Technische Universität München
Fakultät für Informatik
PD Dr. Tobias Lasser
Rüdiger Göbl
Anca Stefanoiu
Walter Simson

WiSe 2017
Exercise Sheet 1
October 24, 2017

Exercises in Basic Mathematical Tools

Assignment 1 Norm – Equality

Consider $\mathbf{x}, \mathbf{y} \in \mathbb{R}^3$ and the *Euclidean norm* as defined in the lecture. Show that

$$\|\mathbf{x} - \mathbf{y}\| = 0 \Leftrightarrow \mathbf{x} = \mathbf{y} \; . \tag{1}$$

Hint Use the definition of the Euclidean norm and the properties of the scalar product.

Assignment 2 Cauchy-Schwarz Inequality

Prove the *Cauchy-Schwarz Inequality*

$$\langle \mathbf{x}, \mathbf{y} \rangle^2 \leq \langle \mathbf{x}, \mathbf{x} \rangle \langle \mathbf{y}, \mathbf{y} \rangle$$

using the properties of the scalar product discussed in the lecture.

Hint Check out $\langle \mathbf{x} + \lambda \mathbf{y}, \mathbf{x} + \lambda \mathbf{y} \rangle$!

Assignment 3 Function Space

Let $X = \mathbb{R}^{\mathbb{R}}$ denote the function space of all functions $f : \mathbb{R} \to \mathbb{R}$.

a) Show that X is a vector space!

b) For each of the following subsets $S \subset X$, decide whether it is a subspace, and justify your statement!

   (i) $S := \{f \in X | f(0) = 1 + f(1)\} \subset X$

   (ii) $S := \{f \in X | 2f(-1) = f(1)\} \subset X$

   (iii) $S := \{f \in X | f(-1) = 0\} \subset X$

   (iv) $S := \{f \in X | f(x) = f(1 - x) \quad \forall x \in \mathbb{R}\} \subset X$

   (v) $S := \{f \in X | f(x^3) = f(x)^5 \quad \forall x \in \mathbb{R}\} \subset X$

   (vi) $S := \{f \in X | f \text{ is continuous}\} \subset X$

Assignment 4 Linear Dependence

Let $\mathbf{x},\mathbf{y},\mathbf{z} \in \mathbb{R}^3$. Is it possible to write the vector $\mathbf{v} = (3,0,2)^\top \in \mathbb{R}$ as linear combination of the three other vectors, i. e.

$$\mathbf{v} = \lambda\mathbf{x} + \mu\mathbf{y} + \nu\mathbf{z}$$

for some scalars $\lambda,\mu,\nu \in \mathbb{R}$, given the following values?

a) $\mathbf{x} = (1,1,0)^\top$, $\mathbf{y} = (1,0,1)^\top$, $\mathbf{z} = (0,1,0)^\top$

b) $\mathbf{x} = (1,1,0)^\top$, $\mathbf{y} = (-2,-3,1)^\top$, $\mathbf{z} = (1,0,1)^\top$

c) $\mathbf{x} = (1,0,0)^\top$, $\mathbf{y} = (0,0,1)^\top$, $\mathbf{z} = (1,0,1)^\top$

d) Which of these sets form a basis of $\mathbb{R}^3$?

Assignment 5 Properties of the Matrix Product

In the following we will discuss certain properties of the matrix product.

a) *No commutativity:* Find two matrices $A,B \in \mathbb{R}^{2\times 2}$ such that $AB \neq BA$.

b) Matrix multiplication is not commutative in general. However, there exists a subset of matrices, such that $AB = BA$ holds: Consider two matrices $A,B \in \mathbb{R}^{n\times n}$ with $A^\top = A$ and $B^\top = B$ and $(BA)^\top = BA$ (such matrices are called symmetric). Show that – in this case – $AB = BA$ holds.

   Hint $AB = (B^\top A^\top)^\top \quad \forall A,B \in \mathbb{R}^{n\times n}$

c) *Distributivity:* Consider three matrices $A,B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{m\times k}$. Show:

$$(A+B)C = AC + BC$$

   Hint For two matrices $A \in \mathbb{R}^{n\times m}$, $B \in \mathbb{R}^{m\times k}$, the elements of the product $C = AB$ are given by:

$$c_{ij} = \sum_{l=1}^{m} a_{il} b_{lj}$$