.380 -2.685 0.007402 ** 
HWY_state_1_matrix14 -79.898    15.240 -5.243 2.03e-07 *** 
HWY_state_1_matrix15 113.898    14.769  7.712 3.69e-14 *** 
HWY_state_1_matrix16 -81.165    10.640 -7.628 6.77e-14 *** 
HWY_state_1_matrix17  30.621     6.225  4.919 1.05e-06 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 9428 on 798 degrees of freedom
Multiple R-squared:  0.9834,    Adjusted R-squared:  0.9831 
F-statistic: 2787 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 47: Performance of Multivariate Multiple Linear Regression MODEL(GSP-HWY)

```

Response Y13 :

Call:
lm(formula = Y13 ~ HWY_state_1_matrix)

Residuals:
    Min      1Q Median      3Q     Max 
-19007 -5176 -1156   4233  21633 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) -5961.669    787.210 -7.573 1.01e-13 ***
HWY_state_1_matrix1 -11.730     4.915 -2.387  0.0172 *  
HWY_state_1_matrix2  11.640    11.500  1.012  0.3118    
HWY_state_1_matrix3 -14.117    10.326 -1.367  0.1720    
HWY_state_1_matrix4  45.945     8.588  5.350 1.15e-07 *** 
HWY_state_1_matrix5 -24.798    12.369 -2.005  0.0453 *  
HWY_state_1_matrix6 -41.515    17.153 -2.420  0.0157 *  
HWY_state_1_matrix7  78.863    18.949  4.162 3.50e-05 *** 
HWY_state_1_matrix8  13.888    20.200  0.688  0.4920    
HWY_state_1_matrix9 -146.204   15.048 -9.716 < 2e-16 *** 
HWY_state_1_matrix10 240.082   23.203 10.347 < 2e-16 *** 
HWY_state_1_matrix11 -287.872   23.115 -12.454 < 2e-16 *** 
HWY_state_1_matrix12  186.387   19.052  9.783 < 2e-16 *** 
HWY_state_1_matrix13 -22.315    12.858 -1.736  0.0830 .  
HWY_state_1_matrix14 -97.280    15.827 -6.146 1.25e-09 *** 
HWY_state_1_matrix15  134.136   15.338  8.745 < 2e-16 *** 
HWY_state_1_matrix16 -90.118    11.050 -8.155 1.35e-15 *** 
HWY_state_1_matrix17  31.649     6.465  4.896 1.19e-06 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 9791 on 798 degrees of freedom
Multiple R-squared:  0.9816,   Adjusted R-squared:  0.9812
F-statistic: 2507 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 48: Performance of Multivariate Multiple Linear Regression MODEL(GSP-HWY)

```

Response Y14 :

Call:
lm(formula = Y14 ~ HWY_state_1_matrix)

Residuals:
    Min      1Q Median      3Q     Max 
-21426 -4861 -1255  5055 22994 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept) -6347.571   834.278 -7.608 7.82e-14 ***
HWY_state_1_matrix1 -5.548     5.209 -1.065 0.28715  
HWY_state_1_matrix2  6.364    12.187  0.522 0.60170  
HWY_state_1_matrix3 -17.748   10.944 -1.622 0.10526  
HWY_state_1_matrix4  47.620    9.101  5.232 2.14e-07 ***
HWY_state_1_matrix5 -31.766   13.109 -2.423 0.01560 *  
HWY_state_1_matrix6 -31.892   18.179 -1.754 0.07976 .  
HWY_state_1_matrix7  90.972   20.082  4.530 6.80e-06 *** 
HWY_state_1_matrix8  1.736    21.408  0.081 0.93538  
HWY_state_1_matrix9 -166.292  15.947 -10.428 < 2e-16 *** 
HWY_state_1_matrix10 269.697  24.591  10.968 < 2e-16 *** 
HWY_state_1_matrix11 -302.851  24.497 -12.363 < 2e-16 *** 
HWY_state_1_matrix12  201.052  20.192  9.957 < 2e-16 *** 
HWY_state_1_matrix13 -35.506   13.626 -2.606 0.00934 ** 
HWY_state_1_matrix14 -91.533   16.774 -5.457 6.47e-08 *** 
HWY_state_1_matrix15  143.121  16.255  8.804 < 2e-16 *** 
HWY_state_1_matrix16 -113.283  11.711 -9.673 < 2e-16 *** 
HWY_state_1_matrix17  42.847   6.851   6.254 6.52e-10 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 10380 on 798 degrees of freedom
Multiple R-squared:  0.9809,    Adjusted R-squared:  0.9805 
F-statistic:  2411 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 49: Performance of Multivariate Multiple Linear Regression MODEL(GSP-HWY)

```

Response Y15 :

Call:
lm(formula = Y15 ~ HWY_state_1_matrix)

Residuals:
    Min      1Q Median      3Q     Max 
-23245 -5880 -1202  6056 26118 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) -7204.435   903.504 -7.974 5.32e-15 ***
HWY_state_1_matrix1 -5.050    5.641 -0.895  0.37096  
HWY_state_1_matrix2  8.593   13.199  0.651  0.51519  
HWY_state_1_matrix3 -23.144   11.852 -1.953  0.05120 .  
HWY_state_1_matrix4  54.074    9.857  5.486 5.52e-08 *** 
HWY_state_1_matrix5 -38.617   14.196 -2.720  0.00667 ** 
HWY_state_1_matrix6 -26.215   19.688 -1.332  0.18338  
HWY_state_1_matrix7  91.464   21.749  4.206 2.90e-05 *** 
HWY_state_1_matrix8   5.184   23.185  0.224  0.82314  
HWY_state_1_matrix9 -190.916   17.271 -11.054 < 2e-16 *** 
HWY_state_1_matrix10 310.559   26.631 11.662 < 2e-16 *** 
HWY_state_1_matrix11 -349.662   26.530 -13.180 < 2e-16 *** 
HWY_state_1_matrix12  239.418   21.867 10.949 < 2e-16 *** 
HWY_state_1_matrix13 -48.183   14.757 -3.265  0.00114 ** 
HWY_state_1_matrix14 -92.001   18.166 -5.065 5.09e-07 *** 
HWY_state_1_matrix15  152.258   17.604  8.649 < 2e-16 *** 
HWY_state_1_matrix16 -135.725   12.683 -10.702 < 2e-16 *** 
HWY_state_1_matrix17  55.548    7.420  7.487 1.87e-13 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 11240 on 798 degrees of freedom
Multiple R-squared:  0.9805,   Adjusted R-squared:  0.9801
F-statistic: 2363 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 50: Performance of Multivariate Multiple Linear Regression MODEL(GSP-HWY)

```

Response Y16 :

Call:
lm(formula = Y16 ~ HWY_state_1_matrix)

Residuals:
    Min      1Q Median      3Q     Max 
-24980 -6237 -1137   7562  29092 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept) -7689.147   959.943 -8.010 4.06e-15 ***
HWY_state_1_matrix1 -2.525     5.993 -0.421  0.67371  
HWY_state_1_matrix2  6.517    14.023  0.465  0.64225  
HWY_state_1_matrix3 -25.399    12.592 -2.017  0.04402 *  
HWY_state_1_matrix4  57.973    10.472  5.536  4.21e-08 *** 
HWY_state_1_matrix5 -43.350    15.083 -2.874  0.00416 ** 
HWY_state_1_matrix6 -27.175    20.917 -1.299  0.19426  
HWY_state_1_matrix7 109.830    23.107  4.753  2.38e-06 *** 
HWY_state_1_matrix8 -9.072     24.633 -0.368  0.71274  
HWY_state_1_matrix9 -211.095   18.349 -11.504 < 2e-16 *** 
HWY_state_1_matrix10 353.728   28.295 12.502 < 2e-16 *** 
HWY_state_1_matrix11 -392.826   28.187 -13.936 < 2e-16 *** 
HWY_state_1_matrix12 272.384    23.233 11.724 < 2e-16 *** 
HWY_state_1_matrix13 -63.936    15.679 -4.078 5.00e-05 *** 
HWY_state_1_matrix14 -84.454    19.300 -4.376 1.37e-05 *** 
HWY_state_1_matrix15 158.382    18.704  8.468 < 2e-16 *** 
HWY_state_1_matrix16 -157.793   13.475 -11.710 < 2e-16 *** 
HWY_state_1_matrix17  66.809     7.883  8.475 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 11940 on 798 degrees of freedom
Multiple R-squared:  0.98,    Adjusted R-squared:  0.9795 
F-statistic:  2296 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 51: Performance of Multivariate Multiple Linear Regression MODEL(GSP-HWY)

```

Response Y17 :

Call:
lm(formula = Y17 ~ HWY_state_1_matrix)

Residuals:
    Min      1Q Median      3Q     Max 
-25996 -6861   -399   9328  32378 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) -8547.263    990.287 -8.631 < 2e-16 ***
HWY_state_1_matrix1  8.063     6.183   1.304   0.1926    
HWY_state_1_matrix2 -4.956    14.466  -0.343   0.7320    
HWY_state_1_matrix3 -22.876   12.990  -1.761   0.0786 .  
HWY_state_1_matrix4 45.793    10.803   4.239  2.51e-05 ***
HWY_state_1_matrix5 -38.078   15.560  -2.447   0.0146 *  
HWY_state_1_matrix6 -10.760   21.579  -0.499   0.6182    
HWY_state_1_matrix7 105.372   23.838   4.420  1.12e-05 *** 
HWY_state_1_matrix8 -14.027   25.412  -0.552   0.5811    
HWY_state_1_matrix9 -228.348  18.929 -12.063 < 2e-16 *** 
HWY_state_1_matrix10 377.524   29.189  12.934 < 2e-16 *** 
HWY_state_1_matrix11 -398.084  29.078 -13.690 < 2e-16 *** 
HWY_state_1_matrix12 277.319   23.967  11.571 < 2e-16 *** 
HWY_state_1_matrix13 -71.080   16.174  -4.395  1.26e-05 *** 
HWY_state_1_matrix14 -81.003   19.910  -4.068  5.20e-05 *** 
HWY_state_1_matrix15 171.937   19.295   8.911 < 2e-16 *** 
HWY_state_1_matrix16 -193.091  13.901 -13.891 < 2e-16 *** 
HWY_state_1_matrix17  84.779    8.132   10.425 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 12320 on 798 degrees of freedom
Multiple R-squared:  0.98,    Adjusted R-squared:  0.9796 
F-statistic: 2304 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 52: Performance of Multivariate Multiple Linear Regression MODEL(GSP-HWY)

Response: GSP, Covariate: WATER We have now taken GSP as response variable and WATER as covariate. Summary for the model is described below. There are 17 summary outputs for this session of response and covariates.

```

> summary(model_3)
Response Y1 :

Call:
lm(formula = Y1 ~ WATER_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-8621.2 -4871.7 -532.5  3031.5 21457.3 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 1766.791   352.640   5.010 6.70e-07 ***
WATER_state_1_matrix1 -13.449    10.977  -1.225 0.220854    
WATER_state_1_matrix2  43.673    17.106   2.553 0.010865 *  
WATER_state_1_matrix3 11.846    11.251   1.053 0.292712    
WATER_state_1_matrix4 -34.222    8.747  -3.913 9.91e-05 *** 
WATER_state_1_matrix5 -23.307    9.474  -2.460 0.014104 *  
WATER_state_1_matrix6 -20.031   10.879  -1.841 0.065945 .  
WATER_state_1_matrix7  44.292   12.080   3.666 0.000262 *** 
WATER_state_1_matrix8  46.163   15.913   2.901 0.003822 ** 
WATER_state_1_matrix9 -156.989   12.962 -12.111 < 2e-16 *** 
WATER_state_1_matrix10 70.396    8.457   8.324 3.66e-16 *** 
WATER_state_1_matrix11 113.771   11.342  10.031 < 2e-16 *** 
WATER_state_1_matrix12 -200.100   10.929 -18.310 < 2e-16 *** 
WATER_state_1_matrix13 385.812   13.819  27.920 < 2e-16 *** 
WATER_state_1_matrix14 -341.474   15.421 -22.144 < 2e-16 *** 
WATER_state_1_matrix15 174.361   10.705  16.288 < 2e-16 *** 
WATER_state_1_matrix16 -199.324   10.397 -19.171 < 2e-16 *** 
WATER_state_1_matrix17 109.170    6.557  16.650 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 6150 on 798 degrees of freedom
Multiple R-squared:  0.9885,    Adjusted R-squared:  0.9882 
F-statistic:  4027 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 53: Performance of Multivariate Multiple Linear Regression MODEL(GSP-WATER)

```

Response Y2 :

Call:
lm(formula = Y2 ~ WATER_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-8632.1 -5029.3 -698.8  2539.8 22875.1 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept)  2121.892   356.928   5.945 4.13e-09 *** 
WATER_state_1_matrix1 -15.150    11.110  -1.364 0.173092    
WATER_state_1_matrix2  49.149    17.314   2.839 0.004646 **  
WATER_state_1_matrix3  2.758    11.387   0.242 0.808688    
WATER_state_1_matrix4 -27.058    8.853  -3.056 0.002315 **  
WATER_state_1_matrix5 -21.396    9.590  -2.231 0.025945 *   
WATER_state_1_matrix6 -27.261   11.011  -2.476 0.013502 *   
WATER_state_1_matrix7  46.512   12.227   3.804 0.000153 ***  
WATER_state_1_matrix8  47.932   16.107   2.976 0.003009 **  
WATER_state_1_matrix9 -157.207  13.120 -11.983 < 2e-16 *** 
WATER_state_1_matrix10 73.121    8.559   8.543 < 2e-16 *** 
WATER_state_1_matrix11 107.265  11.480   9.343 < 2e-16 *** 
WATER_state_1_matrix12 -198.622 11.061 -17.956 < 2e-16 *** 
WATER_state_1_matrix13 387.552  13.987  27.709 < 2e-16 *** 
WATER_state_1_matrix14 -350.747 15.608 -22.472 < 2e-16 *** 
WATER_state_1_matrix15 185.445  10.835  17.115 < 2e-16 *** 
WATER_state_1_matrix16 -196.462 10.524 -18.669 < 2e-16 *** 
WATER_state_1_matrix17 104.862   6.637  15.801 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 6225 on 798 degrees of freedom
Multiple R-squared:  0.9884,   Adjusted R-squared:  0.9882
F-statistic:  4014 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 54: Performance of Multivariate Multiple Linear Regression MODEL(GSP-WATER)

```

Response Y3 :

Call:
lm(formula = Y3 ~ WATER_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-9056.3 -5056.6 - 993.8 2327.0 23224.9 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) 2629.435   361.228   7.279 8.05e-13 ***  
WATER_state_1_matrix1 -24.651    11.244  -2.192 0.028642 *  
WATER_state_1_matrix2  67.885    17.523   3.874 0.000116 ***  
WATER_state_1_matrix3 -13.371    11.525  -1.160 0.246317    
WATER_state_1_matrix4 -9.785     8.960  -1.092 0.275097    
WATER_state_1_matrix5 -35.677    9.705  -3.676 0.000253 ***  
WATER_state_1_matrix6 -23.421    11.144  -2.102 0.035893 *  
WATER_state_1_matrix7 39.408    12.375   3.185 0.001506 **  
WATER_state_1_matrix8 58.887    16.301   3.613 0.000322 ***  
WATER_state_1_matrix9 -161.918   13.278 -12.195 < 2e-16 ***  
WATER_state_1_matrix10 76.929    8.662   8.881 < 2e-16 ***  
WATER_state_1_matrix11 104.001   11.619   8.951 < 2e-16 ***  
WATER_state_1_matrix12 -202.247   11.195 -18.066 < 2e-16 ***  
WATER_state_1_matrix13 391.947   14.155   27.690 < 2e-16 ***  
WATER_state_1_matrix14 -360.422   15.796 -22.817 < 2e-16 ***  
WATER_state_1_matrix15 203.404   10.966   18.549 < 2e-16 ***  
WATER_state_1_matrix16 -204.194   10.650 -19.172 < 2e-16 ***  
WATER_state_1_matrix17 104.283    6.716   15.527 < 2e-16 ***  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 6300 on 798 degrees of freedom
Multiple R-squared:  0.9891,   Adjusted R-squared:  0.9889
F-statistic: 4276 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 55: Performance of Multivariate Multiple Linear Regression MODEL(GSP-WATER)

```

Response Y4 :

Call:
lm(formula = Y4 ~ WATER_state_1_matrix)

Residuals:
    Min      1Q Median      3Q     Max 
 -8823  -4928  -1328   2421  21087 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) 2941.750   351.412   8.371 2.54e-16 ***
WATER_state_1_matrix1 -31.873    10.939  -2.914 0.003671 ** 
WATER_state_1_matrix2  82.212    17.047   4.823 1.70e-06 *** 
WATER_state_1_matrix3 -21.092    11.211  -1.881 0.060294 .  
WATER_state_1_matrix4 -8.257     8.716  -0.947 0.343791    
WATER_state_1_matrix5 -33.230    9.441  -3.520 0.000457 *** 
WATER_state_1_matrix6 -25.038    10.841  -2.310 0.021165 *  
WATER_state_1_matrix7  30.812    12.038   2.560 0.010665 *  
WATER_state_1_matrix8  70.202    15.858   4.427 1.09e-05 *** 
WATER_state_1_matrix9 -167.631   12.917 -12.978 < 2e-16 *** 
WATER_state_1_matrix10 79.227    8.427   9.401 < 2e-16 *** 
WATER_state_1_matrix11 95.966    11.303   8.490 < 2e-16 *** 
WATER_state_1_matrix12 -196.026   10.890 -18.000 < 2e-16 *** 
WATER_state_1_matrix13 393.225   13.770  28.556 < 2e-16 *** 
WATER_state_1_matrix14 -372.308   15.367 -24.228 < 2e-16 *** 
WATER_state_1_matrix15 221.818   10.668  20.794 < 2e-16 *** 
WATER_state_1_matrix16 -206.236   10.361 -19.905 < 2e-16 *** 
WATER_state_1_matrix17  99.818    6.534  15.277 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 6129 on 798 degrees of freedom
Multiple R-squared:  0.9906,    Adjusted R-squared:  0.9904 
F-statistic: 4953 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 56: Performance of Multivariate Multiple Linear Regression MODEL(GSP-WATER)

```
Response Y5 :  
  
Call:  
lm(formula = Y5 ~ WATER_state_1_matrix)  
  
Residuals:  
    Min      1Q Median      3Q     Max  
-9002 -4253 -1133  2167 18590  
  
Coefficients:  
              Estimate Std. Error t value Pr(>|t|)  
(Intercept) 2808.251   324.595   8.652 < 2e-16 ***  
WATER_state_1_matrix1 -33.079   10.104  -3.274 0.00111 **  
WATER_state_1_matrix2  79.966   15.746   5.079 4.74e-07 ***  
WATER_state_1_matrix3 -12.969   10.356  -1.252 0.21083  
WATER_state_1_matrix4 -12.710    8.051  -1.579 0.11480  
WATER_state_1_matrix5 -34.972    8.721  -4.010 6.64e-05 ***  
WATER_state_1_matrix6 -24.511   10.014  -2.448 0.01459 *  
WATER_state_1_matrix7  35.486   11.120   3.191 0.00147 **  
WATER_state_1_matrix8  59.713   14.648   4.077 5.03e-05 ***  
WATER_state_1_matrix9 -143.981  11.931 -12.068 < 2e-16 ***  
WATER_state_1_matrix10 51.978    7.784   6.678 4.55e-11 ***  
WATER_state_1_matrix11 111.402   10.440  10.670 < 2e-16 ***  
WATER_state_1_matrix12 -195.535  10.059 -19.438 < 2e-16 ***  
WATER_state_1_matrix13 368.402   12.720  28.963 < 2e-16 ***  
WATER_state_1_matrix14 -341.777  14.194 -24.078 < 2e-16 ***  
WATER_state_1_matrix15 204.405   9.853  20.744 < 2e-16 ***  
WATER_state_1_matrix16 -201.701   9.570 -21.076 < 2e-16 ***  
WATER_state_1_matrix17 101.502   6.035  16.818 < 2e-16 ***  
---  
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1  
  
Residual standard error: 5661 on 798 degrees of freedom  
Multiple R-squared:  0.9919,   Adjusted R-squared:  0.9917  
F-statistic:  5748 on 17 and 798 DF,  p-value: < 2.2e-16
```

Figure 57: Performance of Multivariate Multiple Linear Regression MODEL(GSP-WATER)

```

Response Y6 :

Call:
lm(formula = Y6 ~ WATER_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-8968.8 -3483.8 -579.8 2410.2 16212.2 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 2543.142   303.010   8.393 < 2e-16 ***
WATER_state_1_matrix1 -40.665     9.432  -4.311 1.83e-05 ***
WATER_state_1_matrix2  87.142    14.699   5.928 4.55e-09 ***
WATER_state_1_matrix3 -13.857     9.667  -1.433   0.1521  
WATER_state_1_matrix4 -9.668     7.516  -1.286   0.1987  
WATER_state_1_matrix5 -34.888     8.141  -4.286 2.05e-05 ***
WATER_state_1_matrix6 -36.558     9.348  -3.911 9.98e-05 ***
WATER_state_1_matrix7  58.274    10.380   5.614 2.73e-08 ***
WATER_state_1_matrix8  33.621    13.674   2.459   0.0141 *  
WATER_state_1_matrix9 -109.668    11.138  -9.847 < 2e-16 ***
WATER_state_1_matrix10 22.134     7.266   3.046   0.0024 ** 
WATER_state_1_matrix11 127.507     9.746  13.083 < 2e-16 ***
WATER_state_1_matrix12 -193.467    9.390 -20.602 < 2e-16 ***
WATER_state_1_matrix13 338.817    11.874  28.535 < 2e-16 ***
WATER_state_1_matrix14 -310.630    13.251 -23.443 < 2e-16 ***
WATER_state_1_matrix15 188.856     9.198  20.532 < 2e-16 ***
WATER_state_1_matrix16 -201.131    8.934 -22.513 < 2e-16 ***
WATER_state_1_matrix17 105.773     5.634  18.774 < 2e-16 ***

---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 5284 on 798 degrees of freedom
Multiple R-squared:  0.9928,    Adjusted R-squared:  0.9927 
F-statistic:  6502 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 58: Performance of Multivariate Multiple Linear Regression MODEL(GSP-WATER)

```

Response Y7 :

Call:
lm(formula = Y7 ~ WATER_state_1_matrix)

Residuals:
    Min      1Q Median      3Q     Max 
-9943 -3248   -717   3040  16916 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) 3084.664    315.954   9.763 < 2e-16 ***
WATER_state_1_matrix1 -48.470     9.835  -4.928 1.01e-06 ***
WATER_state_1_matrix2  98.541    15.327   6.429 2.20e-10 ***
WATER_state_1_matrix3 -19.953    10.080  -1.979  0.0481 *  
WATER_state_1_matrix4 -3.783     7.837  -0.483  0.6294  
WATER_state_1_matrix5 -33.795     8.489  -3.981 7.48e-05 *** 
WATER_state_1_matrix6 -43.116     9.747  -4.424 1.11e-05 *** 
WATER_state_1_matrix7  63.001    10.824   5.821 8.49e-09 *** 
WATER_state_1_matrix8  32.836    14.258   2.303  0.0215 *  
WATER_state_1_matrix9 -102.012    11.613  -8.784 < 2e-16 *** 
WATER_state_1_matrix10  7.984     7.577   1.054  0.2923  
WATER_state_1_matrix11 133.032    10.162  13.091 < 2e-16 *** 
WATER_state_1_matrix12 -191.842    9.792  -19.593 < 2e-16 *** 
WATER_state_1_matrix13 333.036    12.381  26.899 < 2e-16 *** 
WATER_state_1_matrix14 -314.375    13.817 -22.754 < 2e-16 *** 
WATER_state_1_matrix15 201.214     9.591  20.979 < 2e-16 *** 
WATER_state_1_matrix16 -203.823    9.316  -21.880 < 2e-16 *** 
WATER_state_1_matrix17 103.999     5.875  17.703 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 5510 on 798 degrees of freedom
Multiple R-squared:  0.9928,   Adjusted R-squared:  0.9927
F-statistic:  6481 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 59: Performance of Multivariate Multiple Linear Regression MODEL(GSP-WATER)

```

Response Y8 :

Call:
lm(formula = Y8 ~ WATER_state_1_matrix)

Residuals:
    Min      1Q Median      3Q     Max 
-10318 -2990 -1215  2919 16482 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) 3345.286   323.417 10.344 < 2e-16 ***
WATER_state_1_matrix1 -54.792    10.067 -5.443 6.99e-08 ***
WATER_state_1_matrix2 108.051    15.689  6.887 1.15e-11 ***
WATER_state_1_matrix3 -27.309    10.318 -2.647 0.008288 ** 
WATER_state_1_matrix4  2.695     8.022  0.336 0.737002  
WATER_state_1_matrix5 -30.511    8.689 -3.511 0.000471 *** 
WATER_state_1_matrix6 -50.402    9.977 -5.052 5.43e-07 *** 
WATER_state_1_matrix7 64.568    11.079  5.828 8.15e-09 *** 
WATER_state_1_matrix8 35.550    14.594  2.436 0.015075 *  
WATER_state_1_matrix9 -98.358   11.888 -8.274 5.42e-16 *** 
WATER_state_1_matrix10 -2.238    7.756 -0.289 0.773035  
WATER_state_1_matrix11 138.030   10.402 13.269 < 2e-16 *** 
WATER_state_1_matrix12 -189.442   10.023 -18.901 < 2e-16 *** 
WATER_state_1_matrix13 323.078   12.673 25.493 < 2e-16 *** 
WATER_state_1_matrix14 -312.006   14.143 -22.061 < 2e-16 *** 
WATER_state_1_matrix15 210.852   9.818 21.477 < 2e-16 *** 
WATER_state_1_matrix16 -210.799   9.536 -22.107 < 2e-16 *** 
WATER_state_1_matrix17 106.278    6.013 17.673 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 5640 on 798 degrees of freedom
Multiple R-squared:  0.9931,    Adjusted R-squared:  0.993 
F-statistic: 6782 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 60: Performance of Multivariate Multiple Linear Regression MODEL(GSP-WATER)

```

Response Y9 :

Call:
  lm(formula = Y9 ~ WATER_state_1_matrix)

Residuals:
    Min      1Q   Median      3Q     Max 
 -11048.4 -2815.1 - 932.1  2889.7 17253.2 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) 3925.945   334.621 11.733 < 2e-16 ***
WATER_state_1_matrix1 -64.868    10.416 -6.228 7.65e-10 ***
WATER_state_1_matrix2 121.071    16.232 7.459 2.28e-13 ***
WATER_state_1_matrix3 -33.520    10.676 -3.140 0.00175 ** 
WATER_state_1_matrix4 12.698     8.300  1.530 0.12642  
WATER_state_1_matrix5 -36.278    8.990 -4.035 5.98e-05 *** 
WATER_state_1_matrix6 -50.045    10.323 -4.848 1.50e-06 *** 
WATER_state_1_matrix7 61.443     11.463  5.360 1.09e-07 *** 
WATER_state_1_matrix8 35.295     15.100  2.337 0.01967 *  
WATER_state_1_matrix9 -85.062    12.300 -6.916 9.54e-12 *** 
WATER_state_1_matrix10 -16.884    8.024 -2.104 0.03568 * 
WATER_state_1_matrix11 142.428    10.763 13.233 < 2e-16 *** 
WATER_state_1_matrix12 -186.276   10.370 -17.963 < 2e-16 *** 
WATER_state_1_matrix13 306.453    13.112 23.371 < 2e-16 *** 
WATER_state_1_matrix14 -298.487   14.633 -20.398 < 2e-16 *** 
WATER_state_1_matrix15 210.995    10.158 20.772 < 2e-16 *** 
WATER_state_1_matrix16 -212.679   9.866 -21.557 < 2e-16 *** 
WATER_state_1_matrix17 107.669    6.222 17.305 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 5836 on 798 degrees of freedom
Multiple R-squared:  0.9933,    Adjusted R-squared:  0.9931 
F-statistic:  6924 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 61: Performance of Multivariate Multiple Linear Regression MODEL(GSP-WATER)

```

Response Y10 :

Call:
lm(formula = Y10 ~ WATER_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-11746.4 -3055.7  -715.9   2552.3 16641.3 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 4327.409   330.059 13.111 < 2e-16 ***
WATER_state_1_matrix1 -73.423    10.274 -7.146 2.01e-12 ***
WATER_state_1_matrix2 133.204    16.011  8.319 3.81e-16 ***
WATER_state_1_matrix3 -41.622    10.530 -3.953 8.41e-05 *** 
WATER_state_1_matrix4  25.180     8.187  3.076 0.002171 ** 
WATER_state_1_matrix5 -48.373    8.868 -5.455 6.54e-08 *** 
WATER_state_1_matrix6 -45.866    10.182 -4.505 7.65e-06 *** 
WATER_state_1_matrix7  60.805    11.307  5.378 9.91e-08 *** 
WATER_state_1_matrix8  31.064    14.894  2.086 0.037326 *  
WATER_state_1_matrix9 -71.223    12.132 -5.871 6.36e-09 *** 
WATER_state_1_matrix10 -29.512    7.915 -3.729 0.000206 *** 
WATER_state_1_matrix11 151.580    10.616 14.278 < 2e-16 *** 
WATER_state_1_matrix12 -188.198   10.229 -18.399 < 2e-16 *** 
WATER_state_1_matrix13 288.045    12.934 22.271 < 2e-16 *** 
WATER_state_1_matrix14 -277.864   14.433 -19.251 < 2e-16 *** 
WATER_state_1_matrix15 203.962    10.019 20.357 < 2e-16 *** 
WATER_state_1_matrix16 -215.708   9.731 -22.166 < 2e-16 *** 
WATER_state_1_matrix17 112.070    6.137 18.262 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 5756 on 798 degrees of freedom
Multiple R-squared:  0.9937,    Adjusted R-squared:  0.9936 
F-statistic:  7456 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 62: Performance of Multivariate Multiple Linear Regression MODEL(GSP-WATER)

```

Response Y11 :

Call:
lm(formula = Y11 ~ WATER_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-12405.6 -3093.3 - 715.5  2583.1 18104.8 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) 4300.391   333.771 12.884 < 2e-16 ***
WATER_state_1_matrix1 -74.823    10.390 -7.202 1.38e-12 ***
WATER_state_1_matrix2 130.174    16.191  8.040 3.24e-15 ***
WATER_state_1_matrix3 -42.695    10.649 -4.009 6.66e-05 *** 
WATER_state_1_matrix4  42.334     8.279  5.114 3.96e-07 *** 
WATER_state_1_matrix5 -66.396    8.967 -7.404 3.36e-13 *** 
WATER_state_1_matrix6 -45.334    10.297 -4.403 1.21e-05 *** 
WATER_state_1_matrix7  67.402     11.434  5.895 5.53e-09 *** 
WATER_state_1_matrix8  21.135    15.062  1.403   0.161  
WATER_state_1_matrix9 -51.094    12.268 -4.165 3.46e-05 *** 
WATER_state_1_matrix10 -49.671    8.004 -6.206 8.74e-10 *** 
WATER_state_1_matrix11 170.142    10.735 15.849 < 2e-16 *** 
WATER_state_1_matrix12 -191.903   10.344 -18.553 < 2e-16 *** 
WATER_state_1_matrix13 261.846    13.079 20.020 < 2e-16 *** 
WATER_state_1_matrix14 -248.138   14.596 -17.001 < 2e-16 *** 
WATER_state_1_matrix15 186.352    10.132 18.392 < 2e-16 *** 
WATER_state_1_matrix16 -211.731   9.841 -21.516 < 2e-16 *** 
WATER_state_1_matrix17 115.901    6.206 18.676 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 5821 on 798 degrees of freedom
Multiple R-squared:  0.9936,   Adjusted R-squared:  0.9934
F-statistic:  7244 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 63: Performance of Multivariate Multiple Linear Regression MODEL(GSP-WATER)

```

Response Y12 :

Call:
lm(formula = Y12 ~ WATER_state_1_matrix)

Residuals:
    Min      1Q Median      3Q     Max 
-12632 -3600 -1320  2962  20010 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) 4701.029   358.500 13.113 < 2e-16 ***
WATER_state_1_matrix1 -71.504    11.159 -6.408 2.53e-10 ***
WATER_state_1_matrix2 120.153    17.391  6.909 9.98e-12 ***
WATER_state_1_matrix3 -25.796    11.438 -2.255 0.024381 *  
WATER_state_1_matrix4  30.666     8.892  3.449 0.000593 *** 
WATER_state_1_matrix5 -66.029    9.632 -6.855 1.42e-11 *** 
WATER_state_1_matrix6 -50.196    11.059 -4.539 6.53e-06 *** 
WATER_state_1_matrix7  72.937    12.281  5.939 4.28e-09 *** 
WATER_state_1_matrix8  24.218    16.178  1.497 0.134777    
WATER_state_1_matrix9 -55.696    13.177 -4.227 2.65e-05 *** 
WATER_state_1_matrix10 -59.456    8.597 -6.916 9.54e-12 *** 
WATER_state_1_matrix11 182.007   11.531 15.784 < 2e-16 *** 
WATER_state_1_matrix12 -192.388   11.110 -17.316 < 2e-16 *** 
WATER_state_1_matrix13 272.272   14.048 19.381 < 2e-16 *** 
WATER_state_1_matrix14 -270.398   15.677 -17.248 < 2e-16 *** 
WATER_state_1_matrix15 195.572   10.883 17.971 < 2e-16 *** 
WATER_state_1_matrix16 -209.446   10.570 -19.815 < 2e-16 *** 
WATER_state_1_matrix17 116.219    6.666 17.435 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 6252 on 798 degrees of freedom
Multiple R-squared:  0.9927,    Adjusted R-squared:  0.9926 
F-statistic:  6396 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 64: Performance of Multivariate Multiple Linear Regression MODEL(GSP-WATER)

```

Response Y13 :

Call:
lm(formula = Y13 ~ WATER_state_1_matrix)

Residuals:
    Min      1Q Median      3Q     Max 
-12519 -3902 -1605  3745 20496 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) 4631.802   369.436 12.537 < 2e-16 ***
WATER_state_1_matrix1 -73.401    11.500 -6.383 2.95e-10 ***
WATER_state_1_matrix2 119.968    17.921  6.694 4.09e-11 ***
WATER_state_1_matrix3 -28.634    11.786 -2.429 0.01534 *  
WATER_state_1_matrix4  39.309     9.163  4.290 2.01e-05 *** 
WATER_state_1_matrix5 -80.963    9.926 -8.157 1.33e-15 *** 
WATER_state_1_matrix6 -37.504    11.397 -3.291 0.00104 ** 
WATER_state_1_matrix7  74.067    12.656  5.852 7.07e-09 *** 
WATER_state_1_matrix8 12.995    16.671  0.779 0.43592  
WATER_state_1_matrix9 -40.522    13.579 -2.984 0.00293 ** 
WATER_state_1_matrix10 -75.887    8.859 -8.566 < 2e-16 *** 
WATER_state_1_matrix11 196.269   11.883 16.517 < 2e-16 *** 
WATER_state_1_matrix12 -192.389   11.449 -16.804 < 2e-16 *** 
WATER_state_1_matrix13 255.302   14.477 17.635 < 2e-16 *** 
WATER_state_1_matrix14 -252.067   16.155 -15.603 < 2e-16 *** 
WATER_state_1_matrix15 178.610   11.215 15.926 < 2e-16 *** 
WATER_state_1_matrix16 -197.008   10.892 -18.087 < 2e-16 *** 
WATER_state_1_matrix17 114.073    6.869 16.607 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 6443 on 798 degrees of freedom
Multiple R-squared:  0.992,    Adjusted R-squared:  0.9919 
F-statistic:  5852 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 65: Performance of Multivariate Multiple Linear Regression MODEL(GSP-WATER)

```

Response Y14 :

Call:
lm(formula = Y14 ~ WATER_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-12915.7 -4364.1 - 958.7 2969.7 17535.2 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 5096.219   363.844 14.007 < 2e-16 ***
WATER_state_1_matrix1 -104.022    11.326 -9.185 < 2e-16 ***
WATER_state_1_matrix2 167.082    17.650  9.466 < 2e-16 ***
WATER_state_1_matrix3 -74.719    11.608 -6.437 2.10e-10 ***
WATER_state_1_matrix4  85.636     9.025  9.489 < 2e-16 ***
WATER_state_1_matrix5 -109.568    9.775 -11.209 < 2e-16 ***
WATER_state_1_matrix6 -17.030    11.224 -1.517 0.129607  
WATER_state_1_matrix7  47.630    12.464  3.821 0.000143 *** 
WATER_state_1_matrix8  33.171    16.419  2.020 0.043681 *  
WATER_state_1_matrix9 -43.135    13.374 -3.225 0.001310 ** 
WATER_state_1_matrix10 -71.132    8.725 -8.153 1.38e-15 *** 
WATER_state_1_matrix11 187.932   11.703 16.059 < 2e-16 *** 
WATER_state_1_matrix12 -184.284   11.276 -16.343 < 2e-16 *** 
WATER_state_1_matrix13 222.104   14.258 15.578 < 2e-16 *** 
WATER_state_1_matrix14 -212.701   15.911 -13.368 < 2e-16 *** 
WATER_state_1_matrix15 168.908   11.045 15.293 < 2e-16 *** 
WATER_state_1_matrix16 -188.901   10.728 -17.609 < 2e-16 *** 
WATER_state_1_matrix17 104.531    6.765 15.451 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 6345 on 798 degrees of freedom
Multiple R-squared:  0.9929,    Adjusted R-squared:  0.9927 
F-statistic:  6526 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 66: Performance of Multivariate Multiple Linear Regression MODEL(GSP-WATER)

```

Response Y15 :

Call:
lm(formula = Y15 ~ WATER_state_1_matrix)

Residuals:
    Min      1Q Median      3Q     Max 
-13629 -4745 -1255  3011 16624 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) 5651.481   380.370 14.858 < 2e-16 ***
WATER_state_1_matrix1 -126.027   11.840 -10.644 < 2e-16 ***
WATER_state_1_matrix2  203.929   18.452 11.052 < 2e-16 ***
WATER_state_1_matrix3 -101.107   12.135 -8.332 3.46e-16 ***
WATER_state_1_matrix4  103.106   9.435 10.929 < 2e-16 ***
WATER_state_1_matrix5 -114.154   10.219 -11.170 < 2e-16 ***
WATER_state_1_matrix6 -21.250   11.734 -1.811  0.07053 .  
WATER_state_1_matrix7  42.507   13.030  3.262  0.00115 ** 
WATER_state_1_matrix8  49.199   17.164  2.866  0.00426 ** 
WATER_state_1_matrix9 -58.845   13.981 -4.209 2.86e-05 *** 
WATER_state_1_matrix10 -66.627   9.122 -7.304 6.75e-13 *** 
WATER_state_1_matrix11 202.221   12.234 16.529 < 2e-16 *** 
WATER_state_1_matrix12 -200.476   11.788 -17.007 < 2e-16 *** 
WATER_state_1_matrix13 220.682   14.905 14.806 < 2e-16 *** 
WATER_state_1_matrix14 -213.309   16.633 -12.824 < 2e-16 *** 
WATER_state_1_matrix15 185.970   11.547 16.106 < 2e-16 *** 
WATER_state_1_matrix16 -197.733   11.215 -17.631 < 2e-16 *** 
WATER_state_1_matrix17 103.927    7.072 14.695 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 6634 on 798 degrees of freedom
Multiple R-squared:  0.9932,   Adjusted R-squared:  0.9931
F-statistic: 6868 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 67: Performance of Multivariate Multiple Linear Regression MODEL(GSP-WATER)

```

Response Y16 :

Call:
lm(formula = Y16 ~ WATER_state_1_matrix)

Residuals:
    Min      1Q Median      3Q     Max 
-14075 -4469 -1620  3106 14960 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) 5824.103   379.334 15.353 < 2e-16 ***
WATER_state_1_matrix1 -150.815    11.808 -12.772 < 2e-16 ***
WATER_state_1_matrix2  243.539    18.401 13.235 < 2e-16 ***
WATER_state_1_matrix3 -137.331    12.102 -11.348 < 2e-16 ***
WATER_state_1_matrix4  136.737     9.409 14.533 < 2e-16 ***
WATER_state_1_matrix5 -127.599    10.191 -12.520 < 2e-16 ***
WATER_state_1_matrix6 -22.054    11.702 -1.885 0.059849 .
WATER_state_1_matrix7  35.258    12.995  2.713 0.006806 ** 
WATER_state_1_matrix8  60.694    17.118  3.546 0.000414 *** 
WATER_state_1_matrix9 -60.617    13.943 -4.347 1.56e-05 *** 
WATER_state_1_matrix10 -65.743    9.097 -7.227 1.15e-12 *** 
WATER_state_1_matrix11 201.693   12.201 16.531 < 2e-16 *** 
WATER_state_1_matrix12 -199.228   11.756 -16.947 < 2e-16 *** 
WATER_state_1_matrix13 200.489   14.865 13.488 < 2e-16 *** 
WATER_state_1_matrix14 -191.793   16.588 -11.562 < 2e-16 *** 
WATER_state_1_matrix15 188.336   11.515 16.356 < 2e-16 *** 
WATER_state_1_matrix16 -201.802   11.184 -18.043 < 2e-16 *** 
WATER_state_1_matrix17 102.198    7.053 14.490 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 6616 on 798 degrees of freedom
Multiple R-squared:  0.9938,    Adjusted R-squared:  0.9937 
F-statistic: 7584 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 68: Performance of Multivariate Multiple Linear Regression MODEL(GSP-WATER)

```

Response Y17 :

Call:
lm(formula = Y17 ~ WATER_state_1_matrix)

Residuals:
    Min      1Q Median      3Q     Max 
-14318 -4971 -1506  4172 14706 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) 6714.133   382.368 17.559 < 2e-16 ***
WATER_state_1_matrix1 -180.885   11.902 -15.197 < 2e-16 ***
WATER_state_1_matrix2  300.938   18.549 16.224 < 2e-16 ***
WATER_state_1_matrix3 -200.173   12.199 -16.409 < 2e-16 ***
WATER_state_1_matrix4  190.846   9.484 20.123 < 2e-16 ***
WATER_state_1_matrix5 -166.068   10.273 -16.165 < 2e-16 ***
WATER_state_1_matrix6  13.131   11.796  1.113  0.266  
WATER_state_1_matrix7 -0.247   13.099 -0.019  0.985  
WATER_state_1_matrix8  82.055   17.255  4.756 2.35e-06 ***
WATER_state_1_matrix9 -71.533   14.055 -5.090 4.48e-07 *** 
WATER_state_1_matrix10 -43.430   9.169 -4.736 2.57e-06 *** 
WATER_state_1_matrix11 194.913   12.299 15.848 < 2e-16 *** 
WATER_state_1_matrix12 -208.923   11.850 -17.631 < 2e-16 *** 
WATER_state_1_matrix13 177.809   14.983 11.867 < 2e-16 *** 
WATER_state_1_matrix14 -149.747   16.721 -8.956 < 2e-16 *** 
WATER_state_1_matrix15 179.554   11.607 15.469 < 2e-16 *** 
WATER_state_1_matrix16 -198.808   11.274 -17.635 < 2e-16 *** 
WATER_state_1_matrix17  92.006    7.109 12.941 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 6668 on 798 degrees of freedom
Multiple R-squared:  0.9941,    Adjusted R-squared:  0.994 
F-statistic: 7973 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 69: Performance of Multivariate Multiple Linear Regression MODEL(GSP-WATER)

Response: GSP, Covariate: UTIL We have now taken GSP as response variable and UTIL as covariate. Summary for the model is described below. There are 17 summary outputs for this session of response and covariates.

```

> summary(model_4)
Response Y1 :

Call:
lm(formula = Y1 ~ UTIL_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-20651.3 -5857.7   416.8  4115.7  20573.1 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 1987.0163   482.9770   4.114 4.29e-05 ***
UTIL_state_1_matrix1 -25.2135    3.7038  -6.808 1.95e-11 ***
UTIL_state_1_matrix2  25.2750    6.5085   3.883 0.000112 *** 
UTIL_state_1_matrix3 12.1480    5.5494   2.189 0.028882 *  
UTIL_state_1_matrix4 -18.2390    4.9486  -3.686 0.000243 *** 
UTIL_state_1_matrix5 12.2406    6.4738   1.891 0.059016 .  
UTIL_state_1_matrix6 33.4706    9.3552   3.578 0.000367 *** 
UTIL_state_1_matrix7 -84.4911    7.4603  -11.325 < 2e-16 *** 
UTIL_state_1_matrix8  0.4361    8.6925   0.050 0.960004    
UTIL_state_1_matrix9 110.1057   10.9240   10.079 < 2e-16 *** 
UTIL_state_1_matrix10 -46.0790    7.5767  -6.082 1.84e-09 *** 
UTIL_state_1_matrix11 -40.2677    6.3163  -6.375 3.09e-10 *** 
UTIL_state_1_matrix12 -14.5121    9.9426  -1.460 0.144797    
UTIL_state_1_matrix13 67.2302   11.3564   5.920 4.78e-09 *** 
UTIL_state_1_matrix14 -12.7389    7.5342  -1.691 0.091265 .  
UTIL_state_1_matrix15 -56.3083    5.9582  -9.451 < 2e-16 *** 
UTIL_state_1_matrix16 60.4796    6.3364   9.545 < 2e-16 *** 
UTIL_state_1_matrix17 -19.6791    3.1419  -6.263 6.15e-10 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 7666 on 798 degrees of freedom
Multiple R-squared:  0.9821,    Adjusted R-squared:  0.9817 
F-statistic: 2575 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 70: Performance of Multivariate Multiple Linear Regression MODEL(GSP-UTIL)

```

Response Y2 :

Call:
lm(formula = Y2 ~ UTIL_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-21418.3 -5425.2   534.9  4730.7 21996.3 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept)  2041.257   498.054   4.098 4.58e-05 *** 
UTIL_state_1_matrix1 -26.114    3.819  -6.837 1.61e-11 *** 
UTIL_state_1_matrix2  26.405    6.712   3.934 9.08e-05 *** 
UTIL_state_1_matrix3 11.793    5.723   2.061 0.039645 *  
UTIL_state_1_matrix4 -15.820    5.103  -3.100 0.002003 ** 
UTIL_state_1_matrix5  8.461    6.676   1.267 0.205410    
UTIL_state_1_matrix6 35.674    9.647   3.698 0.000232 *** 
UTIL_state_1_matrix7 -83.622   7.693  -10.870 < 2e-16 *** 
UTIL_state_1_matrix8 -7.654    8.964  -0.854 0.393457    
UTIL_state_1_matrix9 121.334   11.265  10.771 < 2e-16 *** 
UTIL_state_1_matrix10 -54.343   7.813  -6.955 7.33e-12 *** 
UTIL_state_1_matrix11 -35.918   6.513  -5.514 4.73e-08 *** 
UTIL_state_1_matrix12 -17.021   10.253  -1.660 0.097285 .  
UTIL_state_1_matrix13  70.110   11.711   5.987 3.23e-09 *** 
UTIL_state_1_matrix14 -8.635    7.769  -1.111 0.266733    
UTIL_state_1_matrix15 -65.425   6.144  -10.648 < 2e-16 *** 
UTIL_state_1_matrix16  64.538    6.534   9.877 < 2e-16 *** 
UTIL_state_1_matrix17 -19.833   3.240  -6.121 1.45e-09 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 7905 on 798 degrees of freedom
Multiple R-squared:  0.9814,   Adjusted R-squared:  0.981
F-statistic:  2471 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 71: Performance of Multivariate Multiple Linear Regression MODEL(GSP-UTIL)

```

Response Y3 :

Call:
lm(formula = Y3 ~ UTIL_state_1_matrix)

Residuals:
    Min      1Q   Median      3Q     Max 
-22938.3 -5765.8   506.4  5126.5 23168.5 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept)  2136.459    521.910   4.094 4.68e-05 *** 
UTIL_state_1_matrix1 -27.658     4.002  -6.911 9.88e-12 *** 
UTIL_state_1_matrix2  28.303     7.033   4.024 6.26e-05 *** 
UTIL_state_1_matrix3 12.463     5.997   2.078  0.03800 *  
UTIL_state_1_matrix4 -14.371     5.347  -2.687  0.00735 ** 
UTIL_state_1_matrix5  6.732     6.996   0.962  0.33619    
UTIL_state_1_matrix6 33.319     10.109   3.296  0.00102 ** 
UTIL_state_1_matrix7 -83.112     8.062  -10.309 < 2e-16 *** 
UTIL_state_1_matrix8 -8.390     9.393  -0.893  0.37203    
UTIL_state_1_matrix9 127.042    11.805   10.762 < 2e-16 *** 
UTIL_state_1_matrix10 -60.886     8.187  -7.437 2.67e-13 *** 
UTIL_state_1_matrix11 -32.170     6.825  -4.713 2.87e-06 *** 
UTIL_state_1_matrix12 -20.696     10.744  -1.926  0.05443 .  
UTIL_state_1_matrix13  76.507    12.272   6.234 7.35e-10 *** 
UTIL_state_1_matrix14 -11.399     8.142  -1.400  0.16186    
UTIL_state_1_matrix15 -69.558     6.438  -10.804 < 2e-16 *** 
UTIL_state_1_matrix16  67.803     6.847   9.902 < 2e-16 *** 
UTIL_state_1_matrix17 -19.775     3.395  -5.824 8.31e-09 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 8284 on 798 degrees of freedom
Multiple R-squared:  0.9812,    Adjusted R-squared:  0.9808 
F-statistic: 2453 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 72: Performance of Multivariate Multiple Linear Regression MODEL(GSP-UTIL)

```

Response Y4 :

Call:
lm(formula = Y4 ~ UTIL_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-24083.1 -5747.3   512.3  5180.4 25037.1 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) 2175.092    552.740   3.935 9.04e-05 *** 
UTIL_state_1_matrix1 -36.883     4.239  -8.701 < 2e-16 *** 
UTIL_state_1_matrix2  42.840     7.449   5.751 1.26e-08 *** 
UTIL_state_1_matrix3  7.212     6.351   1.136   0.2565    
UTIL_state_1_matrix4 -13.034     5.663  -2.301   0.0216 *  
UTIL_state_1_matrix5  7.339     7.409   0.991   0.3222    
UTIL_state_1_matrix6 27.490    10.707   2.568   0.0104 *  
UTIL_state_1_matrix7 -81.967     8.538  -9.600 < 2e-16 *** 
UTIL_state_1_matrix8 -5.201     9.948  -0.523   0.6012    
UTIL_state_1_matrix9 129.116   12.502  10.328 < 2e-16 *** 
UTIL_state_1_matrix10 -61.313     8.671  -7.071 3.36e-12 *** 
UTIL_state_1_matrix11 -37.465     7.229  -5.183 2.77e-07 *** 
UTIL_state_1_matrix12 -16.809    11.379  -1.477   0.1400    
UTIL_state_1_matrix13 74.391    12.997   5.724  1.47e-08 *** 
UTIL_state_1_matrix14 -9.308     8.623  -1.080   0.2807    
UTIL_state_1_matrix15 -74.930     6.819 -10.989 < 2e-16 *** 
UTIL_state_1_matrix16  74.828    7.252  10.319 < 2e-16 *** 
UTIL_state_1_matrix17 -22.084     3.596  -6.142 1.29e-09 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 8773 on 798 degrees of freedom
Multiple R-squared:  0.9808,    Adjusted R-squared:  0.9804 
F-statistic:  2393 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 73: Performance of Multivariate Multiple Linear Regression MODEL(GSP-UTIL)

```

Response Y5 :

Call:
lm(formula = Y5 ~ UTIL_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-23834.8 -5822.9   455.6  5429.7 24511.0 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) 2248.696   548.083   4.103 4.50e-05 *** 
UTIL_state_1_matrix1 -34.241    4.203  -8.147 1.44e-15 *** 
UTIL_state_1_matrix2  39.124    7.386   5.297 1.52e-07 *** 
UTIL_state_1_matrix3 11.200    6.298   1.779  0.0757 .  
UTIL_state_1_matrix4 -19.110    5.616  -3.403  0.0007 *** 
UTIL_state_1_matrix5 11.989    7.347   1.632  0.1031  
UTIL_state_1_matrix6 25.725    10.616   2.423  0.0156 *  
UTIL_state_1_matrix7 -83.763    8.466  -9.894 < 2e-16 *** 
UTIL_state_1_matrix8  1.881    9.864   0.191  0.8488  
UTIL_state_1_matrix9 118.997   12.397   9.599 < 2e-16 *** 
UTIL_state_1_matrix10 -51.757    8.598  -6.020 2.66e-09 *** 
UTIL_state_1_matrix11 -47.252    7.168  -6.592 7.87e-11 *** 
UTIL_state_1_matrix12 -7.006    11.283  -0.621  0.5348  
UTIL_state_1_matrix13 68.765    12.887   5.336 1.24e-07 *** 
UTIL_state_1_matrix14 -12.812    8.550  -1.498  0.1344  
UTIL_state_1_matrix15 -65.940    6.761  -9.753 < 2e-16 *** 
UTIL_state_1_matrix16 66.541    7.191   9.254 < 2e-16 *** 
UTIL_state_1_matrix17 -18.118    3.565  -5.082 4.66e-07 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 8699 on 798 degrees of freedom
Multiple R-squared:  0.9809,    Adjusted R-squared:  0.9805 
F-statistic:  2407 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 74: Performance of Multivariate Multiple Linear Regression MODEL(GSP-UTIL)

```

Response Y6 :

Call:
lm(formula = Y6 ~ UTIL_state_1_matrix)

Residuals:
    Min      1Q Median      3Q     Max 
-22938 -6251    309   5139  22258 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) 2260.434    533.102   4.240 2.50e-05 *** 
UTIL_state_1_matrix1 -28.168     4.088  -6.890 1.13e-11 *** 
UTIL_state_1_matrix2  31.845     7.184   4.433 1.06e-05 *** 
UTIL_state_1_matrix3 13.954     6.125   2.278  0.02298 *  
UTIL_state_1_matrix4 -22.995     5.462  -4.210 2.85e-05 *** 
UTIL_state_1_matrix5 12.613     7.146   1.765  0.07792 .  
UTIL_state_1_matrix6 29.737    10.326   2.880  0.00409 ** 
UTIL_state_1_matrix7 -85.866    8.235 -10.428 < 2e-16 *** 
UTIL_state_1_matrix8  7.564     9.595   0.788  0.43073  
UTIL_state_1_matrix9 105.503   12.058   8.750 < 2e-16 *** 
UTIL_state_1_matrix10 -41.386    8.363  -4.949 9.12e-07 *** 
UTIL_state_1_matrix11 -54.592    6.972  -7.830 1.55e-14 *** 
UTIL_state_1_matrix12  1.186    10.975   0.108  0.91396  
UTIL_state_1_matrix13 65.238    12.535   5.204 2.48e-07 *** 
UTIL_state_1_matrix14 -12.894    8.316  -1.550  0.12143  
UTIL_state_1_matrix15 -65.086    6.577  -9.897 < 2e-16 *** 
UTIL_state_1_matrix16 63.180     6.994   9.033 < 2e-16 *** 
UTIL_state_1_matrix17 -15.569    3.468  -4.489 8.19e-06 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 8462 on 798 degrees of freedom
Multiple R-squared:  0.9816,    Adjusted R-squared:  0.9812 
F-statistic:  2507 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 75: Performance of Multivariate Multiple Linear Regression MODEL(GSP-UTIL)

```

Response Y7 :

Call:
lm(formula = Y7 ~ UTIL_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-24259.1 -5515.8   248.5  6181.9 22525.1 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept)  2283.1182   550.4826   4.147 3.72e-05 *** 
UTIL_state_1_matrix1 -31.4281    4.2214  -7.445 2.52e-13 *** 
UTIL_state_1_matrix2  39.1136    7.4182   5.273 1.73e-07 *** 
UTIL_state_1_matrix3  9.6814    6.3251   1.531 0.126256    
UTIL_state_1_matrix4 -21.3312    5.6402  -3.782 0.000167 *** 
UTIL_state_1_matrix5 11.3992    7.3787   1.545 0.122771    
UTIL_state_1_matrix6 31.8276   10.6628   2.985 0.002923 **  
UTIL_state_1_matrix7 -87.8498   8.5030 -10.332 < 2e-16 *** 
UTIL_state_1_matrix8  1.4309    9.9074   0.144 0.885198    
UTIL_state_1_matrix9 119.3903   12.4509   9.589 < 2e-16 *** 
UTIL_state_1_matrix10 -52.0816   8.6357  -6.031 2.49e-09 *** 
UTIL_state_1_matrix11 -55.3955   7.1991  -7.695 4.18e-14 *** 
UTIL_state_1_matrix12  0.6835   11.3323   0.060 0.951920    
UTIL_state_1_matrix13 73.3645   12.9437   5.668 2.02e-08 *** 
UTIL_state_1_matrix14 -12.2964   8.5873  -1.432 0.152555    
UTIL_state_1_matrix15 -74.2818   6.7909 -10.938 < 2e-16 *** 
UTIL_state_1_matrix16 69.6770   7.2221   9.648 < 2e-16 *** 
UTIL_state_1_matrix17 -17.3568   3.5811  -4.847 1.51e-06 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 8738 on 798 degrees of freedom
Multiple R-squared:  0.9819,   Adjusted R-squared:  0.9815
F-statistic:  2549 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 76: Performance of Multivariate Multiple Linear Regression MODEL(GSP-UTIL)

```

Response Y8 :

Call:
lm(formula = Y8 ~ UTIL_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-24696.8 -5701.7   411.2  5977.5 22598.4 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 2205.410   569.211   3.875 0.000116 ***
UTIL_state_1_matrix1 -34.287    4.365  -7.855 1.29e-14 ***
UTIL_state_1_matrix2  46.315    7.671   6.038 2.39e-09 ***
UTIL_state_1_matrix3  4.799    6.540   0.734 0.463270  
UTIL_state_1_matrix4 -19.956    5.832  -3.422 0.000654 ***
UTIL_state_1_matrix5 11.517    7.630   1.509 0.131572  
UTIL_state_1_matrix6 30.687   11.026   2.783 0.005508 ** 
UTIL_state_1_matrix7 -87.597   8.792  -9.963 < 2e-16 ***
UTIL_state_1_matrix8 -1.524    10.245  -0.149 0.881768  
UTIL_state_1_matrix9 128.927   12.874  10.014 < 2e-16 ***
UTIL_state_1_matrix10 -59.820   8.929  -6.699 3.96e-11 ***
UTIL_state_1_matrix11 -58.110   7.444  -7.806 1.85e-14 ***
UTIL_state_1_matrix12  1.018    11.718   0.087 0.930776  
UTIL_state_1_matrix13 79.756   13.384   5.959 3.80e-09 *** 
UTIL_state_1_matrix14 -12.045   8.879  -1.356 0.175335  
UTIL_state_1_matrix15 -80.326   7.022 -11.439 < 2e-16 *** 
UTIL_state_1_matrix16 72.989   7.468   9.774 < 2e-16 *** 
UTIL_state_1_matrix17 -17.519   3.703  -4.731 2.64e-06 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 9035 on 798 degrees of freedom
Multiple R-squared:  0.9824,    Adjusted R-squared:  0.982 
F-statistic:  2614 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 77: Performance of Multivariate Multiple Linear Regression MODEL(GSP-UTIL)

```

Response Y9 :

Call:
lm(formula = Y9 ~ UTIL_state_1_matrix)

Residuals:
    Min      1Q Median      3Q     Max 
-25602   -6194    793   6093  23603 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) 2592.233   596.502   4.346 1.57e-05 *** 
UTIL_state_1_matrix1 -32.636    4.574  -7.135 2.18e-12 *** 
UTIL_state_1_matrix2  45.161    8.038   5.618 2.67e-08 *** 
UTIL_state_1_matrix3  5.507    6.854   0.803  0.42197    
UTIL_state_1_matrix4 -19.772    6.112  -3.235  0.00127 **  
UTIL_state_1_matrix5 10.931    7.996   1.367  0.17196    
UTIL_state_1_matrix6 29.999   11.554   2.596  0.00959 **  
UTIL_state_1_matrix7 -89.730    9.214  -9.739 < 2e-16 *** 
UTIL_state_1_matrix8 -1.388    10.736  -0.129  0.89720    
UTIL_state_1_matrix9 135.118   13.492  10.015 < 2e-16 *** 
UTIL_state_1_matrix10 -63.678    9.358  -6.805 1.98e-11 *** 
UTIL_state_1_matrix11 -60.980    7.801  -7.817 1.71e-14 *** 
UTIL_state_1_matrix12 -1.051    12.280  -0.086  0.93182    
UTIL_state_1_matrix13 87.878   14.026   6.265 6.08e-10 *** 
UTIL_state_1_matrix14 -14.934    9.305  -1.605  0.10891    
UTIL_state_1_matrix15 -82.712    7.359 -11.240 < 2e-16 *** 
UTIL_state_1_matrix16 74.611    7.826   9.534 < 2e-16 *** 
UTIL_state_1_matrix17 -17.149    3.880  -4.419 1.13e-05 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 9468 on 798 degrees of freedom
Multiple R-squared:  0.9823,    Adjusted R-squared:  0.9819 
F-statistic:  2601 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 78: Performance of Multivariate Multiple Linear Regression MODEL(GSP-UTIL)

```

Response Y10 :

Call:
lm(formula = Y10 ~ UTIL_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-24618.1 -7609.4   829.6  6221.8 24481.9 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) 2967.0900   606.0176   4.896 1.18e-06 ***  
UTIL_state_1_matrix1 -28.8998    4.6473  -6.219 8.08e-10 ***  
UTIL_state_1_matrix2  39.9593    8.1666   4.893 1.20e-06 ***  
UTIL_state_1_matrix3  7.2421    6.9632   1.040 0.298633    
UTIL_state_1_matrix4 -18.5339    6.2092  -2.985 0.002923 **  
UTIL_state_1_matrix5  7.9895    8.1231   0.984 0.325629    
UTIL_state_1_matrix6  33.4707   11.7385   2.851 0.004466 **  
UTIL_state_1_matrix7 -92.7915    9.3609  -9.913 < 2e-16 ***  
UTIL_state_1_matrix8   0.6994   10.9070   0.064 0.948885    
UTIL_state_1_matrix9  132.7172   13.7070   9.682 < 2e-16 ***  
UTIL_state_1_matrix10 -60.3446    9.5069  -6.347 3.67e-10 ***  
UTIL_state_1_matrix11 -63.4768    7.9253  -8.009 4.08e-15 ***  
UTIL_state_1_matrix12 -3.4443    12.4756  -0.276 0.782556    
UTIL_state_1_matrix13  93.5322   14.2496   6.564 9.43e-11 ***  
UTIL_state_1_matrix14 -20.4092    9.4536  -2.159 0.031157 *  
UTIL_state_1_matrix15 -78.3115    7.4760 -10.475 < 2e-16 ***  
UTIL_state_1_matrix16  70.0531    7.9506   8.811 < 2e-16 ***  
UTIL_state_1_matrix17 -14.0826    3.9424  -3.572 0.000375 ***  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 9619 on 798 degrees of freedom
Multiple R-squared:  0.9825,   Adjusted R-squared:  0.9822
F-statistic: 2640 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 79: Performance of Multivariate Multiple Linear Regression MODEL(GSP-UTIL)

```

Response Y11 :

Call:
lm(formula = Y11 ~ UTIL_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-27089.3 -7260.2   748.1  6193.3 25840.6 

Coefficients:
                                         Estimate Std. Error t value Pr(>|t|)    
(Intercept)           3109.553    620.212   5.014 6.58e-07 ***  
UTIL_state_1_matrix1 -17.477     4.756   -3.675 0.000254 ***  
UTIL_state_1_matrix2  20.431     8.358    2.445 0.014719 *    
UTIL_state_1_matrix3 17.998     7.126    2.526 0.011741 *    
UTIL_state_1_matrix4 -20.374     6.355   -3.206 0.001399 **  
UTIL_state_1_matrix5  2.755     8.313    0.331 0.740402    
UTIL_state_1_matrix6 39.807     12.013   3.314 0.000963 ***  
UTIL_state_1_matrix7 -93.945     9.580   -9.806 < 2e-16 ***  
UTIL_state_1_matrix8  1.733     11.162   0.155 0.876691    
UTIL_state_1_matrix9 128.925    14.028   9.191 < 2e-16 ***  
UTIL_state_1_matrix10 -55.978     9.730   -5.753 1.25e-08 ***  
UTIL_state_1_matrix11 -66.861     8.111   -8.243 6.86e-16 ***  
UTIL_state_1_matrix12 -4.515     12.768   -0.354 0.723726    
UTIL_state_1_matrix13 97.849     14.583   6.710 3.70e-11 ***  
UTIL_state_1_matrix14 -26.958     9.675   -2.786 0.005457 **  
UTIL_state_1_matrix15 -68.359     7.651   -8.934 < 2e-16 ***  
UTIL_state_1_matrix16  57.136     8.137    7.022 4.69e-12 ***  
UTIL_state_1_matrix17 -6.728     4.035   -1.668 0.095787 .    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 9844 on 798 degrees of freedom
Multiple R-squared:  0.9816,   Adjusted R-squared:  0.9812
F-statistic: 2502 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 80: Performance of Multivariate Multiple Linear Regression MODEL(GSP-UTIL)

```

Response Y12 :

Call:
lm(formula = Y12 ~ UTIL_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-29537.5 -7101.9   680.1  6322.9 27181.8 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 3206.627   652.830   4.912 1.09e-06 ***
UTIL_state_1_matrix1 -17.853    5.006  -3.566 0.000384 *** 
UTIL_state_1_matrix2  19.267    8.797   2.190 0.028809 *  
UTIL_state_1_matrix3 18.799    7.501   2.506 0.012405 *  
UTIL_state_1_matrix4 -19.119    6.689  -2.858 0.004369 ** 
UTIL_state_1_matrix5 -2.398    8.751  -0.274 0.784163    
UTIL_state_1_matrix6  50.449   12.645   3.990 7.23e-05 *** 
UTIL_state_1_matrix7 -100.668   10.084  -9.983 < 2e-16 *** 
UTIL_state_1_matrix8 -4.518    11.749  -0.385 0.700674    
UTIL_state_1_matrix9 141.049   14.766   9.552 < 2e-16 *** 
UTIL_state_1_matrix10 -59.067   10.241  -5.768 1.15e-08 *** 
UTIL_state_1_matrix11 -72.679   8.538  -8.513 < 2e-16 *** 
UTIL_state_1_matrix12 -1.715    13.439  -0.128 0.898473    
UTIL_state_1_matrix13 96.952    15.350   6.316 4.46e-10 *** 
UTIL_state_1_matrix14 -21.698   10.184  -2.131 0.033429 *  
UTIL_state_1_matrix15 -72.524   8.054  -9.005 < 2e-16 *** 
UTIL_state_1_matrix16  54.477   8.565   6.361 3.38e-10 *** 
UTIL_state_1_matrix17 -3.368    4.247  -0.793 0.428025    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 10360 on 798 degrees of freedom
Multiple R-squared:  0.98,    Adjusted R-squared:  0.9796 
F-statistic: 2299 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 81: Performance of Multivariate Multiple Linear Regression MODEL(GSP-UTIL)

```

Response Y13 :

Call:
lm(formula = Y13 ~ UTIL_state_1_matrix)

Residuals:
    Min      1Q Median      3Q     Max 
-32050 -6007    752   5715  27776 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) 2976.510   669.268   4.447 9.92e-06 ***  
UTIL_state_1_matrix1 -12.237    5.132  -2.384 0.017348 *  
UTIL_state_1_matrix2   7.222    9.019   0.801 0.423528    
UTIL_state_1_matrix3  26.330    7.690   3.424 0.000649 ***  
UTIL_state_1_matrix4 -20.180    6.857  -2.943 0.003346 **  
UTIL_state_1_matrix5 -8.342    8.971  -0.930 0.352691    
UTIL_state_1_matrix6  59.943   12.964   4.624 4.39e-06 ***  
UTIL_state_1_matrix7 -102.498   10.338  -9.915 < 2e-16 ***  
UTIL_state_1_matrix8  -5.732   12.045  -0.476 0.634295    
UTIL_state_1_matrix9  137.822   15.138   9.105 < 2e-16 ***  
UTIL_state_1_matrix10 -53.745   10.499  -5.119 3.85e-07 ***  
UTIL_state_1_matrix11 -74.925   8.753  -8.560 < 2e-16 ***  
UTIL_state_1_matrix12 -5.984    13.778  -0.434 0.664168    
UTIL_state_1_matrix13  101.139   15.737   6.427 2.24e-10 ***  
UTIL_state_1_matrix14 -21.269   10.440  -2.037 0.041961 *  
UTIL_state_1_matrix15 -69.386   8.256  -8.404 < 2e-16 ***  
UTIL_state_1_matrix16  43.650    8.780   4.971 8.15e-07 ***  
UTIL_state_1_matrix17   3.401    4.354   0.781 0.435010    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 10620 on 798 degrees of freedom
Multiple R-squared:  0.9784,   Adjusted R-squared:  0.9779
F-statistic:  2123 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 82: Performance of Multivariate Multiple Linear Regression MODEL(GSP-UTIL)

```

Response Y14 :

Call:
lm(formula = Y14 ~ UTIL_state_1_matrix)

Residuals:
    Min      1Q Median      3Q     Max 
-34301 -7116   1515   6071  28176 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept)  2452.021   682.981   3.590 0.000351 ***
UTIL_state_1_matrix1 -15.036     5.238  -2.871 0.004203 ** 
UTIL_state_1_matrix2   9.097     9.204   0.988 0.323253  
UTIL_state_1_matrix3  31.258    7.848   3.983 7.42e-05 *** 
UTIL_state_1_matrix4 -21.379    6.998  -3.055 0.002325 ** 
UTIL_state_1_matrix5 -16.886    9.155  -1.845 0.065474 .  
UTIL_state_1_matrix6  62.349   13.229   4.713 2.88e-06 *** 
UTIL_state_1_matrix7 -95.360   10.550  -9.039 < 2e-16 *** 
UTIL_state_1_matrix8  -7.827   12.292  -0.637 0.524481  
UTIL_state_1_matrix9 139.568   15.448   9.035 < 2e-16 *** 
UTIL_state_1_matrix10 -58.513   10.714  -5.461 6.32e-08 *** 
UTIL_state_1_matrix11 -70.918   8.932  -7.940 6.86e-15 *** 
UTIL_state_1_matrix12 -15.217   14.060  -1.082 0.279452  
UTIL_state_1_matrix13 109.430   16.059   6.814 1.87e-11 *** 
UTIL_state_1_matrix14 -25.281   10.654  -2.373 0.017885 *  
UTIL_state_1_matrix15 -59.129   8.425  -7.018 4.82e-12 *** 
UTIL_state_1_matrix16  27.508   8.960   3.070 0.002213 ** 
UTIL_state_1_matrix17  11.634   4.443   2.619 0.008998 ** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 10840 on 798 degrees of freedom
Multiple R-squared:  0.9792,    Adjusted R-squared:  0.9787 
F-statistic: 2205 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 83: Performance of Multivariate Multiple Linear Regression MODEL(GSP-UTIL)

```

Response Y15 :

Call:
lm(formula = Y15 ~ UTIL_state_1_matrix)

Residuals:
    Min      1Q Median      3Q     Max 
-37603 -7756  1915  7091  29616 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) 2276.832   730.055   3.119 0.001882 ** 
UTIL_state_1_matrix1 -18.856     5.599  -3.368 0.000793 *** 
UTIL_state_1_matrix2  13.637     9.838   1.386 0.166100  
UTIL_state_1_matrix3  31.345     8.388   3.737 0.000200 *** 
UTIL_state_1_matrix4 -20.814     7.480  -2.783 0.005521 ** 
UTIL_state_1_matrix5 -20.171     9.786  -2.061 0.039599 *  
UTIL_state_1_matrix6  68.337    14.141   4.832 1.62e-06 *** 
UTIL_state_1_matrix7 -103.987   11.277  -9.221 < 2e-16 *** 
UTIL_state_1_matrix8  -4.475    13.139  -0.341 0.733509  
UTIL_state_1_matrix9  143.957   16.512   8.718 < 2e-16 *** 
UTIL_state_1_matrix10 -59.941   11.453  -5.234 2.13e-07 *** 
UTIL_state_1_matrix11 -72.933    9.547  -7.639 6.27e-14 *** 
UTIL_state_1_matrix12 -23.180   15.029  -1.542 0.123389  
UTIL_state_1_matrix13 122.692   17.166   7.147 2.00e-12 *** 
UTIL_state_1_matrix14 -25.677   11.389  -2.255 0.024426 *  
UTIL_state_1_matrix15 -63.852    9.006  -7.090 2.96e-12 *** 
UTIL_state_1_matrix16  21.634    9.578   2.259 0.024172 *  
UTIL_state_1_matrix17  17.852    4.749   3.759 0.000183 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 11590 on 798 degrees of freedom
Multiple R-squared:  0.9793,   Adjusted R-squared:  0.9788
F-statistic: 2219 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 84: Performance of Multivariate Multiple Linear Regression MODEL(GSP-UTIL)

```

Response Y16 :

Call:
lm(formula = Y16 ~ UTIL_state_1_matrix)

Residuals:
    Min      1Q Median      3Q     Max 
-40109 -7904   1825   5745  30302 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 1851.042   759.388  2.438 0.015005 *  
UTIL_state_1_matrix1 -23.071    5.823 -3.962 8.11e-05 *** 
UTIL_state_1_matrix2  20.049   10.233  1.959 0.050443 .  
UTIL_state_1_matrix3 30.840    8.725  3.535 0.000432 *** 
UTIL_state_1_matrix4 -20.201    7.781 -2.596 0.009598 ** 
UTIL_state_1_matrix5 -26.965   10.179 -2.649 0.008231 ** 
UTIL_state_1_matrix6  71.242   14.709  4.843 1.53e-06 *** 
UTIL_state_1_matrix7 -101.468   11.730 -8.650 < 2e-16 *** 
UTIL_state_1_matrix8 -1.533    13.667 -0.112 0.910711    
UTIL_state_1_matrix9 138.513   17.176  8.064 2.69e-15 *** 
UTIL_state_1_matrix10 -56.514   11.913 -4.744 2.48e-06 *** 
UTIL_state_1_matrix11 -74.874   9.931 -7.539 1.28e-13 *** 
UTIL_state_1_matrix12 -26.804   15.633 -1.715 0.086813 .  
UTIL_state_1_matrix13 129.514   17.856  7.253 9.63e-13 *** 
UTIL_state_1_matrix14 -30.807   11.846 -2.601 0.009479 ** 
UTIL_state_1_matrix15 -60.399   9.368 -6.447 1.97e-10 *** 
UTIL_state_1_matrix16  16.875   9.963  1.694 0.090684 .  
UTIL_state_1_matrix17  21.300   4.940  4.312 1.82e-05 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 12050 on 798 degrees of freedom
Multiple R-squared:  0.9796,    Adjusted R-squared:  0.9791 
F-statistic:  2252 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 85: Performance of Multivariate Multiple Linear Regression MODEL(GSP-UTIL)

```

Response Y17 :

Call:
lm(formula = Y17 ~ UTIL_state_1_matrix)

Residuals:
    Min      1Q Median      3Q     Max 
-41374 -7198   1517   5559  31091 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) 1771.5184   785.5853   2.255  0.02440 *  
UTIL_state_1_matrix1 -27.2250    6.0244  -4.519 7.15e-06 *** 
UTIL_state_1_matrix2  22.1675   10.5865   2.094  0.03658 *  
UTIL_state_1_matrix3 36.0386    9.0264   3.993 7.14e-05 *** 
UTIL_state_1_matrix4 -19.9251    8.0491  -2.475  0.01351 *  
UTIL_state_1_matrix5 -32.5053   10.5300  -3.087  0.00209 ** 
UTIL_state_1_matrix6  65.1929   15.2167   4.284 2.06e-05 *** 
UTIL_state_1_matrix7 -91.1475   12.1346  -7.511 1.57e-13 *** 
UTIL_state_1_matrix8  8.4885   14.1388   0.600  0.54843    
UTIL_state_1_matrix9 119.9363   17.7684   6.750 2.84e-11 *** 
UTIL_state_1_matrix10 -51.1034   12.3238  -4.147 3.73e-05 *** 
UTIL_state_1_matrix11 -65.5656   10.2737  -6.382 2.96e-10 *** 
UTIL_state_1_matrix12 -43.1808   16.1722  -2.670  0.00774 ** 
UTIL_state_1_matrix13 141.4486   18.4718   7.658 5.48e-14 *** 
UTIL_state_1_matrix14 -39.6432   12.2548  -3.235  0.00127 ** 
UTIL_state_1_matrix15 -45.4316   9.6912  -4.688 3.25e-06 *** 
UTIL_state_1_matrix16 -0.9876   10.3065  -0.096  0.92368    
UTIL_state_1_matrix17 29.1294    5.1105   5.700 1.69e-08 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 12470 on 798 degrees of freedom
Multiple R-squared:  0.9795,   Adjusted R-squared:  0.9791
F-statistic:  2247 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 86: Performance of Multivariate Multiple Linear Regression MODEL(GSP-UTIL)

Response: GSP, Covariate: PC We have now taken GSP as response variable and PC as covariate. Summary for the model is described below. There are 17 summary outputs for this session of response and covariates.

```
> summary(model_5)
Response Y1 :

Call:
lm(formula = Y1 ~ PC_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-9018.5 -1917.9 -446.4  2056.9 8137.9 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) 87.13375  316.00073  0.276  0.78282  
PC_state_1_matrix1 0.03666   0.56349  0.065  0.94815  
PC_state_1_matrix2 38.14535   7.04896  5.411 8.27e-08 *** 
PC_state_1_matrix3 -38.39759   6.92033 -5.549 3.92e-08 *** 
PC_state_1_matrix4 -74.57467   6.15407 -12.118 < 2e-16 *** 
PC_state_1_matrix5 83.02236   6.56038 12.655 < 2e-16 *** 
PC_state_1_matrix6 -8.29455   3.78341 -2.192  0.02864 *  
PC_state_1_matrix7 -5.31130   0.61200 -8.679 < 2e-16 *** 
PC_state_1_matrix8 47.26142   3.88840 12.154 < 2e-16 *** 
PC_state_1_matrix9 -94.85458   9.49196 -9.993 < 2e-16 *** 
PC_state_1_matrix10 73.18835   9.97987 7.334 5.51e-13 *** 
PC_state_1_matrix11 -20.67089   3.90600 -5.292 1.56e-07 *** 
PC_state_1_matrix12 -15.80136   1.51286 -10.445 < 2e-16 *** 
PC_state_1_matrix13 6.44005   2.45620  2.622  0.00891 ** 
PC_state_1_matrix14 14.37416   5.16738  2.782  0.00553 ** 
PC_state_1_matrix15 -1.26451   6.92405 -0.183  0.85514  
PC_state_1_matrix16 -5.06157   2.81510 -1.798  0.07255 . 
PC_state_1_matrix17  0.46510   0.05940  7.831 1.55e-14 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3654 on 798 degrees of freedom
Multiple R-squared:  0.9959,    Adjusted R-squared:  0.9958 
F-statistic: 1.149e+04 on 17 and 798 DF,  p-value: < 2.2e-16
```

Figure 87: Performance of Multivariate Multiple Linear Regression MODEL(GSP-PC)

```

Response Y2 :

Call:
lm(formula = Y2 ~ PC_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-9040.0 -1910.6 -427.8 1850.9 8240.1 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 11.15207 293.57689  0.038  0.96971    
PC_state_1_matrix1 0.35346  0.52350  0.675  0.49976    
PC_state_1_matrix2 29.40877  6.54876  4.491 8.14e-06 ***  
PC_state_1_matrix3 -30.02927  6.42925 -4.671 3.52e-06 ***  
PC_state_1_matrix4 -64.89242  5.71737 -11.350 < 2e-16 ***  
PC_state_1_matrix5  74.73222  6.09484 12.262 < 2e-16 ***  
PC_state_1_matrix6 -9.59367  3.51494 -2.729  0.00648 **  
PC_state_1_matrix7 -4.09028  0.56857 -7.194 1.45e-12 ***  
PC_state_1_matrix8 48.66001  3.61247 13.470 < 2e-16 ***  
PC_state_1_matrix9 -101.95564 8.81839 -11.562 < 2e-16 ***  
PC_state_1_matrix10 71.83516  9.27169  7.748 2.84e-14 ***  
PC_state_1_matrix11 -14.88550  3.62883 -4.102 4.52e-05 ***  
PC_state_1_matrix12 -12.66364  1.40550 -9.010 < 2e-16 ***  
PC_state_1_matrix13  4.72615  2.28191  2.071  0.03867 *  
PC_state_1_matrix14 12.57641  4.80069  2.620  0.00897 **  
PC_state_1_matrix15 -2.12082  6.43271 -0.330  0.74172    
PC_state_1_matrix16 -3.78729  2.61534 -1.448  0.14798    
PC_state_1_matrix17  0.44799  0.05518  8.119 1.78e-15 ***  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 3395 on 798 degrees of freedom
Multiple R-squared:  0.9966,    Adjusted R-squared:  0.9965 
F-statistic: 1.361e+04 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 88: Performance of Multivariate Multiple Linear Regression MODEL(GSP-PC)

```

Response Y3 :

Call:
lm(formula = Y3 ~ PC_state_1_matrix)

Residuals:
    Min      1Q Median      3Q     Max 
 -8994   -1978   -273   1641   8348 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) -139.46828  291.38515 -0.479  0.63233  
PC_state_1_matrix1  0.72117   0.51959   1.388  0.16554  
PC_state_1_matrix2  33.95772   6.49987   5.224 2.23e-07 *** 
PC_state_1_matrix3 -35.19740   6.38126  -5.516 4.69e-08 *** 
PC_state_1_matrix4 -68.22578   5.67468 -12.023 < 2e-16 *** 
PC_state_1_matrix5  73.16676   6.04934  12.095 < 2e-16 *** 
PC_state_1_matrix6 -4.79018   3.48870  -1.373  0.17012  
PC_state_1_matrix7 -3.65215   0.56433  -6.472 1.69e-10 *** 
PC_state_1_matrix8  49.86877   3.58551  13.908 < 2e-16 *** 
PC_state_1_matrix9 -101.31956   8.75256 -11.576 < 2e-16 *** 
PC_state_1_matrix10 65.90817   9.20247   7.162 1.81e-12 *** 
PC_state_1_matrix11 -11.12168   3.60174  -3.088  0.00209 **  
PC_state_1_matrix12 -12.57680   1.39501  -9.016 < 2e-16 *** 
PC_state_1_matrix13  4.52122   2.26487   1.996  0.04625 *  
PC_state_1_matrix14 12.59492   4.76485   2.643  0.00837 ** 
PC_state_1_matrix15 -0.92142   6.38469  -0.144  0.88529  
PC_state_1_matrix16 -4.90299   2.59582  -1.889  0.05928 .  
PC_state_1_matrix17  0.50714   0.05477   9.260 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3369 on 798 degrees of freedom
Multiple R-squared:  0.9969,   Adjusted R-squared:  0.9968
F-statistic: 1.506e+04 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 89: Performance of Multivariate Multiple Linear Regression MODEL(GSP-PC)

```

Response Y4 :

Call:
lm(formula = Y4 ~ PC_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-7744.1 -1671.7 -239.9 1845.6 8390.0 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -538.46360 278.29440 -1.935 0.05336 .  
PC_state_1_matrix1  1.12322  0.49625  2.263 0.02388 * 
PC_state_1_matrix2  35.51618  6.20785  5.721 1.50e-08 *** 
PC_state_1_matrix3 -38.83041  6.09457 -6.371 3.16e-10 *** 
PC_state_1_matrix4 -66.21551  5.41974 -12.217 < 2e-16 *** 
PC_state_1_matrix5  68.67383  5.77757 11.886 < 2e-16 *** 
PC_state_1_matrix6 -0.93257  3.33196 -0.280 0.77964  
PC_state_1_matrix7 -3.94745  0.53898 -7.324 5.89e-13 *** 
PC_state_1_matrix8  57.94493  3.42442 16.921 < 2e-16 *** 
PC_state_1_matrix9 -117.45691 8.35934 -14.051 < 2e-16 *** 
PC_state_1_matrix10 74.39569  8.78904  8.465 < 2e-16 *** 
PC_state_1_matrix11 -11.16603  3.43993 -3.246 0.00122 ** 
PC_state_1_matrix12 -12.60145  1.33234 -9.458 < 2e-16 *** 
PC_state_1_matrix13  5.08308  2.16312  2.350 0.01902 *  
PC_state_1_matrix14  8.35096  4.55079  1.835 0.06687 .  
PC_state_1_matrix15  5.99610  6.09785  0.983 0.32575  
PC_state_1_matrix16 -8.00796  2.47920 -3.230 0.00129 ** 
PC_state_1_matrix17  0.46861  0.05231  8.959 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 3218 on 798 degrees of freedom
Multiple R-squared:  0.9974,    Adjusted R-squared:  0.9974 
F-statistic: 1.809e+04 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 90: Performance of Multivariate Multiple Linear Regression MODEL(GSP-PC)

```

Response Y5 :

Call:
lm(formula = Y5 ~ PC_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-7870.2 -1767.0 -260.3 1698.5 7999.3 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -721.29836 286.83664 -2.515   0.0121 *  
PC_state_1_matrix1  0.15973  0.51148  0.312   0.7549    
PC_state_1_matrix2  40.73082  6.39840  6.366 3.28e-10 *** 
PC_state_1_matrix3 -42.54784  6.28164 -6.773 2.44e-11 *** 
PC_state_1_matrix4 -68.20825  5.58610 -12.210 < 2e-16 *** 
PC_state_1_matrix5  70.30920  5.95491 11.807 < 2e-16 *** 
PC_state_1_matrix6 -1.08663  3.43424 -0.316   0.7518    
PC_state_1_matrix7 -5.63722  0.55552 -10.148 < 2e-16 *** 
PC_state_1_matrix8  55.96063  3.52954 15.855 < 2e-16 *** 
PC_state_1_matrix9 -113.46287 8.61593 -13.169 < 2e-16 *** 
PC_state_1_matrix10 80.22351  9.05882  8.856 < 2e-16 *** 
PC_state_1_matrix11 -17.36670  3.54551 -4.898 1.17e-06 *** 
PC_state_1_matrix12 -13.94138  1.37323 -10.152 < 2e-16 *** 
PC_state_1_matrix13  9.15458  2.22952  4.106 4.44e-05 *** 
PC_state_1_matrix14  0.09756  4.69047  0.021   0.9834    
PC_state_1_matrix15 14.01464  6.28502  2.230   0.0260 *  
PC_state_1_matrix16 -10.35389  2.55529 -4.052 5.58e-05 *** 
PC_state_1_matrix17  0.49948  0.05391  9.264 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 3317 on 798 degrees of freedom
Multiple R-squared:  0.9972,    Adjusted R-squared:  0.9972 
F-statistic: 1.683e+04 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 91: Performance of Multivariate Multiple Linear Regression MODEL(GSP-PC)

```

Response Y6 :

Call:
lm(formula = Y6 ~ PC_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-8410.1 -1979.2 -303.3 1937.0 9256.1 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -808.5618   306.4598 -2.638  0.00849 ** 
PC_state_1_matrix1 -0.3087    0.5465 -0.565  0.57228  
PC_state_1_matrix2  33.9772   6.8361  4.970 8.19e-07 *** 
PC_state_1_matrix3 -33.5657   6.7114 -5.001 7.01e-07 *** 
PC_state_1_matrix4 -66.8834   5.9683 -11.207 < 2e-16 *** 
PC_state_1_matrix5  72.7920   6.3623 11.441 < 2e-16 *** 
PC_state_1_matrix6 -6.5507   3.6692 -1.785  0.07459 .    
PC_state_1_matrix7 -6.4482   0.5935 -10.864 < 2e-16 *** 
PC_state_1_matrix8  55.5312   3.7710 14.726 < 2e-16 *** 
PC_state_1_matrix9 -113.6106  9.2054 -12.342 < 2e-16 *** 
PC_state_1_matrix10 83.5186   9.6785  8.629 < 2e-16 *** 
PC_state_1_matrix11 -19.4611   3.7881 -5.137 3.50e-07 *** 
PC_state_1_matrix12 -14.8574   1.4672 -10.126 < 2e-16 *** 
PC_state_1_matrix13 11.6338   2.3820  4.884 1.26e-06 *** 
PC_state_1_matrix14  3.7828   5.0114  0.755  0.45056  
PC_state_1_matrix15  4.6886   6.7150  0.698  0.48523  
PC_state_1_matrix16 -6.1619   2.7301 -2.257  0.02428 *   
PC_state_1_matrix17  0.5467   0.0576  9.490 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 3544 on 798 degrees of freedom
Multiple R-squared:  0.9968,    Adjusted R-squared:  0.9967 
F-statistic: 1.452e+04 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 92: Performance of Multivariate Multiple Linear Regression MODEL(GSP-PC)

```

Response Y7 :

Call:
lm(formula = Y7 ~ PC_state_1_matrix)

Residuals:
    Min      1Q Median      3Q     Max 
 -8973  -1696   -220   2025   8677 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -922.44187  307.30382 -3.002  0.00277 ** 
PC_state_1_matrix1  0.42330   0.54798  0.772  0.44007    
PC_state_1_matrix2  20.43879   6.85496  2.982  0.00295 ** 
PC_state_1_matrix3 -18.61305   6.72987 -2.766  0.00581 ** 
PC_state_1_matrix4 -60.85514   5.98470 -10.168 < 2e-16 *** 
PC_state_1_matrix5  66.05315   6.37982  10.353 < 2e-16 *** 
PC_state_1_matrix6 -8.04544   3.67929 -2.187  0.02906 *  
PC_state_1_matrix7 -6.28645   0.59516 -10.563 < 2e-16 *** 
PC_state_1_matrix8  69.44971   3.78139  18.366 < 2e-16 *** 
PC_state_1_matrix9 -142.61901  9.23072 -15.450 < 2e-16 *** 
PC_state_1_matrix10 95.88153   9.70521  9.879 < 2e-16 *** 
PC_state_1_matrix11 -16.91269  3.79850 -4.452  9.70e-06 *** 
PC_state_1_matrix12 -14.64321  1.47122 -9.953 < 2e-16 *** 
PC_state_1_matrix13 11.93931   2.38860  4.998 7.11e-07 *** 
PC_state_1_matrix14  6.37557   5.02516  1.269  0.20491    
PC_state_1_matrix15  0.24248   6.73349  0.036  0.97128    
PC_state_1_matrix16 -4.81913   2.73763 -1.760  0.07873 .  
PC_state_1_matrix17  0.54938   0.05776  9.511 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 3554 on 798 degrees of freedom
Multiple R-squared:  0.997,    Adjusted R-squared:  0.9969 
F-statistic: 1.565e+04 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 93: Performance of Multivariate Multiple Linear Regression MODEL(GSP-PC)

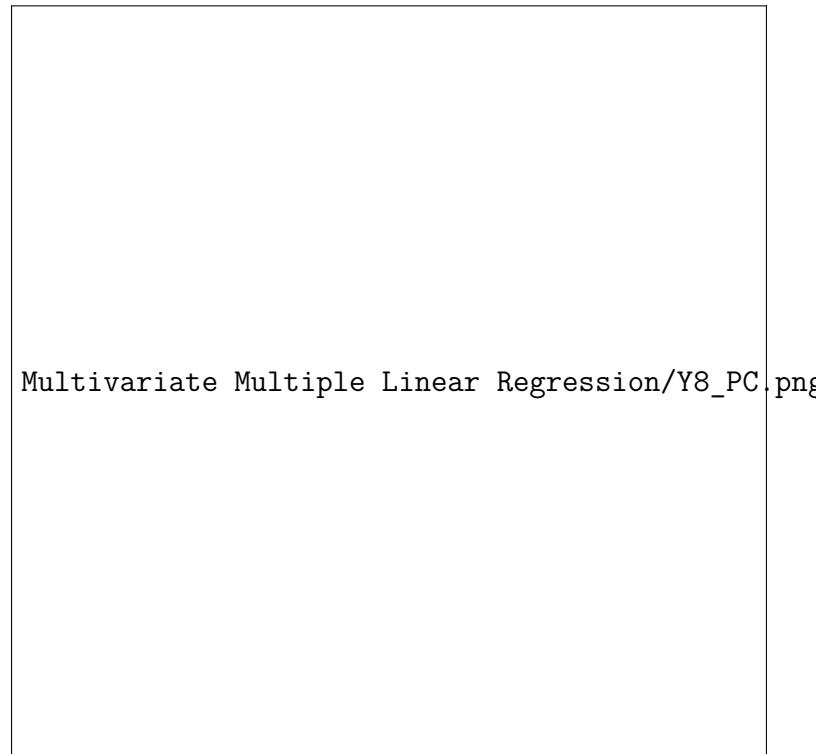


Figure 94: Performance of Multivariate Multiple Linear Regression MODEL(GSP-PC)

```

Response Y9 :

Call:
lm(formula = Y9 ~ PC_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-9597.3 -1910.2   296.8  2632.2  7335.7 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) -1.344e+03  3.230e+02 -4.160 3.53e-05 *** 
PC_state_1_matrix1 1.290e+00  5.760e-01  2.240  0.02537 *  
PC_state_1_matrix2 9.035e+00  7.205e+00  1.254  0.21022  
PC_state_1_matrix3 -6.593e+00  7.074e+00 -0.932  0.35160  
PC_state_1_matrix4 -5.827e+01  6.290e+00 -9.263 < 2e-16 *** 
PC_state_1_matrix5 6.249e+01  6.706e+00  9.319 < 2e-16 *** 
PC_state_1_matrix6 -8.801e+00  3.867e+00 -2.276  0.02312 *  
PC_state_1_matrix7 -7.383e+00  6.256e-01 -11.802 < 2e-16 *** 
PC_state_1_matrix8 8.754e+01  3.975e+00 22.025 < 2e-16 *** 
PC_state_1_matrix9 -1.733e+02  9.702e+00 -17.861 < 2e-16 *** 
PC_state_1_matrix10 1.063e+02  1.020e+01 10.422 < 2e-16 *** 
PC_state_1_matrix11 -1.385e+01  3.993e+00 -3.469  0.00055 *** 
PC_state_1_matrix12 -1.438e+01  1.546e+00 -9.296 < 2e-16 *** 
PC_state_1_matrix13 1.149e+01  2.511e+00  4.578 5.44e-06 *** 
PC_state_1_matrix14 5.248e+00  5.282e+00  0.994  0.32077  
PC_state_1_matrix15 5.334e+00  7.077e+00  0.754  0.45128  
PC_state_1_matrix16 -8.235e+00  2.877e+00 -2.862  0.00432 ** 
PC_state_1_matrix17 5.533e-01  6.071e-02  9.114 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 3735 on 798 degrees of freedom
Multiple R-squared:  0.9972,   Adjusted R-squared:  0.9972
F-statistic: 1.697e+04 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 95: Performance of Multivariate Multiple Linear Regression MODEL(GSP-PC)

```

Response Y10 :

Call:
lm(formula = Y10 ~ PC_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-10023.1 -1796.9     7.5  3199.9  8229.0 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -1.467e+03 3.391e+02 -4.327 1.70e-05 ***
PC_state_1_matrix1 9.993e-01 6.047e-01  1.652 0.098828 .  
PC_state_1_matrix2 1.145e+01 7.565e+00  1.513 0.130594    
PC_state_1_matrix3 -8.930e+00 7.427e+00 -1.202 0.229561    
PC_state_1_matrix4 -5.946e+01 6.604e+00 -9.002 < 2e-16 ***
PC_state_1_matrix5 6.388e+01 7.041e+00  9.074 < 2e-16 *** 
PC_state_1_matrix6 -8.710e+00 4.060e+00 -2.145 0.032247 *  
PC_state_1_matrix7 -8.689e+00 6.568e-01 -13.229 < 2e-16 *** 
PC_state_1_matrix8 8.565e+01 4.173e+00  20.524 < 2e-16 *** 
PC_state_1_matrix9 -1.658e+02 1.019e+01 -16.272 < 2e-16 *** 
PC_state_1_matrix10 1.038e+02 1.071e+01  9.693 < 2e-16 *** 
PC_state_1_matrix11 -1.598e+01 4.192e+00 -3.813 0.000148 *** 
PC_state_1_matrix12 -1.482e+01 1.624e+00 -9.128 < 2e-16 *** 
PC_state_1_matrix13 1.379e+01 2.636e+00  5.231 2.16e-07 *** 
PC_state_1_matrix14 -1.443e+00 5.546e+00 -0.260 0.794791    
PC_state_1_matrix15 1.380e+01 7.431e+00  1.857 0.063623 .  
PC_state_1_matrix16 -1.155e+01 3.021e+00 -3.825 0.000141 *** 
PC_state_1_matrix17 6.173e-01 6.374e-02  9.685 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 3922 on 798 degrees of freedom
Multiple R-squared:  0.9971,    Adjusted R-squared:  0.997 
F-statistic: 1.612e+04 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 96: Performance of Multivariate Multiple Linear Regression MODEL(GSP-PC)

```

Response Y11 :

Call:
lm(formula = Y11 ~ PC_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-9696.8 -1933.7 -351.5 2863.9 9657.2 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) -1.286e+03 3.432e+02 -3.748 0.000191 *** 
PC_state_1_matrix1 -1.763e-01 6.120e-01 -0.288 0.773342  
PC_state_1_matrix2 2.488e+01 7.655e+00 3.250 0.001204 **  
PC_state_1_matrix3 -2.280e+01 7.516e+00 -3.033 0.002497 ** 
PC_state_1_matrix4 -6.305e+01 6.683e+00 -9.434 < 2e-16 *** 
PC_state_1_matrix5 6.825e+01 7.125e+00 9.579 < 2e-16 *** 
PC_state_1_matrix6 -7.686e+00 4.109e+00 -1.871 0.061768 .  
PC_state_1_matrix7 -9.746e+00 6.646e-01 -14.663 < 2e-16 *** 
PC_state_1_matrix8 7.378e+01 4.223e+00 17.471 < 2e-16 *** 
PC_state_1_matrix9 -1.429e+02 1.031e+01 -13.862 < 2e-16 *** 
PC_state_1_matrix10 9.848e+01 1.084e+01 9.086 < 2e-16 *** 
PC_state_1_matrix11 -2.095e+01 4.242e+00 -4.938 9.60e-07 *** 
PC_state_1_matrix12 -1.357e+01 1.643e+00 -8.262 5.94e-16 *** 
PC_state_1_matrix13 1.263e+01 2.667e+00 4.735 2.59e-06 *** 
PC_state_1_matrix14 -8.265e+00 5.612e+00 -1.473 0.141213  
PC_state_1_matrix15 2.409e+01 7.520e+00 3.204 0.001410 **  
PC_state_1_matrix16 -1.489e+01 3.057e+00 -4.870 1.34e-06 *** 
PC_state_1_matrix17 7.588e-01 6.451e-02 11.763 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 3968 on 798 degrees of freedom
Multiple R-squared:  0.997,   Adjusted R-squared:  0.9969
F-statistic: 1.564e+04 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 97: Performance of Multivariate Multiple Linear Regression MODEL(GSP-PC)

```

Response Y12 :

Call:
lm(formula = Y12 ~ PC_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-8880.1 -2093.0 -401.3  2755.0  8149.4 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -739.39235  314.17828 -2.353 0.018843 *  
PC_state_1_matrix1 -1.14950   0.56024 -2.052 0.040514 *  
PC_state_1_matrix2  41.27944   7.00831  5.890 5.69e-09 *** 
PC_state_1_matrix3 -39.77698   6.88042 -5.781 1.06e-08 *** 
PC_state_1_matrix4 -65.22984   6.11857 -10.661 < 2e-16 *** 
PC_state_1_matrix5  68.29282   6.52254 10.470 < 2e-16 *** 
PC_state_1_matrix6 -3.81503   3.76159 -1.014 0.310792    
PC_state_1_matrix7 -9.21064   0.60847 -15.137 < 2e-16 *** 
PC_state_1_matrix8  61.62663   3.86598 15.941 < 2e-16 *** 
PC_state_1_matrix9 -129.42735  9.43721 -13.715 < 2e-16 *** 
PC_state_1_matrix10 103.29859  9.92231 10.411 < 2e-16 *** 
PC_state_1_matrix11 -27.74918   3.88348 -7.145 2.02e-12 *** 
PC_state_1_matrix12 -10.25792  1.50413 -6.820 1.80e-11 *** 
PC_state_1_matrix13  9.02093   2.44204  3.694 0.000236 *** 
PC_state_1_matrix14 -12.34495  5.13758 -2.403 0.016494 *  
PC_state_1_matrix15 28.09214   6.88412  4.081 4.94e-05 *** 
PC_state_1_matrix16 -14.50886  2.79887 -5.184 2.76e-07 *** 
PC_state_1_matrix17  0.80267   0.05905 13.592 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 3633 on 798 degrees of freedom
Multiple R-squared:  0.9975,    Adjusted R-squared:  0.9975 
F-statistic: 1.904e+04 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 98: Performance of Multivariate Multiple Linear Regression MODEL(GSP-PC)

```

Response Y13 :

Call:
lm(formula = Y13 ~ PC_state_1_matrix)

Residuals:
    Min      1Q Median      3Q     Max 
 -7909   -2393   -266   2978   7704 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -416.06492  307.08036 -1.355 0.175831  
PC_state_1_matrix1 -1.88068   0.54758 -3.435 0.000624 *** 
PC_state_1_matrix2  48.25089   6.84998  7.044 4.04e-12 *** 
PC_state_1_matrix3 -46.06280   6.72498 -6.850 1.48e-11 *** 
PC_state_1_matrix4 -62.88134   5.98034 -10.515 < 2e-16 *** 
PC_state_1_matrix5  68.39160   6.37518  10.728 < 2e-16 *** 
PC_state_1_matrix6 -5.75033   3.67661 -1.564 0.118207  
PC_state_1_matrix7 -8.74600   0.59473 -14.706 < 2e-16 *** 
PC_state_1_matrix8  49.69927   3.77864  13.153 < 2e-16 *** 
PC_state_1_matrix9 -120.18333  9.22401 -13.029 < 2e-16 *** 
PC_state_1_matrix10 111.97940  9.69815  11.546 < 2e-16 *** 
PC_state_1_matrix11 -34.59057   3.79574 -9.113 < 2e-16 *** 
PC_state_1_matrix12 -5.98475   1.47015 -4.071 5.15e-05 *** 
PC_state_1_matrix13  7.03376   2.38687  2.947 0.003304 ** 
PC_state_1_matrix14 -21.25435   5.02151 -4.233 2.58e-05 *** 
PC_state_1_matrix15  34.20500   6.72860  5.084 4.62e-07 *** 
PC_state_1_matrix16 -13.89181   2.73564 -5.078 4.75e-07 *** 
PC_state_1_matrix17   0.86734   0.05772  15.027 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 3551 on 798 degrees of freedom
Multiple R-squared:  0.9976,    Adjusted R-squared:  0.9975 
F-statistic: 1.937e+04 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 99: Performance of Multivariate Multiple Linear Regression MODEL(GSP-PC)

```

Response Y14 :

Call:
lm(formula = Y14 ~ PC_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-7368.4 -1958.5 -514.9  2611.9  5982.6 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -296.7173   286.7454 -1.035 0.301087    
PC_state_1_matrix1 -1.8357    0.5113 -3.590 0.000351 ***  
PC_state_1_matrix2  51.8082    6.3964  8.100 2.06e-15 ***  
PC_state_1_matrix3 -49.6122    6.2797 -7.900 9.20e-15 ***  
PC_state_1_matrix4 -65.1123    5.5843 -11.660 < 2e-16 ***  
PC_state_1_matrix5  68.9880    5.9530 11.589 < 2e-16 ***  
PC_state_1_matrix6 -4.2357    3.4331 -1.234 0.217653    
PC_state_1_matrix7 -8.0535    0.5553 -14.502 < 2e-16 ***  
PC_state_1_matrix8  51.8889    3.5284 14.706 < 2e-16 ***  
PC_state_1_matrix9 -134.7574   8.6132 -15.645 < 2e-16 ***  
PC_state_1_matrix10 127.0226   9.0559 14.026 < 2e-16 ***  
PC_state_1_matrix11 -38.1027   3.5444 -10.750 < 2e-16 ***  
PC_state_1_matrix12 -5.0344    1.3728 -3.667 0.000261 ***  
PC_state_1_matrix13  7.3033    2.2288  3.277 0.001095 **  
PC_state_1_matrix14 -27.8788   4.6890 -5.946 4.12e-09 ***  
PC_state_1_matrix15  42.0247   6.2830  6.689 4.24e-11 ***  
PC_state_1_matrix16 -16.4227   2.5545 -6.429 2.21e-10 ***  
PC_state_1_matrix17  1.0313    0.0539 19.135 < 2e-16 ***  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3316 on 798 degrees of freedom
Multiple R-squared:  0.998,    Adjusted R-squared:  0.998 
F-statistic: 2.403e+04 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 100: Performance of Multivariate Multiple Linear Regression MODEL(GSP-PC)

```

Response Y15 :

Call:
lm(formula = Y15 ~ PC_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-7654.8 -2042.6 -426.7 2584.4 6987.7 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) -378.96594  279.22173 -1.357 0.175094  
PC_state_1_matrix1 -1.87220   0.49790 -3.760 0.000182 *** 
PC_state_1_matrix2  51.93499   6.22854  8.338 3.29e-16 *** 
PC_state_1_matrix3 -49.45545   6.11488 -8.088 2.25e-15 *** 
PC_state_1_matrix4 -66.07847   5.43780 -12.152 < 2e-16 *** 
PC_state_1_matrix5  70.46366   5.79682 12.156 < 2e-16 *** 
PC_state_1_matrix6 -4.98449   3.34307 -1.491 0.136358  
PC_state_1_matrix7 -9.10283   0.54077 -16.833 < 2e-16 *** 
PC_state_1_matrix8  58.32911   3.43583 16.977 < 2e-16 *** 
PC_state_1_matrix9 -153.86857   8.38720 -18.346 < 2e-16 *** 
PC_state_1_matrix10 144.64896   8.81832 16.403 < 2e-16 *** 
PC_state_1_matrix11 -42.21941   3.45139 -12.233 < 2e-16 *** 
PC_state_1_matrix12 -5.02561   1.33678 -3.759 0.000183 *** 
PC_state_1_matrix13  8.54247   2.17033  3.936 9.01e-05 *** 
PC_state_1_matrix14 -29.86563   4.56595 -6.541 1.09e-10 *** 
PC_state_1_matrix15  42.53617   6.11817  6.952 7.47e-12 *** 
PC_state_1_matrix16 -16.30267   2.48746 -6.554 1.00e-10 *** 
PC_state_1_matrix17  1.20994   0.05248 23.054 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3229 on 798 degrees of freedom
Multiple R-squared:  0.9984,    Adjusted R-squared:  0.9984 
F-statistic: 2.914e+04 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 101: Performance of Multivariate Multiple Linear Regression MODEL(GSP-PC)

```

Response Y16 :

Call:
lm(formula = Y16 ~ PC_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-7926.6 -2074.1 - 598.1  2709.4  7345.6 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -530.2144   283.0347 -1.873  0.06139 .  
PC_state_1_matrix1 -1.3362    0.5047 -2.647  0.00827 ** 
PC_state_1_matrix2  50.1789   6.3136  7.948 6.47e-15 *** 
PC_state_1_matrix3 -48.7632   6.1984 -7.867 1.18e-14 *** 
PC_state_1_matrix4 -67.0255   5.5121 -12.160 < 2e-16 *** 
PC_state_1_matrix5  72.1848   5.8760 12.285 < 2e-16 *** 
PC_state_1_matrix6 -5.3781    3.3887 -1.587  0.11289    
PC_state_1_matrix7 -9.5668    0.5482 -17.453 < 2e-16 *** 
PC_state_1_matrix8  65.6322   3.4827 18.845 < 2e-16 *** 
PC_state_1_matrix9 -173.4584   8.5017 -20.403 < 2e-16 *** 
PC_state_1_matrix10 160.6266   8.9387 17.970 < 2e-16 *** 
PC_state_1_matrix11 -45.6038   3.4985 -13.035 < 2e-16 *** 
PC_state_1_matrix12 -6.4691    1.3550 -4.774 2.15e-06 *** 
PC_state_1_matrix13 10.8006    2.2000  4.909 1.11e-06 *** 
PC_state_1_matrix14 -32.1523   4.6283 -6.947 7.75e-12 *** 
PC_state_1_matrix15 45.5476    6.2017  7.344 5.11e-13 *** 
PC_state_1_matrix16 -17.8867   2.5214 -7.094 2.88e-12 *** 
PC_state_1_matrix17  1.3421    0.0532 25.227 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 3273 on 798 degrees of freedom
Multiple R-squared:  0.9985,    Adjusted R-squared:  0.9985 
F-statistic: 3.113e+04 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 102: Performance of Multivariate Multiple Linear Regression MODEL(GSP-PC)

```

Response Y17 :

Call:
lm(formula = Y17 ~ PC_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-8187.8 -2246.0  -607.3  3125.5  6882.2 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) -440.35880  293.02341 -1.503   0.1333    
PC_state_1_matrix1 -1.19266   0.52251 -2.283   0.0227 *  
PC_state_1_matrix2  49.10957   6.53641  7.513 1.55e-13 *** 
PC_state_1_matrix3 -48.06650   6.41713 -7.490 1.82e-13 *** 
PC_state_1_matrix4 -68.43416   5.70659 -11.992 < 2e-16 *** 
PC_state_1_matrix5  75.47186   6.08335 12.406 < 2e-16 *** 
PC_state_1_matrix6 -6.97442   3.50831 -1.988   0.0472 *  
PC_state_1_matrix7 -10.43879   0.56750 -18.394 < 2e-16 *** 
PC_state_1_matrix8  73.96781   3.60566 20.514 < 2e-16 *** 
PC_state_1_matrix9 -192.95791   8.80177 -21.923 < 2e-16 *** 
PC_state_1_matrix10 175.30906   9.25421 18.944 < 2e-16 *** 
PC_state_1_matrix11 -48.50921   3.62199 -13.393 < 2e-16 *** 
PC_state_1_matrix12 -6.45987   1.40285 -4.605 4.80e-06 *** 
PC_state_1_matrix13  13.02829   2.27760  5.720 1.50e-08 *** 
PC_state_1_matrix14 -38.69064   4.79164 -8.075 2.49e-15 *** 
PC_state_1_matrix15  52.34506   6.42059  8.153 1.37e-15 *** 
PC_state_1_matrix16 -20.46460   2.61041 -7.840 1.45e-14 *** 
PC_state_1_matrix17  1.54616   0.05508 28.073 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3388 on 798 degrees of freedom
Multiple R-squared:  0.9985,   Adjusted R-squared:  0.9985
F-statistic: 3.101e+04 on 17 and 798 DF,  p-value: < 2.2e-16
```

Figure 103: Performance of Multivariate Multiple Linear Regression MODEL(GSP-PC)

Response: GSP, Covariate: UNEP We have now taken GSP as response variable and UNEP as covariate. Summary for the model is described below. There are 17 summary outputs for this session of response and covariates.

```
> summary(model_6)
Response Y1 :

Call:
lm(formula = Y1 ~ UNEMP_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-9018.5 -1917.9  -446.4  2056.9  8137.9 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 87.13375  316.00073   0.276  0.78282    
UNEMP_state_1_matrix1  0.03666   0.56349   0.065  0.94815    
UNEMP_state_1_matrix2  38.14535   7.04896   5.411 8.27e-08 ***  
UNEMP_state_1_matrix3 -38.39759   6.92033  -5.549 3.92e-08 ***  
UNEMP_state_1_matrix4 -74.57467   6.15407 -12.118 < 2e-16 ***  
UNEMP_state_1_matrix5  83.02236   6.56038  12.655 < 2e-16 ***  
UNEMP_state_1_matrix6  -8.29455   3.78341  -2.192  0.02864 *  
UNEMP_state_1_matrix7  -5.31130   0.61200  -8.679 < 2e-16 ***  
UNEMP_state_1_matrix8  47.26142   3.88840  12.154 < 2e-16 ***  
UNEMP_state_1_matrix9  -94.85458   9.49196  -9.993 < 2e-16 ***  
UNEMP_state_1_matrix10 73.18835   9.97987   7.334 5.51e-13 ***  
UNEMP_state_1_matrix11 -20.67089   3.90600  -5.292 1.56e-07 ***  
UNEMP_state_1_matrix12 -15.80136   1.51286 -10.445 < 2e-16 ***  
UNEMP_state_1_matrix13  6.44005   2.45620   2.622  0.00891 **  
UNEMP_state_1_matrix14 14.37416   5.16738   2.782  0.00553 **  
UNEMP_state_1_matrix15 -1.26451   6.92405  -0.183  0.85514    
UNEMP_state_1_matrix16 -5.06157   2.81510  -1.798  0.07255 .  
UNEMP_state_1_matrix17  0.46510   0.05940   7.831 1.55e-14 ***  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3654 on 798 degrees of freedom
Multiple R-squared:  0.9959,    Adjusted R-squared:  0.9958 
F-statistic: 1.149e+04 on 17 and 798 DF,  p-value: < 2.2e-16
```

Figure 104: Performance of Multivariate Multiple Linear Regression MODEL(GSP-UNEMP)

```

Response Y2 :

Call:
lm(formula = Y2 ~ UNEMP_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-9040.0 -1910.6 -427.8 1850.9 8240.1 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) 11.15207 293.57689  0.038  0.96971    
UNEMP_state_1_matrix1 0.35346  0.52350  0.675  0.49976    
UNEMP_state_1_matrix2 29.40877  6.54876  4.491 8.14e-06 ***  
UNEMP_state_1_matrix3 -30.02927  6.42925 -4.671 3.52e-06 ***  
UNEMP_state_1_matrix4 -64.89242  5.71737 -11.350 < 2e-16 ***  
UNEMP_state_1_matrix5 74.73222  6.09484 12.262 < 2e-16 ***  
UNEMP_state_1_matrix6 -9.59367  3.51494 -2.729  0.00648 **  
UNEMP_state_1_matrix7 -4.09028  0.56857 -7.194 1.45e-12 ***  
UNEMP_state_1_matrix8 48.66001  3.61247 13.470 < 2e-16 ***  
UNEMP_state_1_matrix9 -101.95564 8.81839 -11.562 < 2e-16 ***  
UNEMP_state_1_matrix10 71.83516  9.27169  7.748 2.84e-14 ***  
UNEMP_state_1_matrix11 -14.88550  3.62883 -4.102 4.52e-05 ***  
UNEMP_state_1_matrix12 -12.66364  1.40550 -9.010 < 2e-16 ***  
UNEMP_state_1_matrix13 4.72615   2.28191  2.071  0.03867 *  
UNEMP_state_1_matrix14 12.57641  4.80069  2.620  0.00897 **  
UNEMP_state_1_matrix15 -2.12082  6.43271 -0.330  0.74172    
UNEMP_state_1_matrix16 -3.78729  2.61534 -1.448  0.14798    
UNEMP_state_1_matrix17  0.44799  0.05518  8.119 1.78e-15 ***  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 3395 on 798 degrees of freedom
Multiple R-squared:  0.9966,   Adjusted R-squared:  0.9965
F-statistic: 1.361e+04 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 105: Performance of Multivariate Multiple Linear Regression MODEL(GSP-UNEMP)

```

Response Y3 :

Call:
lm(formula = Y3 ~ UNEMP_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-8994 -1978   -273   1641   8348 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept) -139.46828  291.38515 -0.479  0.63233  
UNEMP_state_1_matrix1  0.72117   0.51959   1.388  0.16554  
UNEMP_state_1_matrix2 33.95772   6.49987   5.224 2.23e-07 ***
UNEMP_state_1_matrix3 -35.19740   6.38126  -5.516 4.69e-08 *** 
UNEMP_state_1_matrix4 -68.22578   5.67468 -12.023 < 2e-16 ***
UNEMP_state_1_matrix5 73.16676   6.04934  12.095 < 2e-16 *** 
UNEMP_state_1_matrix6 -4.79018   3.48870  -1.373  0.17012  
UNEMP_state_1_matrix7 -3.65215   0.56433  -6.472 1.69e-10 *** 
UNEMP_state_1_matrix8 49.86877   3.58551  13.908 < 2e-16 *** 
UNEMP_state_1_matrix9 -101.31956   8.75256 -11.576 < 2e-16 *** 
UNEMP_state_1_matrix10 65.90817   9.20247   7.162 1.81e-12 *** 
UNEMP_state_1_matrix11 -11.12168   3.60174  -3.088  0.00209 ** 
UNEMP_state_1_matrix12 -12.57680   1.39501  -9.016 < 2e-16 *** 
UNEMP_state_1_matrix13  4.52122   2.26487   1.996  0.04625 *  
UNEMP_state_1_matrix14 12.59492   4.76485   2.643  0.00837 ** 
UNEMP_state_1_matrix15 -0.92142   6.38469  -0.144  0.88529  
UNEMP_state_1_matrix16 -4.90299   2.59582  -1.889  0.05928 . 
UNEMP_state_1_matrix17  0.50714   0.05477   9.260 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3369 on 798 degrees of freedom
Multiple R-squared:  0.9969,    Adjusted R-squared:  0.9968 
F-statistic: 1.506e+04 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 106: Performance of Multivariate Multiple Linear Regression MODEL(GSP-UNEMP)

```

Response Y4 :

Call:
lm(formula = Y4 ~ UNEMP_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-7744.1 -1671.7 -239.9 1845.6 8390.0 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -538.46360 278.29440 -1.935 0.05336 .  
UNEMP_state_1_matrix1 1.12322 0.49625 2.263 0.02388 *  
UNEMP_state_1_matrix2 35.51618 6.20785 5.721 1.50e-08 *** 
UNEMP_state_1_matrix3 -38.83041 6.09457 -6.371 3.16e-10 *** 
UNEMP_state_1_matrix4 -66.21551 5.41974 -12.217 < 2e-16 *** 
UNEMP_state_1_matrix5 68.67383 5.77757 11.886 < 2e-16 *** 
UNEMP_state_1_matrix6 -0.93257 3.33196 -0.280 0.77964  
UNEMP_state_1_matrix7 -3.94745 0.53898 -7.324 5.89e-13 *** 
UNEMP_state_1_matrix8 57.94493 3.42442 16.921 < 2e-16 *** 
UNEMP_state_1_matrix9 -117.45691 8.35934 -14.051 < 2e-16 *** 
UNEMP_state_1_matrix10 74.39569 8.78904 8.465 < 2e-16 *** 
UNEMP_state_1_matrix11 -11.16603 3.43993 -3.246 0.00122 ** 
UNEMP_state_1_matrix12 -12.60145 1.33234 -9.458 < 2e-16 *** 
UNEMP_state_1_matrix13 5.08308 2.16312 2.350 0.01902 *  
UNEMP_state_1_matrix14 8.35096 4.55079 1.835 0.06687 .  
UNEMP_state_1_matrix15 5.99610 6.09785 0.983 0.32575  
UNEMP_state_1_matrix16 -8.00796 2.47920 -3.230 0.00129 ** 
UNEMP_state_1_matrix17 0.46861 0.05231 8.959 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 3218 on 798 degrees of freedom
Multiple R-squared:  0.9974,    Adjusted R-squared:  0.9974 
F-statistic: 1.809e+04 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 107: Performance of Multivariate Multiple Linear Regression MODEL(GSP-UNEMP)

```

Response Y5 :

Call:
lm(formula = Y5 ~ UNEMP_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-7870.2 -1767.0 -260.3 1698.5 7999.3 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept) -721.29836  286.83664 -2.515   0.0121 *  
UNEMP_state_1_matrix1  0.15973   0.51148   0.312   0.7549  
UNEMP_state_1_matrix2  40.73082   6.39840   6.366 3.28e-10 *** 
UNEMP_state_1_matrix3 -42.54784   6.28164  -6.773 2.44e-11 *** 
UNEMP_state_1_matrix4 -68.20825   5.58610 -12.210 < 2e-16 *** 
UNEMP_state_1_matrix5  70.30920   5.95491  11.807 < 2e-16 *** 
UNEMP_state_1_matrix6 -1.08663   3.43424  -0.316   0.7518  
UNEMP_state_1_matrix7 -5.63722   0.55552 -10.148 < 2e-16 *** 
UNEMP_state_1_matrix8  55.96063   3.52954  15.855 < 2e-16 *** 
UNEMP_state_1_matrix9 -113.46287  8.61593 -13.169 < 2e-16 *** 
UNEMP_state_1_matrix10 80.22351   9.05882  8.856 < 2e-16 *** 
UNEMP_state_1_matrix11 -17.36670  3.54551 -4.898 1.17e-06 *** 
UNEMP_state_1_matrix12 -13.94138  1.37323 -10.152 < 2e-16 *** 
UNEMP_state_1_matrix13  9.15458   2.22952  4.106 4.44e-05 *** 
UNEMP_state_1_matrix14  0.09756   4.69047  0.021   0.9834  
UNEMP_state_1_matrix15 14.01464   6.28502  2.230   0.0260 *  
UNEMP_state_1_matrix16 -10.35389  2.55529 -4.052 5.58e-05 *** 
UNEMP_state_1_matrix17  0.49948   0.05391  9.264 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 3317 on 798 degrees of freedom
Multiple R-squared:  0.9972,    Adjusted R-squared:  0.9972 
F-statistic: 1.683e+04 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 108: Performance of Multivariate Multiple Linear Regression MODEL(GSP-UNEMP)

```

Response Y6 :

Call:
lm(formula = Y6 ~ UNEMP_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-8410.1 -1979.2 -303.3 1937.0  9256.1 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) -808.5618   306.4598 -2.638  0.00849 **  
UNEMP_state_1_matrix1 -0.3087    0.5465 -0.565  0.57228  
UNEMP_state_1_matrix2  33.9772   6.8361  4.970 8.19e-07 *** 
UNEMP_state_1_matrix3 -33.5657   6.7114 -5.001 7.01e-07 *** 
UNEMP_state_1_matrix4 -66.8834   5.9683 -11.207 < 2e-16 *** 
UNEMP_state_1_matrix5  72.7920   6.3623 11.441 < 2e-16 *** 
UNEMP_state_1_matrix6 -6.5507    3.6692 -1.785  0.07459 .    
UNEMP_state_1_matrix7 -6.4482    0.5935 -10.864 < 2e-16 *** 
UNEMP_state_1_matrix8  55.5312   3.7710 14.726 < 2e-16 *** 
UNEMP_state_1_matrix9 -113.6106  9.2054 -12.342 < 2e-16 *** 
UNEMP_state_1_matrix10 83.5186   9.6785  8.629 < 2e-16 *** 
UNEMP_state_1_matrix11 -19.4611   3.7881 -5.137 3.50e-07 *** 
UNEMP_state_1_matrix12 -14.8574   1.4672 -10.126 < 2e-16 *** 
UNEMP_state_1_matrix13  11.6338   2.3820  4.884 1.26e-06 *** 
UNEMP_state_1_matrix14  3.7828    5.0114  0.755  0.45056  
UNEMP_state_1_matrix15  4.6886    6.7150  0.698  0.48523  
UNEMP_state_1_matrix16 -6.1619    2.7301 -2.257  0.02428 *  
UNEMP_state_1_matrix17  0.5467    0.0576  9.490 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 3544 on 798 degrees of freedom
Multiple R-squared:  0.9968,   Adjusted R-squared:  0.9967
F-statistic: 1.452e+04 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 109: Performance of Multivariate Multiple Linear Regression MODEL(GSP-UNEMP)

```

Response Y7 :

Call:
lm(formula = Y7 ~ UNEMP_state_1_matrix)

Residuals:
    Min      1Q Median      3Q     Max 
 -8973   -1696   -220   2025   8677 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) -922.44187  307.30382 -3.002  0.00277 ** 
UNEMP_state_1_matrix1  0.42330   0.54798  0.772  0.44007  
UNEMP_state_1_matrix2  20.43879   6.85496  2.982  0.00295 ** 
UNEMP_state_1_matrix3 -18.61305   6.72987 -2.766  0.00581 ** 
UNEMP_state_1_matrix4 -60.85514   5.98470 -10.168 < 2e-16 *** 
UNEMP_state_1_matrix5  66.05315   6.37982 10.353 < 2e-16 *** 
UNEMP_state_1_matrix6 -8.04544   3.67929 -2.187  0.02906 *  
UNEMP_state_1_matrix7 -6.28645   0.59516 -10.563 < 2e-16 *** 
UNEMP_state_1_matrix8  69.44971   3.78139 18.366 < 2e-16 *** 
UNEMP_state_1_matrix9 -142.61901   9.23072 -15.450 < 2e-16 *** 
UNEMP_state_1_matrix10 95.88153   9.70521  9.879 < 2e-16 *** 
UNEMP_state_1_matrix11 -16.91269   3.79850 -4.452 9.70e-06 *** 
UNEMP_state_1_matrix12 -14.64321   1.47122 -9.953 < 2e-16 *** 
UNEMP_state_1_matrix13 11.93931   2.38860  4.998 7.11e-07 *** 
UNEMP_state_1_matrix14  6.37557   5.02516  1.269  0.20491  
UNEMP_state_1_matrix15  0.24248   6.73349  0.036  0.97128  
UNEMP_state_1_matrix16 -4.81913   2.73763 -1.760  0.07873 .  
UNEMP_state_1_matrix17  0.54938   0.05776  9.511 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3554 on 798 degrees of freedom
Multiple R-squared:  0.997,    Adjusted R-squared:  0.9969 
F-statistic: 1.565e+04 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 110: Performance of Multivariate Multiple Linear Regression MODEL(GSP-UNEMP)

```

Response Y8 :

Call:
lm(formula = Y8 ~ UNEMP_state_1_matrix)

Residuals:
    Min      1Q Median      3Q     Max 
 -8815   -1914     307   2188   7653 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) -1.254e+03  3.081e+02 -4.070 5.17e-05 *** 
UNEMP_state_1_matrix1  9.058e-01  5.494e-01   1.649  0.09962 .  
UNEMP_state_1_matrix2  1.054e+01  6.873e+00   1.533  0.12570  
UNEMP_state_1_matrix3 -8.466e+00  6.748e+00  -1.255  0.21001  
UNEMP_state_1_matrix4 -5.785e+01  6.001e+00  -9.641 < 2e-16 *** 
UNEMP_state_1_matrix5  6.593e+01  6.397e+00  10.306 < 2e-16 *** 
UNEMP_state_1_matrix6 -1.169e+01  3.689e+00  -3.168  0.00159 ** 
UNEMP_state_1_matrix7 -6.606e+00  5.968e-01 -11.070 < 2e-16 *** 
UNEMP_state_1_matrix8  8.168e+01  3.792e+00  21.542 < 2e-16 *** 
UNEMP_state_1_matrix9 -1.691e+02  9.255e+00 -18.265 < 2e-16 *** 
UNEMP_state_1_matrix10 1.105e+02  9.731e+00  11.351 < 2e-16 *** 
UNEMP_state_1_matrix11 -1.708e+01  3.809e+00  -4.484 8.39e-06 *** 
UNEMP_state_1_matrix12 -1.415e+01  1.475e+00  -9.590 < 2e-16 *** 
UNEMP_state_1_matrix13  1.044e+01  2.395e+00   4.358 1.48e-05 *** 
UNEMP_state_1_matrix14  8.839e+00  5.039e+00   1.754  0.07976 .  
UNEMP_state_1_matrix15 -4.629e-01  6.752e+00  -0.069  0.94535  
UNEMP_state_1_matrix16 -5.475e+00  2.745e+00  -1.995  0.04643 *  
UNEMP_state_1_matrix17  5.593e-01  5.792e-02   9.657 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 3563 on 798 degrees of freedom
Multiple R-squared:  0.9973,   Adjusted R-squared:  0.9972
F-statistic: 1.707e+04 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 111: Performance of Multivariate Multiple Linear Regression MODEL(GSP-UNEMP)

```

Response Y9 :

Call:
lm(formula = Y9 ~ UNEMP_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-9597.3 -1910.2   296.8  2632.2  7335.7 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) -1.344e+03 3.230e+02 -4.160 3.53e-05 ***  
UNEMP_state_1_matrix1 1.290e+00 5.760e-01  2.240 0.02537 *  
UNEMP_state_1_matrix2 9.035e+00 7.205e+00  1.254 0.21022    
UNEMP_state_1_matrix3 -6.593e+00 7.074e+00 -0.932 0.35160    
UNEMP_state_1_matrix4 -5.827e+01 6.290e+00 -9.263 < 2e-16 ***  
UNEMP_state_1_matrix5 6.249e+01 6.706e+00  9.319 < 2e-16 ***  
UNEMP_state_1_matrix6 -8.801e+00 3.867e+00 -2.276 0.02312 *  
UNEMP_state_1_matrix7 -7.383e+00 6.256e-01 -11.802 < 2e-16 ***  
UNEMP_state_1_matrix8 8.754e+01 3.975e+00 22.025 < 2e-16 ***  
UNEMP_state_1_matrix9 -1.733e+02 9.702e+00 -17.861 < 2e-16 ***  
UNEMP_state_1_matrix10 1.063e+02 1.020e+01 10.422 < 2e-16 ***  
UNEMP_state_1_matrix11 -1.385e+01 3.993e+00 -3.469 0.00055 ***  
UNEMP_state_1_matrix12 -1.438e+01 1.546e+00 -9.296 < 2e-16 ***  
UNEMP_state_1_matrix13 1.149e+01 2.511e+00  4.578 5.44e-06 ***  
UNEMP_state_1_matrix14 5.248e+00 5.282e+00  0.994 0.32077    
UNEMP_state_1_matrix15 5.334e+00 7.077e+00  0.754 0.45128    
UNEMP_state_1_matrix16 -8.235e+00 2.877e+00 -2.862 0.00432 **  
UNEMP_state_1_matrix17 5.533e-01 6.071e-02  9.114 < 2e-16 ***  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3735 on 798 degrees of freedom
Multiple R-squared:  0.9972,   Adjusted R-squared:  0.9972
F-statistic: 1.697e+04 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 112: Performance of Multivariate Multiple Linear Regression MODEL(GSP-UNEMP)

```

Response Y10 :

Call:
lm(formula = Y10 ~ UNEMP_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-10023.1 -1796.9     7.5  3199.9  8229.0 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) -1.467e+03  3.391e+02 -4.327 1.70e-05 ***
UNEMP_state_1_matrix1 9.993e-01  6.047e-01  1.652  0.098828 .  
UNEMP_state_1_matrix2 1.145e+01  7.565e+00  1.513  0.130594  
UNEMP_state_1_matrix3 -8.930e+00  7.427e+00 -1.202  0.229561  
UNEMP_state_1_matrix4 -5.946e+01  6.604e+00 -9.002 < 2e-16 ***
UNEMP_state_1_matrix5 6.388e+01  7.041e+00  9.074 < 2e-16 *** 
UNEMP_state_1_matrix6 -8.710e+00  4.060e+00 -2.145  0.032247 *  
UNEMP_state_1_matrix7 -8.689e+00  6.568e-01 -13.229 < 2e-16 *** 
UNEMP_state_1_matrix8 8.565e+01  4.173e+00  20.524 < 2e-16 *** 
UNEMP_state_1_matrix9 -1.658e+02  1.019e+01 -16.272 < 2e-16 *** 
UNEMP_state_1_matrix10 1.038e+02  1.071e+01  9.693 < 2e-16 *** 
UNEMP_state_1_matrix11 -1.598e+01  4.192e+00 -3.813  0.000148 *** 
UNEMP_state_1_matrix12 -1.482e+01  1.624e+00 -9.128 < 2e-16 *** 
UNEMP_state_1_matrix13 1.379e+01  2.636e+00  5.231 2.16e-07 *** 
UNEMP_state_1_matrix14 -1.443e+00  5.546e+00 -0.260  0.794791  
UNEMP_state_1_matrix15 1.380e+01  7.431e+00  1.857  0.063623 .  
UNEMP_state_1_matrix16 -1.155e+01  3.021e+00 -3.825  0.000141 *** 
UNEMP_state_1_matrix17 6.173e-01  6.374e-02  9.685 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3922 on 798 degrees of freedom
Multiple R-squared:  0.9971,   Adjusted R-squared:  0.997 
F-statistic: 1.612e+04 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 113: Performance of Multivariate Multiple Linear Regression MODEL(GSP-UNEMP)

```

Response Y11 :

Call:
lm(formula = Y11 ~ UNEMP_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-9696.8 -1933.7 -351.5  2863.9  9657.2 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) -1.286e+03  3.432e+02 -3.748 0.000191 *** 
UNEMP_state_1_matrix1 -1.763e-01  6.120e-01 -0.288 0.773342  
UNEMP_state_1_matrix2  2.488e+01  7.655e+00  3.250 0.001204 ** 
UNEMP_state_1_matrix3 -2.280e+01  7.516e+00 -3.033 0.002497 ** 
UNEMP_state_1_matrix4 -6.305e+01  6.683e+00 -9.434 < 2e-16 *** 
UNEMP_state_1_matrix5  6.825e+01  7.125e+00  9.579 < 2e-16 *** 
UNEMP_state_1_matrix6 -7.686e+00  4.109e+00 -1.871 0.061768 .  
UNEMP_state_1_matrix7 -9.746e+00  6.646e-01 -14.663 < 2e-16 *** 
UNEMP_state_1_matrix8  7.378e+01  4.223e+00  17.471 < 2e-16 *** 
UNEMP_state_1_matrix9 -1.429e+02  1.031e+01 -13.862 < 2e-16 *** 
UNEMP_state_1_matrix10 9.848e+01  1.084e+01  9.086 < 2e-16 *** 
UNEMP_state_1_matrix11 -2.095e+01  4.242e+00 -4.938 9.60e-07 *** 
UNEMP_state_1_matrix12 -1.357e+01  1.643e+00 -8.262 5.94e-16 *** 
UNEMP_state_1_matrix13  1.263e+01  2.667e+00  4.735 2.59e-06 *** 
UNEMP_state_1_matrix14 -8.265e+00  5.612e+00 -1.473 0.141213  
UNEMP_state_1_matrix15  2.409e+01  7.520e+00  3.204 0.001410 ** 
UNEMP_state_1_matrix16 -1.489e+01  3.057e+00 -4.870 1.34e-06 *** 
UNEMP_state_1_matrix17  7.588e-01  6.451e-02 11.763 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 3968 on 798 degrees of freedom
Multiple R-squared:  0.997,    Adjusted R-squared:  0.9969 
F-statistic: 1.564e+04 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 114: Performance of Multivariate Multiple Linear Regression MODEL(GSP-UNEMP)

```

Response Y12 :

Call:
lm(formula = Y12 ~ UNEMP_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-8880.1 -2093.0 -401.3  2755.0  8149.4 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -739.39235  314.17828 -2.353 0.018843 *  
UNEMP_state_1_matrix1 -1.14950   0.56024 -2.052 0.040514 *  
UNEMP_state_1_matrix2  41.27944   7.00831  5.890 5.69e-09 *** 
UNEMP_state_1_matrix3 -39.77698   6.88042 -5.781 1.06e-08 *** 
UNEMP_state_1_matrix4 -65.22984   6.11857 -10.661 < 2e-16 *** 
UNEMP_state_1_matrix5  68.29282   6.52254  10.470 < 2e-16 *** 
UNEMP_state_1_matrix6 -3.81503   3.76159 -1.014 0.310792    
UNEMP_state_1_matrix7 -9.21064   0.60847 -15.137 < 2e-16 *** 
UNEMP_state_1_matrix8  61.62663   3.86598  15.941 < 2e-16 *** 
UNEMP_state_1_matrix9 -129.42735   9.43721 -13.715 < 2e-16 *** 
UNEMP_state_1_matrix10 103.29859   9.92231  10.411 < 2e-16 *** 
UNEMP_state_1_matrix11 -27.74918   3.88348 -7.145 2.02e-12 *** 
UNEMP_state_1_matrix12 -10.25792   1.50413 -6.820 1.80e-11 *** 
UNEMP_state_1_matrix13  9.02893   2.44204  3.694 0.000236 *** 
UNEMP_state_1_matrix14 -12.34495   5.13758 -2.403 0.016494 *  
UNEMP_state_1_matrix15  28.09214   6.88412  4.081 4.94e-05 *** 
UNEMP_state_1_matrix16 -14.50886   2.79887 -5.184 2.76e-07 *** 
UNEMP_state_1_matrix17   0.80267   0.05905  13.592 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 3633 on 798 degrees of freedom
Multiple R-squared:  0.9975,    Adjusted R-squared:  0.9975 
F-statistic: 1.904e+04 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 115: Performance of Multivariate Multiple Linear Regression MODEL(GSP-UNEMP)

```

Response Y13 :

Call:
lm(formula = Y13 ~ UNEMP_state_1_matrix)

Residuals:
    Min      1Q Median      3Q     Max 
 -7909   -2393   -266   2978   7704 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) -416.06492 307.08036 -1.355 0.175831  
UNEMP_state_1_matrix1 -1.88068 0.54758 -3.435 0.000624 *** 
UNEMP_state_1_matrix2 48.25089 6.84998 7.044 4.04e-12 *** 
UNEMP_state_1_matrix3 -46.06280 6.72498 -6.850 1.48e-11 *** 
UNEMP_state_1_matrix4 -62.88134 5.98034 -10.515 < 2e-16 *** 
UNEMP_state_1_matrix5 68.39160 6.37518 10.728 < 2e-16 *** 
UNEMP_state_1_matrix6 -5.75033 3.67661 -1.564 0.118207  
UNEMP_state_1_matrix7 -8.74600 0.59473 -14.706 < 2e-16 *** 
UNEMP_state_1_matrix8 49.69927 3.77864 13.153 < 2e-16 *** 
UNEMP_state_1_matrix9 -120.18333 9.22401 -13.029 < 2e-16 *** 
UNEMP_state_1_matrix10 111.97940 9.69815 11.546 < 2e-16 *** 
UNEMP_state_1_matrix11 -34.59057 3.79574 -9.113 < 2e-16 *** 
UNEMP_state_1_matrix12 -5.98475 1.47015 -4.071 5.15e-05 *** 
UNEMP_state_1_matrix13 7.03376 2.38687 2.947 0.003304 ** 
UNEMP_state_1_matrix14 -21.25435 5.02151 -4.233 2.58e-05 *** 
UNEMP_state_1_matrix15 34.20500 6.72860 5.084 4.62e-07 *** 
UNEMP_state_1_matrix16 -13.89181 2.73564 -5.078 4.75e-07 *** 
UNEMP_state_1_matrix17 0.86734 0.05772 15.027 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3551 on 798 degrees of freedom
Multiple R-squared:  0.9976,   Adjusted R-squared:  0.9975
F-statistic: 1.937e+04 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 116: Performance of Multivariate Multiple Linear Regression MODEL(GSP UNEMP)

```

Response Y14 :

Call:
lm(formula = Y14 ~ UNEMP_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-7368.4 -1958.5 -514.9  2611.9  5982.6 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) -296.7173   286.7454 -1.035 0.301087    
UNEMP_state_1_matrix1 -1.8357    0.5113 -3.590 0.000351 ***  
UNEMP_state_1_matrix2  51.8082    6.3964  8.100 2.06e-15 ***  
UNEMP_state_1_matrix3 -49.6122    6.2797 -7.900 9.20e-15 ***  
UNEMP_state_1_matrix4 -65.1123    5.5843 -11.660 < 2e-16 *** 
UNEMP_state_1_matrix5  68.9880    5.9530 11.589 < 2e-16 *** 
UNEMP_state_1_matrix6 -4.2357    3.4331 -1.234 0.217653    
UNEMP_state_1_matrix7 -8.0535    0.5553 -14.502 < 2e-16 *** 
UNEMP_state_1_matrix8  51.8889    3.5284 14.706 < 2e-16 *** 
UNEMP_state_1_matrix9 -134.7574   8.6132 -15.645 < 2e-16 *** 
UNEMP_state_1_matrix10 127.0226   9.0559 14.026 < 2e-16 *** 
UNEMP_state_1_matrix11 -38.1027   3.5444 -10.750 < 2e-16 *** 
UNEMP_state_1_matrix12 -5.0344    1.3728 -3.667 0.000261 *** 
UNEMP_state_1_matrix13  7.3033    2.2288  3.277 0.001095 **  
UNEMP_state_1_matrix14 -27.8788   4.6890 -5.946 4.12e-09 *** 
UNEMP_state_1_matrix15  42.0247   6.2830  6.689 4.24e-11 *** 
UNEMP_state_1_matrix16 -16.4227   2.5545 -6.429 2.21e-10 *** 
UNEMP_state_1_matrix17  1.0313    0.0539 19.135 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 3316 on 798 degrees of freedom
Multiple R-squared:  0.998,    Adjusted R-squared:  0.998 
F-statistic: 2.403e+04 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 117: Performance of Multivariate Multiple Linear Regression MODEL(GSP-UNEMP)

```

Response Y15 :

Call:
lm(formula = Y15 ~ UNEMP_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-7654.8 -2042.6  -426.7  2584.4  6987.7 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept) -378.96594  279.22173 -1.357 0.175094  
UNEMP_state_1_matrix1 -1.87220   0.49790 -3.760 0.000182 *** 
UNEMP_state_1_matrix2  51.93499   6.22854  8.338 3.29e-16 *** 
UNEMP_state_1_matrix3 -49.45545   6.11488 -8.088 2.25e-15 *** 
UNEMP_state_1_matrix4 -66.07847   5.43780 -12.152 < 2e-16 *** 
UNEMP_state_1_matrix5  70.46366   5.79682 12.156 < 2e-16 *** 
UNEMP_state_1_matrix6 -4.98449   3.34307 -1.491 0.136358  
UNEMP_state_1_matrix7 -9.10283   0.54077 -16.833 < 2e-16 *** 
UNEMP_state_1_matrix8  58.32911   3.43583 16.977 < 2e-16 *** 
UNEMP_state_1_matrix9 -153.86857  8.38720 -18.346 < 2e-16 *** 
UNEMP_state_1_matrix10 144.64896  8.81832 16.403 < 2e-16 *** 
UNEMP_state_1_matrix11 -42.21941  3.45139 -12.233 < 2e-16 *** 
UNEMP_state_1_matrix12 -5.02561   1.33678 -3.759 0.000183 *** 
UNEMP_state_1_matrix13  8.54247   2.17033  3.936 9.01e-05 *** 
UNEMP_state_1_matrix14 -29.86563  4.56595 -6.541 1.09e-10 *** 
UNEMP_state_1_matrix15  42.53617  6.11817  6.952 7.47e-12 *** 
UNEMP_state_1_matrix16 -16.30267  2.48746 -6.554 1.00e-10 *** 
UNEMP_state_1_matrix17  1.20994   0.05248 23.054 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 3229 on 798 degrees of freedom
Multiple R-squared:  0.9984,    Adjusted R-squared:  0.9984 
F-statistic: 2.914e+04 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 118: Performance of Multivariate Multiple Linear Regression MODEL(GSP-UNEMP)

```

Response Y16 :

Call:
lm(formula = Y16 ~ UNEMP_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-7926.6 -2074.1 - 598.1  2709.4  7345.6 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept) -530.2144   283.0347 - 1.873  0.06139 .  
UNEMP_state_1_matrix1 -1.3362    0.5047 - 2.647  0.00827 ** 
UNEMP_state_1_matrix2  50.1789   6.3136   7.948 6.47e-15 *** 
UNEMP_state_1_matrix3 -48.7632   6.1984 - 7.867 1.18e-14 *** 
UNEMP_state_1_matrix4 -67.0255   5.5121 - 12.160 < 2e-16 *** 
UNEMP_state_1_matrix5  72.1848   5.8760   12.285 < 2e-16 *** 
UNEMP_state_1_matrix6 -5.3781    3.3887 - 1.587  0.11289    
UNEMP_state_1_matrix7 -9.5668    0.5482 - 17.453 < 2e-16 *** 
UNEMP_state_1_matrix8  65.6322   3.4827   18.845 < 2e-16 *** 
UNEMP_state_1_matrix9 -173.4584   8.5017 - 20.403 < 2e-16 *** 
UNEMP_state_1_matrix10 160.6266   8.9387   17.970 < 2e-16 *** 
UNEMP_state_1_matrix11 -45.6038   3.4985 - 13.035 < 2e-16 *** 
UNEMP_state_1_matrix12 -6.4691    1.3550 - 4.774 2.15e-06 *** 
UNEMP_state_1_matrix13  10.8006   2.2000   4.909 1.11e-06 *** 
UNEMP_state_1_matrix14 -32.1523   4.6283 - 6.947 7.75e-12 *** 
UNEMP_state_1_matrix15  45.5476   6.2017   7.344 5.11e-13 *** 
UNEMP_state_1_matrix16 -17.8867   2.5214 - 7.094 2.88e-12 *** 
UNEMP_state_1_matrix17  1.3421    0.0532   25.227 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 3273 on 798 degrees of freedom
Multiple R-squared:  0.9985,    Adjusted R-squared:  0.9985 
F-statistic: 3.113e+04 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 119: Performance of Multivariate Multiple Linear Regression MODEL(GSP-UNEMP)

```

Response Y17 :

Call:
lm(formula = Y17 ~ UNEMP_state_1_matrix)

Residuals:
    Min      1Q  Median      3Q     Max 
-8187.8 -2246.0 -607.3  3125.5 6882.2 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept) -440.35880  293.02341 -1.503   0.1333    
UNEMP_state_1_matrix1 -1.19266  0.52251 -2.283   0.0227 *  
UNEMP_state_1_matrix2 49.10957  6.53641  7.513 1.55e-13 *** 
UNEMP_state_1_matrix3 -48.06650  6.41713 -7.490 1.82e-13 *** 
UNEMP_state_1_matrix4 -68.43416  5.70659 -11.992 < 2e-16 *** 
UNEMP_state_1_matrix5 75.47186  6.08335 12.406 < 2e-16 *** 
UNEMP_state_1_matrix6 -6.97442  3.50831 -1.988   0.0472 *  
UNEMP_state_1_matrix7 -10.43879  0.56750 -18.394 < 2e-16 *** 
UNEMP_state_1_matrix8 73.96781  3.60566 20.514 < 2e-16 *** 
UNEMP_state_1_matrix9 -192.95791 8.80177 -21.923 < 2e-16 *** 
UNEMP_state_1_matrix10 175.30906 9.25421 18.944 < 2e-16 *** 
UNEMP_state_1_matrix11 -48.50921  3.62199 -13.393 < 2e-16 *** 
UNEMP_state_1_matrix12 -6.45987  1.40285 -4.605 4.80e-06 *** 
UNEMP_state_1_matrix13 13.02829  2.27760  5.720 1.50e-08 *** 
UNEMP_state_1_matrix14 -38.69064  4.79164 -8.075 2.49e-15 *** 
UNEMP_state_1_matrix15 52.34506  6.42059  8.153 1.37e-15 *** 
UNEMP_state_1_matrix16 -20.46460  2.61041 -7.840 1.45e-14 *** 
UNEMP_state_1_matrix17  1.54616  0.05508 28.073 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 3388 on 798 degrees of freedom
Multiple R-squared:  0.9985,    Adjusted R-squared:  0.9985 
F-statistic: 3.101e+04 on 17 and 798 DF,  p-value: < 2.2e-16

```

Figure 120: Performance of Multivariate Multiple Linear Regression MODEL(GSP-UNEMP)