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WiSe 2017
Assignment 2
November 7, 2017

Assignment for Basic Mathematical Tools

Exercise 1 Linear Functions

For each of the following functions, decide whether it is linear, and provide a justification!

a) $f : \mathbb{R} \to \mathbb{R}, x \mapsto x^2$

b) $f : \mathbb{R}^2 \to \mathbb{R}^2, \mathbf{x} \mapsto \begin{pmatrix} 42a^2 & \frac{\pi}{3c} \\ b & 23 \end{pmatrix} \begin{pmatrix} ax_1 + bx_2 \\ cx_1 + ax_2 \end{pmatrix}$ with $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, $a, b, c \in \mathbb{R}$

c) $f : \mathbb{R}^3 \to \mathbb{R}^3, \mathbf{x} \mapsto A\mathbf{x} + \mathbf{t}$ with $A \in \mathbb{R}^{3\times 3}$, $\mathbf{t} \in \mathbb{R}^3$

d) $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}, (a, b) \mapsto a \sin(b)$ with $a, b \in \mathbb{R}$

Exercise 2 Invertible Matrices

For each of the following matrices, compute the rank and the determinant. Decide whether the matrix is invertible, and – if so – compute the inverse.

a) $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

b) $\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$

c) $\begin{pmatrix} 3 & 1 & 3 \\ 9 & 4 & 9 \\ 15 & 7 & 15 \end{pmatrix}$

d) $\begin{pmatrix} 132 & 165 & 198 \\ 348 & 435 & 522 \\ 564 & 705 & 846 \end{pmatrix}$

Exercise 3 Linear System

Let $\lambda \in \mathbb{R}$ and

$$A = \begin{pmatrix} 3 & \lambda + 1 \\ \lambda + 2 & \lambda + 9 \end{pmatrix} \in \mathbb{R}^{2\times 2}.$$

For which λ does the linear system $Ax = b$ with

$$b = \begin{pmatrix} 6 \\ -6 \end{pmatrix} \in \mathbb{R}^2.$$

have none / exactly one / more than one solution(s) $x \in \mathbb{R}^2$? Use only approaches and rules from the lecture!

Exercise 4 (P) Algebraic Reconstruction Technique

The goal of this assignment is the implementation of the Algebraic Reconstruction Technique (ART), better known as the Kaczmarz Method, as discussed in the lecture. We provide an exemplary system of equations corresponding to a computed tomography problem to test your implementation. In order to complete this assignment please go through the following steps:

a) Download the file `art.zip` from the lecture homepage. It contains a the following files: `main.py`, `helper.py`, `art.py`, and a data file `system.mat` containing the system of equations as well as the true solution.

b) Complete the implementation of `art.py`.

c) Play around with the number of iterations and try to apply the ART to other systems of equations as well.

d) Compare the obtained result to Numpy's standard solver, i.e. `np.linalg.solve(A,b)`. What are your conclusions? What is a singular matrix?