CS 6501 Natural Language Processing

Word Embeddings

Yangfeng Ji

October 22, 2018

Department of Computer Science
University of Virginia

Overview

1. Distributional Hypothesis
2. Latent Semantic Analysis
3. Word Embeddings

Distributional Hypothesis

Distributional Hypothesis

words that occur in the same contexts tend to have similar meanings

- `to have a splendid time in Rome`
- `to have a wonderful time in Rome`

Generalized Hypotheses

- **Statistical semantics hypothesis**: Statistical patterns of human word usage can be used to figure out what people mean.
- **Bag of words hypothesis**: The frequencies of words in a document tend to indicate the relevance of the document to a query

[Turney and Pantel, 2010]

Latent Semantic Analysis

Word-document Matrix

For a corpus of d documents over a vocabulary $\mathcal{V}$, the cooccurance matrix is defined as $\mathbf{C}$,

$$\mathbf{C} = [C_{ij}] \in \mathbb{R}^{v \times d}, \tag{1}$$

where $v = |\mathcal{V}|$ is the size of vocab, and C_{ij} is the count of word i in document j.

Word-document Matrix

Word	Documents							
	1	2	3	4	5	6	7	8
w_1	0	1	0	0	0	0	0	0
w_2	0	0	1	0	0	3	0	0
w_3	1	0	0	2	0	0	5	0
w_4	3	0	0	1	1	0	2	0
w_5	0	1	3	0	1	2	1	0
w_6	1	2	0	0	0	0	1	0
w_7	0	1	0	1	0	1	0	1
w_8	0	0	0	0	0	7	0	0

Similarity

If we consider a document as a context, we can use row vectors $\boldsymbol{c}_i$ to represent words and hence measure similarity between words as

$$\text{sim}(\boldsymbol{c}_i, \boldsymbol{c}_j) = \frac{\langle \boldsymbol{c}_i, \boldsymbol{c}_j \rangle}{\|\boldsymbol{c}_i\| \|\boldsymbol{c}_j\|} \tag{2}$$

Data Sparsity

Word	Documents							
	1	2	3	4	5	6	7	8
w_1	0	1	0	0	0	0	0	0
w_2	0	0	1	0	0	3	0	0
w_3	1	0	0	2	0	0	5	0
w_4	3	0	0	1	1	0	2	0
w_5	0	1	3	0	1	2	1	0
w_6	1	2	0	0	0	0	1	0
w_7	0	1	0	1	0	1	0	1
w_8	0	0	0	0	0	7	0	0

- Vocab size: 50K, 100K, etc.

Context Window Size

Word	Documents							
	1	2	3	4	5	6	7	8
w_1	0	1	0	0	0	0	0	0
w_2	0	0	1	0	0	3	0	0
w_3	1	0	0	2	0	0	5	0
w_4	3	0	0	1	1	0	2	0
w_5	0	1	3	0	1	2	1	0
w_6	1	2	0	0	0	0	1	0
w_7	0	1	0	1	0	1	0	1
w_8	0	0	0	0	0	7	0	0

Are w_i and w_j similar to each other, when they appear in the same documents but far away from each other?

Solution to Data Sparsity

Using SVD, the matrix $\mathbf{C}$ is decomposed into a multiplication of three matrices

$$\mathbf{C} = \mathbf{U}_0 \mathbf{\Sigma}_0 \mathbf{V}_0^\top. \tag{3}$$

where $\mathbf{U}_0 \in \mathbb{R}^{v \times v}$, $\mathbf{\Sigma}_0 \in \mathbb{R}^{v \times d}$ is a diagonal matrix and $\mathbf{V}_0 \in \mathbb{R}^{d \times d}$.

The columns of $\mathbf{U}_0$ (and $\mathbf{V}_0$) are orthonormal.

SVD

the approximation can be wrotten as

$$\mathbf{C} \approx \mathbf{U}\mathbf{\Sigma}\mathbf{V}^\top \tag{4}$$

where $\mathbf{U} \in \mathbb{R}^{v \times k}$, $\mathbf{V} \in \mathbb{R}^{k \times d}$ and $\mathbf{\Sigma} \in \mathbb{R}^{k \times k}$.

Optimization

Mathematically, if we are looking for a rank-k approximation of $\mathbf{C}$ as,

$$\arg\min_{\mathbf{M}} \|\mathbf{M} - \mathbf{C}\|_2 \tag{5}$$

where $\mathbf{M}$ is rank-k,

Based on 4, the word representation can be constructed as

$$\mathbf{W} = \mathbf{U}\Sigma \tag{6}$$

and document representation as

$$\mathbf{D} = \Sigma\mathbf{V} \tag{7}$$

Reweighting

- TF-IDF
- Pointwise Mutual Information: the definition of $\text{PMI}(w_i, w_j)$ is

$$\text{PMI}(w_i, w_j) = \log \frac{P(w_i, w_j)}{P(w_i)P(w_j)} = \log \frac{P(w_j \mid w_i)}{P(w_j)} \quad (8)$$

Word Embeddings

Skip-gram

One way of finding a better word representation is to make sure it has the potential to predict its surrounding words

$$P(w_{t+i} \mid w_t; \boldsymbol{\theta}) = \frac{\exp(\boldsymbol{u}_{w_{t+i}}^\top \boldsymbol{v}_{w_t})}{\sum_{w' \in \mathcal{V}} \exp(\boldsymbol{u}_{w'}^\top \boldsymbol{v}_{w_t})} \tag{9}$$

where $i \in \{-c, \ldots, -1, 1, \ldots, c\}$ and c is the window size. Usually, larger window size c gives better quality of word representations, but it also causes large computational complexity.

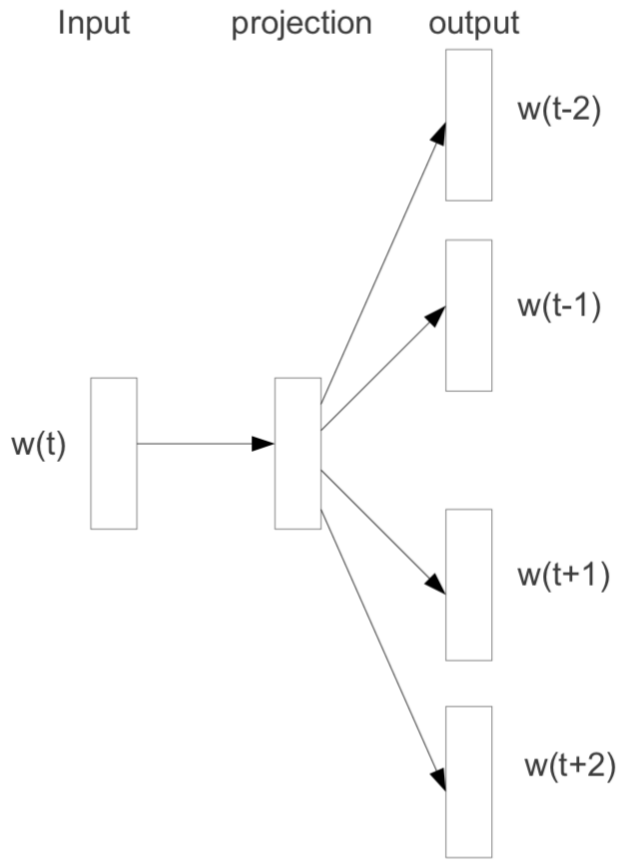


Figure: The skip-gram model.

Word Vectors

$$P(w_{t+i} \mid w_t; \boldsymbol{\theta}) = \frac{\exp(\boldsymbol{u}_{w_{t+i}}^\top \boldsymbol{v}_{w_t})}{\sum_{w' \in \mathcal{V}} \exp(\boldsymbol{u}_{w'}^\top \boldsymbol{v}_{w_t})} \tag{10}$$

- $\boldsymbol{v}_w$: word vector (as input)
- $\boldsymbol{u}_w$: context vector (as output)

Question

Why we need two vectors for a word?

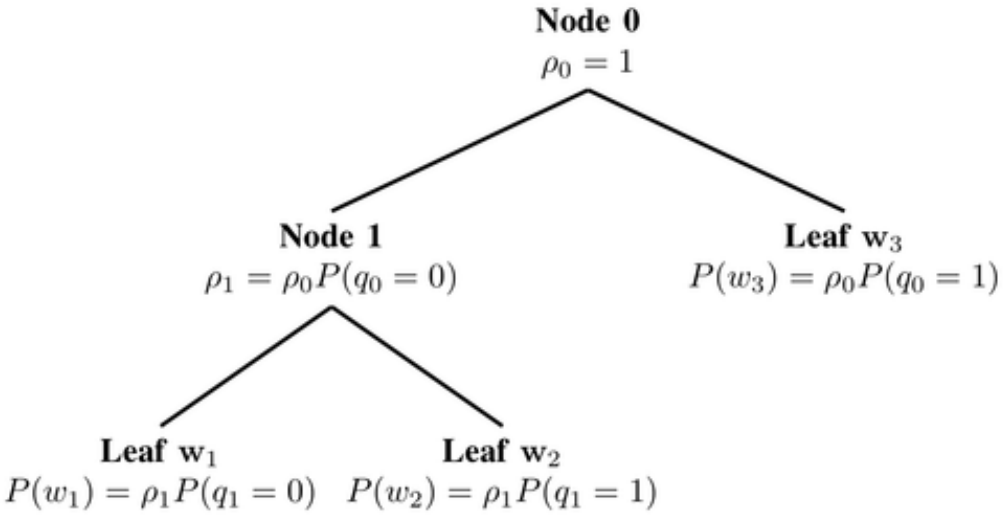
Objective Function

The objective function of a skip-gram model is defined as

$$\frac{1}{T}\sum_{t=1}^{T}\sum_{-c\leq i\leq c; i\neq 0}\log P(w_{t+i}\mid w_t) \tag{11}$$

In practice, it could be 10K, 50K or even bigger and the computation of $\sum_{w'\in\mathcal{V}}\exp(\boldsymbol{v}_{w'}^{\top}\boldsymbol{u}_{w_t})$ can be very expensive.

Hierarchical Softmax



- Huffman tree based word frequency

Negative Sampling

Negative sampling of the skip-gram model is defined as, for a specific w_{t+i} to predict, we have

$$\log \sigma(\boldsymbol{u}_{w_{t+i}}^{\top} \boldsymbol{v}_{w_t}) + \sum_{i=1}^{k} E_{w' \sim P_n(w)} \Big[\log \sigma(-\boldsymbol{u}_{w'}^{\top} \boldsymbol{v}_{w_t}) \Big] \qquad (12)$$

Two Factors in Negative Sampling

$$\log \sigma(\boldsymbol{u}_{w_{t+i}}^{\top} \boldsymbol{v}_{w_t}) + \sum_{i=1}^{k} E_{w' \sim P_n(w)} \left[\log \sigma(-\boldsymbol{u}_{w'}^{\top}, \boldsymbol{v}_{w_t}) \right] \qquad (13)$$

Two factors [Mikolov et al., 2013]

- k =?
  - $5 \leq k \leq 20$ works better for small datasets, while for large datasets, $2 \leq k \leq 5$ is enough
- noisy distribution $P(w)$
  - $P(w) \propto U(w)^{\frac{3}{4}}$

Glove

The motivation of GloVe [Pennington et al., 2014] is to find a balance between the methods based on

- global matrix factorization (e.g., LSA) and
- local context windows (e.g., Skip-gram).

Word-to-word Co-occurrence Matrix

- $\mathbf{X}$ with X_{ij} denotes the frequency of word j appears in the context of word i

Empirical probability estimation of w_j given w_i

$$Q(w_j \mid w_i) = \frac{X_{ij}}{X_i} \tag{14}$$

Probability Estimation via Word Embeddings

$$P(w_j \mid w_i) = \frac{\exp(\boldsymbol{u}_{w_j}^\top \boldsymbol{v}_{w_i})}{\sum_{w' \in \mathcal{V}} \exp(\boldsymbol{u}_{w'}^\top \boldsymbol{v}_{w_i})} \tag{15}$$

The basic idea is to learn $\boldsymbol{v}_w$ and $\boldsymbol{u}_w$, such that

$$Q(w_j \mid w_i) \approx P(w_j \mid w_i) \tag{16}$$

The basic idea is to learn $\boldsymbol{v}_w$ and $\boldsymbol{u}_w$, such that

$$Q(w_j \mid w_i) \approx P(w_j \mid w_i) \tag{16}$$

Or more specific

$$\log(X_{ij}) - \log(X_i) \approx \boldsymbol{u}_{w_j}^\top \boldsymbol{v}_{w_i} - \log \sum_{w' \in \mathcal{V}} \exp(\boldsymbol{u}_{w'}^\top \boldsymbol{v}_{w_i}) \tag{17}$$

In order to find the best approximation, we could formulate this as a optimization problem

$$(\log(X_{ij}) - \log(X_i) - \boldsymbol{u}_{w_j}^\top \boldsymbol{v}_{w_i} + \log \sum_{w' \in \mathcal{V}} \exp(\boldsymbol{u}_{w'}^\top \boldsymbol{v}_{w_i})))^2 \quad (18)$$

In order to find the best approximation, we could formulate this as a optimization problem

$$(\log(X_{ij}) - \log(X_i) - \boldsymbol{u}_{w_j}^\top \boldsymbol{v}_{w_i} + \log \sum_{w' \in \mathcal{V}} \exp(\boldsymbol{u}_{w'}^\top \boldsymbol{v}_{w_i})))^2 \quad (18)$$

It can be further simplified as (Eq. 16 in [Pennington et al., 2014])

$$(\log(X_{ij}) - \boldsymbol{u}_{w_j}^\top \boldsymbol{v}_{w_i})^2 \quad (19)$$

if we only consider the *unnormalized* version of P and Q.

Objective Function

The overall objective function is defined as

$$\sum_{w_i} \sum_{w_j} (\log(X_{ij}) - \boldsymbol{u}_{w_j}^{\top} \boldsymbol{v}_{w_i})^2 \tag{20}$$

Objective Function

The overall objective function is defined as

$$\sum_{w_i} \sum_{w_j} (\log(X_{ij}) - \boldsymbol{u}_{w_j}^\top \boldsymbol{v}_{w_i})^2 \tag{20}$$

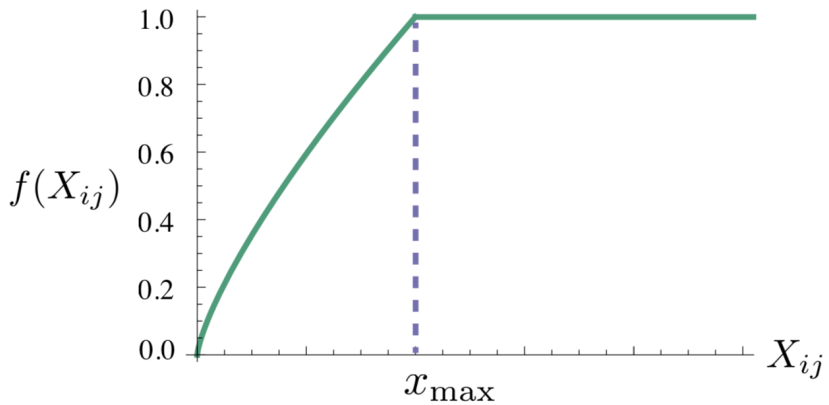
The objective function is further refined by discouraging high-frequency words as

$$\sum_{w_i} \sum_{w_j} f(X_{ij})(\log(X_{ij}) - \boldsymbol{u}_{w_j}^\top \boldsymbol{v}_{w_i})^2 \tag{21}$$

Down-weighting

$$f(x) = \begin{cases} (\frac{x}{x_{\max}})^a & \text{if } x < x_{\max} \\ 1 & \text{otherwise} \end{cases} \tag{22}$$

where $a = 3/4$.



Skip-gram as Implicit Matrix Factorization

[Levy and Goldberg, 2014] shows that skip-gram with negative sampling can be viewed as an implicit matrix factorization over a word-word co-occurrance matrix weighted by pointwise mutual information (PMI).

$$\boldsymbol{u}_{w_j}^{\top} \boldsymbol{v}_{w_i} \approx \text{PMI}(w_i, w_j) - \log k \tag{23}$$

where $\text{PMI}(w_i, w_j)$ is the mutual information of $P(w_i)$ and $P(w_j)$ with *a given window size* and k is the number of negative samples.

Skip-gram as Implicit Matrix Factorization (II)

The definition of $\text{PMI}(w_i, w_j)$ is

$$\text{PMI}(w_i, w_j) = \log \frac{P(w_i, w_j)}{P(w_i)P(w_j)} = \log P(w_j \mid w_i) - \log P(w_j) \tag{24}$$

Combine 23 and 24, we have

$$\begin{aligned} \boldsymbol{u}_{w_j}^\top \boldsymbol{v}_{w_i} &\approx \log \frac{P(w_i, w_j)}{P(w_i)P(w_j)} - \log k \\ &= \log P(w_j \mid w_i) - \log P(w_j) - \log k \\ &= \log(X_{ij}) - \log(X_i) - \log(X_j) + \log D - \log k \end{aligned} \tag{25}$$

Very similar to Eq. 8 in [Pennington et al., 2014].

Essentially,

$$\boldsymbol{u}_{w_j}^{\top} \boldsymbol{v}_{w_i} \approx \log(X_{ij}) + g(\mathbf{X}) \tag{26}$$

Essentially,

$$\boldsymbol{u}_{w_j}^{\top} \boldsymbol{v}_{w_i} \approx \log(X_{ij}) + g(\mathbf{X}) \tag{26}$$

Which one matters?

- $g(\mathbf{X})$, or
- Implicit/explicit optimization, or
- Other tricks (downsampling, hyper-parameters, etc.)

Summary

1. Distributional Hypothesis
2. Latent Semantic Analysis
3. Word Embeddings

Reference

Levy, O. and Goldberg, Y. (2014).
Neural word embedding as implicit matrix factorization.
In *Advances in neural information processing systems*, pages 2177–2185.

Mikolov, T., Sutskever, I., Chen, K., Corrado, G. S., and Dean, J. (2013).
Distributed representations of words and phrases and their compositionality.
In *Advances in neural information processing systems*, pages 3111–3119.

Pennington, J., Socher, R., and Manning, C. (2014).
Glove: Global vectors for word representation.
In *Proceedings of the 2014 conference on empirical methods in natural language processing (EMNLP)*, pages 1532–1543.

Turney, P. D. and Pantel, P. (2010).
From frequency to meaning: Vector space models of semantics.
Journal of artificial intelligence research, 37:141–188.