CS 6501 Natural Language Processing

Dependency Parsing

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Overview

1. Noisy Channel Model
2. IBM Model 1
3. IBM Model 2
4. Parameter Estimation

Based on slides from [Collins, 2017]

Noisy Channel Model

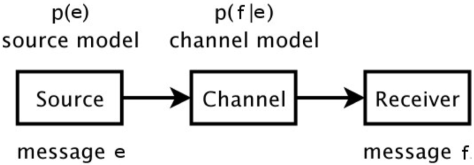
Problem

- Goal: translate French to English
- Mathematical formulation

$$P(\boldsymbol{e} \mid \boldsymbol{f}) \tag{1}$$

where $\boldsymbol{f} = (f_1, \ldots, f_m)$ is a French sentence and $\boldsymbol{e} = (e_1, \ldots, e_l)$ is an English translation.

Noisy Channel Model: Definition



- Bayes theorem

$$P(e \mid f) = \frac{P(e, f)}{P(f)} = \frac{P(e)P(f \mid e)}{\sum_e P(e)P(f \mid e)} \tag{2}$$

Two Components

$$P(\boldsymbol{e} \mid \boldsymbol{f}) = \frac{P(\boldsymbol{e}, \boldsymbol{f})}{P(\boldsymbol{f})} = \frac{P(\boldsymbol{e})P(\boldsymbol{f} \mid \boldsymbol{e})}{\sum_{\boldsymbol{e}} P(\boldsymbol{e})P(\boldsymbol{f} \mid \boldsymbol{e})} \tag{3}$$

- $P(\boldsymbol{e})$: the language model
- $P(\boldsymbol{f} \mid \boldsymbol{e})$: the translation model

Two Components

$$P(\boldsymbol{e} \mid \boldsymbol{f}) = \frac{P(\boldsymbol{e}, \boldsymbol{f})}{P(\boldsymbol{f})} = \frac{P(\boldsymbol{e})P(\boldsymbol{f} \mid \boldsymbol{e})}{\sum_{\boldsymbol{e}} P(\boldsymbol{e})P(\boldsymbol{f} \mid \boldsymbol{e})} \tag{3}$$

- $P(\boldsymbol{e})$: the language model
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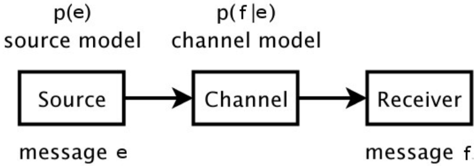
Translation:

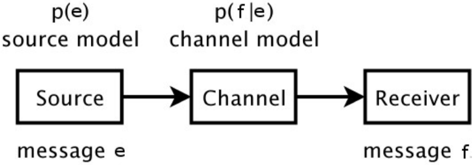
$$\hat{\boldsymbol{e}} = \arg\max_{e} P(\boldsymbol{e} \mid \boldsymbol{f}) = \arg\max_{\boldsymbol{e}} P(\boldsymbol{e})P(\boldsymbol{f} \mid \boldsymbol{e}) \tag{4}$$

Why Noisy Channel Model?



- Divide one big problem into two subproblems
- Solve them separately (with extra resources)

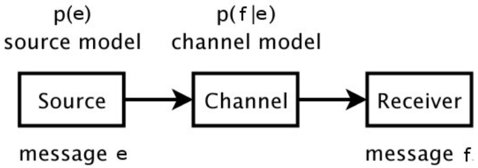
Question



Central question

How do we model $P(f \mid e)$?

Question



Central question

How do we model $P(f \mid e)$?

Examples:

- IBM Model 1
- IBM Model 2

What about $P(e)$?

IBM Model 1

Challenge of Probability Definition

Given

- an English sentence $\boldsymbol{e}$ with l words $(e_1, \ldots, e_l)$ and
- a French sentence $\boldsymbol{f}$ with m words $(f_1, \ldots, f_m)$,

directly modeling

$$P(f_1, \ldots, f_m \mid e_1, \ldots, e_l) \tag{5}$$

is a challenging task.

Example

$$e = \text{And the program has been implemented}$$
$$f = \text{Le programme a ete mis en application}$$

- $P(\boldsymbol{f} \mid \boldsymbol{e})$ defines a probability on a 13-dimensional space
- There are alignments between French and English words

Example (Cont.)

	1	2	3	4	5	6	7
e	And	the	program	has	been	implemented	
f	Le	programme	a	ete	mis	en	application

Example (Cont.)

	1	2	3	4	5	6	7
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Example (Cont.)

	1	2	3	4	5	6	7
e	And	the	program	has	been	implemented	
f	Le	programme	a	ete	mis	en	application

$$a_j = i$$

the j-th French word is aligned with the i-th word in English.

Examples

$$a_1 = 2, \quad a_6 = 6$$

Alignments

$$P(f_1, \ldots, f_m, a_1, \cdots, a_m \mid e_1, \ldots, e_l) \tag{6}$$

where $a_j \in \{0, 1, \ldots, l\}$

Alignments

$$P(f_1, \ldots, f_m, a_1, \cdots, a_m \mid e_1, \ldots, e_l) \tag{6}$$

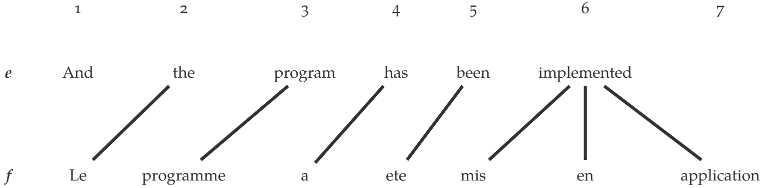
where $a_j \in \{0, 1, \ldots, l\}$

Marginalizing over $(a_1, \ldots, a_m)$ gives the translation probability

$$P(f_1, \ldots, f_m \mid e_1, \ldots, e_l) = \sum_{a_1, \ldots, a_m} P(f_1, \ldots, f_m, a_1, \cdots, a_m \mid e_1, \ldots, e_l) \tag{7}$$

Example (Cont.)

An example of alignment



$$
\begin{aligned}
a_2 &= 3 \\
a_3 &= 4 \\
a_4 &= 5 \\
a_5 &= 6 \\
a_6 &= 6 \\
a_7 &= 6
\end{aligned}
$$

Further Factorization

Factorization

$$P(\boldsymbol{f}, \boldsymbol{a} \mid \boldsymbol{e}) = P(\boldsymbol{a} \mid \boldsymbol{e}) P(\boldsymbol{f} \mid \boldsymbol{a}, \boldsymbol{e}) \tag{8}$$

- Alignment $P(\boldsymbol{a} \mid \boldsymbol{e})$
- Translation with a given alignment $\boldsymbol{a}$, $P(\boldsymbol{f} \mid \boldsymbol{a}, \boldsymbol{e})$

IBM Model 1: $P(\boldsymbol{a} \mid \boldsymbol{e})$

In IBM Model 1, all alignments are equally likely

$$P(\boldsymbol{a} \mid \boldsymbol{e}) = \frac{1}{(l+1)^m} \tag{9}$$

- Major simplification, great starting point
- Independent of words in $\boldsymbol{f}$ and $\boldsymbol{e}$

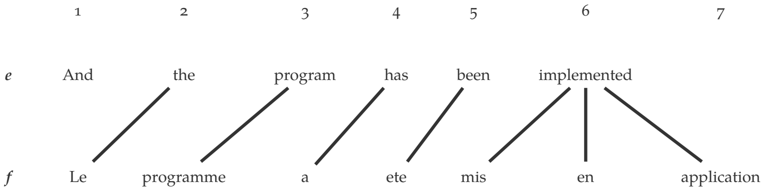
IBM Model 1: $P(\boldsymbol{f} \mid \boldsymbol{a}, \boldsymbol{e})$

Translation probabilities

$$P(\boldsymbol{f} \mid \boldsymbol{a}, \boldsymbol{e}) = \prod_{j=1}^{m} t(f_j \mid e_{a_j}) \tag{10}$$

- f_j: the j-th French word
- $a_j = i$: the alignment of the j-th French word
- e_{a_j}: the aligned English word of the j-th French word
- $t(f_j \mid e_{a_j})$: the translation probability from the a_j-th English word to the j-th French word

Example (Cont.)



$$
\begin{aligned}
P(\boldsymbol{f} \mid \boldsymbol{a}, \boldsymbol{e}) = & \, t(\text{Le} \mid \text{the}) \cdot t(\text{programme} \mid \text{program}) \cdot \\
& t(\text{a} \mid \text{has}) \cdot t(\text{ete} \mid \text{been}) \cdot \\
& t(\text{mis} \mid \text{implemented}) \cdot t(\text{en} \mid \text{implemented}) \cdot \\
& t(\text{application} \mid \text{implemented})
\end{aligned}
$$

IBM Model 1: Final result

$$\begin{aligned} P(\boldsymbol{f} \mid \boldsymbol{e}) &= \sum_{\boldsymbol{a}} P(\boldsymbol{f}, \boldsymbol{a} \mid \boldsymbol{e}) \\ &= \sum_{\boldsymbol{a}} P(\boldsymbol{a} \mid \boldsymbol{e}) P(\boldsymbol{f} \mid \boldsymbol{a}, \boldsymbol{e}) \\ &= \sum_{\boldsymbol{a}} \frac{1}{(l+1)^m} \prod_{j=1}^{m} t(f_j \mid e_{a_j}) \end{aligned} \tag{11}$$

Break a big conditional probability into small pieces on word pairs

IBM Model 2

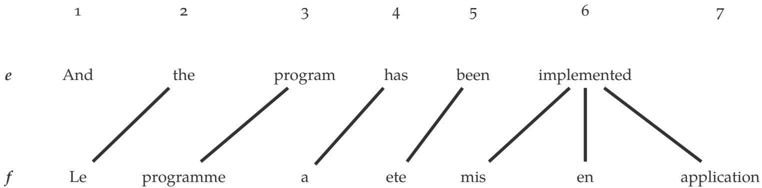
Alignments: Non-uniform distribution

Instead of uniform distribution, IBM 2 defines

$$P(\boldsymbol{a} \mid \boldsymbol{e}) = \prod_{j=1}^{m} q(a_j = i \mid j, l, m), \tag{12}$$

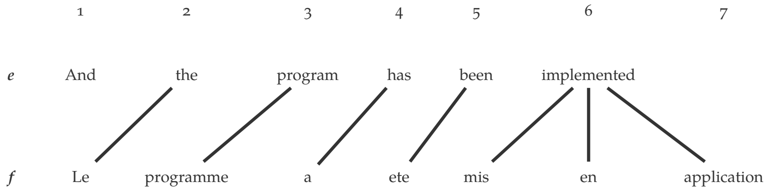
the probability that the j-th French word is connected to the i-th English word, given sentence lengths as l and m in English and French respectively

An Example



$$
\begin{aligned}
P(\boldsymbol{a} \mid \boldsymbol{e}) = & q(2 \mid 1,6,7) \cdot q(3 \mid 2,6,7) \cdot \\
& q(4 \mid 3,6,7) \cdot q(5 \mid 4,6,7) \cdot \\
& q(6 \mid 5,6,7) \cdot q(6 \mid 6,6,7) \cdot \\
& q(6 \mid 7,6,7)
\end{aligned}
$$

An Example (Cont.)



$$
\begin{aligned}
P(\boldsymbol{f} \mid \boldsymbol{a}, \boldsymbol{e}) = & \, t(\text{Le} \mid \text{the}) \cdot t(\text{programme} \mid \text{program}) \cdot \\
& t(\text{a} \mid \text{has}) \cdot t(\text{ete} \mid \text{been}) \cdot \\
& t(\text{mis} \mid \text{implemented}) \cdot t(\text{en} \mid \text{implemented}) \cdot \\
& t(\text{application} \mid \text{implemented})
\end{aligned}
$$

IBM Model 2: Final result

$$
\begin{aligned}
P(\boldsymbol{f} \mid \boldsymbol{e}) &= \sum_{\boldsymbol{a}} P(\boldsymbol{f}, \boldsymbol{a} \mid \boldsymbol{e}) \\
&= \sum_{\boldsymbol{a}} P(\boldsymbol{a} \mid \boldsymbol{e}) P(\boldsymbol{f} \mid \boldsymbol{a}, \boldsymbol{e}) \\
&= \sum_{\boldsymbol{a}} \prod_{j=1}^{m} \{q(i \mid j, l, m) t(f_j \mid e_{a_j})\}
\end{aligned} \tag{13}
$$

Break a big conditional probability into small pieces on word pairs

Parameter Estimation

Problem Definition

- Input: $\{(\boldsymbol{e}^{(k)}, \boldsymbol{f}^{(k)}, \boldsymbol{a}^{(k)})\}$
- Output:

$$
\begin{aligned}
t(\boldsymbol{f} \mid \boldsymbol{e}) &= ? \\
q(i \mid j, l, m) &= ?
\end{aligned}
$$

- Method: Maximum-likelihood estimation, parameter estimation used in
  - HMMs with fully observed sentence-tag pairs
  - PCFGs with fully observed sentence-parse paris

Estimation with Alignments Observed

- An example sentence pair with alignment

$$\begin{aligned} \boldsymbol{e}^{(100)} &= \text{And the program has been implemented} \\ \boldsymbol{f}^{(100)} &= \text{Le programme a ete mis en application} \\ \boldsymbol{a}^{(100)} &= (2,3,4,5,6,6,6) \end{aligned}$$

- Maximum-likelihood parameter estimates

$$\begin{aligned} t_{ML}(f_j \mid e_i) &= \frac{c(e_i, f_j)}{c(e_i)} \\ q_{ML}(i \mid j, l, m) &= \frac{c(i, j, l, m)}{c(j, l, m)} \end{aligned}$$

Input: A training corpus $(f^{(k)}, e^{(k)}, a^{(k)})$ for $k = 1 \ldots n$, where $f^{(k)} = f_1^{(k)} \ldots f_{m_k}^{(k)}$, $e^{(k)} = e_1^{(k)} \ldots e_{l_k}^{(k)}$, $a^{(k)} = a_1^{(k)} \ldots a_{m_k}^{(k)}$.

Algorithm:

- Set all counts $c(\ldots) = 0$
- For $k = 1 \ldots n$
  - For $i = 1 \ldots m_k$
    - For $j = 0 \ldots l_k$

$$\begin{aligned}
c(e_j^{(k)}, f_i^{(k)}) &\leftarrow c(e_j^{(k)}, f_i^{(k)}) + \delta(k,i,j) \\
c(e_j^{(k)}) &\leftarrow c(e_j^{(k)}) + \delta(k,i,j) \\
c(j|i,l,m) &\leftarrow c(j|i,l,m) + \delta(k,i,j) \\
c(i,l,m) &\leftarrow c(i,l,m) + \delta(k,i,j)
\end{aligned}$$

where $\delta(k,i,j) = 1$ if $a_i^{(k)} = j$, 0 otherwise.

Output:

$$t_{ML}(f|e) = \frac{c(e,f)}{c(e)} \qquad q_{ML}(j|i,l,m) = \frac{c(j|i,l,m)}{c(i,l,m)}$$

Comments: (1) i and j are exchanged; (2) the equation of q_{ML} is wrong.

Estimation without Alignments Observed

- Input: $\{(\boldsymbol{e}^{(k)}, \boldsymbol{f}^{(k)})\}$
- Output:

$$
\begin{aligned}
q(i \mid j, l, m) &= ? \\
t(f_j \mid e_{a_j}) &= ?
\end{aligned}
$$

- Challenge:

We do not have alignments $\delta(k, i, j)$ on our training examples

Basic Idea

- Initialize $\{P(f_i \mid e_j)\}$ and $\{P(i \mid j, l, m)\}$
- Iterate between the following
  - $\forall k$, compute the expected alignment $\delta(k, i, j)$ where $\sum_j \delta(k, i, j) = 1$, and use it to update $c(\cdots)$,

$$c(\cdots) \leftarrow c(\cdots) + \delta$$

  - Update $\{P(f_i \mid e_j)\}$ and $\{P(i \mid j, l, m)\}$ with $c(\cdots)$

Comments: i and j are exchanged.

- For $s = 1 \ldots S$
  - Set all counts $c(\ldots) = 0$
  - For $k = 1 \ldots n$
    * For $i = 1 \ldots m_k$
      · For $j = 0 \ldots l_k$

$$
\begin{aligned}
c(e_j^{(k)}, f_i^{(k)}) &\leftarrow c(e_j^{(k)}, f_i^{(k)}) + \delta(k, i, j) \\
c(e_j^{(k)}) &\leftarrow c(e_j^{(k)}) + \delta(k, i, j) \\
c(j|i, l, m) &\leftarrow c(j|i, l, m) + \delta(k, i, j) \\
c(i, l, m) &\leftarrow c(i, l, m) + \delta(k, i, j)
\end{aligned}
$$

where

$$
\delta(k, i, j) = \frac{q(j|i, l_k, m_k) t(f_i^{(k)}|e_j^{(k)})}{\sum_{j=0}^{l_k} q(j|i, l_k, m_k) t(f_i^{(k)}|e_j^{(k)})}
$$

  - Set

$$
t(f|e) = \frac{c(e, f)}{c(e)} \qquad q(j|i, l, m) = \frac{c(j|i, l, m)}{c(i, l, m)}
$$

Comments: (1) i and j are exchanged; (2) the equation of q_{ML} is wrong; (3) the only difference is how to compute δ.

EM Algorithm

E-step $\forall k$, compute its expected alignment δ to $c(\cdots)$

$$c(\cdots) \leftarrow c(\cdots) + \delta$$

M-step Maximize the likelihood with the total expected count $c(\cdots)$

[Eisenstein, 2018, Sec. 5.1]

Summary

1. Noisy Channel Model
2. IBM Model 1
3. IBM Model 2
4. Parameter Estimation

Reference

Collins, M. (2017).
Natural language processing: Lecture notes.

Eisenstein, J. (2018).
Natural Language Processing.
MIT Press.