CS 6501 Natural Language Processing

Probabilistic Context-Free Grammars

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Overview

1. Probabilistic CFGs
2. Probabilistic CKY Algorithm

Based on slides from [Collins, 2017, Smith, 2017]

Probabilistic CFGs

A Probabilistic Context-Free Grammar (PCFG)

- N: a set of non-terminal symbols
- $S \in N$: a distinguished start symbol
- Σ: a set of terminal symbols

S	$\Rightarrow$	NP	VP	1.0
VP	$\Rightarrow$	Vi		0.4
VP	$\Rightarrow$	Vt	NP	0.4
VP	$\Rightarrow$	VP	PP	0.2
NP	$\Rightarrow$	DT	NN	0.3
NP	$\Rightarrow$	NP	PP	0.7
PP	$\Rightarrow$	P	NP	1.0

Vi	$\Rightarrow$	sleeps	1.0
Vt	$\Rightarrow$	saw	1.0
NN	$\Rightarrow$	man	0.7
NN	$\Rightarrow$	woman	0.2
NN	$\Rightarrow$	telescope	0.1
DT	$\Rightarrow$	the	1.0
IN	$\Rightarrow$	with	0.5
IN	$\Rightarrow$	in	0.5

Probability of a Tree

The probability of a tree $\boldsymbol{t}$ with rules $\{\alpha_i \rightarrow \beta_i\}$, such as

$$\mathrm{S} \rightarrow \mathrm{NP\ VP}, \mathrm{NP} \rightarrow \mathrm{DT\ NN}, \ldots, \mathrm{Vi} \rightarrow \mathrm{sleeps}$$

is

$$\begin{aligned} P(\boldsymbol{t}) &= \underbrace{\prod_{i=1}^{n} P(\alpha_i \rightarrow \beta_i)}_{\text{production rule form}} \\ &= \underbrace{\prod_{i=1}^{n} P(\beta_i \mid \alpha_i)}_{\text{Standard conditional prob form}} \end{aligned} \tag{1}$$

An Example

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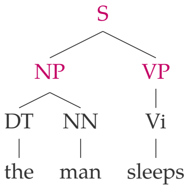

$$P(t) = P(\text{NP VP} \mid \text{S}) \tag{2}$$

An Example

S	⇒	NP	VP	1.0
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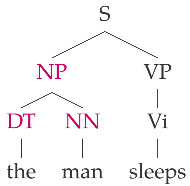

$$P(t) = P(\text{NP VP} \mid \text{S}) \cdot P(\text{DT NN} \mid \text{NP}) \qquad (2)$$

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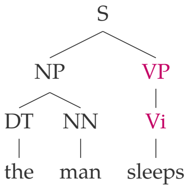

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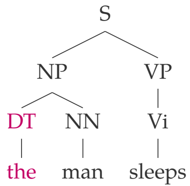

$$
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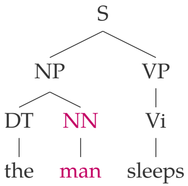

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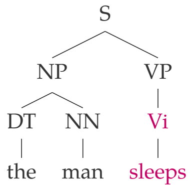

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Properties of PCFGs

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- Assigns a probability to each derivation, or parse-tree, allowed by the underlying CFG
- If one sentence has more than one derivations, we can rank them based on their probabilities
- The most likely parse tree for a sentence is

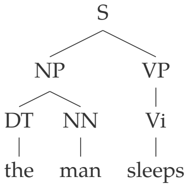
$$\arg\max_{\boldsymbol{t}\in\mathcal{T}(s)} P(\boldsymbol{t}|s) \tag{3}$$

where $\mathcal{T}(s)$ is the set of all possible parse trees of sentence s.

Probabilistic CKY Algorithm

Score of Parse Trees: An example

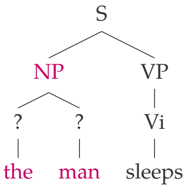
$$P(\boldsymbol{t} \mid s) = P(\text{NP VP} \mid \text{S}) \cdot P(\text{DT NN} \mid \text{NP}) \cdot P(\text{Vi} \mid \text{VP}) \cdot P(\text{the} \mid \text{DT}) \cdot P(\text{man} \mid \text{NN}) \cdot P(\text{sleeps} \mid \text{Vi}) \tag{4}$$



- Decoding $\arg\max_{\boldsymbol{t}} P(\boldsymbol{t} \mid s)$
- Effect of change on non-terminal node
- Similar phonemenon is handled by Viterbi decoding in HMM and CRF

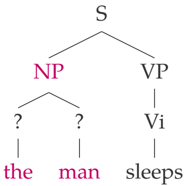
Notations (I)

- X: non-terminal node
- i, j: word indices, $1 \leq i \leq j \leq n$
- $\mathcal{T}(i, j, X)$: the set of all parse trees for words $x_i, \ldots, x_j$ with X as the root
- Example: $\mathcal{T}(1, 2, \mathrm{NP})$



Notations (II)

- X: non-terminal node
- i, j: word indices, $1 \leq i \leq j \leq n$
- $\pi(i, j, X) = \max_{\boldsymbol{t} \in \mathcal{T}(i,j,X)} P(\boldsymbol{t})$
- Example: $\pi(1, 2, \mathrm{NP})$



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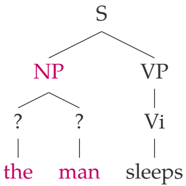
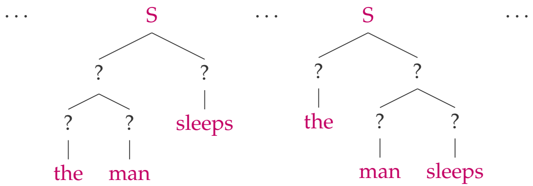


- $\pi(i, j, X) = 0$, if $\mathcal{T}(i, j, X) = \emptyset$, for example $\pi(1, 2, \mathrm{VP})$

Special Cases of $\mathcal{T}(i, j, X)$ and $\pi(i, j, X)$

- $\mathcal{T}(1, n, S)$



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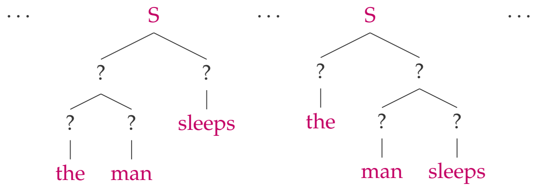
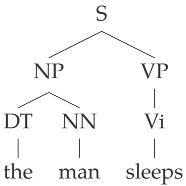


- $\pi(1, n, S)$: the score of the optimal tree



Special Cases of $\mathcal{T}(i, j, X)$ and $\pi(i, j, X)$

For the example sentence `the man sleeps`

- $\mathcal{T}(1, 1, X)$ if $X = \text{DT}$

DT
|
the

- $\pi(1, 1, \text{DT}) = 1$

S	$\Rightarrow$	NP	VP	1.0
VP	$\Rightarrow$	Vi		0.4
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$$\begin{array}{c} \mathrm{DT} \\ | \\ \text{the} \end{array}$$

- $\pi(1, 1, \mathrm{DT}) = 1$
- What if $X = \mathrm{NN}$?

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Special Cases of $\mathcal{T}(i, j, X)$ and $\pi(i, j, X)$

For the example sentence `the man sleeps`

- $\mathcal{T}(1, 1, X)$ if $X = \text{DT}$

DT
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the

- $\pi(1, 1, \text{DT}) = 1$
- What if $X = \text{NN}$?

$$\mathcal{T}(1, 1, \text{NN}) = \emptyset$$

$$\pi(1, 1, \text{NN}) = 0$$

because there is no such rule
$\text{NN} \rightarrow \text{the}$

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Notation Summary

- $\mathcal{T}(i, j, X)$. Special cases
  - $\mathcal{T}(1, n, S)$
  - $\mathcal{T}(i, i, X)$
- $\pi(i, j, X)$. Special cases
  - $\pi(1, n, S)$
  - $\pi(i, i, X)$

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  - $\pi(1, n, S)$
  - $\pi(i, i, X)$
- Parsing:
  - from $\mathcal{T}(1, n, S)$, find the tree with score $\pi(1, n, S)$
  - starting points $\mathcal{T}(i, i, X)$, $\forall i \in \{1, \ldots, n\}$

An Example

$S \rightarrow NP\ VP$	1.0		$NP \rightarrow NP\ PP$	0.4
$PP \rightarrow P\ NP$	1.0		$NP \rightarrow$ *astronomers*	0.1
$VP \rightarrow V\ NP$	0.7		$NP \rightarrow$ *ears*	0.18
$VP \rightarrow VP\ PP$	0.3		$NP \rightarrow$ *saw*	0.04
$P \rightarrow$ *with*	1.0		$NP \rightarrow$ *stars*	0.18
$V \rightarrow$ *saw*	1.0		$NP \rightarrow$ *telescopes*	0.1

An Example

S → NP VP	1.0	NP → NP PP	0.4
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Sentence

`astronomers saw stars with ears`

Parse Chart

Parsing:

- from $\mathcal{T}(1, n, S)$, find the tree with score $\pi(1, n, S)$
- starting points $\mathcal{T}(i, i, X)$, $\forall i \in \{1, \ldots, n\}$

	1	2	3	4	5	
astronomers						1
	saw					2
		stars				3
			with			4
				ears		5

Parse Chart

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	1	2	3	4	5	
astronomers	$(1,1,X)$					1
	saw	$(2,2,X)$				2
		stars	$(3,3,X)$			3
			with	$(4,4,X)$		4
				ears	$(5,5,X)$	5

Parse Chart

Parsing:

- from $\mathcal{T}(1, n, S)$, find the tree with score $\pi(1, n, S)$
- starting points $\mathcal{T}(i, i, X)$, $\forall i \in \{1, \ldots, n\}$

	1	2	3	4	5	
astronomers	$(1, 1, X)$				$(1, 5, S)$	1
	saw	$(2, 2, X)$				2
		stars	$(3, 3, X)$			3
			with	$(4, 4, X)$		4
				ears	$(5, 5, X)$	5

Probabilistic CKY: Base cases

Sentence

```
astronomers saw stars with ears
```

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- For $i \in \{1, \ldots, n\}$ and for each $X \in \mathcal{N}$

$$\pi(i, i, X) = P(x_i \mid X)$$

Probabilistic CKY: Base cases

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`astronomers saw stars with ears`

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- For $i \in \{1, \ldots, n\}$ and for each $X \in \mathcal{N}$

$$\pi(i, i, X) = P(x_i \mid X)$$

- Example: $x_2 = \text{saw}$

$$\begin{aligned} \pi(2,2,\mathrm{V}) &= P(\mathrm{V} \longrightarrow \text{saw}) = 1.0 \\ \pi(2,2,\mathrm{NP}) &= P(\mathrm{NP} \longrightarrow \text{saw}) = 0.04 \end{aligned} \tag{5}$$

Probabilistic CKY: Base cases

S → NP VP	1.0		NP → NP PP	0.4
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	1	2	3	4	5	
astronomers	NP, 0.1					1
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		stars	NP, 0.18			3
			with	P, 1.0		4
				ears	NP, 0.18	5

Probabilistic CKY: Recursive cases

For each i, j such that $1 \leq i < j \leq n$ and each $X \in \mathcal{N}$

$$\pi(i, j, X) = \max_{Y,Z \in \mathcal{N}, k \in \{i, \ldots, j-1\}} P(\mathrm{X} \rightarrow \mathrm{Y}\ \mathrm{Z}) \cdot \pi(i, k, Y) \cdot \pi(k+1, j, Z) \quad (6)$$

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Example with $i = 2$ and $j = 3$

$$\pi(2, 3, X) = \max_{Y,Z \in \mathcal{N}, k \in \{2\}} P(\mathrm{X} \rightarrow \mathrm{Y\ Z}) \cdot \pi(2, k, Y) \cdot \pi(k+1, 3, Z) \tag{7}$$

Probabilistic CKY: Recursive cases

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- $\pi(2,3,\mathrm{NP}) = P(\mathrm{NP} \rightarrow \mathrm{Y\ Z}) \cdot \pi(2,2,Y) \cdot \pi(3,3,Z)$, $\mathrm{Y,Z} \in \mathcal{N}$

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- $\pi(2,3,\mathrm{NP}) = P(\mathrm{NP} \rightarrow \mathrm{Y\ Z}) \cdot \pi(2,2,Y) \cdot \pi(3,3,Z)$, $\mathrm{Y,Z} \in \mathcal{N}$
- $\pi(2,3,\mathrm{VP}) = P(\mathrm{VP} \rightarrow \mathrm{Y\ Z}) \cdot \pi(2,2,Y) \cdot \pi(3,3,Z)$, $\mathrm{Y,Z} \in \mathcal{N}$

S → NP VP	1.0	NP → NP PP	0.4
PP → P NP	1.0	NP → *astronomers*	0.1
VP → V NP	0.7	NP → *ears*	0.18
VP → VP PP	0.3	NP → *saw*	0.04
P → *with*	1.0	NP → *stars*	0.18
V → *saw*	1.0	NP → *telescopes*	0.1

	1	2	3	4	5	
astronomers	NP, 0.1					1
	saw	V, 1.0 NP, 0.04				2
		stars	NP, 0.18			3
			with	P, 1.0		4
				ears	NP, 0.18	5

Recursive Cases: Example (I)

$$\pi(2,3,X) = \max_{Y,Z\in\mathcal{N},k\in\{2\}} P(\mathrm{X} \longrightarrow \mathrm{Y\ Z}) \cdot \pi(2,k,Y) \cdot \pi(k+1,3,Z) \qquad (9)$$

S → NP VP	1.0	NP → NP PP	0.4
PP → P NP	1.0	NP → *astronomers*	0.1
VP → V NP	0.7	NP → *ears*	0.18
VP → VP PP	0.3	NP → *saw*	0.04
P → *with*	1.0	NP → *stars*	0.18
V → *saw*	1.0	NP → *telescopes*	0.1

	1	2	3	4	5	
astronomers	NP, 0.1					1
	saw	*V*, 1.0 *NP*, 0.04	VP, 0.126			2
		stars	NP, 0.18			3
			with	P, 1.0		4
				ears	NP, 0.18	5

Recursive Cases: Example (II)

$$\pi(1,2,X) = \max_{L,R \in \mathcal{N}, k \in \{1\}} P(\mathrm{X} \longrightarrow \mathrm{Y\ Z}) \cdot \pi(1,1,Y) \cdot \pi(2,2,Z) \qquad (10)$$

S → NP VP	1.0	NP → NP PP	0.4
PP → P NP	1.0	NP → *astronomers*	0.1
VP → V NP	0.7	NP → *ears*	0.18
VP → VP PP	0.3	NP → *saw*	0.04
P → *with*	1.0	NP → *stars*	0.18
V → *saw*	1.0	NP → *telescopes*	0.1

	1	2	3	4	5	
astronomers	NP, 0.1	∅				1
	saw	$V, 1.0$ $NP, 0.04$	VP, 0.126			2
		stars	NP, 0.18			3
			with	P, 1.0		4
				ears	NP, 0.18	5

Recursive Cases

$$\pi(i,j,X) = \max_{Y,Z\in\mathcal{N},k\in\{i,\ldots,j-1\}} P(\mathrm{X} \longrightarrow \mathrm{Y\ Z}) \cdot \pi(i,k,Y) \cdot \pi(k+1,j,Z)$$

S → NP VP	1.0	NP → NP PP	0.4
PP → P NP	1.0	NP → *astronomers*	0.1
VP → V NP	0.7	NP → *ears*	0.18
VP → VP PP	0.3	NP → *saw*	0.04
P → *with*	1.0	NP → *stars*	0.18
V → *saw*	1.0	NP → *telescopes*	0.1

	1	2	3	4	5	
astronomers	NP, 0.1	∅				1
	saw	$V, 1.0$ $NP, 0.04$	VP, 0.126			2
		stars	NP, 0.18	∅		3
			with	P, 1.0	PP, 0.18	4
				ears	NP, 0.18	5

Probabilistic CKY: Recursive cases

$$\pi(i,j,X) = \max_{Y,Z\in\mathcal{N},k\in\{i,\ldots,j-1\}} P(\mathrm{X} \longrightarrow \mathrm{Y\ Z}) \cdot \pi(i,k,Y) \cdot \pi(k+1,j,Z)$$

S → NP VP	1.0		NP → NP PP	0.4
PP → P NP	1.0		NP → *astronomers*	0.1
VP → V NP	0.7		NP → *ears*	0.18
VP → VP PP	0.3		NP → *saw*	0.04
P → *with*	1.0		NP → *stars*	0.18
V → *saw*	1.0		NP → *telescopes*	0.1

	1	2	3	4	5	
astronomers	NP, 0.1	∅	S, 0.0126			1
	saw	V, 1.0 NP, 0.04	VP, 0.126	∅		2
		stars	NP, 0.18	∅	NP, 0.01296	3
			with	P, 1.0	PP, 0.18	4
				ears	NP, 0.18	5

Probabilistic CKY: Recursive cases

$$\pi(i,j,X) = \max_{Y,Z\in\mathcal{N},k\in\{i,\ldots,j-1\}} P(X \rightarrow Y\ Z) \cdot \pi(i,k,Y) \cdot \pi(k+1,j,Z)$$

S → NP VP	1.0		NP → NP PP	0.4
PP → P NP	1.0		NP → *astronomers*	0.1
VP → V NP	0.7		NP → *ears*	0.18
VP → VP PP	0.3		NP → *saw*	0.04
P → *with*	1.0		NP → *stars*	0.18
V → *saw*	1.0		NP → *telescopes*	0.1

	1	2	3	4	5	
astronomers	NP, 0.1	∅	S, 0.0126	∅		1
	saw	*V*, 1.0 *NP*, 0.04	VP, 0.126	∅	VP, 0.015876	2
		stars	NP, 0.18	∅	NP, 0.01296	3
			with	P, 1.0	PP, 0.18	4
				ears	NP, 0.18	5

Probabilistic CKY: Recursive cases

$$\pi(i,j,X) = \max_{Y,Z\in\mathcal{N},k\in\{i,\ldots,j-1\}} P(\mathrm{X} \longrightarrow \mathrm{Y\ Z}) \cdot \pi(i,k,Y) \cdot \pi(k+1,j,Z)$$

S → NP VP	1.0		NP → NP PP	0.4
PP → P NP	1.0		NP → *astronomers*	0.1
VP → V NP	0.7		NP → *ears*	0.18
VP → VP PP	0.3		NP → *saw*	0.04
P → *with*	1.0		NP → *stars*	0.18
V → *saw*	1.0		NP → *telescopes*	0.1

	1	2	3	4	5	
astronomers	NP, 0.1	∅	S, 0.0126	∅	S, 0.0015876	1
	saw	V, 1.0 NP, 0.04	VP, 0.126	∅	VP, 0.015876	2
		stars	NP, 0.18	∅	NP, 0.01296	3
			with	P, 1.0	PP, 0.18	4
				ears	NP, 0.18	5

Probabilistic CKY

Input: a sentence $s = x_1 \ldots x_n$, a PCFG $G = (N, \Sigma, S, R, q)$.
Initialization:
For all $i \in \{1 \ldots n\}$, for all $X \in N$,

$$\pi(i, i, X) = \begin{cases} q(X \rightarrow x_i) & \text{if } X \rightarrow x_i \in R \\ 0 & \text{otherwise} \end{cases}$$

Algorithm:

- For $l = 1 \ldots (n-1)$
  - For $i = 1 \ldots (n-l)$
    - Set $j = i + l$
    - For all $X \in N$, calculate

$$\pi(i, j, X) = \max_{\substack{X \rightarrow Y Z \in R, \\ s \in \{i \ldots (j-1)\}}} (q(X \rightarrow Y Z) \times \pi(i, s, Y) \times \pi(s+1, j, Z))$$

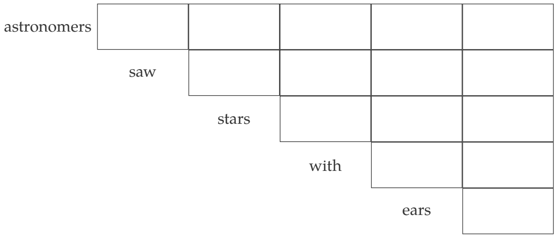
and

$$bp(i, j, X) = \arg \max_{\substack{X \rightarrow Y Z \in R, \\ s \in \{i \ldots (j-1)\}}} (q(X \rightarrow Y Z) \times \pi(i, s, Y) \times \pi(s+1, j, Z))$$

Output: Return $\pi(1, n, S) = \max_{t \in \mathcal{T}(s)} p(t)$, and backpointers bp which allow recovery of $\arg\max_{t \in \mathcal{T}(s)} p(t)$.

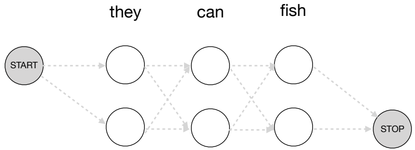
Remarks

- Space and runtime requirements
  - Space: $\mathcal{O}(|\mathcal{N}|n^2)$
  - Time: $\mathcal{O}(|\mathcal{N}|n^3)$

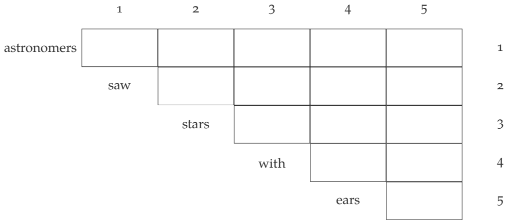


Probabilistic CKY vs. Viterbi Decoding

- Viterbi decoding



- Probabilistic CKY



Keywords: conditional independence, forward enumerating, backward tracing, dynamic programming

Summary

1. Probabilistic CFGs

2. Probabilistic CKY Algorithm

Reference

Collins, M. (2017).
Natural language processing: Lecture notes.

Smith, N. A. (2017).
Natural language processing: Lecture notes.