

Final Project of Regression Analysis

Regression Analysis of Happiness Index in 2016

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CONTENTS:

SUMMARY.....	1
MOTIVATIONS AND GOALS.....	1
DATA ANALYSIS.....	3
RESEARCH METHOD.....	5
MODEL FITTING AND ANALYSIS.....	5
CONCLUSION.....	17
APPENDIX.....	18

1. Summary

There are many factors that affect the happiness index. I mainly explore the relationship between the happiness index (response variable) of each country in 2016 and whether the country is in sub-Saharan Africa, human development index, economy, consumption of alcoholic beverages (independent variable), etc.

After many transformations and adaptations, a multiple regression model is finally constructed with , , , and interaction terms as explanatory variables and as response variables, and the achievement indicates that the model has good explanatory ability. At the same time, indicates that the model has reliable inference ability.

2. Motivations and Goals

Researchers at the University of Kent in the United Kingdom used a software recorder to observe users' activity and happiness. The study found that when the user was drinking, the happiness index increased by 10.79 points. However, this data may be biased, because the user may be doing other things while drinking, so it may be other simultaneous activities that make people happy, while Non-drinking itself.

While researchers found that drinking alcohol can increase people's well-being, but it's temporary, and drinking has no significant effect on people's long-term well-being. According to other scientific studies, alcohol stimulates dopamine, which is used to help cells transmit erotic, exciting, and happy messages to make people happy.

Generally speaking, the Happiness Index is related to the country's economy and human development level, so I added two variables to ensure the explanatory power of the model.

This study will analyze data from countries around the world, hoping to build a simpler multiple regression model to explain the relationship between variables. The goal is that 75% of the variance of the response

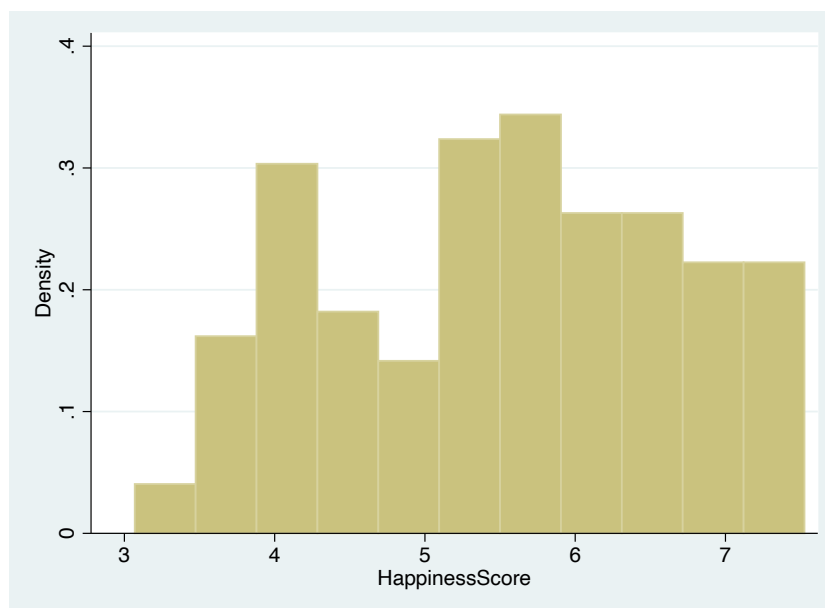
variable can be explained by the variance of the explanatory variable, and at the same time it has inference ability.

3. Data Analysis

Dependent Variable (Y)(happiness score) :

There are a total of 122 records. According to the figure below, it can be seen that there is a bimodal distribution. After in-depth discussion, we found that it may be related to sub-Saharan Africa, and this bimodal problem can be solved. Therefore, we choose to add whether it is Saharan Africa in the explanatory variables. Dummy variables for sub-Latin Africa.

Variable	N	Average	Median	Std Dev	Minimum	Maximum
Y (Happiness)	122	5.524828	5.542	0.1040017	3.069	7.526



We considered the following factors that may be related to the Fortune Index as our explanatory variables:

1. GDP per Capita

If a country's GDP growth is higher, its relative economic development is also better, and the overall people's lives will be more prosperous, and we believe that the people of that country will be happier.

On the contrary, some countries with relatively backward economic development are also relatively

lacking in material life, so in order to make a living, they may be busy running around without leisure and entertainment time, and they will also be less happy. Therefore, we believe that the average GDP of buyers in various countries may be one of the parameters affecting the happiness index.

2. Human Development Index (HDI)

HDI is an index released by the United Nations to measure the social and economic development of countries. The index value is calculated based on average life expectancy at birth, years of education (including average and expected years of education), and per capita gross national income, and can be used as a comparison between countries worldwide.

3. Southern than Saharan Africa (SA)

Since the histogram of the happiness index shows a bimodal distribution, and after observing the original data, it is found that most of the countries with low happiness index are located in sub-Saharan Africa, so this variable is added.

4. Beer per Capita (Unit: liter)

5. Spirit per Capita (Unit: liter)

6. Wine per Capita (Unit: liter)

Among the above explanatory variables, we set the numerical data per capita GDP as GDPpc, the human development index as HDI, and the per capita alcoholic beverage consumption as BEERpc, WINEpc, SPIRITpc by category; and because of whether the country is located in The sub-Saharan region is a categorical variable, and we set it as SA, 1 if the country is located in sub-Saharan Africa, and 0 otherwise. The response variable happiness index is set to HappinessScore.

4. Research Method

We use the most primitive data to fit Model 1. Since alcohol data and GDP data are right-biased data, we choose to take the logarithm to fit Model 2, and add interaction terms in sequence to fit Models 3 and 4, and then use the SBC criterion to find suitable independent variables, and the Cook's Distance criterion to judge whether there is an influential point that seriously affects the regression line, and finally explain the relationship between variables by fitting the regression model method and results, hoping to finally simplify the complex judgment of the model. The coefficient can reach 75%. Note: In the following verification process, we all consider the confidence level to be 95%.

5. Model Fitting and Analysis

(1) Process of model fitting

Model 1:

$$E(Y) = \beta_0 + \beta_1 SA + \beta_2 HDI + \beta_3 GDP_{pc} + \beta_4 BEER_{pc} + \beta_5 SPIRIT_{pc} + \beta_6 WINE_{pc}$$

Since $R^2 = 0.6918$ did not reach our goal of 0.75, and the residual plot of in the appendix seems to have a trend. By looking at the plot of each variable against the residuals, we try to transform the explanatory variables GDP_{pc}, BEER_{pc}, SPIRIT_{pc}, WINE_{pc} by taking the logarithm.

模型 1 之估計結果

變異數的分析					
來源	DF	平方和	均方	F 值	Pr > F
模型	6	110.46018	18.41003	43.02	<.0001
誤差	115	49.21076	0.42792		
已校正的總計	121	159.67094			

根 MSE	0.65416	R 平方	0.6918
應變平均值	5.52483	調整 R 平方	0.6757
變異係數	11.84029		

參數估計值						
變數	DF	參數估計值	標準誤差	t 值	Pr > t	變異數膨脹
Intercept	1	1.47025	0.61175	2.40	0.0178	0
Sa	1	-0.59554	0.25037	-2.38	0.0190	3.16021
HDI	1	0.00577	0.00083088	6.94	<.0001	4.38315
GDP	1	0.00047006	0.00036933	1.27	0.2057	1.69406
Beer	1	0.00131	0.00078068	1.68	0.0955	1.89337
Spirit	1	-0.00227	0.00090353	-2.51	0.0133	1.53084
Wine	1	-0.00127	0.00090424	-1.40	0.1641	1.79536

Model 2:

$$E(Y) = \beta_0 + \beta_1 SA + \beta_2 HDI + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \beta_6 X_6$$

$$X_3 = \log(GDP\ pc) \cdot X_4 = \log(BEERpc) \cdot X_5 = \log(WINEpc) \cdot X_6 = \log(SPIRITpc)$$

The R-Squared of this model is $R^2 = 0.7240$, is better than model 1, and the residual plot in the appendix is also improved, but it still does not reach the goal we set, so we choose to add an interaction term to see if there will be better results .

模型 2 之估計結果

變異數的分析					
來源	DF	平方和	均方	F 值	Pr > F
模型	6	115.59951	19.26659	50.27	<.0001
誤差	115	44.07143	0.38323		
已校正的總計	121	159.67094			

根 MSE	0.61906	R 平方	0.7240
應變平均值	5.52483	調整 R 平方	0.7096
變異係數	11.20498		

參數估計值						
變數	DF	參數估計值	標準誤差	t 值	Pr > t	變異數膨脹
Intercept	1	0.61096	0.56388	1.08	0.2809	0
Sa	1	-0.52970	0.25490	-2.08	0.0399	3.65765
HDI	1	0.00635	0.00084869	7.49	<.0001	5.10645
logGDP	1	0.11642	0.03686	3.16	0.0020	1.32355
logBeer	1	0.17289	0.06342	2.73	0.0074	2.18412
logSpirit	1	-0.05884	0.05404	-1.09	0.2785	2.26571
logWine	1	-0.17995	0.04814	-3.74	0.0003	2.53829

Model 3:

$$E(Y) = \beta_0 + \beta_1 SA + \beta_2 HDI + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \beta_6 X_6 + \beta_7 (SA * HDI) \\ + \beta_8 (SA * X_3) + \beta_9 (SA * X_4) + \beta_{10} (SA * X_5) + \beta_{11} (SA * X_6)$$

$$X_3 = \log(GDP\ pc) \cdot X_4 = \log(BEERpc) \cdot X_5 = \log(WINEpc) \cdot X_6 = \log(SPIRITpc)$$

模型 3-1 之估計結果

變異數的分析					
來源	DF	平方和	均方	F 值	Pr > F
模型	12	125.46236	10.45520	33.31	<.0001
誤差	109	34.20858	0.31384		
已校正的總計	121	159.67094			

根 MSE	0.56021	R 平方	0.7858
應變平均值	5.52483	調整 R 平方	0.7622
變異係數	10.13994		

參數估計值						
變數	DF	參數估計值	標準誤差	t 值	Pr > t	變異數膨脹
Intercept	1	1.55036	0.81074	1.91	0.0585	0
Sa	1	1.77121	1.14185	1.55	0.1238	89.62580
HDI	1	0.00446	0.00121	3.70	0.0003	12.58894
logGDP	1	0.16523	0.09146	1.81	0.0736	9.95095
logBeer	1	0.23969	0.06854	3.50	0.0007	3.11522
logSpirit	1	-0.03611	0.06718	-0.54	0.5920	4.27569
logWine	1	-0.32733	0.07086	-4.62	<.0001	6.71656
SaHDI	1	-0.00059538	0.00208	-0.29	0.7751	89.54727
LogGdpLogWine	1	0.05774	0.02400	2.41	0.0178	11.36412
SaLogGDP	1	-0.31354	0.09237	-3.39	0.0010	13.63874
SaLogBeer	1	-0.37397	0.14584	-2.56	0.0117	22.42979
SaLogSpirit	1	0.04409	0.10465	0.42	0.6744	4.92924
SaLogWine	1	0.00098938	0.12279	0.01	0.9936	4.93384

The variables are the same as in Model 2, here we add dummy variables and other variables (HDI、 $\log(GDP\ pc)$ 、 $\log(BEERpc)$ 、 $\log(WINEpc)$ 、 $\log(SPIRITpc)$) . According to the F-test results, at least one interaction term was helpful in explaining the happiness index. Finally, the SBC criterion is used to find a suitable variable model. (SBC results are in the appendix)

Model 3-2

$$E(Y) = \beta_0 + \beta_1 SA + \beta_2 HDI + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \beta_8 (SA * X_3) + \beta_9 (SA * X_4)$$

$$X_3 = \log(GDP\ pc) \cdot X_4 = \log(BEERpc) \cdot X_5 = \log(WINEpc)$$

Model 3-2 is the result found after adding SA and interacting with other variables. Although the $R^2 = 0.7696$ has achieved our goal. But due to the collinearity problem of this model, we choose to use VIF. The SA variable with the largest value is removed and the fitting is done again.

模型 3-2 之估計結果

變異數的分析					
來源	DF	平方和	均方	F 值	Pr > F
模型	7	122.88003	17.55429	54.39	<.0001
誤差	114	36.79091	0.32273		
已校正的總計	121	159.67094			

根 MSE	0.56809	R 平方	0.7696
應變平均值	5.52483	調整 R 平方	0.7554
變異係數	10.28251		

參數估計值						
變數	DF	參數估計值	標準誤差	t 值	Pr > t	變異數膨脹
Intercept	1	0.72779	0.56598	1.29	0.2011	0
Sa	1	1.73601	0.56835	3.05	0.0028	21.59323
HDI	1	0.00487	0.00094201	5.17	<.0001	7.47050
logGDP	1	0.29865	0.06806	4.39	<.0001	5.35818
logBeer	1	0.25995	0.06105	4.26	<.0001	2.40357
logWine	1	-0.19313	0.04406	-4.38	<.0001	2.52526
SaLogGDP	1	-0.35596	0.08580	-4.15	<.0001	11.44525
SaLogBeer	1	-0.37624	0.11085	-3.39	0.0009	12.60210

Model 3-3

$$E(Y) = \beta_0 + \beta_2 HDI + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \beta_8 (SA * X_3) + \beta_9 (SA * X_4)$$

$$X_3 = \log(GDP\ pc) \cdot X_4 = \log(BEERpc) \cdot X_5 = \log(WINEpc)$$

After removing the SA variable with collinearity, although the R^2 dropped to 0.7507, the collinearity problem was slightly less than that of model 3-2. However, with Cook's Distance, we found the 108th data to be a very serious influential point (listed in the appendix), so we chose to remove this data and re-fit the new model.

模型 3-3 之估計結果

變異數的分析					
來源	DF	平方和	均方	F 值	Pr > F
模型	6	119.86901	19.97817	57.72	<.0001
誤差	115	39.80193	0.34610		
已校正的總計	121	159.67094			

根 MSE	0.58831	R 平方	0.7507
應變平均值	5.52483	調整 R 平方	0.7377
變異係數	10.64840		

參數估計值						
變數	DF	參數估計值	標準誤差	t 值	Pr > t	變異數膨脹
Intercept	1	1.40559	0.53920	2.61	0.0103	0
HDI	1	0.00425	0.00095242	4.46	<.0001	7.12083
logGDP	1	0.29946	0.07048	4.25	<.0001	5.35810
logBeer	1	0.21546	0.06140	3.51	0.0006	2.26677
logWine	1	-0.17984	0.04541	-3.96	0.0001	2.50066
SaLogGDP	1	-0.25792	0.08241	-3.13	0.0022	9.84384
SaLogBeer	1	-0.06804	0.04754	-1.43	0.1551	2.16124

Model 3-4

$$E(Y) = \beta_0 + \beta_2 HDI + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \beta_8 (SA * X_3) + \beta_9 (SA * X_4)$$

$$X_3 = \log(GDP\ pc) \cdot X_4 = \log(BEERpc) \cdot X_5 = \log(WINEpc)$$

After removing the 108th outlier, we found that there was still a collinearity problem after doing it again, so we deleted the interaction term between SA and log(GDP) and did it again.

模型 3-4 之估計結果

變異數的分析					
來源	DF	平方和	均方	F 值	Pr > F
模型	6	123.05018	20.50836	68.04	<.0001
誤差	114	34.36175	0.30142		
已校正的總計	120	157.41193			

根 MSE	0.54902	R 平方	0.7817
應變平均值	5.53720	調整 R 平方	0.7702
變異係數	9.91506		

參數估計值						
變數	DF	參數估計值	標準誤差	t 值	Pr > t	變異數膨脹
Intercept	1	3.10954	0.64348	4.83	<.0001	0
HDI	1	0.00137	0.00112	1.23	0.2213	10.99139
logGDP	1	0.60054	0.09669	6.21	<.0001	11.11562
logBeer	1	0.18779	0.05767	3.26	0.0015	2.08074
logWine	1	-0.17580	0.04239	-4.15	<.0001	2.45082
SaLogGDP	1	-0.62060	0.11490	-5.40	<.0001	21.94232
SaLogBeer	1	-0.01667	0.04598	-0.36	0.7177	2.31697

Model 3-5

$$E(Y) = \beta_0 + \beta_2 HDI + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \beta_9 (SA * X_4)$$

$$X_3 = \log(GDP\ pc) \cdot X_4 = \log(BEERpc) \cdot X_5 = \log(WINEpc)$$

After removing the interaction term between SA and log(GDP), there is no longer the problem of collinearity. However, the R^2 of this model drops to 0.7258, which is not in line with our expectations, so we choose to further improve this model.

模型 3-5 之估計結果

變異數的分析					
來源	DF	平方和	均方	F 值	Pr > F
模型	5	114.25682	22.85136	60.89	<.0001
誤差	115	43.15511	0.37526		
已校正的總計	120	157.41193			

根 MSE	0.61259	R 平方	0.7258
應變平均值	5.53720	調整 R 平方	0.7139
變異係數	11.06311		

參數估計值						
變數	DF	參數估計值	標準誤差	t 值	Pr > t	變異數膨脹
Intercept	1	0.45544	0.46356	0.98	0.3279	0
HDI	1	0.00621	0.00074449	8.34	<.0001	3.92361
logGDP	1	0.10586	0.03458	3.06	0.0027	1.14173
logBeer	1	0.18408	0.06434	2.86	0.0050	2.08044
logWine	1	-0.18192	0.04728	-3.85	0.0002	2.44907
SaLogBeer	1	-0.12185	0.04648	-2.62	0.0099	1.90146

Model 4-1

$$\begin{aligned} E(Y) = & \beta_0 + \beta_1 SA + \beta_2 HDI + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \beta_6 X_6 + \beta_7 (SA * HDI) \\ & + \beta_8 (SA * X_3) + \beta_9 (SA * X_4) + \beta_{10} (SA * X_5) + \beta_{11} (SA * X_6) + \beta_{12} (X_3 * X_5) \\ X_3 = & \log(GDP\ pc) \ , \ X_4 = \log(BEERpc) \ , \ X_5 = \log(WINEpc) \ , \ X_6 = \log(SPIRITpc) \end{aligned}$$

Since the regression model after removing the influential point did not meet our expectations, we conjectured that there may be other interaction terms, and we believe that there may be an interaction between red wine and national GDP per capita, because red wine is relatively The cost of obtaining other wines is higher, and it is possible that countries with higher GDP per capita will have relatively higher consumption of red wine per capita, so we choose to add the interaction term of $\log(\text{wine}) * \log(\text{GDP pc})$. The -test result proves that this interaction term has a significant marginal contribution, and then a suitable model is found through the SBC criterion. (SBC results are in the appendix)

模型 4-1 之估計結果

變異數的分析					
來源	DF	平方和	均方	F 值	Pr > F
模型	12	125.46236	10.45520	33.31	<.0001
誤差	109	34.20858	0.31384		
已校正的總計	121	159.67094			

根 MSE	0.56021	R 平方	0.7858
應變平均值	5.52483	調整 R 平方	0.7622
變異係數	10.13994		

參數估計值						
變數	DF	參數估計值	標準誤差	t 值	Pr > t	變異數膨脹
Intercept	1	1.55036	0.81074	1.91	0.0585	0
Sa	1	1.77121	1.14185	1.55	0.1238	89.62580
HDI	1	0.00446	0.00121	3.70	0.0003	12.58894
logGDP	1	0.16523	0.09146	1.81	0.0736	9.95095
logBeer	1	0.23969	0.06854	3.50	0.0007	3.11522
logSpirit	1	-0.03611	0.06718	-0.54	0.5920	4.27569
logWine	1	-0.32733	0.07086	-4.62	<.0001	6.71656
SaHDI	1	-0.00059538	0.00208	-0.29	0.7751	89.54727
LogGdpLogWine	1	0.05774	0.02400	2.41	0.0178	11.36412
SaLogGDP	1	-0.31354	0.09237	-3.39	0.0010	13.63874
SaLogBeer	1	-0.37397	0.14584	-2.56	0.0117	22.42979
SaLogSpirit	1	0.04409	0.10465	0.42	0.6744	4.92924
SaLogWine	1	0.00098938	0.12279	0.01	0.9936	4.93384

Model 4-2

$$E(Y) = \beta_0 + \beta_1 SA + \beta_2 HDI + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 \\ + \beta_8 (SA * X_3) + \beta_9 (SA * X_4) + \beta_{12} (X_3 * X_5)$$

$$X_3 = \log(GDP\ pc) \cdot X_4 = \log(BEERpc) \cdot X_5 = \log(WINEpc)$$

After entering the new interaction term ($\log(\text{wine}) * \log(\text{GDP pc})$), the appropriate model is obtained by SBC, but due to the problem of collinearity in this model, we again take the SA variable with the highest VIF value as the drop and do another return.

模型 4-2 之估計結果

變異數的分析					
來源	DF	平方和	均方	F 值	Pr > F
模型	8	125.33127	15.66641	51.55	<.0001
誤差	113	34.33967	0.30389		
已校正的總計	121	159.67094			

根 MSE	0.55126	R 平方	0.7849
應變平均值	5.52483	調整 R 平方	0.7697
變異係數	9.97792		

參數估計值						
變數	DF	參數估計值	標準誤差	t 值	Pr > t	變異數膨脹
Intercept	1	1.53643	0.61863	2.48	0.0145	0
Sa	1	1.64182	0.55251	2.97	0.0036	21.67130
HDI	1	0.00434	0.00093306	4.65	<.0001	7.78349
logGDP	1	0.15830	0.08248	1.92	0.0575	8.35849
logBeer	1	0.23227	0.06004	3.87	0.0002	2.46864
logWine	1	-0.33646	0.06614	-5.09	<.0001	6.04307
LogGdpLogWine	1	0.06165	0.02171	2.84	0.0054	9.59814
SaLogGDP	1	-0.31039	0.08479	-3.66	0.0004	11.87025
SaLogBeer	1	-0.38922	0.10767	-3.62	0.0004	12.62485

Model 4-3

$$E(Y) = \beta_0 + \beta_2 HDI + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \beta_8 (SA * X_3) + \beta_9 (SA * X_4) + \beta_{12} (X_3 * X_5)$$

$$X_3 = \log(GDP\ pc) \cdot X_4 = \log(BEERpc) \cdot X_5 = \log(WINEpc)$$

However, after removing the SA variable, there is still a problem of collinearity, so we choose to remove the interaction term between SA and log(GDP) with the highest VIF value and fit a new model.

模型 4-3 之估計結果

變異數的分析					
來源	DF	平方和	均方	F 值	Pr > F
模型	7	122.64782	17.52112	53.95	<.0001
誤差	114	37.02312	0.32476		
已校正的總計	121	159.67094			

根 MSE	0.56988	R 平方	0.7681
應變平均值	5.52483	調整 R 平方	0.7539
變異係數	10.31491		

參數估計值						
變數	DF	參數估計值	標準誤差	t 值	Pr > t	變異數膨脹
Intercept	1	2.22593	0.59284	3.75	0.0003	0
HDI	1	0.00372	0.00094014	3.95	0.0001	7.39431
logGDP	1	0.15025	0.08522	1.76	0.0806	8.34947
logBeer	1	0.18861	0.06018	3.13	0.0022	2.32079
logWine	1	-0.33294	0.06837	-4.87	<.0001	6.04114
LogGdpLogWine	1	0.06552	0.02240	2.93	0.0042	9.56356
SaLogGDP	1	-0.21515	0.08116	-2.65	0.0092	10.17422
SaLogBeer	1	-0.09961	0.04730	-2.11	0.0374	2.27994

Model 4-4

$$E(Y) = \beta_0 + \beta_2 HDI + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \beta_9 (SA * X_4) + \beta_{12} (X_3 * X_5)$$

$$X_3 = \log(GDP\ pc) \cdot X_4 = \log(BEERpc) \cdot X_5 = \log(WINEpc)$$

After removing the interaction term of SA and log(GDP), the VIF of each term is less than 10, but the average is about 4.5, so must be paid attention to collinearity. Furthermore, Cook's Distance has no significant influential point, and R^2 reaches our expected 0.75, so our final model is model 4-4

模型 4-4 之估計結果

變異數的分析					
來源	DF	平方和	均方	F 值	Pr > F
模型	6	120.36534	20.06089	58.69	<.0001
誤差	115	39.30560	0.34179		
已校正的總計	121	159.67094			

根 MSE	0.58463	R 平方	0.7538
應變平均值	5.52483	調整 R 平方	0.7410
變異係數	10.58180		

參數估計值						
變數	DF	參數估計值	標準誤差	t 值	Pr > t	變異數膨脹
Intercept	1	1.57782	0.55406	2.85	0.0052	0
HDI	1	0.00522	0.00076919	6.79	<.0001	4.70312
logGDP	1	-0.03156	0.05190	-0.61	0.5443	2.94248
logBeer	1	0.16187	0.06086	2.66	0.0089	2.25562
logWine	1	-0.35966	0.06937	-5.18	<.0001	5.90978
LogGdpLogWine	1	0.07623	0.02260	3.37	0.0010	9.25301
SaLogBeer	1	-0.14718	0.04490	-3.28	0.0014	1.95174

(2) Model Validation and Analysis

1. Lack of Fit Test

Because there are no two or more identical values for each variable in each country, the lack of fitness test cannot be performed. Therefore, we use residual analysis and normal probability plot to determine that the complex regression function can reasonably fit the data.

2. General Regression Test

We test whether the above variables have sufficient explanatory power for the happiness index, and the test result p-value<0.05, so we reject the null hypothesis, indicating that the above variables are indeed helpful for explaining the happiness index.

3. Residual Analysis

By observing the residual scatter plot, it can be found that the residuals are roughly randomly beating within a certain range above and below 0, and no trend can be seen, and the variation is roughly the same. From the residual normal probability plot, it can be seen that most of the points fall on the 45-degree line, and it can be inferred that the residual should obey a normal distribution.

6. Conclusion

Although happiness is difficult to quantify, we can know that the happiness index can be calculated from the human development index, the log value of per capita GDP, the log value of per capita beer consumption, the log value of per capita red wine consumption, and the per capita production Interaction terms of the log value of the total value and the log value of the per capita red wine consumption, the log value of the per capita beer consumption and the interaction terms of the attribute variables and other variables explained.

The final model reached 0.7538 and at the same time it reached 0.741, indicating that the model has reliable interpretation and inference ability, all in line with my original goals

Let's review the final fitted model again:

$$E(Y) = \beta_0 + \beta_2 HDI + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \beta_9 (SA * X_4) + \beta_{12} (X_3 * X_5)$$

It shows that given other variables unchanged, the human development index, per capita GDP and per capita beer consumption are positively related to the happiness index, and the per capita red wine consumption is negatively related to the happiness index. The average happiness index of sub-Saharan Africa is higher than the average. decrease in other areas.

The results of the above studies are not completely consistent with the original speculation. Although some scientific studies have shown that alcohol can stimulate the secretion of dopamine and make people feel happy in the moment, in the long run, the relationship between alcohol and people's happiness is not

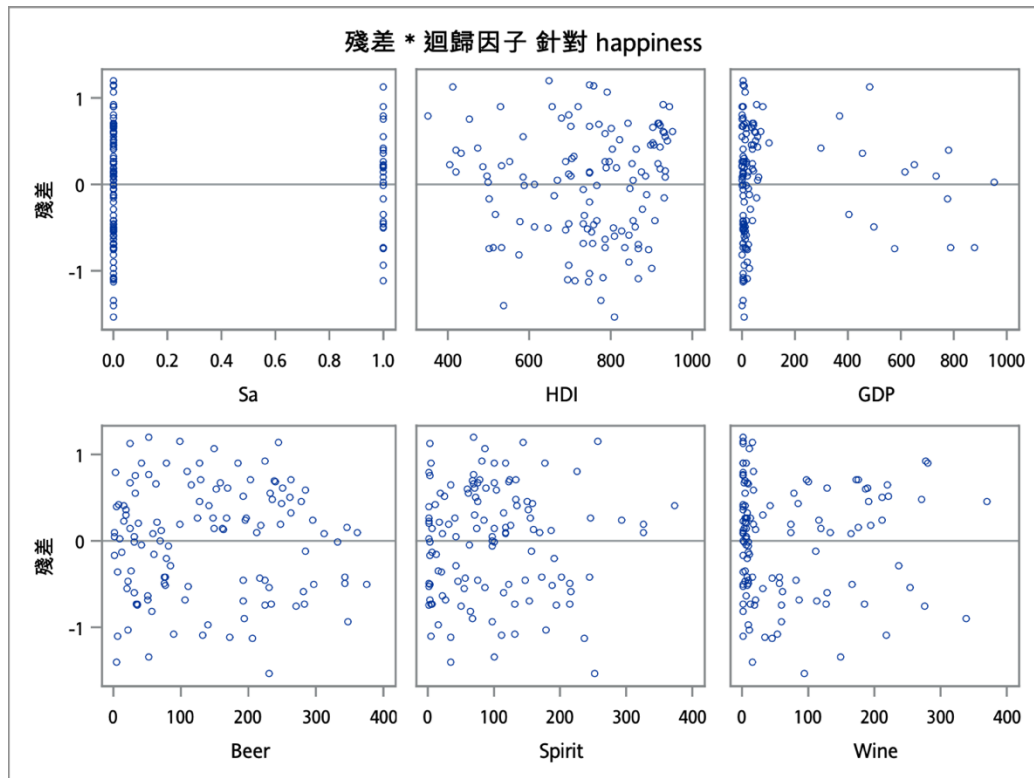
necessarily positive, or even According to the model, the per capita consumption of red wine is negatively related to happiness, the per capita consumption of spirits is not strongly related to happiness, and only the per capita consumption of beer has a positive correlation with it. Therefore, according to the research mentioned in the preface, there is a high probability that the improvement of happiness when drinking is due to the activities and participants at the moment, rather than the drinking itself. Because if the increase in happiness is due to drinking, the happiness index should not have different effects due to differences in alcohol. In addition, according to life experience, most people choose low-alcohol drinks such as beer to help them enjoy entertainment activities, so the happiness index is positively correlated with the per capita beer consumption. While the interaction between sub-Saharan and red wine was positively correlated with happiness index.

Other variables such as GDP per capita and social development index are as expected, and are positively correlated with happiness, representing the average life expectancy at birth, years of education, and per capita gross national income. The higher the overall social benefit, the higher the happiness index. .

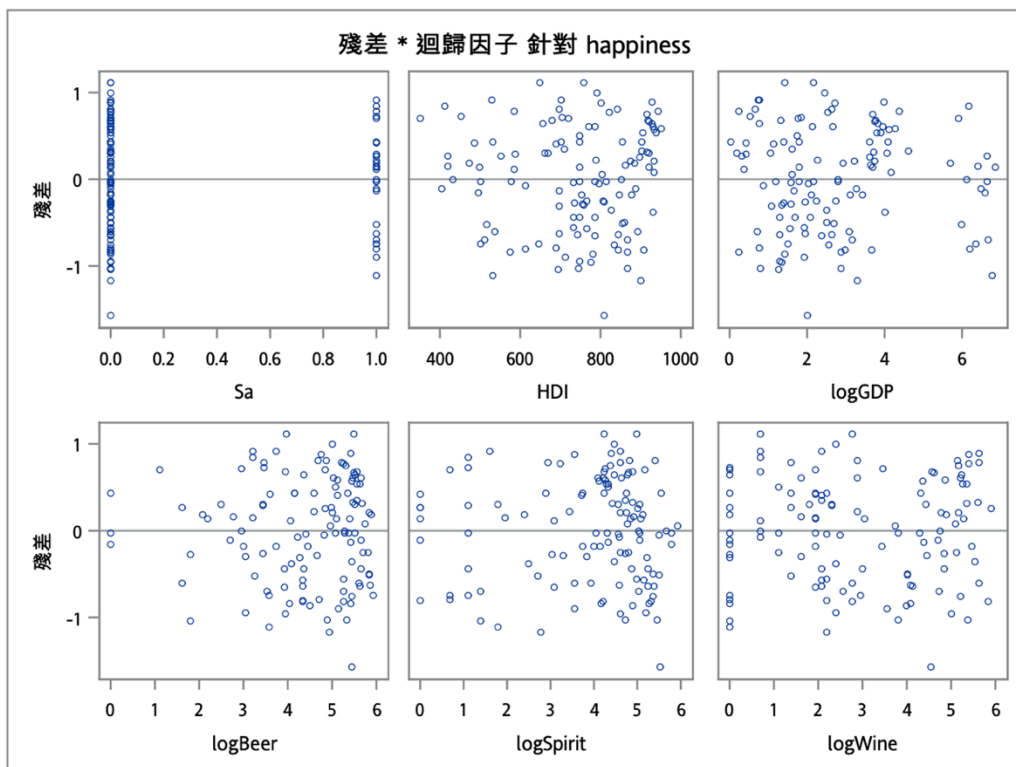
The differences caused by regions are manifested in that the overall average happiness index of sub-Saharan Africa is lower than that of other regions, which may be caused by regional problems such as climate, economic environment, natural resources and other factors.

7. Appendix

Model 1 Residual Plot



Model 2 Residual Plot



Model 3-1 SBC result

模型中的 數目	R 平方	調整的 R 平方	C(p)	AIC	SBC	模型中的變數
1	0.6645	0.6617	45.5786	-96.4071	-90.79909	HDI
1	0.4293	0.4245	160.2583	-31.5939	-25.98583	Sa
1	0.3824	0.3772	183.1347	-21.9549	-16.34685	SaHDI
1	0.3344	0.3289	206.4962	-12.8395	-7.23149	SaLogBeer
1	0.3200	0.3143	213.5338	-10.2219	-4.61387	SaLogGDP
1	0.2647	0.2586	240.4857	-0.6865	4.92151	logBeer
1	0.2492	0.2429	248.0740	1.8690	7.47704	logSpirit
1	0.2286	0.2222	258.0790	5.1586	10.76661	logWine
1	0.2180	0.2115	263.2476	6.8238	12.43186	SaLogSpirit
1	0.2087	0.2022	267.7776	8.2649	13.87295	SaLogWine
1	0.0000	-.0083	369.5286	36.8236	42.43167	logGDP
2	0.6912	0.6860	34.5445	-104.5374	-96.12531	HDI SaLogWine
2	0.6907	0.6855	34.8176	-104.3163	-95.90422	HDI logWine
2	0.6788	0.6734	40.5805	-99.7414	-91.32934	HDI SaLogSpirit
2	0.6788	0.6734	40.6124	-99.7166	-91.30450	HDI SaLogBeer
2	0.6759	0.6704	42.0308	-98.6166	-90.20452	HDI SaHDI
2	0.6704	0.6649	44.6779	-96.5899	-88.17789	HDI logGDP
2	0.6692	0.6636	45.2814	-96.1325	-87.72047	Sa HDI
2	0.6651	0.6595	47.2699	-94.6376	-86.22551	HDI logSpirit
2	0.6648	0.6592	47.4185	-94.5266	-86.11452	HDI logBeer
2	0.6648	0.6591	47.4440	-94.5075	-86.09547	HDI SaLogGDP
2	0.6286	0.6223	65.0933	-81.9974	-73.58538	logGDP SaLogGDP
3	0.7032	0.6956	30.7296	-107.3431	-96.12705	HDI logWine SaLogWine
3	0.7025	0.6949	31.0505	-107.0729	-95.85685	HDI logGDP logWine
3	0.6999	0.6923	32.2965	-106.0294	-94.81331	HDI logGDP SaLogGDP
3	0.6985	0.6908	33.0206	-105.4271	-94.21098	HDI logWine SaLogSpirit
3	0.6977	0.6900	33.4028	-105.1103	-93.89425	HDI logGDP SaLogWine
3	0.6973	0.6896	33.5767	-104.9665	-93.75041	HDI logWine SaLogBeer
3	0.6959	0.6881	34.2791	-104.3872	-93.17112	HDI logBeer SaLogWine
3	0.6957	0.6880	34.3396	-104.3374	-93.12133	Sa HDI SaHDI
3	0.6954	0.6877	34.5063	-104.2004	-92.98436	HDI logWine SaHDI
3	0.6951	0.6873	34.6593	-104.0748	-92.85868	HDI logBeer logWine

模型中的 數目	R 平方	調整的 R 平方	C(p)	AIC	SBC	模型中的變數
3	0.6948	0.6870	34.8077	-103.9531	-92.73697	Sa HDI SaLogWine
4	0.7265	0.7171	21.3548	-115.3293	-101.30919	Sa HDI logWine SaHDI
4	0.7236	0.7141	22.7606	-114.0500	-100.02991	HDI logGDP logWine SaLogGDP
4	0.7137	0.7040	27.5622	-109.7787	-95.75857	HDI logGDP logWine SaLogWine
4	0.7136	0.7038	27.6574	-109.6955	-95.67542	HDI logGDP logBeer logWine
4	0.7115	0.7017	28.6389	-108.8411	-94.82099	HDI logBeer logWine SaLogWine
4	0.7101	0.7002	29.3295	-108.2435	-94.22339	HDI logGDP SaLogGDP SaLogWine
4	0.7099	0.7000	29.4417	-108.1467	-94.12655	HDI logGDP logWine SaLogSpirit
4	0.7096	0.6997	29.5953	-108.0142	-93.99410	HDI logGDP logWine SaHDI
4	0.7090	0.6990	29.8897	-107.7609	-93.74077	HDI logGDP logBeer SaLogGDP
4	0.7088	0.6988	29.9815	-107.6819	-93.66181	HDI logGDP logWine SaLogBeer
4	0.7081	0.6981	30.3407	-107.3737	-93.35358	Sa HDI logWine SaLogBeer
5	0.7463	0.7353	13.6989	-122.4992	-105.67505	HDI logGDP logBeer logWine SaLogGDP
5	0.7379	0.7266	17.8021	-118.5180	-101.69386	Sa HDI logBeer logWine SaHDI
5	0.7304	0.7188	21.4452	-115.0890	-98.26484	HDI logGDP logBeer logWine SaLogWine
5	0.7295	0.7178	21.8871	-114.6795	-97.85541	HDI logGDP logBeer logWine SaLogBeer
5	0.7293	0.7177	21.9630	-114.6093	-97.78519	Sa HDI logWine SaHDI SaLogGDP
5	0.7288	0.7171	22.2402	-114.3533	-97.52921	Sa HDI logBeer logWine SaLogBeer
5	0.7268	0.7151	23.1776	-113.4916	-96.66746	HDI logGDP logBeer logWine SaHDI
5	0.7268	0.7150	23.2120	-113.4600	-96.63588	Sa HDI logWine SaHDI SaLogSpirit
5	0.7267	0.7149	23.2383	-113.4359	-96.61181	Sa HDI logGDP logWine SaHDI
5	0.7265	0.7148	23.3271	-113.3547	-96.53060	Sa HDI logSpirit logWine SaHDI
5	0.7265	0.7147	23.3274	-113.3544	-96.53024	Sa HDI logWine SaHDI SaLogWine
6	0.7508	0.7378	13.4894	-122.6981	-103.06995	HDI logGDP logBeer logWine SaLogGDP SaLogWine
6	0.7507	0.7377	13.5344	-122.6529	-103.02475	HDI logGDP logBeer logWine SaLogGDP SaLogBeer
6	0.7503	0.7372	13.7592	-122.4274	-102.79930	HDI logGDP logBeer logSpirit logWine SaLogGDP
6	0.7492	0.7361	14.2878	-121.8989	-102.27078	HDI logGDP logBeer logWine SaLogGDP SaLogSpirit
6	0.7474	0.7342	15.1501	-121.0416	-101.41350	HDI logGDP logBeer logWine SaHDI SaLogGDP
6	0.7463	0.7331	15.6920	-120.5061	-100.87790	Sa HDI logGDP logBeer logWine SaLogGDP
6	0.7414	0.7279	18.0633	-118.1893	-98.56118	Sa HDI logGDP logWine SaHDI SaLogGDP
6	0.7412	0.7277	18.1866	-118.0700	-98.44189	Sa HDI logBeer logWine SaHDI SaLogBeer
6	0.7408	0.7273	18.3593	-117.9032	-98.27503	Sa HDI logGDP logBeer logWine SaHDI

模型中的 數目	R 平方	調整的 R 平方	C(p)	AIC	SBC	模型中的變數
6	0.7390	0.7254	19.2579	-117.0387	-97.41052	Sa HDI logBeer logWine SaHDI SaLogGDP
6	0.7383	0.7247	19.5856	-116.7248	-97.09669	Sa HDI logBeer logWine SaHDI SaLogSpirit
7	0.7696	0.7554	6.3403	-130.2500	-107.81779	Sa HDI logGDP logBeer logWine SaLogGDP SaLogBeer
7	0.7614	0.7468	10.3216	-126.0011	-103.56896	Sa HDI logGDP logBeer logWine SaHDI SaLogGDP
7	0.7584	0.7436	11.7824	-124.4786	-102.04646	HDI logGDP logBeer logSpirit logWine SaLogGDP SaLogBeer
7	0.7584	0.7436	11.7900	-124.4707	-102.03854	HDI logGDP logBeer logWine SaHDI SaLogGDP SaLogBeer
7	0.7556	0.7406	13.1386	-123.0818	-100.64967	HDI logGDP logBeer logSpirit logWine SaLogGDP SaLogWine
7	0.7534	0.7382	14.2488	-121.9503	-99.51809	HDI logGDP logBeer logSpirit logWine SaHDI SaLogGDP
7	0.7531	0.7379	14.3807	-121.8166	-99.38438	Sa HDI logGDP logBeer logWine SaLogGDP SaLogWine
7	0.7519	0.7367	14.9550	-121.2359	-98.80373	HDI logGDP logBeer logSpirit logWine SaLogGDP SaLogSpirit
7	0.7514	0.7362	15.1904	-120.9987	-98.56653	HDI logGDP logBeer logWine SaLogGDP SaLogBeer SaLogWine
7	0.7512	0.7360	15.2878	-120.9007	-98.46854	HDI logGDP logBeer logWine SaHDI SaLogGDP SaLogWine
7	0.7511	0.7359	15.3312	-120.8571	-98.42490	Sa HDI logGDP logBeer logSpirit logWine SaLogGDP
8	0.7727	0.7566	6.8207	-129.9115	-104.67529	Sa HDI logGDP logBeer logSpirit logWine SaLogGDP SaLogBeer
8	0.7712	0.7550	7.5483	-129.1131	-103.87692	Sa HDI logGDP logBeer logWine SaHDI SaLogGDP SaLogBeer
8	0.7702	0.7539	8.0626	-128.5520	-103.31578	Sa HDI logGDP logBeer logWine SaLogGDP SaLogBeer SaLogSpirit
8	0.7696	0.7533	8.3304	-128.2607	-103.02448	Sa HDI logGDP logBeer logWine SaLogGDP SaLogBeer SaLogWine
8	0.7652	0.7486	10.4553	-125.9744	-100.73823	HDI logGDP logBeer logSpirit logWine SaHDI SaLogGDP SaLogBeer
8	0.7630	0.7462	11.5405	-124.8232	-99.58697	Sa HDI logGDP logBeer logSpirit logWine SaHDI SaLogGDP
8	0.7620	0.7452	12.0254	-124.3122	-99.07600	Sa HDI logGDP logBeer logWine SaHDI SaLogGDP SaLogSpirit
8	0.7615	0.7447	12.2603	-124.0655	-98.82932	Sa HDI logGDP logBeer logWine SaHDI SaLogGDP SaLogWine
8	0.7611	0.7442	12.4791	-123.8361	-98.59992	HDI logGDP logBeer logSpirit logWine SaLogGDP SaLogBeer SaLogSpirit
8	0.7594	0.7423	13.3169	-122.9617	-97.72554	HDI logGDP logBeer logWine SaHDI SaLogGDP SaLogBeer SaLogSpirit
8	0.7593	0.7423	13.3411	-122.9365	-97.70034	HDI logGDP logBeer logWine SaHDI SaLogGDP SaLogBeer SaLogWine
9	0.7735	0.7552	8.4530	-128.3170	-100.27677	Sa HDI logGDP logBeer logSpirit logWine SaHDI SaLogGDP SaLogBeer
9	0.7730	0.7547	8.6888	-128.0568	-100.01656	Sa HDI logGDP logBeer logSpirit logWine SaLogGDP SaLogBeer SaLogSpirit
9	0.7729	0.7547	8.7217	-128.0205	-99.98032	Sa HDI logGDP logBeer logSpirit logWine SaLogGDP SaLogBeer SaLogWine
9	0.7717	0.7534	9.3078	-127.3764	-99.33617	Sa HDI logGDP logBeer logWine SaHDI SaLogGDP SaLogBeer SaLogWine
9	0.7714	0.7531	9.4431	-127.2283	-99.18805	Sa HDI logGDP logBeer logWine SaHDI SaLogGDP SaLogBeer SaLogSpirit
9	0.7703	0.7518	10.0052	-126.6144	-98.57423	Sa HDI logGDP logBeer logWine SaLogGDP SaLogBeer SaLogSpirit SaLogWine
9	0.7660	0.7472	12.0794	-124.3758	-96.33559	HDI logGDP logBeer logSpirit logWine SaHDI SaLogGDP SaLogBeer SaLogSpirit
9	0.7654	0.7465	12.3794	-124.0553	-96.01513	HDI logGDP logBeer logSpirit logWine SaHDI SaLogGDP SaLogBeer SaLogWine

模型中的 數目	R 平方	調整的 R 平方	C(p)	AIC	SBC	模型中的變數
9	0.7631	0.7441	13.5007	-122.8653	-94.82504	Sa HDI logGDP logBeer logSpirit logWine SaHDI SaLogGDP SaLogWine
9	0.7630	0.7440	13.5405	-122.8232	-94.78295	Sa HDI logGDP logBeer logSpirit logWine SaHDI SaLogGDP SaLogSpirit
9	0.7621	0.7429	14.0052	-122.3335	-94.29327	Sa HDI logGDP logBeer logWine SaHDI SaLogGDP SaLogSpirit SaLogWine
10	0.7741	0.7538	10.1170	-126.6886	-95.84438	Sa HDI logGDP logBeer logSpirit logWine SaHDI SaLogGDP SaLogBeer SaLogWine
10	0.7738	0.7535	10.2677	-126.5218	-95.67759	Sa HDI logGDP logBeer logSpirit logWine SaHDI SaLogGDP SaLogBeer SaLogSpirit
10	0.7731	0.7527	10.6258	-126.1263	-95.28204	Sa HDI logGDP logBeer logSpirit logWine SaLogGDP SaLogBeer SaLogSpirit SaLogWine
10	0.7721	0.7515	11.1336	-125.5675	-94.72330	Sa HDI logGDP logBeer logWine SaHDI SaLogGDP SaLogBeer SaLogSpirit SaLogWine
10	0.7663	0.7452	13.9609	-122.5026	-91.65833	HDI logGDP logBeer logSpirit logWine SaHDI SaLogGDP SaLogBeer SaLogSpirit SaLogWine
10	0.7631	0.7418	15.4987	-120.8673	-90.02312	Sa HDI logGDP logBeer logSpirit logWine SaHDI SaLogGDP SaLogSpirit SaLogWine
10	0.7453	0.7223	24.1872	-112.0185	-81.17431	Sa HDI logBeer logSpirit logWine SaHDI SaLogGDP SaLogBeer SaLogSpirit SaLogWine
10	0.7435	0.7204	25.0416	-111.1820	-80.33780	Sa HDI logGDP logBeer logSpirit logWine SaHDI SaLogBeer SaLogSpirit SaLogWine
10	0.7427	0.7195	25.4373	-110.7966	-79.95235	Sa HDI logGDP logSpirit logWine SaHDI SaLogGDP SaLogBeer SaLogSpirit SaLogWine
10	0.7395	0.7161	26.9923	-109.2935	-78.44926	Sa HDI logGDP logBeer logSpirit SaHDI SaLogGDP SaLogBeer SaLogSpirit SaLogWine
10	0.7343	0.7103	29.5580	-106.8533	-76.00904	Sa logGDP logBeer logSpirit logWine SaHDI SaLogGDP SaLogBeer SaLogSpirit SaLogWine
11	0.7744	0.7518	12.0000	-124.8183	-91.17009	Sa HDI logGDP logBeer logSpirit logWine SaHDI SaLogGDP SaLogBeer SaLogSpirit SaLogWine

Model 4-1 SBCresult

模型中的 數目	R 平方	調整的 R 平方	C(p)	AIC	SBC	模型中的變數
1	0.6645	0.6617	52.6958	-96.4071	-90.79909	HDI
1	0.4293	0.4245	172.3652	-31.5939	-25.98583	Sa
1	0.3824	0.3772	196.2370	-21.9549	-16.34685	SaHDI
1	0.3344	0.3289	220.6149	-12.8395	-7.23149	SaLogBeer
1	0.3200	0.3143	227.9587	-10.2219	-4.61387	SaLogGDP
1	0.2992	0.2933	238.5638	-6.5383	-0.93025	LogGdpLogWine
1	0.2647	0.2586	256.0833	-0.6865	4.92151	logBeer
1	0.2492	0.2429	264.0018	1.8690	7.47704	logSpirit
1	0.2286	0.2222	274.4421	5.1586	10.76661	logWine
1	0.2180	0.2115	279.8355	6.8238	12.43186	SaLogSpirit
1	0.2087	0.2022	284.5627	8.2649	13.87295	SaLogWine
1	0.0000	-.0083	390.7408	36.8236	42.43167	logGDP
2	0.6912	0.6860	41.0946	-104.5374	-96.12531	HDI SaLogWine
2	0.6907	0.6855	41.3796	-104.3163	-95.90422	HDI logWine
2	0.6788	0.6734	47.3932	-99.7414	-91.32934	HDI SaLogSpirit
2	0.6788	0.6734	47.4265	-99.7166	-91.30450	HDI SaLogBeer
2	0.6759	0.6704	48.9066	-98.6166	-90.20452	HDI SaHDI
2	0.6704	0.6649	51.6689	-96.5899	-88.17789	HDI logGDP
2	0.6692	0.6636	52.2987	-96.1325	-87.72047	Sa HDI
2	0.6651	0.6595	54.3737	-94.6376	-86.22551	HDI logSpirit
2	0.6648	0.6592	54.5288	-94.5266	-86.11452	HDI logBeer
2	0.6648	0.6591	54.5554	-94.5075	-86.09547	HDI SaLogGDP
2	0.6647	0.6590	54.6019	-94.4743	-86.06222	HDI LogGdpLogWine
2	0.6286	0.6223	72.9726	-81.9974	-73.58538	logGDP SaLogGDP
3	0.7228	0.7157	27.0398	-115.6897	-104.47365	HDI logWine LogGdpLogWine
3	0.7032	0.6956	37.0268	-107.3431	-96.12705	HDI logWine SaLogWine
3	0.7025	0.6949	37.3616	-107.0729	-95.85685	HDI logGDP logWine
3	0.6999	0.6923	38.6619	-106.0294	-94.81331	HDI logGDP SaLogGDP
3	0.6985	0.6908	39.4174	-105.4271	-94.21098	HDI logWine SaLogSpirit
3	0.6977	0.6900	39.8162	-105.1103	-93.89425	HDI logGDP SaLogWine
3	0.6973	0.6896	39.9977	-104.9665	-93.75041	HDI logWine SaLogBeer
3	0.6959	0.6881	40.7307	-104.3872	-93.17112	HDI logBeer SaLogWine

模型中的 數目	R 平方	調整的 R 平方	C(p)	AIC	SBC	模型中的變數
3	0.6958	0.6881	40.7490	-104.3728	-93.15670	HDI LogGdpLogWine SaLogWine
3	0.6957	0.6880	40.7938	-104.3374	-93.12133	Sa HDI SaHDI
3	0.6954	0.6877	40.9677	-104.2004	-92.98436	HDI logWine SaHDI
3	0.6951	0.6873	41.1274	-104.0748	-92.85868	HDI logBeer logWine
4	0.7387	0.7298	20.9216	-120.9222	-106.90212	HDI logWine LogGdpLogWine SaLogWine
4	0.7387	0.7297	20.9603	-120.8868	-106.86666	HDI logWine LogGdpLogWine SaLogGDP
4	0.7345	0.7254	23.0846	-118.9530	-104.93286	HDI logWine SaHDI LogGdpLogWine
4	0.7341	0.7250	23.3002	-118.7583	-104.73823	HDI logWine LogGdpLogWine SaLogBeer
4	0.7306	0.7214	25.0593	-117.1824	-103.16228	Sa HDI logWine LogGdpLogWine
4	0.7305	0.7213	25.0895	-117.1556	-103.13546	HDI logBeer logWine LogGdpLogWine
4	0.7303	0.7211	25.2212	-117.0384	-103.01828	HDI logWine LogGdpLogWine SaLogSpirit
4	0.7265	0.7171	27.1571	-115.3293	-101.30919	Sa HDI logWine SaHDI
4	0.7249	0.7155	27.9560	-114.6309	-100.61079	HDI logGDP logWine LogGdpLogWine
4	0.7246	0.7152	28.1035	-114.5024	-100.48228	HDI logSpirit logWine LogGdpLogWine
4	0.7236	0.7141	28.6239	-114.0500	-100.02991	HDI logGDP logWine SaLogGDP
4	0.7137	0.7040	33.6345	-109.7787	-95.75857	HDI logGDP logWine SaLogWine
5	0.7530	0.7424	15.6437	-125.7920	-108.96790	HDI logBeer logWine LogGdpLogWine SaLogBeer
5	0.7528	0.7421	15.7757	-125.6639	-108.83974	HDI logBeer logWine LogGdpLogWine SaLogWine
5	0.7508	0.7401	16.7882	-124.6856	-107.86152	HDI logWine LogGdpLogWine SaLogGDP SaLogWine
5	0.7481	0.7372	18.1733	-123.3601	-106.53598	HDI logBeer logWine SaHDI LogGdpLogWine
5	0.7467	0.7358	18.8503	-122.7175	-105.89332	HDI logWine LogGdpLogWine SaLogGDP SaLogBeer
5	0.7463	0.7353	19.0810	-122.4992	-105.67505	HDI logGDP logBeer logWine SaLogGDP
5	0.7447	0.7337	19.8636	-121.7617	-104.93761	HDI logBeer logWine LogGdpLogWine SaLogGDP
5	0.7442	0.7332	20.1428	-121.4997	-104.67559	HDI logGDP logWine LogGdpLogWine SaLogGDP
5	0.7439	0.7328	20.3064	-121.3465	-104.52238	HDI logWine SaHDI LogGdpLogWine SaLogGDP
5	0.7436	0.7326	20.4259	-121.2347	-104.41053	HDI logGDP logWine LogGdpLogWine SaLogWine
5	0.7427	0.7316	20.9112	-120.7815	-103.95739	Sa HDI logBeer logWine LogGdpLogWine
5	0.7424	0.7313	21.0759	-120.6282	-103.80404	HDI logWine LogGdpLogWine SaLogGDP SaLogSpirit
6	0.7621	0.7497	13.0167	-128.3696	-108.74142	HDI logBeer logWine LogGdpLogWine SaLogGDP SaLogWine
6	0.7618	0.7494	13.1844	-128.2006	-108.57249	HDI logBeer logWine LogGdpLogWine SaLogGDP SaLogBeer
6	0.7591	0.7465	14.5571	-126.8265	-107.19830	HDI logGDP logBeer logWine LogGdpLogWine SaLogGDP
6	0.7562	0.7435	16.0386	-125.3605	-105.73240	Sa HDI logBeer logWine LogGdpLogWine SaLogBeer

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6	0.7561	0.7433	16.1132	-125.2872	-105.65907	Sa HDI logWine SaHDI LogGdpLogWine SaLogGDP
6	0.7558	0.7430	16.2500	-125.1528	-105.52468	HDI logBeer logWine LogGdpLogWine SaLogBeer SaLogWine
6	0.7553	0.7426	16.4743	-124.9328	-105.30463	Sa HDI logWine LogGdpLogWine SaLogGDP SaLogBeer
6	0.7550	0.7423	16.6229	-124.7872	-105.15905	Sa HDI logBeer logWine SaHDI LogGdpLogWine
6	0.7542	0.7414	17.0470	-124.3728	-104.74465	HDI logBeer logWine SaHDI LogGdpLogWine SaLogGDP
6	0.7539	0.7411	17.1836	-124.2396	-104.61145	HDI logGDP logBeer logWine LogGdpLogWine SaLogWine
6	0.7538	0.7410	17.2408	-124.1838	-104.55564	HDI logGDP logBeer logWine LogGdpLogWine SaLogBeer
6	0.7538	0.7410	17.2445	-124.1802	-104.55210	HDI logBeer logWine SaHDI LogGdpLogWine SaLogWine
7	0.7779	0.7643	6.9842	-134.7486	-112.31643	Sa HDI logBeer logWine LogGdpLogWine SaLogGDP SaLogBeer
7	0.7696	0.7554	11.2282	-130.2500	-107.81779	Sa HDI logGDP logBeer logWine SaLogGDP SaLogBeer
7	0.7687	0.7545	11.6584	-129.8031	-107.37092	HDI logBeer logWine SaHDI LogGdpLogWine SaLogGDP SaLogBeer
7	0.7681	0.7539	11.9681	-129.4824	-107.05019	HDI logGDP logBeer logWine LogGdpLogWine SaLogGDP SaLogBeer
7	0.7678	0.7535	12.1380	-129.3068	-106.87459	HDI logGDP logBeer logWine LogGdpLogWine SaLogGDP SaLogWine
7	0.7664	0.7521	12.8499	-128.5738	-106.14161	Sa HDI logBeer logWine SaHDI LogGdpLogWine SaLogGDP
7	0.7643	0.7498	13.9391	-127.4608	-105.02864	HDI logBeer logWine LogGdpLogWine SaLogGDP SaLogBeer SaLogWine
7	0.7626	0.7480	14.7817	-126.6067	-104.17455	Sa HDI logBeer logWine LogGdpLogWine SaLogGDP SaLogWine
7	0.7625	0.7480	14.8118	-126.5764	-104.14421	HDI logBeer logSpirit logWine LogGdpLogWine SaLogGDP SaLogBeer
7	0.7623	0.7477	14.9463	-126.4406	-104.00847	HDI logGDP logBeer logWine SaHDI LogGdpLogWine SaLogGDP
7	0.7622	0.7476	14.9881	-126.3984	-103.96624	HDI logBeer logWine LogGdpLogWine SaLogGDP SaLogSpirit SaLogWine
7	0.7621	0.7475	15.0164	-126.3699	-103.93776	HDI logBeer logWine SaHDI LogGdpLogWine SaLogGDP SaLogWine
8	0.7849	0.7697	5.4177	-136.6618	-111.42561	Sa HDI logGDP logBeer logWine LogGdpLogWine SaLogGDP SaLogBeer
8	0.7791	0.7634	8.3943	-133.3873	-108.15109	Sa HDI logBeer logWine SaHDI LogGdpLogWine SaLogGDP SaLogBeer
8	0.7784	0.7627	8.7338	-133.0193	-107.78316	Sa HDI logBeer logWine LogGdpLogWine SaLogGDP SaLogBeer SaLogWine
8	0.7780	0.7622	8.9687	-132.7654	-107.52920	Sa HDI logBeer logWine LogGdpLogWine SaLogGDP SaLogBeer SaLogSpirit
8	0.7779	0.7622	8.9785	-132.7548	-107.51859	Sa HDI logBeer logSpirit logWine LogGdpLogWine SaLogGDP SaLogBeer
8	0.7776	0.7619	9.1330	-132.5881	-107.35192	HDI logGDP logBeer logWine SaHDI LogGdpLogWine SaLogGDP SaLogBeer
8	0.7727	0.7566	11.6425	-129.9115	-104.67529	Sa HDI logGDP logBeer logSpirit logWine SaLogGDP SaLogBeer
8	0.7719	0.7557	12.0581	-129.4737	-104.23756	HDI logBeer logWine SaHDI LogGdpLogWine SaLogGDP SaLogBeer SaLogWine
8	0.7716	0.7554	12.2017	-129.3229	-104.08670	Sa HDI logGDP logBeer logWine SaHDI LogGdpLogWine SaLogGDP
8	0.7712	0.7550	12.4017	-129.1131	-103.87692	Sa HDI logGDP logBeer logWine SaHDI SaLogGDP SaLogBeer
8	0.7705	0.7542	12.7770	-128.7204	-103.48418	HDI logGDP logBeer logSpirit logWine LogGdpLogWine SaLogGDP SaLogBeer
8	0.7702	0.7539	12.9383	-128.5520	-103.31578	Sa HDI logGDP logBeer logWine SaLogGDP SaLogBeer SaLogSpirit

模型中的 數目	R 平方	調整的 R 平方	C(p)	AIC	SBC	模型中的變數
9	0.7853	0.7680	7.2527	-134.8460	-106.80575	Sa HDI logGDP logBeer logSpirit logWine LogGdpLogWine SaLogGDP SaLogBeer
9	0.7852	0.7679	7.3043	-134.7883	-106.74812	Sa HDI logGDP logBeer logWine SaHDI LogGdpLogWine SaLogGDP SaLogBeer
9	0.7850	0.7677	7.4042	-134.6769	-106.63666	Sa HDI logGDP logBeer logWine LogGdpLogWine SaLogGDP SaLogBeer SaLogWine
9	0.7849	0.7677	7.4149	-134.6649	-106.62471	Sa HDI logGDP logBeer logWine LogGdpLogWine SaLogGDP SaLogBeer SaLogSpirit
9	0.7792	0.7615	10.3156	-131.4728	-103.43255	HDI logGDP logBeer logSpirit logWine SaHDI LogGdpLogWine SaLogGDP SaLogBeer
9	0.7792	0.7615	10.3189	-131.4691	-103.42892	Sa HDI logBeer logWine SaHDI LogGdpLogWine SaLogGDP SaLogBeer SaLogSpirit
9	0.7792	0.7614	10.3390	-131.4473	-103.40708	HDI logGDP logBeer logWine SaHDI LogGdpLogWine SaLogGDP SaLogBeer SaLogWine
9	0.7791	0.7614	10.3639	-131.4203	-103.38008	Sa HDI logBeer logWine SaHDI LogGdpLogWine SaLogGDP SaLogBeer SaLogWine
9	0.7791	0.7614	10.3644	-131.4197	-103.37949	Sa HDI logBeer logSpirit logWine SaHDI LogGdpLogWine SaLogGDP SaLogBeer
9	0.7786	0.7608	10.6638	-131.0951	-103.05489	Sa HDI logBeer logWine LogGdpLogWine SaLogGDP SaLogBeer SaLogSpirit SaLogWine
9	0.7785	0.7607	10.7114	-131.0436	-103.00337	Sa HDI logBeer logSpirit logWine LogGdpLogWine SaLogGDP SaLogBeer SaLogWine
9	0.7780	0.7601	10.9687	-130.7654	-102.72519	Sa HDI logBeer logSpirit logWine LogGdpLogWine SaLogGDP SaLogBeer SaLogSpirit
10	0.7856	0.7663	9.0944	-133.0229	-102.17863	Sa HDI logGDP logBeer logSpirit logWine LogGdpLogWine SaLogGDP SaLogBeer SaLogSpirit
10	0.7854	0.7661	9.1835	-132.9232	-102.07899	Sa HDI logGDP logBeer logSpirit logWine SaHDI LogGdpLogWine SaLogGDP SaLogBeer
10	0.7853	0.7659	9.2516	-132.8472	-102.00296	Sa HDI logGDP logBeer logSpirit logWine LogGdpLogWine SaLogGDP SaLogBeer SaLogWine
10	0.7852	0.7658	9.2889	-132.8055	-101.96125	Sa HDI logGDP logBeer logWine SaHDI LogGdpLogWine SaLogGDP SaLogBeer SaLogSpirit
10	0.7852	0.7658	9.3035	-132.7892	-101.94499	Sa HDI logGDP logBeer logWine SaHDI LogGdpLogWine SaLogGDP SaLogBeer SaLogWine
10	0.7850	0.7656	9.3964	-132.6856	-101.84132	Sa HDI logGDP logBeer logWine LogGdpLogWine SaLogGDP SaLogBeer SaLogSpirit SaLogWine
10	0.7802	0.7604	11.8490	-129.9806	-99.13637	HDI logGDP logBeer logSpirit logWine SaHDI LogGdpLogWine SaLogGDP SaLogBeer SaLogWine
10	0.7799	0.7601	11.9699	-129.8489	-99.00463	HDI logGDP logBeer logSpirit logWine SaHDI LogGdpLogWine SaLogGDP SaLogBeer SaLogSpirit
10	0.7793	0.7595	12.2640	-129.5288	-98.68456	Sa HDI logBeer logWine SaHDI LogGdpLogWine SaLogGDP SaLogBeer SaLogSpirit SaLogWine
10	0.7792	0.7593	12.3189	-129.4692	-98.62494	Sa HDI logBeer logSpirit logWine SaHDI LogGdpLogWine SaLogGDP SaLogBeer SaLogSpirit
10	0.7792	0.7593	12.3243	-129.4633	-98.61908	HDI logGDP logBeer logWine SaHDI LogGdpLogWine SaLogGDP SaLogBeer SaLogSpirit SaLogWine
10	0.7792	0.7593	12.3273	-129.4600	-98.61576	Sa HDI logBeer logSpirit logWine SaHDI LogGdpLogWine SaLogGDP SaLogBeer SaLogWine
11	0.7858	0.7643	11.0001	-131.1284	-97.48012	Sa HDI logGDP logBeer logSpirit logWine SaHDI LogGdpLogWine SaLogGDP SaLogBeer SaLogSpirit

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11	0.7856	0.7642	11.0820	-131.0367	-97.38842	Sa HDI logGDP logBeer logSpirit logWine LogGdpLogWine SaLogGDP SaLogBeer SaLogSpirit SaLogWine
11	0.7854	0.7639	11.1775	-130.9300	-97.28170	Sa HDI logGDP logBeer logSpirit logWine SaHDI LogGdpLogWine SaLogGDP SaLogBeer SaLogWine
11	0.7852	0.7637	11.2889	-130.8055	-97.15723	Sa HDI logGDP logBeer logWine SaHDI LogGdpLogWine SaLogGDP SaLogBeer SaLogSpirit SaLogWine
11	0.7810	0.7591	13.4062	-128.4646	-94.81635	HDI logGDP logBeer logSpirit logWine SaHDI LogGdpLogWine SaLogGDP SaLogBeer SaLogSpirit SaLogWine
11	0.7793	0.7573	14.2640	-127.5288	-93.88054	Sa HDI logBeer logSpirit logWine SaHDI LogGdpLogWine SaLogGDP SaLogBeer SaLogSpirit SaLogWine
11	0.7744	0.7518	16.7861	-124.8183	-91.17009	Sa HDI logGDP logBeer logSpirit logWine SaHDI SaLogGDP SaLogBeer SaLogSpirit SaLogWine
11	0.7728	0.7501	17.5754	-123.9823	-90.33407	Sa HDI logGDP logBeer logSpirit logWine SaHDI LogGdpLogWine SaLogGDP SaLogSpirit SaLogWine
11	0.7631	0.7394	22.5220	-118.8693	-85.22106	Sa HDI logGDP logBeer logSpirit logWine SaHDI LogGdpLogWine SaLogBeer SaLogSpirit SaLogWine
11	0.7617	0.7379	23.2297	-118.1550	-84.50677	Sa HDI logGDP logSpirit logWine SaHDI LogGdpLogWine SaLogGDP SaLogBeer SaLogSpirit SaLogWine
11	0.7588	0.7347	24.7045	-116.6798	-83.03156	Sa logGDP logBeer logSpirit logWine SaHDI LogGdpLogWine SaLogGDP SaLogBeer SaLogSpirit SaLogWine
11	0.7438	0.7182	32.3365	-109.3183	-75.67005	Sa HDI logGDP logBeer logSpirit SaHDI LogGdpLogWine SaLogGDP SaLogBeer SaLogSpirit SaLogWine
12	0.7858	0.7622	13.0000	-129.1284	-92.67617	Sa HDI logGDP logBeer logSpirit logWine SaHDI LogGdpLogWine SaLogGDP SaLogBeer SaLogSpirit SaLogWine