UVa ICPC Team Notebook (2019-20)

Contents

1 Combinatorial optimization

1.1 Dense max-flow

```
// Adjacency matrix implementation of Dinic's blocking flow algorithm.
//
// Running time:
//     O(|V|^4)
//
// INPUT:
//     - graph, constructed using AddEdge()
//     - source
//     - sink
//
// OUTPUT:
//     - maximum flow value
//     - To obtain the actual flow, look at positive values only.

#include <cmath>
#include <vector>
#include <iostream>

using namespace std;

typedef vector<int> VI;
typedef vector<VI> VVI;

const int INF = 1000000000;

struct MaxFlow {
  int N;
  VVI cap, flow;
  VI dad, Q;

  MaxFlow(int N) :
    N(N), cap(N, VI(N)), flow(N, VI(N)), dad(N), Q(N) {}

  void AddEdge(int from, int to, int cap) {
    this->cap[from][to] += cap;
  }

  int BlockingFlow(int s, int t) {
    fill(dad.begin(), dad.end(), -1);
    dad[s] = -2;

    int head = 0, tail = 0;
    Q[tail++] = s;
    while (head < tail) {
      int x = Q[head++];
      for (int i = 0; i < N; i++) {
        if (dad[i] == -1 && cap[x][i] - flow[x][i] > 0) {
          dad[i] = x;
          Q[tail++] = i;
        }
      }
    }

    if (dad[t] == -1) return 0;

    int totflow = 0;
    for (int i = 0; i < N; i++) {
      if (dad[i] == -1) continue;
      int amt = cap[i][t] - flow[i][t];
      for (int j = i; amt && j != s; j = dad[j])
        amt = min(amt, cap[dad[j]][j] - flow[dad[j]][j]);
      if (amt == 0) continue;
      flow[i][t] += amt;
      flow[t][i] -= amt;
      for (int j = i; j != s; j = dad[j]) {
        flow[dad[j]][j] += amt;
        flow[j][dad[j]] -= amt;
      }
      totflow += amt;
    }

    return totflow;
  }

  int GetMaxFlow(int source, int sink) {
    int totflow = 0;
    while (int flow = BlockingFlow(source, sink))
      totflow += flow;
    return totflow;
  }
};

int main() {

  MaxFlow mf(5);
  mf.AddEdge(0, 1, 3);
  mf.AddEdge(0, 2, 4);
  mf.AddEdge(0, 3, 5);
  mf.AddEdge(0, 4, 5);
  mf.AddEdge(1, 2, 2);
  mf.AddEdge(2, 3, 4);
  mf.AddEdge(2, 4, 1);
  mf.AddEdge(3, 4, 10);

  // should print out "15"
  cout << mf.GetMaxFlow(0, 4) << endl;
}

// BEGIN CUT
// The following code solves SPOJ problem #203: Potholers (POTHOLE)

#ifdef COMMENT
int main() {
  int t;
  cin >> t;
  for (int i = 0; i < t; i++) {
    int n;
    cin >> n;
    MaxFlow mf(n);
    for (int j = 0; j < n-1; j++) {
      int m;
      cin >> m;
      for (int k = 0; k < m; k++) {
        int p;
        cin >> p;
        p--;
        int cap = (j == 0 || p == n-1) ? 1 : INF;
        mf.AddEdge(j, p, cap);
      }
    }

    cout << mf.GetMaxFlow(0, n-1) << endl;
  }
  return 0;
}
#endif

// END CUT
```

1.2 Sparse max-flow

```
// Adjacency list implementation of Dinic's blocking flow algorithm.
// This is very fast in practice, and only loses to push-relabel flow.
//
// Running time:
```

```
//     O(|V|^2 |E|)
//
// INPUT:
//     - graph, constructed using AddEdge()
//     - source and sink
//
// OUTPUT:
//     - maximum flow value
//     - To obtain actual flow values, look at edges with capacity > 0
//       (zero capacity edges are residual edges).

#include<cstdio>
#include<vector>
#include<queue>
using namespace std;
typedef long long LL;

struct Edge {
  int u, v;
  LL cap, flow;
  Edge() {}
  Edge(int u, int v, LL cap): u(u), v(v), cap(cap), flow(0) {}
};

struct Dinic {
  int N;
  vector<Edge> E;
  vector<vector<int>> g;
  vector<int> d, pt;

  Dinic(int N): N(N), E(0), g(N), d(N), pt(N) {}

  void AddEdge(int u, int v, LL cap) {
    if (u != v) {
      E.emplace_back(u, v, cap);
      g[u].emplace_back(E.size() - 1);
      E.emplace_back(v, u, 0);
      g[v].emplace_back(E.size() - 1);
    }
  }

  bool BFS(int S, int T) {
    queue<int> q({S});
    fill(d.begin(), d.end(), N + 1);
    d[S] = 0;
    while(!q.empty()) {
      int u = q.front(); q.pop();
      if (u == T) break;
      for (int k: g[u]) {
        Edge &e = E[k];
        if (e.flow < e.cap && d[e.v] > d[e.u] + 1) {
          d[e.v] = d[e.u] + 1;
          q.emplace(e.v);
        }
      }
    }
    return d[T] != N + 1;
  }

  LL DFS(int u, int T, LL flow = -1) {
    if (u == T || flow == 0) return flow;
    for (int &i = pt[u]; i < g[u].size(); ++i) {
      Edge &e = E[g[u][i]];
      Edge &oe = E[g[u][i]^1];
      if (d[e.v] == d[e.u] + 1) {
        LL amt = e.cap - e.flow;
        if (flow != -1 && amt > flow) amt = flow;
        if (LL pushed = DFS(e.v, T, amt)) {
          e.flow += pushed;
          oe.flow -= pushed;
          return pushed;
        }
      }
    }
    return 0;
  }

  LL MaxFlow(int S, int T) {
    LL total = 0;
    while (BFS(S, T)) {
      fill(pt.begin(), pt.end(), 0);
      while (LL flow = DFS(S, T))
        total += flow;
    }
    return total;
  }
};

// BEGIN CUT
// The following code solves SPOJ problem #4110: Fast Maximum Flow (
   FASTFLOW)

int main()
{
  int N, E;
  scanf("%d%d", &N, &E);
  Dinic dinic(N);
  for(int i = 0; i < E; i++)
  {
    int u, v;
    LL cap;
    scanf("%d%d%lld", &u, &v, &cap);
    dinic.AddEdge(u - 1, v - 1, cap);
    dinic.AddEdge(v - 1, u - 1, cap);
  }
  printf("%lld\n", dinic.MaxFlow(0, N - 1));
  return 0;
}

// END CUT
```

1.3 Min-cost max-flow

```
// Implementation of min cost max flow algorithm using adjacency
// matrix (Edmonds and Karp 1972).  This implementation keeps track of
// forward and reverse edges separately (so you can set cap[i][j] !=
// cap[j][i]).  For a regular max flow, set all edge costs to 0.
//
// Running time, O(|V|^2) cost per augmentation
//     max flow:           O(|V|^3) augmentations
//     min cost max flow:  O(|V|^4 * MAX_EDGE_COST) augmentations
//
// INPUT:
//     - graph, constructed using AddEdge()
//     - source
//     - sink
//
// OUTPUT:
//     - (maximum flow value, minimum cost value)
//     - To obtain the actual flow, look at positive values only.

#include <cmath>
#include <vector>
#include <iostream>

using namespace std;

typedef vector<int> VI;
typedef vector<VI> VVI;
typedef long long L;
```

```
typedef vector<L> VL;
typedef vector<VL> VVL;
typedef pair<int, int> PII;
typedef vector<PII> VPII;

const L INF = numeric_limits<L>::max() / 4;

struct MinCostMaxFlow {
  int N;
  VVL cap, flow, cost;
  VI found;
  VL dist, pi, width;
  VPII dad;

  MinCostMaxFlow(int N) :
    N(N), cap(N, VL(N)), flow(N, VL(N)), cost(N, VL(N)),
    found(N), dist(N), pi(N), width(N), dad(N) {}

  void AddEdge(int from, int to, L cap, L cost) {
    this->cap[from][to] = cap;
    this->cost[from][to] = cost;
  }

  void Relax(int s, int k, L cap, L cost, int dir) {
    L val = dist[s] + pi[s] - pi[k] + cost;
    if (cap && val < dist[k]) {
      dist[k] = val;
      dad[k] = make_pair(s, dir);
      width[k] = min(cap, width[s]);
    }
  }

  L Dijkstra(int s, int t) {
    fill(found.begin(), found.end(), false);
    fill(dist.begin(), dist.end(), INF);
    fill(width.begin(), width.end(), 0);
    dist[s] = 0;
    width[s] = INF;

    while (s != -1) {
      int best = -1;
      found[s] = true;
      for (int k = 0; k < N; k++) {
        if (found[k]) continue;
        Relax(s, k, cap[s][k] - flow[s][k], cost[s][k], 1);
        Relax(s, k, flow[k][s], -cost[k][s], -1);
        if (best == -1 || dist[k] < dist[best]) best = k;
      }
      s = best;
    }

    for (int k = 0; k < N; k++)
      pi[k] = min(pi[k] + dist[k], INF);
    return width[t];
  }

  pair<L, L> GetMaxFlow(int s, int t) {
    L totflow = 0, totcost = 0;
    while (L amt = Dijkstra(s, t)) {
      totflow += amt;
      for (int x = t; x != s; x = dad[x].first) {
        if (dad[x].second == 1) {
          flow[dad[x].first][x] += amt;
          totcost += amt * cost[dad[x].first][x];
        } else {
          flow[x][dad[x].first] -= amt;
          totcost -= amt * cost[x][dad[x].first];
        }
      }
    }
    return make_pair(totflow, totcost);
  }
};

// BEGIN CUT
// The following code solves UVA problem #10594: Data Flow

int main() {
  int N, M;

  while (scanf("%d%d", &N, &M) == 2) {
    VVL v(M, VL(3));
    for (int i = 0; i < M; i++)
      scanf("%Ld%Ld%Ld", &v[i][0], &v[i][1], &v[i][2]);
    L D, K;
    scanf("%Ld%Ld", &D, &K);

    MinCostMaxFlow mcmf(N+1);
    for (int i = 0; i < M; i++) {
      mcmf.AddEdge(int(v[i][0]), int(v[i][1]), K, v[i][2]);
      mcmf.AddEdge(int(v[i][1]), int(v[i][0]), K, v[i][2]);
    }
    mcmf.AddEdge(0, 1, D, 0);

    pair<L, L> res = mcmf.GetMaxFlow(0, N);

    if (res.first == D) {
      printf("%Ld\n", res.second);
    } else {
      printf("Impossible.\n");
    }
  }

  return 0;
}

// END CUT
```

1.4 Push-relabel max-flow

```
// Adjacency list implementation of FIFO push relabel maximum flow
// with the gap relabeling heuristic.  This implementation is
// significantly faster than straight Ford-Fulkerson.  It solves
// random problems with 10000 vertices and 1000000 edges in a few
// seconds, though it is possible to construct test cases that
// achieve the worst-case.
//
// Running time:
//     O(|V|^3)
//
// INPUT:
//     - graph, constructed using AddEdge()
//     - source
//     - sink
//
// OUTPUT:
//     - maximum flow value
//     - To obtain the actual flow values, look at all edges with
//       capacity > 0 (zero capacity edges are residual edges).

#include <cmath>
#include <vector>
#include <iostream>
#include <queue>

using namespace std;

typedef long long LL;

struct Edge {
  int from, to, cap, flow, index;
```

```cpp
  Edge(int from, int to, int cap, int flow, int index) :
    from(from), to(to), cap(cap), flow(flow), index(index) {}
};

struct PushRelabel {
  int N;
  vector<vector<Edge> > G;
  vector<LL> excess;
  vector<int> dist, active, count;
  queue<int> Q;

  PushRelabel(int N) : N(N), G(N), excess(N), dist(N), active(N),
      count(2*N) {}

  void AddEdge(int from, int to, int cap) {
    G[from].push_back(Edge(from, to, cap, 0, G[to].size()));
    if (from == to) G[from].back().index++;
    G[to].push_back(Edge(to, from, 0, 0, G[from].size() - 1));
  }

  void Enqueue(int v) {
    if (!active[v] && excess[v] > 0) { active[v] = true; Q.push(v); }
  }

  void Push(Edge &e) {
    int amt = int(min(excess[e.from], LL(e.cap - e.flow)));
    if (dist[e.from] <= dist[e.to] || amt == 0) return;
    e.flow += amt;
    G[e.to][e.index].flow -= amt;
    excess[e.to] += amt;
    excess[e.from] -= amt;
    Enqueue(e.to);
  }

  void Gap(int k) {
    for (int v = 0; v < N; v++) {
      if (dist[v] < k) continue;
      count[dist[v]]--;
      dist[v] = max(dist[v], N+1);
      count[dist[v]]++;
      Enqueue(v);
    }
  }

  void Relabel(int v) {
    count[dist[v]]--;
    dist[v] = 2*N;
    for (int i = 0; i < G[v].size(); i++)
      if (G[v][i].cap - G[v][i].flow > 0)
        dist[v] = min(dist[v], dist[G[v][i].to] + 1);
    count[dist[v]]++;
    Enqueue(v);
  }

  void Discharge(int v) {
    for (int i = 0; excess[v] > 0 && i < G[v].size(); i++) Push(G[v][i
        ]);
    if (excess[v] > 0) {
      if (count[dist[v]] == 1)
        Gap(dist[v]);
      else
        Relabel(v);
    }
  }

  LL GetMaxFlow(int s, int t) {
    count[0] = N-1;
    count[N] = 1;
    dist[s] = N;
    active[s] = active[t] = true;
    for (int i = 0; i < G[s].size(); i++) {
      excess[s] += G[s][i].cap;
      Push(G[s][i]);
    }

    while (!Q.empty()) {
      int v = Q.front();
      Q.pop();
      active[v] = false;
      Discharge(v);
    }

    LL totflow = 0;
    for (int i = 0; i < G[s].size(); i++) totflow += G[s][i].flow;
    return totflow;
  }
};

// BEGIN CUT
// The following code solves SPOJ problem #4110: Fast Maximum Flow (
    FASTFLOW)

int main() {
  int n, m;
  scanf("%d%d", &n, &m);

  PushRelabel pr(n);
  for (int i = 0; i < m; i++) {
   int a, b, c;
    scanf("%d%d%d", &a, &b, &c);
    if (a == b) continue;
    pr.AddEdge(a-1, b-1, c);
    pr.AddEdge(b-1, a-1, c);
  }
  printf("%Ld\n", pr.GetMaxFlow(0, n-1));
  return 0;
}

// END CUT
```

1.5 Min-cost matching

```cpp
///////////////////////////////////////////////////////////////////////
// Min cost bipartite matching via shortest augmenting paths
//
// This is an O(n^3) implementation of a shortest augmenting path
// algorithm for finding min cost perfect matchings in dense
// graphs.  In practice, it solves 1000x1000 problems in around 1
// second.
//
//   cost[i][j] = cost for pairing left node i with right node j
//   Lmate[i] = index of right node that left node i pairs with
//   Rmate[j] = index of left node that right node j pairs with
//
// The values in cost[i][j] may be positive or negative.  To perform
// maximization, simply negate the cost[][] matrix.
///////////////////////////////////////////////////////////////////////

#include <algorithm>
#include <cstdio>
#include <cmath>
#include <vector>

using namespace std;

typedef vector<double> VD;
typedef vector<VD> VVD;
typedef vector<int> VI;

double MinCostMatching(const VVD &cost, VI &Lmate, VI &Rmate) {
```

```
  int n = int(cost.size());

  // construct dual feasible solution
  VD u(n);
  VD v(n);
  for (int i = 0; i < n; i++) {
    u[i] = cost[i][0];
    for (int j = 1; j < n; j++) u[i] = min(u[i], cost[i][j]);
  }
  for (int j = 0; j < n; j++) {
    v[j] = cost[0][j] - u[0];
    for (int i = 1; i < n; i++) v[j] = min(v[j], cost[i][j] - u[i]);
  }

  // construct primal solution satisfying complementary slackness
  Lmate = VI(n, -1);
  Rmate = VI(n, -1);
  int mated = 0;
  for (int i = 0; i < n; i++) {
    for (int j = 0; j < n; j++) {
      if (Rmate[j] != -1) continue;
      if (fabs(cost[i][j] - u[i] - v[j]) < 1e-10) {
        Lmate[i] = j;
        Rmate[j] = i;
        mated++;
        break;
      }
    }
  }

  VD dist(n);
  VI dad(n);
  VI seen(n);

  // repeat until primal solution is feasible
  while (mated < n) {

    // find an unmatched left node
    int s = 0;
    while (Lmate[s] != -1) s++;

    // initialize Dijkstra
    fill(dad.begin(), dad.end(), -1);
    fill(seen.begin(), seen.end(), 0);
    for (int k = 0; k < n; k++)
      dist[k] = cost[s][k] - u[s] - v[k];

    int j = 0;
    while (true) {

      // find closest
      j = -1;
      for (int k = 0; k < n; k++) {
        if (seen[k]) continue;
        if (j == -1 || dist[k] < dist[j]) j = k;
      }
      seen[j] = 1;

      // termination condition
      if (Rmate[j] == -1) break;

      // relax neighbors
      const int i = Rmate[j];
      for (int k = 0; k < n; k++) {
        if (seen[k]) continue;
        const double new_dist = dist[j] + cost[i][k] - u[i] - v[k];
        if (dist[k] > new_dist) {
          dist[k] = new_dist;
          dad[k] = j;
        }
      }
    }

    // update dual variables
    for (int k = 0; k < n; k++) {
      if (k == j || !seen[k]) continue;
      const int i = Rmate[k];
      v[k] += dist[k] - dist[j];
      u[i] -= dist[k] - dist[j];
    }
    u[s] += dist[j];

    // augment along path
    while (dad[j] >= 0) {
      const int d = dad[j];
      Rmate[j] = Rmate[d];
      Lmate[Rmate[j]] = j;
      j = d;
    }
    Rmate[j] = s;
    Lmate[s] = j;

    mated++;
  }

  double value = 0;
  for (int i = 0; i < n; i++)
    value += cost[i][Lmate[i]];

  return value;
}
```

1.6 Max bipartite matchine

```
// This code performs maximum bipartite matching.
//
// Running time: O(|E| |V|) -- often much faster in practice
//
//   INPUT: w[i][j] = edge between row node i and column node j
//   OUTPUT: mr[i] = assignment for row node i, -1 if unassigned
//           mc[j] = assignment for column node j, -1 if unassigned
//           function returns number of matches made

#include <vector>

using namespace std;

typedef vector<int> VI;
typedef vector<VI> VVI;

bool FindMatch(int i, const VVI &w, VI &mr, VI &mc, VI &seen) {
  for (int j = 0; j < w[i].size(); j++) {
    if (w[i][j] && !seen[j]) {
      seen[j] = true;
      if (mc[j] < 0 || FindMatch(mc[j], w, mr, mc, seen)) {
        mr[i] = j;
        mc[j] = i;
        return true;
      }
    }
  }
  return false;
}

int BipartiteMatching(const VVI &w, VI &mr, VI &mc) {
  mr = VI(w.size(), -1);
  mc = VI(w[0].size(), -1);

  int ct = 0;
  for (int i = 0; i < w.size(); i++) {
```

```
    VI seen(w[0].size());
    if (FindMatch(i, w, mr, mc, seen)) ct++;
  }
  return ct;
}
```

1.7 Global min-cut

```
// Adjacency matrix implementation of Stoer-Wagner min cut algorithm.
//
// Running time:
//     O(|V|^3)
//
// INPUT:
//     - graph, constructed using AddEdge()
//
// OUTPUT:
//     - (min cut value, nodes in half of min cut)

#include <cmath>
#include <vector>
#include <iostream>

using namespace std;

typedef vector<int> VI;
typedef vector<VI> VVI;

const int INF = 1000000000;

pair<int, VI> GetMinCut(VVI &weights) {
  int N = weights.size();
  VI used(N), cut, best_cut;
  int best_weight = -1;

  for (int phase = N-1; phase >= 0; phase--) {
    VI w = weights[0];
    VI added = used;
    int prev, last = 0;
    for (int i = 0; i < phase; i++) {
      prev = last;
      last = -1;
      for (int j = 1; j < N; j++)
        if (!added[j] && (last == -1 || w[j] > w[last])) last = j;
      if (i == phase-1) {
        for (int j = 0; j < N; j++) weights[prev][j] += weights[last][
            j];
        for (int j = 0; j < N; j++) weights[j][prev] = weights[prev][j
            ];
        used[last] = true;
        cut.push_back(last);
        if (best_weight == -1 || w[last] < best_weight) {
          best_cut = cut;
          best_weight = w[last];
        }
      } else {
        for (int j = 0; j < N; j++)
          w[j] += weights[last][j];
        added[last] = true;
      }
    }
  }
  return make_pair(best_weight, best_cut);
}

// BEGIN CUT
// The following code solves UVA problem #10989: Bomb, Divide and
    Conquer
int main() {
  int N;
  cin >> N;
  for (int i = 0; i < N; i++) {
    int n, m;
    cin >> n >> m;
    VVI weights(n, VI(n));
    for (int j = 0; j < m; j++) {
      int a, b, c;
      cin >> a >> b >> c;
      weights[a-1][b-1] = weights[b-1][a-1] = c;
    }
    pair<int, VI> res = GetMinCut(weights);
    cout << "Case #" << i+1 << ": " << res.first << endl;
  }
}
// END CUT
```

1.8 Graph cut inference

```
// Special-purpose {0,1} combinatorial optimization solver for
// problems of the following by a reduction to graph cuts:
//
//        minimize         sum_i  psi_i(x[i])
//  x[1]...x[n] in {0,1}      + sum_{i < j}  phi_{ij}(x[i], x[j])
//
// where
//      psi_i : {0, 1} --> R
//   phi_{ij} : {0, 1} x {0, 1} --> R
//
// such that
//   phi_{ij}(0,0) + phi_{ij}(1,1) <= phi_{ij}(0,1) + phi_{ij}(1,0)
    (*)
//
// This can also be used to solve maximization problems where the
// direction of the inequality in (*) is reversed.
//
// INPUT: phi -- a matrix such that phi[i][j][u][v] = phi_{ij}(u, v)
//        psi -- a matrix such that psi[i][u] = psi_i(u)
//        x -- a vector where the optimal solution will be stored
//
// OUTPUT: value of the optimal solution
//
// To use this code, create a GraphCutInference object, and call the
// DoInference() method.  To perform maximization instead of
    minimization,
// ensure that #define MAXIMIZATION is enabled.

#include <vector>
#include <iostream>

using namespace std;

typedef vector<int> VI;
typedef vector<VI> VVI;
typedef vector<VVI> VVVI;
typedef vector<VVVI> VVVVI;

const int INF = 1000000000;

// comment out following line for minimization
#define MAXIMIZATION

struct GraphCutInference {
  int N;
  VVI cap, flow;
  VI reached;

  int Augment(int s, int t, int a) {
```

```
    reached[s] = 1;
    if (s == t) return a;
    for (int k = 0; k < N; k++) {
      if (reached[k]) continue;
      if (int aa = min(a, cap[s][k] - flow[s][k])) {
        if (int b = Augment(k, t, aa)) {
          flow[s][k] += b;
          flow[k][s] -= b;
          return b;
        }
      }
    }
    return 0;
  }

  int GetMaxFlow(int s, int t) {
    N = cap.size();
    flow = VVI(N, VI(N));
    reached = VI(N);

    int totflow = 0;
    while (int amt = Augment(s, t, INF)) {
      totflow += amt;
      fill(reached.begin(), reached.end(), 0);
    }
    return totflow;
  }

  int DoInference(const VVVVI &phi, const VVI &psi, VI &x) {
    int M = phi.size();
    cap = VVI(M+2, VI(M+2));
    VI b(M);
    int c = 0;

    for (int i = 0; i < M; i++) {
      b[i] += psi[i][1] - psi[i][0];
      c += psi[i][0];
      for (int j = 0; j < i; j++)
        b[i] += phi[i][j][1][1] - phi[i][j][0][1];
      for (int j = i+1; j < M; j++) {
        cap[i][j] = phi[i][j][0][1] + phi[i][j][1][0] - phi[i][j
            ][0][0] - phi[i][j][1][1];
        b[i] += phi[i][j][1][0] - phi[i][j][0][0];
        c += phi[i][j][0][0];
      }
    }

#ifdef MAXIMIZATION
    for (int i = 0; i < M; i++) {
      for (int j = i+1; j < M; j++)
        cap[i][j] *= -1;
      b[i] *= -1;
    }
    c *= -1;
#endif

    for (int i = 0; i < M; i++) {
      if (b[i] >= 0) {
        cap[M][i] = b[i];
      } else {
        cap[i][M+1] = -b[i];
        c += b[i];
      }
    }

    int score = GetMaxFlow(M, M+1);
    fill(reached.begin(), reached.end(), 0);
    Augment(M, M+1, INF);
    x = VI(M);
    for (int i = 0; i < M; i++) x[i] = reached[i] ? 0 : 1;
    score += c;
#ifdef MAXIMIZATION
    score *= -1;
#endif

    return score;
  }

};

int main() {

  // solver for "Cat vs. Dog" from NWERC 2008

  int numcases;
  cin >> numcases;
  for (int caseno = 0; caseno < numcases; caseno++) {
    int c, d, v;
    cin >> c >> d >> v;

    VVVVI phi(c+d, VVVI(c+d, VVI(2, VI(2))));
    VVI psi(c+d, VI(2));
    for (int i = 0; i < v; i++) {
      char p, q;
      int u, v;
      cin >> p >> u >> q >> v;
      u--; v--;
      if (p == 'C') {
        phi[u][c+v][0][0]++;
        phi[c+v][u][0][0]++;
      } else {
        phi[v][c+u][1][1]++;
        phi[c+u][v][1][1]++;
      }
    }

    GraphCutInference graph;
    VI x;
    cout << graph.DoInference(phi, psi, x) << endl;
  }

  return 0;
}
```

2 Geometry

2.1 Convex hull

```
// Compute the 2D convex hull of a set of points using the monotone
    chain
// algorithm.  Eliminate redundant points from the hull if
    REMOVE_REDUNDANT is
// #defined.
//
// Running time: O(n log n)
//
//   INPUT:   a vector of input points, unordered.
//   OUTPUT:  a vector of points in the convex hull, counterclockwise,
     starting
//            with bottommost/leftmost point

#include <cstdio>
#include <cassert>
#include <vector>
#include <algorithm>
#include <cmath>
// BEGIN CUT
#include <map>
// END CUT

using namespace std;

#define REMOVE_REDUNDANT

typedef double T;
const T EPS = 1e-7;
struct PT {
  T x, y;
  PT() {}
  PT(T x, T y) : x(x), y(y) {}
  bool operator<(const PT &rhs) const { return make_pair(y,x) <
      make_pair(rhs.y,rhs.x); }
  bool operator==(const PT &rhs) const { return make_pair(y,x) ==
      make_pair(rhs.y,rhs.x); }
};

T cross(PT p, PT q) { return p.x*q.y-p.y*q.x; }
T area2(PT a, PT b, PT c) { return cross(a,b) + cross(b,c) + cross(c,a
    ); }

#ifdef REMOVE_REDUNDANT
bool between(const PT &a, const PT &b, const PT &c) {
  return (fabs(area2(a,b,c)) < EPS && (a.x-b.x)*(c.x-b.x) <= 0 && (a.y
      -b.y)*(c.y-b.y) <= 0);
}
#endif

void ConvexHull(vector<PT> &pts) {
  sort(pts.begin(), pts.end());
  pts.erase(unique(pts.begin(), pts.end()), pts.end());
  vector<PT> up, dn;
  for (int i = 0; i < pts.size(); i++) {
    while (up.size() > 1 && area2(up[up.size()-2], up.back(), pts[i])
        >= 0) up.pop_back();
    while (dn.size() > 1 && area2(dn[dn.size()-2], dn.back(), pts[i])
        <= 0) dn.pop_back();
    up.push_back(pts[i]);
    dn.push_back(pts[i]);
  }
  pts = dn;
  for (int i = (int) up.size() - 2; i >= 1; i--) pts.push_back(up[i]);

#ifdef REMOVE_REDUNDANT
  if (pts.size() <= 2) return;
  dn.clear();
  dn.push_back(pts[0]);
  dn.push_back(pts[1]);
  for (int i = 2; i < pts.size(); i++) {
    if (between(dn[dn.size()-2], dn[dn.size()-1], pts[i])) dn.pop_back
        ();
    dn.push_back(pts[i]);
  }
  if (dn.size() >= 3 && between(dn.back(), dn[0], dn[1])) {
    dn[0] = dn.back();
    dn.pop_back();
  }
  pts = dn;
#endif
}

// BEGIN CUT
// The following code solves SPOJ problem #26: Build the Fence (BSHEEP
    )

int main() {
  int t;
  scanf("%d", &t);
  for (int caseno = 0; caseno < t; caseno++) {
    int n;
    scanf("%d", &n);
    vector<PT> v(n);
    for (int i = 0; i < n; i++) scanf("%lf%lf", &v[i].x, &v[i].y);
    vector<PT> h(v);
    map<PT,int> index;
    for (int i = n-1; i >= 0; i--) index[v[i]] = i+1;
    ConvexHull(h);

    double len = 0;
    for (int i = 0; i < h.size(); i++) {
      double dx = h[i].x - h[(i+1)%h.size()].x;
      double dy = h[i].y - h[(i+1)%h.size()].y;
      len += sqrt(dx*dx+dy*dy);
    }

    if (caseno > 0) printf("\n");
    printf("%.2f\n", len);
    for (int i = 0; i < h.size(); i++) {
      if (i > 0) printf(" ");
      printf("%d", index[h[i]]);
    }
    printf("\n");
  }
}

// END CUT
```

2.2 Miscellaneous geometry

```
// C++ routines for computational geometry.

#include <iostream>
#include <vector>
#include <cmath>
#include <cassert>

using namespace std;

double INF = 1e100;
double EPS = 1e-12;

struct PT {
  double x, y;
  PT() {}
```

```
  PT(double x, double y) : x(x), y(y) {}
  PT(const PT &p) : x(p.x), y(p.y)    {}
  PT operator + (const PT &p)  const { return PT(x+p.x, y+p.y); }
  PT operator - (const PT &p)  const { return PT(x-p.x, y-p.y); }
  PT operator * (double c)     const { return PT(x*c,   y*c  ); }
  PT operator / (double c)     const { return PT(x/c,   y/c  ); }
};

double dot(PT p, PT q)     { return p.x*q.x+p.y*q.y; }
double dist2(PT p, PT q)   { return dot(p-q,p-q); }
double cross(PT p, PT q)   { return p.x*q.y-p.y*q.x; }
ostream &operator<<(ostream &os, const PT &p) {
  return os << "(" << p.x << "," << p.y << ")";
}

// rotate a point CCW or CW around the origin
PT RotateCCW90(PT p)   { return PT(-p.y,p.x); }
PT RotateCW90(PT p)    { return PT(p.y,-p.x); }
PT RotateCCW(PT p, double t) {
  return PT(p.x*cos(t)-p.y*sin(t), p.x*sin(t)+p.y*cos(t));
}

// project point c onto line through a and b
// assuming a != b
PT ProjectPointLine(PT a, PT b, PT c) {
  return a + (b-a)*dot(c-a, b-a)/dot(b-a, b-a);
}

// project point c onto line segment through a and b
PT ProjectPointSegment(PT a, PT b, PT c) {
  double r = dot(b-a,b-a);
  if (fabs(r) < EPS) return a;
  r = dot(c-a, b-a)/r;
  if (r < 0) return a;
  if (r > 1) return b;
  return a + (b-a)*r;
}

// compute distance from c to segment between a and b
double DistancePointSegment(PT a, PT b, PT c) {
  return sqrt(dist2(c, ProjectPointSegment(a, b, c)));
}

// compute distance between point (x,y,z) and plane ax+by+cz=d
double DistancePointPlane(double x, double y, double z,
                          double a, double b, double c, double d)
{
  return fabs(a*x+b*y+c*z-d)/sqrt(a*a+b*b+c*c);
}

// determine if lines from a to b and c to d are parallel or collinear
bool LinesParallel(PT a, PT b, PT c, PT d) {
  return fabs(cross(b-a, c-d)) < EPS;
}

bool LinesCollinear(PT a, PT b, PT c, PT d) {
  return LinesParallel(a, b, c, d)
      && fabs(cross(a-b, a-c)) < EPS
      && fabs(cross(c-d, c-a)) < EPS;
}

// determine if line segment from a to b intersects with
// line segment from c to d
bool SegmentsIntersect(PT a, PT b, PT c, PT d) {
  if (LinesCollinear(a, b, c, d)) {
    if (dist2(a, c) < EPS || dist2(a, d) < EPS ||
      dist2(b, c) < EPS || dist2(b, d) < EPS) return true;
    if (dot(c-a, c-b) > 0 && dot(d-a, d-b) > 0 && dot(c-b, d-b) > 0)
      return false;
    return true;
  }
  if (cross(d-a, b-a) * cross(c-a, b-a) > 0) return false;
  if (cross(a-c, d-c) * cross(b-c, d-c) > 0) return false;
  return true;
}

// compute intersection of line passing through a and b
// with line passing through c and d, assuming that unique
// intersection exists; for segment intersection, check if
// segments intersect first
PT ComputeLineIntersection(PT a, PT b, PT c, PT d) {
  b=b-a; d=c-d; c=c-a;
  assert(dot(b, b) > EPS && dot(d, d) > EPS);
  return a + b*cross(c, d)/cross(b, d);
}

// compute center of circle given three points
PT ComputeCircleCenter(PT a, PT b, PT c) {
  b=(a+b)/2;
  c=(a+c)/2;
  return ComputeLineIntersection(b, b+RotateCW90(a-b), c, c+RotateCW90
      (a-c));
}

// determine if point is in a possibly non-convex polygon (by William
// Randolph Franklin); returns 1 for strictly interior points, 0 for
// strictly exterior points, and 0 or 1 for the remaining points.
// Note that it is possible to convert this into an *exact* test using
// integer arithmetic by taking care of the division appropriately
// (making sure to deal with signs properly) and then by writing exact
// tests for checking point on polygon boundary
bool PointInPolygon(const vector<PT> &p, PT q) {
  bool c = 0;
  for (int i = 0; i < p.size(); i++){
    int j = (i+1)%p.size();
    if ((p[i].y <= q.y && q.y < p[j].y ||
      p[j].y <= q.y && q.y < p[i].y) &&
      q.x < p[i].x + (p[j].x - p[i].x) * (q.y - p[i].y) / (p[j].y - p[
          i].y))
      c = !c;
  }
  return c;
}

// determine if point is on the boundary of a polygon
bool PointOnPolygon(const vector<PT> &p, PT q) {
  for (int i = 0; i < p.size(); i++)
    if (dist2(ProjectPointSegment(p[i], p[(i+1)%p.size()], q), q) <
        EPS)
      return true;
    return false;
}

// compute intersection of line through points a and b with
// circle centered at c with radius r > 0
vector<PT> CircleLineIntersection(PT a, PT b, PT c, double r) {
  vector<PT> ret;
  b = b-a;
  a = a-c;
  double A = dot(b, b);
  double B = dot(a, b);
  double C = dot(a, a) - r*r;
  double D = B*B - A*C;
  if (D < -EPS) return ret;
  ret.push_back(c+a+b*(-B+sqrt(D+EPS))/A);
  if (D > EPS)
    ret.push_back(c+a+b*(-B-sqrt(D))/A);
  return ret;
}

// compute intersection of circle centered at a with radius r
// with circle centered at b with radius R
vector<PT> CircleCircleIntersection(PT a, PT b, double r, double R) {
```

```
  vector<PT> ret;
  double d = sqrt(dist2(a, b));
  if (d > r+R || d+min(r, R) < max(r, R)) return ret;
  double x = (d*d-R*R+r*r)/(2*d);
  double y = sqrt(r*r-x*x);
  PT v = (b-a)/d;
  ret.push_back(a+v*x + RotateCCW90(v)*y);
  if (y > 0)
    ret.push_back(a+v*x - RotateCCW90(v)*y);
  return ret;
}

// This code computes the area or centroid of a (possibly nonconvex)
// polygon, assuming that the coordinates are listed in a clockwise or
// counterclockwise fashion.  Note that the centroid is often known as
// the "center of gravity" or "center of mass".
double ComputeSignedArea(const vector<PT> &p) {
  double area = 0;
  for(int i = 0; i < p.size(); i++) {
    int j = (i+1) % p.size();
    area += p[i].x*p[j].y - p[j].x*p[i].y;
  }
  return area / 2.0;
}

double ComputeArea(const vector<PT> &p) {
  return fabs(ComputeSignedArea(p));
}

PT ComputeCentroid(const vector<PT> &p) {
  PT c(0,0);
  double scale = 6.0 * ComputeSignedArea(p);
  for (int i = 0; i < p.size(); i++){
    int j = (i+1) % p.size();
    c = c + (p[i]+p[j])*(p[i].x*p[j].y - p[j].x*p[i].y);
  }
  return c / scale;
}

// tests whether or not a given polygon (in CW or CCW order) is simple
bool IsSimple(const vector<PT> &p) {
  for (int i = 0; i < p.size(); i++) {
    for (int k = i+1; k < p.size(); k++) {
      int j = (i+1) % p.size();
      int l = (k+1) % p.size();
      if (i == l || j == k) continue;
      if (SegmentsIntersect(p[i], p[j], p[k], p[l]))
        return false;
    }
  }
  return true;
}

int main() {

  // expected: (-5,2)
  cerr << RotateCCW90(PT(2,5)) << endl;

  // expected: (5,-2)
  cerr << RotateCW90(PT(2,5)) << endl;

  // expected: (-5,2)
  cerr << RotateCCW(PT(2,5),M_PI/2) << endl;

  // expected: (5,2)
  cerr << ProjectPointLine(PT(-5,-2), PT(10,4), PT(3,7)) << endl;

  // expected: (5,2) (7.5,3) (2.5,1)
  cerr << ProjectPointSegment(PT(-5,-2), PT(10,4), PT(3,7)) << " "
       << ProjectPointSegment(PT(7.5,3), PT(10,4), PT(3,7)) << " "
       << ProjectPointSegment(PT(-5,-2), PT(2.5,1), PT(3,7)) << endl;

  // expected: 6.78903
  cerr << DistancePointPlane(4,-4,3,2,-2,5,-8) << endl;

  // expected: 1 0 1
  cerr << LinesParallel(PT(1,1), PT(3,5), PT(2,1), PT(4,5)) << " "
       << LinesParallel(PT(1,1), PT(3,5), PT(2,0), PT(4,5)) << " "
       << LinesParallel(PT(1,1), PT(3,5), PT(5,9), PT(7,13)) << endl;

  // expected: 0 0 1
  cerr << LinesCollinear(PT(1,1), PT(3,5), PT(2,1), PT(4,5)) << " "
       << LinesCollinear(PT(1,1), PT(3,5), PT(2,0), PT(4,5)) << " "
       << LinesCollinear(PT(1,1), PT(3,5), PT(5,9), PT(7,13)) << endl;

  // expected: 1 1 1 0
  cerr << SegmentsIntersect(PT(0,0), PT(2,4), PT(3,1), PT(-1,3)) << "
    "
       << SegmentsIntersect(PT(0,0), PT(2,4), PT(4,3), PT(0,5)) << " "
       << SegmentsIntersect(PT(0,0), PT(2,4), PT(2,-1), PT(-2,1)) << "
           "
       << SegmentsIntersect(PT(0,0), PT(2,4), PT(5,5), PT(1,7)) <<
           endl;

  // expected: (1,2)
  cerr << ComputeLineIntersection(PT(0,0), PT(2,4), PT(3,1), PT(-1,3))
       << endl;

  // expected: (1,1)
  cerr << ComputeCircleCenter(PT(-3,4), PT(6,1), PT(4,5)) << endl;

  vector<PT> v;
  v.push_back(PT(0,0));
  v.push_back(PT(5,0));
  v.push_back(PT(5,5));
  v.push_back(PT(0,5));

  // expected: 1 1 1 0 0
  cerr << PointInPolygon(v, PT(2,2)) << " "
       << PointInPolygon(v, PT(2,0)) << " "
       << PointInPolygon(v, PT(0,2)) << " "
       << PointInPolygon(v, PT(5,2)) << " "
       << PointInPolygon(v, PT(2,5)) << endl;

  // expected: 0 1 1 1 1
  cerr << PointOnPolygon(v, PT(2,2)) << " "
       << PointOnPolygon(v, PT(2,0)) << " "
       << PointOnPolygon(v, PT(0,2)) << " "
       << PointOnPolygon(v, PT(5,2)) << " "
       << PointOnPolygon(v, PT(2,5)) << endl;

  // expected: (1,6)
  //           (5,4) (4,5)
  //           blank line
  //           (4,5) (5,4)
  //           blank line
  //           (4,5) (5,4)
  vector<PT> u = CircleLineIntersection(PT(0,6), PT(2,6), PT(1,1), 5);
  for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl
    ;
  u = CircleLineIntersection(PT(0,9), PT(9,0), PT(1,1), 5);
  for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl
    ;
  u = CircleCircleIntersection(PT(1,1), PT(10,10), 5, 5);
  for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl
    ;
  u = CircleCircleIntersection(PT(1,1), PT(8,8), 5, 5);
  for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl
    ;
  u = CircleCircleIntersection(PT(1,1), PT(4.5,4.5), 10, sqrt(2.0)
    /2.0);
  for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl
    ;
  u = CircleCircleIntersection(PT(1,1), PT(4.5,4.5), 5, sqrt(2.0)/2.0)
    ;
```

```
    for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl
        ;

    // area should be 5.0
    // centroid should be (1.1666666, 1.166666)
    PT pa[] = { PT(0,0), PT(5,0), PT(1,1), PT(0,5) };
    vector<PT> p(pa, pa+4);
    PT c = ComputeCentroid(p);
    cerr << "Area: " << ComputeArea(p) << endl;
    cerr << "Centroid: " << c << endl;

    return 0;
}
```

2.3 Latitude/longitude

```
/*
Converts from rectangular coordinates to latitude/longitude and vice
versa. Uses degrees (not radians).
*/

#include <iostream>
#include <cmath>

using namespace std;

struct ll
{
  double r, lat, lon;
};

struct rect
{
  double x, y, z;
};

ll convert(rect& P)
{
  ll Q;
  Q.r = sqrt(P.x*P.x+P.y*P.y+P.z*P.z);
  Q.lat = 180/M_PI*asin(P.z/Q.r);
  Q.lon = 180/M_PI*acos(P.x/sqrt(P.x*P.x+P.y*P.y));

  return Q;
}

rect convert(ll& Q)
{
  rect P;
  P.x = Q.r*cos(Q.lon*M_PI/180)*cos(Q.lat*M_PI/180);
  P.y = Q.r*sin(Q.lon*M_PI/180)*cos(Q.lat*M_PI/180);
  P.z = Q.r*sin(Q.lat*M_PI/180);

  return P;
}

int main()
{
  rect A;
  ll B;

  A.x = -1.0; A.y = 2.0; A.z = -3.0;

  B = convert(A);
  cout << B.r << " " << B.lat << " " << B.lon << endl;

  A = convert(B);
  cout << A.x << " " << A.y << " " << A.z << endl;
}
```

2.4 3D geometry

```
public class Geom3D {
  // distance from point (x, y, z) to plane aX + bY + cZ + d = 0
  public static double ptPlaneDist(double x, double y, double z,
      double a, double b, double c, double d) {
    return Math.abs(a*x + b*y + c*z + d) / Math.sqrt(a*a + b*b + c*c);
  }

  // distance between parallel planes aX + bY + cZ + d1 = 0 and
  // aX + bY + cZ + d2 = 0
  public static double planePlaneDist(double a, double b, double c,
      double d1, double d2) {
    return Math.abs(d1 - d2) / Math.sqrt(a*a + b*b + c*c);
  }

  // distance from point (px, py, pz) to line (x1, y1, z1)-(x2, y2, z2
      )
  // (or ray, or segment; in the case of the ray, the endpoint is the
  // first point)
  public static final int LINE = 0;
  public static final int SEGMENT = 1;
  public static final int RAY = 2;
  public static double ptLineDistSq(double x1, double y1, double z1,
      double x2, double y2, double z2, double px, double py, double pz
      ,
      int type) {
    double pd2 = (x1-x2)*(x1-x2) + (y1-y2)*(y1-y2) + (z1-z2)*(z1-z2);

    double x, y, z;
    if (pd2 == 0) {
      x = x1;
      y = y1;
      z = z1;
    } else {
      double u = ((px-x1)*(x2-x1) + (py-y1)*(y2-y1) + (pz-z1)*(z2-z1))
           / pd2;
      x = x1 + u * (x2 - x1);
      y = y1 + u * (y2 - y1);
      z = z1 + u * (z2 - z1);
      if (type != LINE && u < 0) {
        x = x1;
        y = y1;
        z = z1;
      }
      if (type == SEGMENT && u > 1.0) {
        x = x2;
        y = y2;
        z = z2;
      }
    }

    return (x-px)*(x-px) + (y-py)*(y-py) + (z-pz)*(z-pz);
  }

  public static double ptLineDist(double x1, double y1, double z1,
      double x2, double y2, double z2, double px, double py, double pz
      ,
      int type) {
    return Math.sqrt(ptLineDistSq(x1, y1, z1, x2, y2, z2, px, py, pz,
        type));
  }
}
```

2.5 Slow Delaunay triangulation

```
// Slow but simple Delaunay triangulation. Does not handle
```

```
// degenerate cases (from O'Rourke, Computational Geometry in C)
//
// Running time: O(n^4)
//
// INPUT:    x[] = x-coordinates
//           y[] = y-coordinates
//
// OUTPUT:   triples = a vector containing m triples of indices
//                     corresponding to triangle vertices

#include<vector>
using namespace std;

typedef double T;

struct triple {
    int i, j, k;
    triple() {}
    triple(int i, int j, int k) : i(i), j(j), k(k) {}
};

vector<triple> delaunayTriangulation(vector<T>& x, vector<T>& y) {
        int n = x.size();
        vector<T> z(n);
        vector<triple> ret;

        for (int i = 0; i < n; i++)
            z[i] = x[i] * x[i] + y[i] * y[i];

        for (int i = 0; i < n-2; i++) {
            for (int j = i+1; j < n; j++) {
                for (int k = i+1; k < n; k++) {
                    if (j == k) continue;
                    double xn = (y[j]-y[i])*(z[k]-z[i]) - (y[k]-y[i])
                        *(z[j]-z[i]);
                    double yn = (x[k]-x[i])*(z[j]-z[i]) - (x[j]-x[i])
                        *(z[k]-z[i]);
                    double zn = (x[j]-x[i])*(y[k]-y[i]) - (x[k]-x[i])
                        *(y[j]-y[i]);
                    bool flag = zn < 0;
                    for (int m = 0; flag && m < n; m++)
                        flag = flag && ((x[m]-x[i])*xn +
                                        (y[m]-y[i])*yn +
                                        (z[m]-z[i])*zn <= 0);
                    if (flag) ret.push_back(triple(i, j, k));
                }
            }
        }
        return ret;
}

int main()
{
    T xs[]={0, 0, 1, 0.9};
    T ys[]={0, 1, 0, 0.9};
    vector<T> x(&xs[0], &xs[4]), y(&ys[0], &ys[4]);
    vector<triple> tri = delaunayTriangulation(x, y);

    //expected: 0 1 3
    //          0 3 2

    int i;
    for(i = 0; i < tri.size(); i++)
        printf("%d %d %d\n", tri[i].i, tri[i].j, tri[i].k);
    return 0;
}
```

2.6 Java geometry

```
// In this example, we read an input file containing three lines, each
// containing an even number of doubles, separated by commas.  The
    first two
// lines represent the coordinates of two polygons, given in
    counterclockwise
// (or clockwise) order, which we will call "A" and "B".  The last
    line
// contains a list of points, p[1], p[2], ...
//
// Our goal is to determine:
//   (1) whether B - A is a single closed shape (as opposed to
    multiple shapes)
//   (2) the area of B - A
//   (3) whether each p[i] is in the interior of B - A
//
// INPUT:
//   0 0 10 0 0 10
//   0 0 10 10 10 0
//   8 6
//   5 1
//
// OUTPUT:
//   The area is singular.
//   The area is 25.0
//   Point belongs to the area.
//   Point does not belong to the area.

import java.util.*;
import java.awt.geom.*;
import java.io.*;

public class JavaGeometry {

    // make an array of doubles from a string
    static double[] readPoints(String s) {
        String[] arr = s.trim().split("\\s++");
        double[] ret = new double[arr.length];
        for (int i = 0; i < arr.length; i++) ret[i] = Double.
            parseDouble(arr[i]);
        return ret;
    }

    // make an Area object from the coordinates of a polygon
    static Area makeArea(double[] pts) {
        Path2D.Double p = new Path2D.Double();
        p.moveTo(pts[0], pts[1]);
        for (int i = 2; i < pts.length; i += 2) p.lineTo(pts[i], pts[i
            +1]);
        p.closePath();
        return new Area(p);
    }

    // compute area of polygon
    static double computePolygonArea(ArrayList<Point2D.Double> points)
         {
        Point2D.Double[] pts = points.toArray(new Point2D.Double[
            points.size()]);
        double area = 0;
        for (int i = 0; i < pts.length; i++){
            int j = (i+1) % pts.length;
            area += pts[i].x * pts[j].y - pts[j].x * pts[i].y;
        }
        return Math.abs(area)/2;
    }

    // compute the area of an Area object containing several disjoint
        polygons
    static double computeArea(Area area) {
        double totArea = 0;
        PathIterator iter = area.getPathIterator(null);
        ArrayList<Point2D.Double> points = new ArrayList<Point2D.
```

```java
            Double>();

        while (!iter.isDone()) {
            double[] buffer = new double[6];
            switch (iter.currentSegment(buffer)) {
            case PathIterator.SEG_MOVETO:
            case PathIterator.SEG_LINETO:
                points.add(new Point2D.Double(buffer[0], buffer[1]));
                break;
            case PathIterator.SEG_CLOSE:
                totArea += computePolygonArea(points);
                points.clear();
                break;
            }
            iter.next();
        }
        return totArea;
    }

    // notice that the main() throws an Exception -- necessary to
    // avoid wrapping the Scanner object for file reading in a
    // try { ... } catch block.
    public static void main(String args[]) throws Exception {

        Scanner scanner = new Scanner(new File("input.txt"));
        // also,
        //   Scanner scanner = new Scanner (System.in);

        double[] pointsA = readPoints(scanner.nextLine());
        double[] pointsB = readPoints(scanner.nextLine());
        Area areaA = makeArea(pointsA);
        Area areaB = makeArea(pointsB);
        areaB.subtract(areaA);
        // also,
        //   areaB.exclusiveOr (areaA);
        //   areaB.add (areaA);
        //   areaB.intersect (areaA);

        // (1) determine whether B - A is a single closed shape (as
        //     opposed to multiple shapes)
        boolean isSingle = areaB.isSingular();
        // also,
        //   areaB.isEmpty();

        if (isSingle)
            System.out.println("The area is singular.");
        else
            System.out.println("The area is not singular.");

        // (2) compute the area of B - A
        System.out.println("The area is " + computeArea(areaB) + ".");

        // (3) determine whether each p[i] is in the interior of B - A
        while (scanner.hasNextDouble()) {
            double x = scanner.nextDouble();
            assert(scanner.hasNextDouble());
            double y = scanner.nextDouble();

            if (areaB.contains(x,y)) {
                System.out.println ("Point belongs to the area.");
            } else {
                System.out.println ("Point does not belong to the area
                    .");
            }
        }

        // Finally, some useful things we didn't use in this example:
        //
        //   Ellipse2D.Double ellipse = new Ellipse2D.Double (double x
            , double y,
        //                                                    double w
            , double h);
        //
        //     creates an ellipse inscribed in box with bottom-left
            corner (x,y)
        //     and upper-right corner (x+y,w+h)
        //
        //   Rectangle2D.Double rect = new Rectangle2D.Double (double
            x, double y,
        //                                                    double
            w, double h);
        //
        //     creates a box with bottom-left corner (x,y) and upper-
            right
        //     corner (x+y,w+h)
        //
        // Each of these can be embedded in an Area object (e.g., new
            Area (rect)).

    }
}
```

3 Numerical algorithms

3.1 Number theory (modular, Chinese remainder, linear Diophantine)

```
// This is a collection of useful code for solving problems that
// involve modular linear equations.  Note that all of the
// algorithms described here work on nonnegative integers.

#include <iostream>
#include <vector>
#include <algorithm>

using namespace std;

typedef vector<int> VI;
typedef pair<int, int> PII;

// return a % b (positive value)
int mod(int a, int b) {
	return ((a%b) + b) % b;
}

// computes gcd(a,b)
int gcd(int a, int b) {
	while (b) { int t = a%b; a = b; b = t; }
	return a;
}

// computes lcm(a,b)
int lcm(int a, int b) {
	return a / gcd(a, b)*b;
}

// (a^b) mod m via successive squaring
int powermod(int a, int b, int m)
{
	int ret = 1;
	while (b)
	{
		if (b & 1) ret = mod(ret*a, m);
		a = mod(a*a, m);
		b >>= 1;
	}
	return ret;
}

// returns g = gcd(a, b); finds x, y such that d = ax + by
int extended_euclid(int a, int b, int &x, int &y) {
	int xx = y = 0;
	int yy = x = 1;
	while (b) {
		int q = a / b;
		int t = b; b = a%b; a = t;
		t = xx; xx = x - q*xx; x = t;
		t = yy; yy = y - q*yy; y = t;
	}
	return a;
}

// finds all solutions to ax = b (mod n)
VI modular_linear_equation_solver(int a, int b, int n) {
	int x, y;
	VI ret;
	int g = extended_euclid(a, n, x, y);
	if (!(b%g)) {
		x = mod(x*(b / g), n);
		for (int i = 0; i < g; i++)
			ret.push_back(mod(x + i*(n / g), n));
	}
	return ret;
}

// computes b such that ab = 1 (mod n), returns -1 on failure
int mod_inverse(int a, int n) {
	int x, y;
	int g = extended_euclid(a, n, x, y);
	if (g > 1) return -1;
	return mod(x, n);
}

// Chinese remainder theorem (special case): find z such that
// z % m1 = r1, z % m2 = r2.  Here, z is unique modulo M = lcm(m1, m2)
    .
// Return (z, M).  On failure, M = -1.
PII chinese_remainder_theorem(int m1, int r1, int m2, int r2) {
	int s, t;
	int g = extended_euclid(m1, m2, s, t);
	if (r1%g != r2%g) return make_pair(0, -1);
	return make_pair(mod(s*r2*m1 + t*r1*m2, m1*m2) / g, m1*m2 / g)
	    ;
}

// Chinese remainder theorem: find z such that
// z % m[i] = r[i] for all i.  Note that the solution is
// unique modulo M = lcm_i (m[i]).  Return (z, M). On
// failure, M = -1. Note that we do not require the a[i]'s
// to be relatively prime.
PII chinese_remainder_theorem(const VI &m, const VI &r) {
	PII ret = make_pair(r[0], m[0]);
	for (int i = 1; i < m.size(); i++) {
		ret = chinese_remainder_theorem(ret.second, ret.first,
			m[i], r[i]);
		if (ret.second == -1) break;
	}
	return ret;
}

// computes x and y such that ax + by = c
// returns whether the solution exists
bool linear_diophantine(int a, int b, int c, int &x, int &y) {
	if (!a && !b)
	{
		if (c) return false;
		x = 0; y = 0;
		return true;
	}
	if (!a)
	{
		if (c % b) return false;
		x = 0; y = c / b;
		return true;
	}
	if (!b)
	{
		if (c % a) return false;
		x = c / a; y = 0;
		return true;
	}
	int g = gcd(a, b);
	if (c % g) return false;
	x = c / g * mod_inverse(a / g, b / g);
	y = (c - a*x) / b;
	return true;
}

int main() {
	// expected: 2
	cout << gcd(14, 30) << endl;

	// expected: 2 -2 1
```

```
        int x, y;
        int g = extended_euclid(14, 30, x, y);
        cout << g << " " << x << " " << y << endl;

        // expected: 95 451
        VI sols = modular_linear_equation_solver(14, 30, 100);
        for (int i = 0; i < sols.size(); i++) cout << sols[i] << " ";
        cout << endl;

        // expected: 8
        cout << mod_inverse(8, 9) << endl;

        // expected: 23 105
        //           11 12
        PII ret = chinese_remainder_theorem(VI({ 3, 5, 7 }), VI({ 2,
            3, 2 }));
        cout << ret.first << " " << ret.second << endl;
        ret = chinese_remainder_theorem(VI({ 4, 6 }), VI({ 3, 5 }));
        cout << ret.first << " " << ret.second << endl;

        // expected: 5 -15
        if (!linear_diophantine(7, 2, 5, x, y)) cout << "ERROR" <<
            endl;
        cout << x << " " << y << endl;
        return 0;
}
```

3.2 Prime numbers

```
// O(sqrt(x)) Exhaustive Primality Test
#include <cmath>
#define EPS 1e-7
typedef long long ll;
bool IsPrimeSlow (ll x)
{
  if(x<=1) return false;
  if(x<=3) return true;
  if (!(x%2) || !(x%3)) return false;
  ll s=(ll)(sqrt((double)(x))+EPS);
  for(ll i=5;i<=s;i+=6)
  {
    if (!(x%i) || !(x%(i+2))) return false;
  }
  return true;
}

// O(n) fast generate prime number list
const ll limit = 10000000; // prime number upper bound
bool prime[limit+1]; // bool of squence is prime or not
vector<ll> primes; // list of prime in order

void generateprimes(){
    for (ll i = 0; i < limit; i++)
        prime[i] = false;
        prime[2] = true;
        prime[3] = true;
    for (ll x = 1; x * x < limit; x++) {
        for (ll y = 1; y * y < limit; y++) {

            ll n = (4 * x * x) + (y * y);
            if (n <= limit && (n % 12 == 1 || n % 12 == 5))
                prime[n] ^= true;

            n = (3 * x * x) + (y * y);
            if (n <= limit && n % 12 == 7)
                prime[n] ^= true;

            n = (3 * x * x) - (y * y);
            if (x > y && n <= limit && n % 12 == 11)
                prime[n] ^= true;
        }
    }

    for (ll r = 5; r * r < limit; r++) {
        if (prime[r]) {
            for (ll i = r * r; i < limit; i += r * r)
                prime[i] = false;
        }
    }
        for (ll i=2; i<limit; i++) if (prime[i]) primes.push_back(i);
}

// O(nlog(n)) return number of coprime pair in set [a,b]  [c,d]
ll coprime(ll a,b,c,d) {
        N = max(b, d);
        int mu[N];

        for (ll i=0; i<=N; i++) mu[i] = 1;
        for(auto p : primes){
        for(ll i=1; i*p <= N; i++){
                mu[i*p] *= -1;
        }
        ll pp = p*p;
        for(ll i=1; i*pp <= N; i++){
                mu[i*pp] = 0;
        }
    }
        ll sum = 0;
        for (ll i=1; i<=N; i++) {
                sum += mu[i] * (b/i - (a-1)/i) * (d/i - (c-1)/i);
        }
        return ll;
}


// Primes less than 1000:
//      2     3     5     7    11    13    17    19    23    29    31
//     37
//     41    43    47    53    59    61    67    71    73    79    83
//     89
//     97   101   103   107   109   113   127   131   137   139   149
//    151
//    157   163   167   173   179   181   191   193   197   199   211
//    223
//    227   229   233   239   241   251   257   263   269   271   277
//    281
//    283   293   307   311   313   317   331   337   347   349   353
//    359
//    367   373   379   383   389   397   401   409   419   421   431
//    433
//    439   443   449   457   461   463   467   479   487   491   499
//    503
//    509   521   523   541   547   557   563   569   571   577   587
//    593
//    599   601   607   613   617   619   631   641   643   647   653
//    659
//    661   673   677   683   691   701   709   719   727   733   739
//    743
//    751   757   761   769   773   787   797   809   811   821   823
//    827
//    829   839   853   857   859   863   877   881   883   887   907
//    911
//    919   929   937   941   947   953   967   971   977   983   991
//    997

// Other primes:
//    The largest prime smaller than 10 is 7.
//    The largest prime smaller than 100 is 97.
//    The largest prime smaller than 1000 is 997.
```

```
//    The largest prime smaller than 10000 is 9973.
//    The largest prime smaller than 100000 is 99991.
//    The largest prime smaller than 1000000 is 999983.
//    The largest prime smaller than 10000000 is 9999991.
//    The largest prime smaller than 100000000 is 99999989.
//    The largest prime smaller than 1000000000 is 999999937.
//    The largest prime smaller than 10000000000 is 9999999967.
//    The largest prime smaller than 100000000000 is 99999999977.
//    The largest prime smaller than 1000000000000 is 999999999989.
//    The largest prime smaller than 10000000000000 is 9999999999971.
//    The largest prime smaller than 100000000000000 is
    99999999999973.
//    The largest prime smaller than 1000000000000000 is
    999999999999989.
//    The largest prime smaller than 10000000000000000 is
    9999999999999937.
//    The largest prime smaller than 100000000000000000 is
    99999999999999997.
//    The largest prime smaller than 1000000000000000000 is
    999999999999999989.
```

3.3 Systems of linear equations, matrix inverse, determinant

```
// Gauss-Jordan elimination with full pivoting.
//
// Uses:
//   (1) solving systems of linear equations (AX=B)
//   (2) inverting matrices (AX=I)
//   (3) computing determinants of square matrices
//
// Running time: O(n^3)
//
// INPUT:    a[][] = an nxn matrix
//           b[][] = an nxm matrix
//
// OUTPUT:   X      = an nxm matrix (stored in b[][])
//           A^{-1} = an nxn matrix (stored in a[][])
//           returns determinant of a[][]

#include <iostream>
#include <vector>
#include <cmath>

using namespace std;

const double EPS = 1e-10;

typedef vector<int> VI;
typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;

T GaussJordan(VVT &a, VVT &b) {
  const int n = a.size();
  const int m = b[0].size();
  VI irow(n), icol(n), ipiv(n);
  T det = 1;

  for (int i = 0; i < n; i++) {
    int pj = -1, pk = -1;
    for (int j = 0; j < n; j++) if (!ipiv[j])
      for (int k = 0; k < n; k++) if (!ipiv[k])
        if (pj == -1 || fabs(a[j][k]) > fabs(a[pj][pk])) { pj = j; pk
            = k; }
    if (fabs(a[pj][pk]) < EPS) { cerr << "Matrix is singular." << endl
        ; exit(0); }
    ipiv[pk]++;
    swap(a[pj], a[pk]);
    swap(b[pj], b[pk]);
    if (pj != pk) det *= -1;
    irow[i] = pj;
    icol[i] = pk;

    T c = 1.0 / a[pk][pk];
    det *= a[pk][pk];
    a[pk][pk] = 1.0;
    for (int p = 0; p < n; p++) a[pk][p] *= c;
    for (int p = 0; p < m; p++) b[pk][p] *= c;
    for (int p = 0; p < n; p++) if (p != pk) {
      c = a[p][pk];
      a[p][pk] = 0;
      for (int q = 0; q < n; q++) a[p][q] -= a[pk][q] * c;
      for (int q = 0; q < m; q++) b[p][q] -= b[pk][q] * c;
    }
  }

  for (int p = n-1; p >= 0; p--) if (irow[p] != icol[p]) {
    for (int k = 0; k < n; k++) swap(a[k][irow[p]], a[k][icol[p]]);
  }

  return det;
}

int main() {
  const int n = 4;
  const int m = 2;
  double A[n][n] = { {1,2,3,4},{1,0,1,0},{5,3,2,4},{6,1,4,6} };
  double B[n][m] = { {1,2},{4,3},{5,6},{8,7} };
  VVT a(n), b(n);
  for (int i = 0; i < n; i++) {
    a[i] = VT(A[i], A[i] + n);
    b[i] = VT(B[i], B[i] + m);
  }

  double det = GaussJordan(a, b);

  // expected: 60
  cout << "Determinant: " << det << endl;

  // expected: -0.233333 0.166667 0.133333 0.0666667
  //            0.166667 0.166667 0.333333 -0.333333
  //            0.233333 0.833333 -0.133333 -0.0666667
  //            0.05 -0.75 -0.1 0.2
  cout << "Inverse: " << endl;
  for (int i = 0; i < n; i++) {
    for (int j = 0; j < n; j++)
      cout << a[i][j] << ' ';
    cout << endl;
  }

  // expected: 1.63333 1.3
  //           -0.166667 0.5
  //           2.36667 1.7
  //           -1.85 -1.35
  cout << "Solution: " << endl;
  for (int i = 0; i < n; i++) {
    for (int j = 0; j < m; j++)
      cout << b[i][j] << ' ';
    cout << endl;
  }
}
```

3.4 Reduced row echelon form, matrix rank

```
// Reduced row echelon form via Gauss-Jordan elimination
```

```
// with partial pivoting.  This can be used for computing
// the rank of a matrix.
//
// Running time: O(n^3)
//
// INPUT:    a[][] = an nxm matrix
//
// OUTPUT:   rref[][] = an nxm matrix (stored in a[][])
//           returns rank of a[][]

#include <iostream>
#include <vector>
#include <cmath>

using namespace std;

const double EPSILON = 1e-10;

typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;

int rref(VVT &a) {
  int n = a.size();
  int m = a[0].size();
  int r = 0;
  for (int c = 0; c < m && r < n; c++) {
    int j = r;
    for (int i = r + 1; i < n; i++)
      if (fabs(a[i][c]) > fabs(a[j][c])) j = i;
    if (fabs(a[j][c]) < EPSILON) continue;
    swap(a[j], a[r]);

    T s = 1.0 / a[r][c];
    for (int j = 0; j < m; j++) a[r][j] *= s;
    for (int i = 0; i < n; i++) if (i != r) {
      T t = a[i][c];
      for (int j = 0; j < m; j++) a[i][j] -= t * a[r][j];
    }
    r++;
  }
  return r;
}

int main() {
  const int n = 5, m = 4;
  double A[n][m] = {
    {16,  2,  3, 13},
    { 5, 11, 10,  8},
    { 9,  7,  6, 12},
    { 4, 14, 15,  1},
    {13, 21, 21, 13}};
  VVT a(n);
  for (int i = 0; i < n; i++)
    a[i] = VT(A[i], A[i] + m);

  int rank = rref(a);

  // expected: 3
  cout << "Rank: " << rank << endl;

  // expected: 1 0 0 1
  //           0 1 0 3
  //           0 0 1 -3
  //           0 0 0 3.10862e-15
  //           0 0 0 2.22045e-15
  cout << "rref: " << endl;
  for (int i = 0; i < 5; i++) {
    for (int j = 0; j < 4; j++)
      cout << a[i][j] << ' ';
    cout << endl;
  }
}
```

3.5 Fast Fourier transform

```
#include <cassert>
#include <cstdio>
#include <cmath>

struct cpx
{
  cpx(){}
  cpx(double aa):a(aa),b(0){}
  cpx(double aa, double bb):a(aa),b(bb){}
  double a;
  double b;
  double modsq(void) const
  {
    return a * a + b * b;
  }
  cpx bar(void) const
  {
    return cpx(a, -b);
  }
};

cpx operator +(cpx a, cpx b)
{
  return cpx(a.a + b.a, a.b + b.b);
}

cpx operator *(cpx a, cpx b)
{
  return cpx(a.a * b.a - a.b * b.b, a.a * b.b + a.b * b.a);
}

cpx operator /(cpx a, cpx b)
{
  cpx r = a * b.bar();
  return cpx(r.a / b.modsq(), r.b / b.modsq());
}

cpx EXP(double theta)
{
  return cpx(cos(theta),sin(theta));
}

const double two_pi = 4 * acos(0);

// in:     input array
// out:    output array
// step:   {SET TO 1} (used internally)
// size:   length of the input/output {MUST BE A POWER OF 2}
// dir:    either plus or minus one (direction of the FFT)
// RESULT: out[k] = \sum_{j=0}^{size - 1} in[j] * exp(dir * 2pi * i *
     j * k / size)
void FFT(cpx *in, cpx *out, int step, int size, int dir)
{
  if(size < 1) return;
  if(size == 1)
  {
    out[0] = in[0];
    return;
  }
  FFT(in, out, step * 2, size / 2, dir);
  FFT(in + step, out + size / 2, step * 2, size / 2, dir);
  for(int i = 0 ; i < size / 2 ; i++)
  {
    cpx even = out[i];
    cpx odd = out[i + size / 2];
```

```
      out[i] = even + EXP(dir * two_pi * i / size) * odd;
      out[i + size / 2] = even + EXP(dir * two_pi * (i + size / 2) /
          size) * odd;
    }
  }

// Usage:
// f[0...N-1] and g[0..N-1] are numbers
// Want to compute the convolution h, defined by
// h[n] = sum of f[k]g[n-k] (k = 0, ..., N-1).
// Here, the index is cyclic; f[-1] = f[N-1], f[-2] = f[N-2], etc.
// Let F[0...N-1] be FFT(f), and similarly, define G and H.
// The convolution theorem says H[n] = F[n]G[n] (element-wise product)
   .
// To compute h[] in O(N log N) time, do the following:
//   1. Compute F and G (pass dir = 1 as the argument).
//   2. Get H by element-wise multiplying F and G.
//   3. Get h by taking the inverse FFT (use dir = -1 as the argument)
//      and *dividing by N*. DO NOT FORGET THIS SCALING FACTOR.

int main(void)
{
  printf("If rows come in identical pairs, then everything works.\n");

  cpx a[8] = {0, 1, cpx(1,3), cpx(0,5), 1, 0, 2, 0};
  cpx b[8] = {1, cpx(0,-2), cpx(0,1), 3, -1, -3, 1, -2};
  cpx A[8];
  cpx B[8];
  FFT(a, A, 1, 8, 1);
  FFT(b, B, 1, 8, 1);

  for(int i = 0 ; i < 8 ; i++)
  {
    printf("%7.2lf%7.2lf", A[i].a, A[i].b);
  }
  printf("\n");
  for(int i = 0 ; i < 8 ; i++)
  {
    cpx Ai(0,0);
    for(int j = 0 ; j < 8 ; j++)
    {
      Ai = Ai + a[j] * EXP(j * i * two_pi / 8);
    }
    printf("%7.2lf%7.2lf", Ai.a, Ai.b);
  }
  printf("\n");

  cpx AB[8];
  for(int i = 0 ; i < 8 ; i++)
    AB[i] = A[i] * B[i];
  cpx aconvb[8];
  FFT(AB, aconvb, 1, 8, -1);
  for(int i = 0 ; i < 8 ; i++)
    aconvb[i] = aconvb[i] / 8;
  for(int i = 0 ; i < 8 ; i++)
  {
    printf("%7.2lf%7.2lf", aconvb[i].a, aconvb[i].b);
  }
  printf("\n");
  for(int i = 0 ; i < 8 ; i++)
  {
    cpx aconvbi(0,0);
    for(int j = 0 ; j < 8 ; j++)
    {
      aconvbi = aconvbi + a[j] * b[(8 + i - j) % 8];
    }
    printf("%7.2lf%7.2lf", aconvbi.a, aconvbi.b);
  }
  printf("\n");

  return 0;
}
```

3.6 Simplex algorithm

```
// Two-phase simplex algorithm for solving linear programs of the form
//
//     maximize     c^T x
//     subject to   Ax <= b
//                  x >= 0
//
// INPUT: A -- an m x n matrix
//        b -- an m-dimensional vector
//        c -- an n-dimensional vector
//        x -- a vector where the optimal solution will be stored
//
// OUTPUT: value of the optimal solution (infinity if unbounded
//         above, nan if infeasible)
//
// To use this code, create an LPSolver object with A, b, and c as
// arguments.  Then, call Solve(x).

#include <iostream>
#include <iomanip>
#include <vector>
#include <cmath>
#include <limits>

using namespace std;

typedef long double DOUBLE;
typedef vector<DOUBLE> VD;
typedef vector<VD> VVD;
typedef vector<int> VI;

const DOUBLE EPS = 1e-9;

struct LPSolver {
  int m, n;
  VI B, N;
  VVD D;

  LPSolver(const VVD &A, const VD &b, const VD &c) :
    m(b.size()), n(c.size()), N(n + 1), B(m), D(m + 2, VD(n + 2)) {
    for (int i = 0; i < m; i++) for (int j = 0; j < n; j++) D[i][j] =
        A[i][j];
    for (int i = 0; i < m; i++) { B[i] = n + i; D[i][n] = -1; D[i][n +
         1] = b[i]; }
    for (int j = 0; j < n; j++) { N[j] = j; D[m][j] = -c[j]; }
    N[n] = -1; D[m + 1][n] = 1;
  }

  void Pivot(int r, int s) {
    double inv = 1.0 / D[r][s];
    for (int i = 0; i < m + 2; i++) if (i != r)
      for (int j = 0; j < n + 2; j++) if (j != s)
        D[i][j] -= D[r][j] * D[i][s] * inv;
    for (int j = 0; j < n + 2; j++) if (j != s) D[r][j] *= inv;
    for (int i = 0; i < m + 2; i++) if (i != r) D[i][s] *= -inv;
    D[r][s] = inv;
    swap(B[r], N[s]);
  }

  bool Simplex(int phase) {
    int x = phase == 1 ? m + 1 : m;
    while (true) {
      int s = -1;
      for (int j = 0; j <= n; j++) {
        if (phase == 2 && N[j] == -1) continue;
```

```
        if (s == -1 || D[x][j] < D[x][s] || D[x][j] == D[x][s] && N[j]
             < N[s]) s = j;
      }
      if (D[x][s] > -EPS) return true;
      int r = -1;
      for (int i = 0; i < m; i++) {
        if (D[i][s] < EPS) continue;
        if (r == -1 || D[i][n + 1] / D[i][s] < D[r][n + 1] / D[r][s]
            ||
          (D[i][n + 1] / D[i][s]) == (D[r][n + 1] / D[r][s]) && B[i] <
             B[r]) r = i;
      }
      if (r == -1) return false;
      Pivot(r, s);
    }
  }

  DOUBLE Solve(VD &x) {
    int r = 0;
    for (int i = 1; i < m; i++) if (D[i][n + 1] < D[r][n + 1]) r = i;
    if (D[r][n + 1] < -EPS) {
      Pivot(r, n);
      if (!Simplex(1) || D[m + 1][n + 1] < -EPS) return -
        numeric_limits<DOUBLE>::infinity();
      for (int i = 0; i < m; i++) if (B[i] == -1) {
        int s = -1;
        for (int j = 0; j <= n; j++)
          if (s == -1 || D[i][j] < D[i][s] || D[i][j] == D[i][s] && N[
             j] < N[s]) s = j;
        Pivot(i, s);
      }
    }
    if (!Simplex(2)) return numeric_limits<DOUBLE>::infinity();
    x = VD(n);
    for (int i = 0; i < m; i++) if (B[i] < n) x[B[i]] = D[i][n + 1];
    return D[m][n + 1];
  }
};

int main() {

  const int m = 4;
  const int n = 3;
  DOUBLE _A[m][n] = {
    { 6, -1, 0 },
    { -1, -5, 0 },
    { 1, 5, 1 },
    { -1, -5, -1 }
  };
  DOUBLE _b[m] = { 10, -4, 5, -5 };
  DOUBLE _c[n] = { 1, -1, 0 };

  VVD A(m);
  VD b(_b, _b + m);
  VD c(_c, _c + n);
  for (int i = 0; i < m; i++) A[i] = VD(_A[i], _A[i] + n);

  LPSolver solver(A, b, c);
  VD x;
  DOUBLE value = solver.Solve(x);

  cerr << "VALUE: " << value << endl; // VALUE: 1.29032
  cerr << "SOLUTION:"; // SOLUTION: 1.74194 0.451613 1
  for (size_t i = 0; i < x.size(); i++) cerr << " " << x[i];
  cerr << endl;
  return 0;
}
```

3.7 Miller-Rabin Primality Test (C)

```
// Randomized Primality Test (Miller-Rabin):
//   Error rate: 2^(-TRIAL)
//   Almost constant time. srand is needed

#include <stdlib.h>
#define EPS 1e-7

typedef long long LL;

LL ModularMultiplication(LL a, LL b, LL m)
{
        LL ret=0, c=a;
        while(b)
        {
                if(b&1) ret=(ret+c)%m;
                b>>=1; c=(c+c)%m;
        }
        return ret;
}
LL ModularExponentiation(LL a, LL n, LL m)
{
        LL ret=1, c=a;
        while(n)
        {
                if(n&1) ret=ModularMultiplication(ret, c, m);
                n>>=1; c=ModularMultiplication(c, c, m);
        }
        return ret;
}
bool Witness(LL a, LL n)
{
        LL u=n-1;
  int t=0;
        while(!(u&1)){u>>=1; t++;}
        LL x0=ModularExponentiation(a, u, n), x1;
        for(int i=1;i<=t;i++)
        {
                x1=ModularMultiplication(x0, x0, n);
                if(x1==1 && x0!=1 && x0!=n-1) return true;
                x0=x1;
        }
        if(x0!=1) return true;
        return false;
}
LL Random(LL n)
{
  LL ret=rand(); ret*=32768;
        ret+=rand(); ret*=32768;
        ret+=rand(); ret*=32768;
        ret+=rand();
  return ret%n;
}
bool IsPrimeFast(LL n, int TRIAL)
{
  while(TRIAL--)
  {
    LL a=Random(n-2)+1;
    if(Witness(a, n)) return false;
  }
  return true;
}
```

3.8 Fast exponentiation

```
/*
Uses powers of two to exponentiate numbers and matrices. Calculates
n^k in O(log(k)) time when n is a number. If A is an n x n matrix,
calculates A^k in O(n^3*log(k)) time.
*/

#include <iostream>
#include <vector>

using namespace std;

typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;

T power(T x, int k) {
  T ret = 1;

  while(k) {
    if(k & 1) ret *= x;
    k >>= 1; x *= x;
  }
  return ret;
}

VVT multiply(VVT& A, VVT& B) {
  int n = A.size(), m = A[0].size(), k = B[0].size();
  VVT C(n, VT(k, 0));

  for(int i = 0; i < n; i++)
    for(int j = 0; j < k; j++)
      for(int l = 0; l < m; l++)
        C[i][j] += A[i][l] * B[l][j];

  return C;
}

VVT power(VVT& A, int k) {
  int n = A.size();
  VVT ret(n, VT(n)), B = A;
  for(int i = 0; i < n; i++) ret[i][i]=1;

  while(k) {
    if(k & 1) ret = multiply(ret, B);
    k >>= 1; B = multiply(B, B);
  }
  return ret;
}

int main()
{
  /* Expected Output:
     2.37^48 = 9.72569e+17

     376 264 285 220 265
     550 376 529 285 484
     484 265 376 264 285
     285 220 265 156 264
     529 285 484 265 376 */
  double n = 2.37;
  int k = 48;

  cout << n << "^" << k << " = " << power(n, k) << endl;

  double At[5][5] = {
    { 0, 0, 1, 0, 0 },
    { 1, 0, 0, 1, 0 },
    { 0, 0, 0, 0, 1 },
    { 1, 0, 0, 0, 0 },
    { 0, 1, 0, 0, 0 } };

  vector <vector <double> > A(5, vector <double>(5));
  for(int i = 0; i < 5; i++)
    for(int j = 0; j < 5; j++)
      A[i][j] = At[i][j];

  vector <vector <double> > Ap = power(A, k);

  cout << endl;
  for(int i = 0; i < 5; i++) {
    for(int j = 0; j < 5; j++)
      cout << Ap[i][j] << " ";
    cout << endl;
  }
}
```

4 Graph algorithms

4.1 Bellman-Ford shortest paths with negative edge weights

```
// This function runs the Bellman-Ford algorithm for single source
// shortest paths with negative edge weights.  The function returns
// false if a negative weight cycle is detected.  Otherwise, the
// function returns true and dist[i] is the length of the shortest
// path from start to i.
//
// Running time: O(|V|^3)
//
//   INPUT:   start, w[i][j] = cost of edge from i to j
//   OUTPUT:  dist[i] = min weight path from start to i
//            prev[i] = previous node on the best path from the
//                      start node

#include <iostream>
#include <queue>
#include <cmath>
#include <vector>

using namespace std;

typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;

typedef vector<int> VI;
typedef vector<VI> VVI;

bool BellmanFord (const VVT &w, VT &dist, VI &prev, int start){
  int n = w.size();
  prev = VI(n, -1);
  dist = VT(n, 1000000000);
  dist[start] = 0;

  for (int k = 0; k < n; k++){
    for (int i = 0; i < n; i++){
      for (int j = 0; j < n; j++){
        if (dist[j] > dist[i] + w[i][j]){
          if (k == n-1) return false;
          dist[j] = dist[i] + w[i][j];
          prev[j] = i;
        }
      }
    }
  }

  return true;
}
```

4.2 Dijkstra and Floyd's algorithm

```
#include <iostream>
#include <queue>
#include <cmath>
#include <vector>

using namespace std;

typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;

typedef vector<int> VI;
typedef vector<VI> VVI;

// This function runs Dijkstra's algorithm for single source
// shortest paths.  No negative cycles allowed!
//
// Running time: O(|V|^2)
//
//   INPUT:   start, w[i][j] = cost of edge from i to j
//   OUTPUT:  dist[i] = min weight path from start to i
//            prev[i] = previous node on the best path from the
//                      start node

void Dijkstra (const VVT &w, VT &dist, VI &prev, int start){
  int n = w.size();
  VI found (n);
  prev = VI(n, -1);
  dist = VT(n, 1000000000);
  dist[start] = 0;

  while (start != -1){
    found[start] = true;
    int best = -1;
    for (int k = 0; k < n; k++) if (!found[k]){
      if (dist[k] > dist[start] + w[start][k]){
        dist[k] = dist[start] + w[start][k];
        prev[k] = start;
      }
      if (best == -1 || dist[k] < dist[best]) best = k;
    }
    start = best;
  }
}

// This function runs the Floyd-Warshall algorithm for all-pairs
// shortest paths.  Also handles negative edge weights.  Returns true
// if a negative weight cycle is found.
//
// Running time: O(|V|^3)
//
//   INPUT:  w[i][j] = weight of edge from i to j
//   OUTPUT: w[i][j] = shortest path from i to j
//           prev[i][j] = node before j on the best path starting at i

bool FloydWarshall (VVT &w, VVI &prev){
  int n = w.size();
  prev = VVI (n, VI(n, -1));

  for (int k = 0; k < n; k++){
    for (int i = 0; i < n; i++){
      for (int j = 0; j < n; j++){
        if (w[i][j] > w[i][k] + w[k][j]){
          w[i][j] = w[i][k] + w[k][j];
          prev[i][j] = k;
        }
      }
    }
  }

  // check for negative weight cycles
  for(int i=0;i<n;i++)
    if (w[i][i] < 0) return false;
  return true;
}
```

4.3 Fast Dijkstra's algorithm

```
// Implementation of Dijkstra's algorithm using adjacency lists
// and priority queue for efficiency.
```

```
//
// Running time: O(|E| log |V|)

#include <queue>
#include <cstdio>

using namespace std;
const int INF = 2000000000;
typedef pair<int, int> PII;

int main() {

        int N, s, t;
        scanf("%d%d%d", &N, &s, &t);
        vector<vector<PII> > edges(N);
        for (int i = 0; i < N; i++) {
                int M;
                scanf("%d", &M);
                for (int j = 0; j < M; j++) {
                        int vertex, dist;
                        scanf("%d%d", &vertex, &dist);
                        edges[i].push_back(make_pair(dist, vertex));
                            // note order of arguments here
                }
        }

        // use priority queue in which top element has the "smallest"
            priority
        priority_queue<PII, vector<PII>, greater<PII> > Q;
        vector<int> dist(N, INF), dad(N, -1);
        Q.push(make_pair(0, s));
        dist[s] = 0;
        while (!Q.empty()) {
                PII p = Q.top();
                Q.pop();
                int here = p.second;
                if (here == t) break;
                if (dist[here] != p.first) continue;

                for (vector<PII>::iterator it = edges[here].begin();
                    it != edges[here].end(); it++) {
                        if (dist[here] + it->first < dist[it->second])
                             {
                                dist[it->second] = dist[here] + it->
                                    first;
                                dad[it->second] = here;
                                Q.push(make_pair(dist[it->second], it
                                    ->second));
                        }
                }
        }

        printf("%d\n", dist[t]);
        if (dist[t] < INF)
                for (int i = t; i != -1; i = dad[i])
                        printf("%d%c", i, (i == s ? '\n' : ' '));
        return 0;
}

/*
Sample input:
5 0 4
2 1 2 3 1
2 2 4 4 5
3 1 4 3 3 4 1
2 0 1 2 3
2 1 5 2 1

Expected:
5
4 2 3 0
*/
```

4.4 Strongly connected components

```
#include<memory.h>
struct edge{int e, nxt;};
int V, E;
edge e[MAXE], er[MAXE];
int sp[MAXV], spr[MAXV];
int group_cnt, group_num[MAXV];
bool v[MAXV];
int stk[MAXV];
void fill_forward(int x)
{
  int i;
  v[x]=true;
  for(i=sp[x];i;i=e[i].nxt) if(!v[e[i].e]) fill_forward(e[i].e);
  stk[++stk[0]]=x;
}
void fill_backward(int x)
{
  int i;
  v[x]=false;
  group_num[x]=group_cnt;
  for(i=spr[x];i;i=er[i].nxt) if(v[er[i].e]) fill_backward(er[i].e);
}
void add_edge(int v1, int v2) //add edge v1->v2
{
  e [++E].e=v2; e [E].nxt=sp [v1]; sp [v1]=E;
  er[  E].e=v1; er[E].nxt=spr[v2]; spr[v2]=E;
}
void SCC()
{
  int i;
  stk[0]=0;
  memset(v, false, sizeof(v));
  for(i=1;i<=V;i++) if(!v[i]) fill_forward(i);
  group_cnt=0;
  for(i=stk[0];i>=1;i--) if(v[stk[i]]){group_cnt++; fill_backward(stk[
      i]);}
}

// Tarjan's SCC Algorithm
int n, m;
struct Node{vector<int> adj;};
Node graph[MAX_N];
stack<int> Stack;
bool onStack[MAX_N];
int Indices;
int Index[MAX_N];
int LowLink[MAX_N];
int component[MAX_N];
int numComponents;

void tarjanDFS(int i)
{
    Index[i] = ++Indices;
    LowLink[i] = Indices;
    Stack.push(i); onStack[i] = true;
    for (int j=0;j<graph[i].adj.size();j++){
        int w = graph[i].adj[j];
        if (Index[w] == 0){
            tarjanDFS(w);
            LowLink[i] = min(LowLink[i], LowLink[w]);
        }else if (onStack[w]){
            LowLink[i] = min(LowLink[i], Index[w]);
        }
```

```
        }
        if (LowLink[i] == Index[i]){
            int w = 0;
            do{
                w = Stack.top(); Stack.pop();
                component[w] = numComponents;
                onStack[w]=false;
            } while (i != w && !Stack.empty());
            numComponents++;
        }
    }

    void Tarjan()
    {
        Indices = 0;
        while (!Stack.empty()) Stack.pop();
        for (int i=n;i>0;i--) onStack[i] = LowLink[i] = Index[i] = 0;
        numComponents = 0;
        for (int i=n;i>0;i--) if (Index[i] == 0) tarjanDFS(i);
    }

    // add edge i to j
    // graph[i].adj.push_back(j);
```

4.5 Eulerian path

```
struct Edge;
typedef list<Edge>::iterator iter;

struct Edge
{
        int next_vertex;
        iter reverse_edge;

        Edge(int next_vertex)
                :next_vertex(next_vertex)
                { }
};

const int max_vertices = ;
int num_vertices;
list<Edge> adj[max_vertices];           // adjacency list

vector<int> path;

void find_path(int v)
{
        while(adj[v].size() > 0)
        {
                int vn = adj[v].front().next_vertex;
                adj[vn].erase(adj[v].front().reverse_edge);
                adj[v].pop_front();
                find_path(vn);
        }
        path.push_back(v);
}

void add_edge(int a, int b)
{
        adj[a].push_front(Edge(b));
        iter ita = adj[a].begin();
        adj[b].push_front(Edge(a));
        iter itb = adj[b].begin();
        ita->reverse_edge = itb;
        itb->reverse_edge = ita;
}
```

4.6 Kruskal's algorithm

```
/*
Uses Kruskal's Algorithm to calculate the weight of the minimum
    spanning
forest (union of minimum spanning trees of each connected component)
    of
a possibly disjoint graph, given in the form of a matrix of edge
    weights
(-1 if no edge exists). Returns the weight of the minimum spanning
forest (also calculates the actual edges - stored in T). Note: uses a
disjoint-set data structure with amortized (effectively) constant time
     per
union/find. Runs in O(E*log(E)) time.
*/

#include <iostream>
#include <vector>
#include <algorithm>
#include <queue>

using namespace std;

typedef int T;

struct edge
{
  int u, v;
  T d;
};

struct edgeCmp
{
  int operator()(const edge& a, const edge& b) { return a.d > b.d; }
};

int find(vector <int>& C, int x) { return (C[x] == x) ? x : C[x] =
    find(C, C[x]); }

T Kruskal(vector <vector <T> >& w)
{
  int n = w.size();
  T weight = 0;

  vector <int> C(n), R(n);
  for(int i=0; i<n; i++) { C[i] = i; R[i] = 0; }

  vector <edge> T;
  priority_queue <edge, vector <edge>, edgeCmp> E;

  for(int i=0; i<n; i++)
    for(int j=i+1; j<n; j++)
      if(w[i][j] >= 0)
      {
        edge e;
        e.u = i; e.v = j; e.d = w[i][j];
        E.push(e);
      }

  while(T.size() < n-1 && !E.empty())
  {
    edge cur = E.top(); E.pop();

    int uc = find(C, cur.u), vc = find(C, cur.v);
    if(uc != vc)
    {
      T.push_back(cur); weight += cur.d;

      if(R[uc] > R[vc]) C[vc] = uc;
      else if(R[vc] > R[uc]) C[uc] = vc;
```

```
      else { C[vc] = uc; R[uc]++; }
    }
  }

  return weight;
}

int main()
{
  int wa[6][6] = {
    { 0, -1, 2, -1, 7, -1 },
    { -1, 0, -1, 2, -1, -1 },
    { 2, -1, 0, -1, 8, 6 },
    { -1, 2, -1, 0, -1, -1 },
    { 7, -1, 8, -1, 0, 4 },
    { -1, -1, 6, -1, 4, 0 } };

  vector <vector <int> > w(6, vector <int>(6));

  for(int i=0; i<6; i++)
    for(int j=0; j<6; j++)
      w[i][j] = wa[i][j];

  cout << Kruskal(w) << endl;
  cin >> wa[0][0];
}
```

4.7 Minimum spanning trees

```
// This function runs Prim's algorithm for constructing minimum
// weight spanning trees.
//
// Running time: O(|V|^2)
//
//   INPUT:   w[i][j] = cost of edge from i to j
//
//            NOTE: Make sure that w[i][j] is nonnegative and
//            symmetric.  Missing edges should be given -1
//            weight.
//
//   OUTPUT:  edges = list of pair<int,int> in minimum spanning tree
//            return total weight of tree

#include <iostream>
#include <queue>
#include <cmath>
#include <vector>

using namespace std;

typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;

typedef vector<int> VI;
typedef vector<VI> VVI;
typedef pair<int,int> PII;
typedef vector<PII> VPII;

T Prim (const VVT &w, VPII &edges){
  int n = w.size();
  VI found (n);
  VI prev (n, -1);
  VT dist (n, 1000000000);
  int here = 0;
  dist[here] = 0;

  while (here != -1){
    found[here] = true;

    int best = -1;
    for (int k = 0; k < n; k++) if (!found[k]){
      if (w[here][k] != -1 && dist[k] > w[here][k]){
        dist[k] = w[here][k];
        prev[k] = here;
      }
      if (best == -1 || dist[k] < dist[best]) best = k;
    }
    here = best;
  }

  T tot_weight = 0;
  for (int i = 0; i < n; i++) if (prev[i] != -1){
    edges.push_back (make_pair (prev[i], i));
    tot_weight += w[prev[i]][i];
  }
  return tot_weight;
}

int main(){
  int ww[5][5] = {
    {0, 400, 400, 300, 600},
    {400, 0, 3, -1, 7},
    {400, 3, 0, 2, 0},
    {300, -1, 2, 0, 5},
    {600, 7, 0, 5, 0}
  };
  VVT w(5, VT(5));
  for (int i = 0; i < 5; i++)
    for (int j = 0; j < 5; j++)
      w[i][j] = ww[i][j];

  // expected: 305
  //           2 1
  //           3 2
  //           0 3
  //           2 4

  VPII edges;
  cout << Prim (w, edges) << endl;
  for (int i = 0; i < edges.size(); i++)
    cout << edges[i].first << " " << edges[i].second << endl;
}
```

4.8 Topological sort

```
// This function uses performs a non-recursive topological sort.
//
// Running time: O(|V|^2).  If you use adjacency lists (vector<map<int
   > >),
//               the running time is reduced to O(|E|).
//
//   INPUT:   w[i][j] = 1 if i should come before j, 0 otherwise
//   OUTPUT:  a permutation of 0,...,n-1 (stored in a vector)
//            which represents an ordering of the nodes which
//            is consistent with w
//
// If no ordering is possible, false is returned.

#include <iostream>
#include <queue>
#include <cmath>
#include <vector>

using namespace std;

typedef double T;
typedef vector<T> VT;
```

```
typedef vector<VT> VVT;

typedef vector<int> VI;
typedef vector<VI> VVI;

bool TopologicalSort (const VVI &w, VI &order){
  int n = w.size();
  VI parents (n);
  queue<int> q;
  order.clear();

  for (int i = 0; i < n; i++){
    for (int j = 0; j < n; j++)
      if (w[j][i]) parents[i]++;
      if (parents[i] == 0) q.push (i);
  }

  while (q.size() > 0){
    int i = q.front();
    q.pop();
    order.push_back (i);
    for (int j = 0; j < n; j++) if (w[i][j]){
      parents[j]--;
      if (parents[j] == 0) q.push (j);
    }
  }

  return (order.size() == n);
}
```

5 Data structures

5.1 Suffix array

```
// Suffix array construction in O(L log^2 L) time.  Routine for
// computing the length of the longest common prefix of any two
// suffixes in O(log L) time.
//
// INPUT:   string s
//
// OUTPUT:  array suffix[] such that suffix[i] = index (from 0 to L-1)
//          of substring s[i...L-1] in the list of sorted suffixes.
//          That is, if we take the inverse of the permutation suffix[],
//          we get the actual suffix array.

#include <vector>
#include <iostream>
#include <string>

using namespace std;

struct SuffixArray {
  const int L;
  string s;
  vector<vector<int> > P;
  vector<pair<pair<int,int>,int> > M;

  SuffixArray(const string &s) : L(s.length()), s(s), P(1, vector<int>(L, 0)), M(L) {
    for (int i = 0; i < L; i++) P[0][i] = int(s[i]);
    for (int skip = 1, level = 1; skip < L; skip *= 2, level++) {
      P.push_back(vector<int>(L, 0));
      for (int i = 0; i < L; i++)
        M[i] = make_pair(make_pair(P[level-1][i], i + skip < L ? P[level-1][i + skip] : -1000), i);
      sort(M.begin(), M.end());
      for (int i = 0; i < L; i++)
        P[level][M[i].second] = (i > 0 && M[i].first == M[i-1].first) ? P[level][M[i-1].second] : i;
    }
  }

  vector<int> GetSuffixArray() { return P.back(); }

  // returns the length of the longest common prefix of s[i...L-1] and s[j...L-1]
  int LongestCommonPrefix(int i, int j) {
    int len = 0;
    if (i == j) return L - i;
    for (int k = P.size() - 1; k >= 0 && i < L && j < L; k--) {
      if (P[k][i] == P[k][j]) {
        i += 1 << k;
        j += 1 << k;
        len += 1 << k;
      }
    }
    return len;
  }
};

// BEGIN CUT
// The following code solves UVA problem 11512: GATTACA.
#define TESTING
#ifdef TESTING
int main() {
  int T;
  cin >> T;
  for (int caseno = 0; caseno < T; caseno++) {
    string s;
    cin >> s;
    SuffixArray array(s);
    vector<int> v = array.GetSuffixArray();
    int bestlen = -1, bestpos = -1, bestcount = 0;
    for (int i = 0; i < s.length(); i++) {
      int len = 0, count = 0;
      for (int j = i+1; j < s.length(); j++) {
        int l = array.LongestCommonPrefix(i, j);
        if (l >= len) {
          if (l > len) count = 2; else count++;
          len = l;
        }
      }
      if (len > bestlen || len == bestlen && s.substr(bestpos, bestlen) > s.substr(i, len)) {
        bestlen = len;
        bestcount = count;
        bestpos = i;
      }
    }
    if (bestlen == 0) {
      cout << "No repetitions found!" << endl;
    } else {
      cout << s.substr(bestpos, bestlen) << " " << bestcount << endl;
    }
  }
}

#else
// END CUT
int main() {

  // bobocel is the 0'th suffix
  //  obocel is the 5'th suffix
  //   bocel is the 1'st suffix
  //    ocel is the 6'th suffix
  //     cel is the 2'nd suffix
  //      el is the 3'rd suffix
  //       l is the 4'th suffix
  SuffixArray suffix("bobocel");
  vector<int> v = suffix.GetSuffixArray();

  // Expected output: 0 5 1 6 2 3 4
  //                  2
  for (int i = 0; i < v.size(); i++) cout << v[i] << " ";
  cout << endl;
  cout << suffix.LongestCommonPrefix(0, 2) << endl;
}
// BEGIN CUT
#endif
// END CUT
```

5.2 Binary Indexed Tree

```
#include <iostream>
using namespace std;

#define LOGSZ 17

int tree[(1<<LOGSZ)+1];
int N = (1<<LOGSZ);

// add v to value at x
void set(int x, int v) {
  while(x <= N) {
    tree[x] += v;
    x += (x & -x);
```

```
    }
  }

  // get cumulative sum up to and including x
  int get(int x) {
    int res = 0;
    while(x) {
      res += tree[x];
      x -= (x & -x);
    }
    return res;
  }

  // get largest value with cumulative sum less than or equal to x;
  // for smallest, pass x-1 and add 1 to result
  int getind(int x) {
    int idx = 0, mask = N;
    while(mask && idx < N) {
      int t = idx + mask;
      if(x >= tree[t]) {
        idx = t;
        x -= tree[t];
      }
      mask >>= 1;
    }
    return idx;
  }
```

5.3 Union-find set

```
#include <iostream>
#include <vector>
using namespace std;
struct UnionFind {
    vector<int> C;
    UnionFind(int n) : C(n) { for (int i = 0; i < n; i++) C[i] = i; }
    int find(int x) { return (C[x] == x) ? x : C[x] = find(C[x]); }
    void merge(int x, int y) { C[find(x)] = find(y); }
};
int main()
{
    int n = 5;
    UnionFind uf(n);
    uf.merge(0, 2);
    uf.merge(1, 0);
    uf.merge(3, 4);
    for (int i = 0; i < n; i++) cout << i << " " << uf.find(i) << endl
        ;
    return 0;
}
```

5.4 KD-tree

```
// -----------------------------------------------------------------
// A straightforward, but probably sub-optimal KD-tree implmentation
// that's probably good enough for most things (current it's a
// 2D-tree)
//
//  - constructs from n points in O(n lg^2 n) time
//  - handles nearest-neighbor query in O(lg n) if points are well
//    distributed
//  - worst case for nearest-neighbor may be linear in pathological
//    case
//
// Sonny Chan, Stanford University, April 2009
// -----------------------------------------------------------------

#include <iostream>
#include <vector>
#include <limits>
#include <cstdlib>

using namespace std;

// number type for coordinates, and its maximum value
typedef long long ntype;
const ntype sentry = numeric_limits<ntype>::max();

// point structure for 2D-tree, can be extended to 3D
struct point {
    ntype x, y;
    point(ntype xx = 0, ntype yy = 0) : x(xx), y(yy) {}
};

bool operator==(const point &a, const point &b)
{
    return a.x == b.x && a.y == b.y;
}

// sorts points on x-coordinate
bool on_x(const point &a, const point &b)
{
    return a.x < b.x;
}

// sorts points on y-coordinate
bool on_y(const point &a, const point &b)
{
    return a.y < b.y;
}

// squared distance between points
ntype pdist2(const point &a, const point &b)
{
    ntype dx = a.x-b.x, dy = a.y-b.y;
    return dx*dx + dy*dy;
}

// bounding box for a set of points
struct bbox
{
    ntype x0, x1, y0, y1;

    bbox() : x0(sentry), x1(-sentry), y0(sentry), y1(-sentry) {}

    // computes bounding box from a bunch of points
    void compute(const vector<point> &v) {
        for (int i = 0; i < v.size(); ++i) {
            x0 = min(x0, v[i].x);   x1 = max(x1, v[i].x);
            y0 = min(y0, v[i].y);   y1 = max(y1, v[i].y);
        }
    }

    // squared distance between a point and this bbox, 0 if inside
    ntype distance(const point &p) {
        if (p.x < x0) {
            if (p.y < y0)       return pdist2(point(x0, y0), p);
            else if (p.y > y1)  return pdist2(point(x0, y1), p);
            else                return pdist2(point(x0, p.y), p);
        }
        else if (p.x > x1) {
            if (p.y < y0)       return pdist2(point(x1, y0), p);
            else if (p.y > y1)  return pdist2(point(x1, y1), p);
            else                return pdist2(point(x1, p.y), p);
        }
        else {
            if (p.y < y0)       return pdist2(point(p.x, y0), p);
            else if (p.y > y1)  return pdist2(point(p.x, y1), p);
```

```
            else                return 0;
        }
    }
};

// stores a single node of the kd-tree, either internal or leaf
struct kdnode
{
    bool leaf;      // true if this is a leaf node (has one point)
    point pt;       // the single point of this is a leaf
    bbox bound;     // bounding box for set of points in children

    kdnode *first, *second; // two children of this kd-node

    kdnode() : leaf(false), first(0), second(0) {}
    ~kdnode() { if (first) delete first; if (second) delete second; }

    // intersect a point with this node (returns squared distance)
    ntype intersect(const point &p) {
        return bound.distance(p);
    }

    // recursively builds a kd-tree from a given cloud of points
    void construct(vector<point> &vp)
    {
        // compute bounding box for points at this node
        bound.compute(vp);

        // if we're down to one point, then we're a leaf node
        if (vp.size() == 1) {
            leaf = true;
            pt = vp[0];
        }
        else {
            // split on x if the bbox is wider than high (not best
                heuristic...)
            if (bound.x1-bound.x0 >= bound.y1-bound.y0)
                sort(vp.begin(), vp.end(), on_x);
            // otherwise split on y-coordinate
            else
                sort(vp.begin(), vp.end(), on_y);

            // divide by taking half the array for each child
            // (not best performance if many duplicates in the middle)
            int half = vp.size()/2;
            vector<point> vl(vp.begin(), vp.begin()+half);
            vector<point> vr(vp.begin()+half, vp.end());
            first = new kdnode();   first->construct(vl);
            second = new kdnode();  second->construct(vr);
        }
    }
};

// simple kd-tree class to hold the tree and handle queries
struct kdtree
{
    kdnode *root;

    // constructs a kd-tree from a points (copied here, as it sorts
        them)
    kdtree(const vector<point> &vp) {
        vector<point> v(vp.begin(), vp.end());
        root = new kdnode();
        root->construct(v);
    }
    ~kdtree() { delete root; }

    // recursive search method returns squared distance to nearest
        point
    ntype search(kdnode *node, const point &p)
    {
        if (node->leaf) {
            // commented special case tells a point not to find itself
//            if (p == node->pt) return sentry;
//            else
                return pdist2(p, node->pt);
        }

        ntype bfirst = node->first->intersect(p);
        ntype bsecond = node->second->intersect(p);

        // choose the side with the closest bounding box to search
            first
        // (note that the other side is also searched if needed)
        if (bfirst < bsecond) {
            ntype best = search(node->first, p);
            if (bsecond < best)
                best = min(best, search(node->second, p));
            return best;
        }
        else {
            ntype best = search(node->second, p);
            if (bfirst < best)
                best = min(best, search(node->first, p));
            return best;
        }
    }

    // squared distance to the nearest
    ntype nearest(const point &p) {
        return search(root, p);
    }
};

//
    ---------------------------------------------------------------------------

// some basic test code here

int main()
{
    // generate some random points for a kd-tree
    vector<point> vp;
    for (int i = 0; i < 100000; ++i) {
        vp.push_back(point(rand()%100000, rand()%100000));
    }
    kdtree tree(vp);

    // query some points
    for (int i = 0; i < 10; ++i) {
        point q(rand()%100000, rand()%100000);
        cout << "Closest squared distance to (" << q.x << ", " << q.y
            << ")"
             << " is " << tree.nearest(q) << endl;
    }

    return 0;
}

//
    ---------------------------------------------------------------------------
```

5.5 Splay tree

```
#include <cstdio>
#include <algorithm>
using namespace std;

const int N_MAX = 130010;
const int oo = 0x3f3f3f3f;
struct Node
```

```
{
  Node *ch[2], *pre;
  int val, size;
  bool isTurned;
} nodePool[N_MAX], *null, *root;

Node *allocNode(int val)
{
  static int freePos = 0;
  Node *x = &nodePool[freePos ++];
  x->val = val, x->isTurned = false;
  x->ch[0] = x->ch[1] = x->pre = null;
  x->size = 1;
  return x;
}

inline void update(Node *x)
{
  x->size = x->ch[0]->size + x->ch[1]->size + 1;
}

inline void makeTurned(Node *x)
{
  if(x == null)
    return;
  swap(x->ch[0], x->ch[1]);
  x->isTurned ^= 1;
}

inline void pushDown(Node *x)
{
  if(x->isTurned)
  {
    makeTurned(x->ch[0]);
    makeTurned(x->ch[1]);
    x->isTurned ^= 1;
  }
}

inline void rotate(Node *x, int c)
{
  Node *y = x->pre;
  x->pre = y->pre;
  if(y->pre != null)
    y->pre->ch[y == y->pre->ch[1]] = x;
  y->ch[!c] = x->ch[c];
  if(x->ch[c] != null)
    x->ch[c]->pre = y;
  x->ch[c] = y, y->pre = x;
  update(y);
  if(y == root)
    root = x;
}

void splay(Node *x, Node *p)
{
  while(x->pre != p)
  {
    if(x->pre->pre == p)
      rotate(x, x == x->pre->ch[0]);
    else
    {
      Node *y = x->pre, *z = y->pre;
      if(y == z->ch[0])
      {
        if(x == y->ch[0])
          rotate(y, 1), rotate(x, 1);
        else
          rotate(x, 0), rotate(x, 1);
      }
      else
      {
        if(x == y->ch[1])
          rotate(y, 0), rotate(x, 0);
        else
          rotate(x, 1), rotate(x, 0);
      }
    }
  }
  update(x);
}

void select(int k, Node *fa)
{
  Node *now = root;
  while(1)
  {
    pushDown(now);
    int tmp = now->ch[0]->size + 1;
    if(tmp == k)
      break;
    else if(tmp < k)
      now = now->ch[1], k -= tmp;
    else
      now = now->ch[0];
  }
  splay(now, fa);
}

Node *makeTree(Node *p, int l, int r)
{
  if(l > r)
    return null;
  int mid = (l + r) / 2;
  Node *x = allocNode(mid);
  x->pre = p;
  x->ch[0] = makeTree(x, l, mid - 1);
  x->ch[1] = makeTree(x, mid + 1, r);
  update(x);
  return x;
}

int main()
{
  int n, m;
  null = allocNode(0);
  null->size = 0;
  root = allocNode(0);
  root->ch[1] = allocNode(oo);
  root->ch[1]->pre = root;
  update(root);

  scanf("%d%d", &n, &m);
  root->ch[1]->ch[0] = makeTree(root->ch[1], 1, n);
  splay(root->ch[1]->ch[0], null);

  while(m --)
  {
    int a, b;
    scanf("%d%d", &a, &b);
    a ++, b ++;
    select(a - 1, null);
    select(b + 1, root);
    makeTurned(root->ch[1]->ch[0]);
  }

  for(int i = 1; i <= n; i ++)
  {
    select(i + 1, null);
    printf("%d ", root->val);
  }
}
```

5.6 Lowest common ancestor

```
const int max_nodes, log_max_nodes;
int num_nodes, log_num_nodes, root;

vector<int> children[max_nodes];        // children[i] contains the
    children of node i
int A[max_nodes][log_max_nodes+1];      // A[i][j] is the 2^j-th
    ancestor of node i, or -1 if that ancestor does not exist
int L[max_nodes];                       // L[i] is the distance
    between node i and the root

// floor of the binary logarithm of n
int lb(unsigned int n)
{
    if(n==0)
        return -1;
    int p = 0;
    if (n >= 1<<16) { n >>= 16; p += 16; }
    if (n >= 1<< 8) { n >>=  8; p +=  8; }
    if (n >= 1<< 4) { n >>=  4; p +=  4; }
    if (n >= 1<< 2) { n >>=  2; p +=  2; }
    if (n >= 1<< 1) {           p +=  1; }
    return p;
}

void DFS(int i, int l)
{
    L[i] = l;
    for(int j = 0; j < children[i].size(); j++)
        DFS(children[i][j], l+1);
}

int LCA(int p, int q)
{
    // ensure node p is at least as deep as node q
    if(L[p] < L[q])
        swap(p, q);

    // "binary search" for the ancestor of node p situated on the same
       level as q
    for(int i = log_num_nodes; i >= 0; i--)
        if(L[p] - (1<<i) >= L[q])
            p = A[p][i];

    if(p == q)
        return p;

    // "binary search" for the LCA
    for(int i = log_num_nodes; i >= 0; i--)
        if(A[p][i] != -1 && A[p][i] != A[q][i])
        {
            p = A[p][i];
            q = A[q][i];
        }

    return A[p][0];
}

int main(int argc,char* argv[])
{
    // read num_nodes, the total number of nodes
    log_num_nodes=lb(num_nodes);

    for(int i = 0; i < num_nodes; i++)
    {
        int p;
        // read p, the parent of node i or -1 if node i is the root

        A[i][0] = p;
        if(p != -1)
            children[p].push_back(i);
        else
            root = i;
    }

    // precompute A using dynamic programming
    for(int j = 1; j <= log_num_nodes; j++)
        for(int i = 0; i < num_nodes; i++)
            if(A[i][j-1] != -1)
                A[i][j] = A[A[i][j-1]][j-1];
            else
                A[i][j] = -1;

    // precompute L
    DFS(root, 0);


    return 0;
}
```

5.7 Lazy segment tree(Java)

```
public class SegmentTreeRangeUpdate {
        public long[] leaf;
        public long[] update;
        public int origSize;
        public SegmentTreeRangeUpdate(int[] list)       {
                origSize = list.length;
                leaf = new long[4*list.length];
                update = new long[4*list.length];
                build(1,0,list.length-1,list);
        }
        public void build(int curr, int begin, int end, int[] list)
            {
                if(begin == end)
                        leaf[curr] = list[begin];
                else    {
                        int mid = (begin+end)/2;
                        build(2 * curr, begin, mid, list);
                        build(2 * curr + 1, mid+1, end, list);
                        leaf[curr] = leaf[2*curr] + leaf[2*curr+1];
                }
        }
        public void update(int begin, int end, int val) {
                update(1,0,origSize-1,begin,end,val);
        }
        public void update(int curr,  int tBegin, int tEnd, int begin,
             int end, int val)          {
                if(tBegin >= begin && tEnd <= end)
                        update[curr] += val;
                else    {
                        leaf[curr] += (Math.min(end,tEnd)-Math.max(
                            begin,tBegin)+1) * val;
                        int mid = (tBegin+tEnd)/2;
                        if(mid >= begin && tBegin <= end)
                                update(2*curr, tBegin, mid, begin, end
                                    , val);
                        if(tEnd >= begin && mid+1 <= end)
                                update(2*curr+1, mid+1, tEnd, begin,
                                    end, val);
                }
        }
        public long query(int begin, int end)   {
                return query(1,0,origSize-1,begin,end);
        }
        public long query(int curr, int tBegin, int tEnd, int begin,
            int end)    {
```

```
			if(tBegin >= begin && tEnd <= end)	{
				if(update[curr] != 0)	{
					leaf[curr] += (tEnd-tBegin+1) * update
					    [curr];
					if(2*curr < update.length){
						update[2*curr] += update[curr
						    ];
						update[2*curr+1] += update[
						    curr];
					}
					update[curr] = 0;
				}
				return leaf[curr];
			}
			else	{
				leaf[curr] += (tEnd-tBegin+1) * update[curr];
				if(2*curr < update.length){
					update[2*curr] += update[curr];
					update[2*curr+1] += update[curr];
				}
				update[curr] = 0;
				int mid = (tBegin+tEnd)/2;
				long ret = 0;
				if(mid >= begin && tBegin <= end)
					ret += query(2*curr, tBegin, mid,
					    begin, end);
				if(tEnd >= begin && mid+1 <= end)
					ret += query(2*curr+1, mid+1, tEnd,
					    begin, end);
				return ret;
			}
		}
	}
```

6 String

6.1 Longest increasing subsequence

```
// Given a list of numbers of length n, this routine extracts a
// longest increasing subsequence.
//
// Running time: O(n log n)
//
//   INPUT: a vector of integers
//   OUTPUT: a vector containing the longest increasing subsequence

#include <iostream>
#include <vector>
#include <algorithm>

using namespace std;

typedef vector<int> VI;
typedef pair<int,int> PII;
typedef vector<PII> VPII;

#define STRICTLY_INCREASNG

VI LongestIncreasingSubsequence(VI v) {
  VPII best;
  VI dad(v.size(), -1);

  for (int i = 0; i < v.size(); i++) {
#ifdef STRICTLY_INCREASNG
    PII item = make_pair(v[i], 0);
    VPII::iterator it = lower_bound(best.begin(), best.end(), item);
    item.second = i;
#else
    PII item = make_pair(v[i], i);
    VPII::iterator it = upper_bound(best.begin(), best.end(), item);
#endif
    if (it == best.end()) {
      dad[i] = (best.size() == 0 ? -1 : best.back().second);
      best.push_back(item);
    } else {
      dad[i] = it == best.begin() ? -1 : prev(it)->second;
      *it = item;
    }
  }

  VI ret;
  for (int i = best.back().second; i >= 0; i = dad[i])
    ret.push_back(v[i]);
  reverse(ret.begin(), ret.end());
  return ret;
}
```

6.2 Knuth-Morris-Pratt

```
/*
Finds all occurrences of the pattern string p within the
text string t. Running time is O(n + m), where n and m
are the lengths of p and t, respecitvely.
*/

#include <iostream>
#include <string>
#include <vector>

using namespace std;

typedef vector<int> VI;

void buildPi(string& p, VI& pi)
{
  pi = VI(p.length());
  int k = -2;
  for(int i = 0; i < p.length(); i++) {
    while(k >= -1 && p[k+1] != p[i])
      k = (k == -1) ? -2 : pi[k];
    pi[i] = ++k;
  }
}

int KMP(string& t, string& p)
{
  VI pi;
  buildPi(p, pi);
  int k = -1;
  for(int i = 0; i < t.length(); i++) {
    while(k >= -1 && p[k+1] != t[i])
      k = (k == -1) ? -2 : pi[k];
    k++;
    if(k == p.length() - 1) {
      // p matches t[i-m+1, ..., i]
      cout << "matched at index " << i-k << ": ";
      cout << t.substr(i-k, p.length()) << endl;
      k = (k == -1) ? -2 : pi[k];
    }
  }
  return 0;
}

int main()
{
  string a = "AABAACAADAABAABA", b = "AABA";
  KMP(a, b); // expected matches at: 0, 9, 12
  return 0;
}
```

6.3 Constraint satisfaction problems

```
// Constraint satisfaction problems

#include <cstdlib>
#include <iostream>
#include <vector>
#include <set>
using namespace std;

#define DONE   -1
#define FAILED -2

typedef vector<int> VI;
typedef vector<VI> VVI;
typedef vector<VVI> VVVI;

typedef set<int> SI;

// Lists of assigned/unassigned variables.
VI assigned_vars;
SI unassigned_vars;

// For each variable, a list of reductions (each of which a list of
    eliminated
// variables)
VVVI reductions;

// For each variable, a list of the variables whose domains it reduced
     in
// forward-checking.
```

```
VVI forward_mods;

// need to implement ----------------------------
int Value(int var);

void SetValue(int var, int value);
void ClearValue(int var);

int DomainSize(int var);
void ResetDomain(int var);
void AddValue(int var, int value);
void RemoveValue(int var, int value);

int NextVar() {
  if ( unassigned_vars.empty() ) return DONE;

  // could also do most constrained...
  int var = *unassigned_vars.begin();
  return var;
}

int Initialize() {
  // setup here
  return NextVar();
}
// ------------------------- end -- need to implement


void UpdateCurrentDomain(int var) {
  ResetDomain(var);
  for (int i = 0; i < reductions[var].size(); i++) {
    vector<int>& red = reductions[var][i];
    for (int j = 0; j < red.size(); j++) {
      RemoveValue(var, red[j]);
    }
  }
}


void UndoReductions(int var) {
  for (int i = 0; i < forward_mods[var].size(); i++) {
    int other_var = forward_mods[var][i];
    VI& red = reductions[other_var].back();
    for (int j = 0; j < red.size(); j++) {
      AddValue(other_var, red[j]);
    }
    reductions[other_var].pop_back();
  }
  forward_mods[var].clear();
}


bool ForwardCheck(int var, int other_var) {
  vector<int> red;

  foreach value in current_domain(other_var) {
    SetValue(other_var, value);
    if ( !Consistent(var, other_var) ) {
      red.push_back(value);
      RemoveValue(other_var, value);
    }
    ClearValue(other_var);
  }
  if ( !red.empty() ) {
    reductions[other_var].push_back(red);
    forward_mods[var].push_back(other_var);
  }

  return DomainSize(other_var) != 0;
}


pair<int, bool> Unlabel(int var) {
  assigned_vars.pop_back();
  unassigned_vars.insert(var);

  UndoReductions(var);
  UpdateCurrentDomain(var);

  if ( assigned_vars.empty() ) return make_pair(FAILED, true);

  int prev_var = assigned_vars.back();
  RemoveValue(prev_var, Value(prev_var));
  ClearValue(prev_var);
  if ( DomainSize(prev_var) == 0 ) {
    return make_pair(prev_var, false);
  } else {
    return make_pair(prev_var, true);
  }
}


pair<int, bool> Label(int var) {
  unassigned_vars.erase(var);
  assigned_vars.push_back(var);

  bool consistent;
  foreach value in current_domain(var) {
    SetValue(var, value);
    consistent = true;
    for (int j=0; j<unassigned_vars.size(); j++) {
      int other_var = unassigned_vars[j];
      if ( !ForwardCheck(var, other_var) ) {
        RemoveValue(var, value);
        consistent = false;
        UndoReductions(var);
        ClearValue(var);
        break;
      }
    }
    if ( consistent ) return (NextVar(), true);
  }
  return make_pair(var, false);
}


void BacktrackSearch(int num_var) {
  // (next variable to mess with, whether current state is consistent)
  pair<int, bool> var_consistent = make_pair(Initialize(), true);
  while ( true ) {
    if ( var_consistent.second ) var_consistent = Label(var_consistent
        .first);
    else var_consistent = Unlabel(var_consistent.first);

    if ( var_consistent.first == DONE ) return; // solution found
    if ( var_consistent.first == FAILED ) return; // no solution
  }
}
```

6.4 Longest common subsequence

```
/*
Calculates the length of the longest common subsequence of two vectors
  .
Backtracks to find a single subsequence or all subsequences. Runs in
O(m*n) time except for finding all longest common subsequences, which
may be slow depending on how many there are.
*/

#include <iostream>
#include <vector>
#include <set>
```

```
#include <algorithm>

using namespace std;

typedef int T;
typedef vector<T> VT;
typedef vector<VT> VVT;

typedef vector<int> VI;
typedef vector<VI> VVI;

void backtrack(VVI& dp, VT& res, VT& A, VT& B, int i, int j)
{
  if(!i || !j) return;
  if(A[i-1] == B[j-1]) { res.push_back(A[i-1]); backtrack(dp, res, A,
      B, i-1, j-1); }
  else
  {
    if(dp[i][j-1] >= dp[i-1][j]) backtrack(dp, res, A, B, i, j-1);
    else backtrack(dp, res, A, B, i-1, j);
  }
}

void backtrackall(VVI& dp, set<VT>& res, VT& A, VT& B, int i, int j)
{
  if(!i || !j) { res.insert(VI()); return; }
  if(A[i-1] == B[j-1])
  {
    set<VT> tempres;
    backtrackall(dp, tempres, A, B, i-1, j-1);
    for(set<VT>::iterator it=tempres.begin(); it!=tempres.end(); it++)
    {
      VT temp = *it;
      temp.push_back(A[i-1]);
      res.insert(temp);
    }
  }
  else
  {
    if(dp[i][j-1] >= dp[i-1][j]) backtrackall(dp, res, A, B, i, j-1);
    if(dp[i][j-1] <= dp[i-1][j]) backtrackall(dp, res, A, B, i-1, j);
  }
}

VT LCS(VT& A, VT& B)
{
  VVI dp;
  int n = A.size(), m = B.size();
  dp.resize(n+1);
  for(int i=0; i<=n; i++) dp[i].resize(m+1, 0);

  for(int i=1; i<=n; i++)
    for(int j=1; j<=m; j++)
    {
      if(A[i-1] == B[j-1]) dp[i][j] = dp[i-1][j-1]+1;
      else dp[i][j] = max(dp[i-1][j], dp[i][j-1]);
    }

  VT res;
  backtrack(dp, res, A, B, n, m);
  reverse(res.begin(), res.end());
  return res;
}

set<VT> LCSall(VT& A, VT& B)
{
  VVI dp;
  int n = A.size(), m = B.size();
  dp.resize(n+1);
  for(int i=0; i<=n; i++) dp[i].resize(m+1, 0);
  for(int i=1; i<=n; i++)
    for(int j=1; j<=m; j++)
    {
      if(A[i-1] == B[j-1]) dp[i][j] = dp[i-1][j-1]+1;
      else dp[i][j] = max(dp[i-1][j], dp[i][j-1]);
    }
  set<VT> res;
  backtrackall(dp, res, A, B, n, m);
  return res;
}

int main()
{
  int a[] = { 0, 5, 5, 2, 1, 4, 2, 3 }, b[] = { 5, 2, 4, 3, 2, 1, 2,
      1, 3 };
  VI A = VI(a, a+8), B = VI(b, b+9);
  VI C = LCS(A, B);

  for(int i=0; i<C.size(); i++) cout << C[i] << " ";
  cout << endl << endl;

  set <VI> D = LCSall(A, B);
  for(set<VI>::iterator it = D.begin(); it != D.end(); it++)
  {
    for(int i=0; i<(*it).size(); i++) cout << (*it)[i] << " ";
    cout << endl;
  }
}
```

7 Formating, STL

7.1 C++ input/output

```
#include <iostream>
#include <iomanip>

using namespace std;

int main()
{
    // Ouput a specific number of digits past the decimal point,
    // in this case 5
    cout.setf(ios::fixed); cout << setprecision(5);
    cout << 100.0/7.0 << endl;
    cout.unsetf(ios::fixed);

    // Output the decimal point and trailing zeros
    cout.setf(ios::showpoint);
    cout << 100.0 << endl;
    cout.unsetf(ios::showpoint);

    // Output a '+' before positive values
    cout.setf(ios::showpos);
    cout << 100 << " " << -100 << endl;
    cout.unsetf(ios::showpos);

    // Output numerical values in hexadecimal
    cout << hex << 100 << " " << 1000 << " " << 10000 << dec << endl;
}
```

7.2 STL next permutation

```
// Example for using stringstreams and next_permutation

#include <algorithm>
#include <iostream>
#include <sstream>
#include <vector>

using namespace std;

int main(void){
  vector<int> v;

  v.push_back(1);
  v.push_back(2);
  v.push_back(3);
  v.push_back(4);

  // Expected output: 1 2 3 4
  //                  1 2 4 3
  //                  ...
  //                  4 3 2 1
  do {
    ostringstream oss;
    oss << v[0] << " " << v[1] << " " << v[2] << " " << v[3];

    // for input from a string s,
    //   istringstream iss(s);
    //   iss >> variable;

    cout << oss.str() << endl;
  } while (next_permutation (v.begin(), v.end()));

  v.clear();

  v.push_back(1);
  v.push_back(2);
  v.push_back(1);
  v.push_back(3);

  // To use unique, first sort numbers.  Then call
  // unique to place all the unique elements at the beginning
  // of the vector, and then use erase to remove the duplicate
  // elements.

  sort(v.begin(), v.end());
  v.erase(unique(v.begin(), v.end()), v.end());

  // Expected output: 1 2 3
  for (size_t i = 0; i < v.size(); i++)
    cout << v[i] << " ";
  cout << endl;
}
```

7.3 Dates

```
// Routines for performing computations on dates.  In these routines,
// months are expressed as integers from 1 to 12, days are expressed
// as integers from 1 to 31, and years are expressed as 4-digit
// integers.

#include <iostream>
#include <string>

using namespace std;

string dayOfWeek[] = {"Mon", "Tue", "Wed", "Thu", "Fri", "Sat", "Sun"
    };

// converts Gregorian date to integer (Julian day number)
int dateToInt (int m, int d, int y){
  return
    1461 * (y + 4800 + (m - 14) / 12) / 4 +
    367 * (m - 2 - (m - 14) / 12 * 12) / 12 -
    3 * ((y + 4900 + (m - 14) / 12) / 100) / 4 +
    d - 32075;
}

// converts integer (Julian day number) to Gregorian date: month/day/
    year
void intToDate (int jd, int &m, int &d, int &y){
  int x, n, i, j;

  x = jd + 68569;
  n = 4 * x / 146097;
  x -= (146097 * n + 3) / 4;
  i = (4000 * (x + 1)) / 1461001;
  x -= 1461 * i / 4 - 31;
  j = 80 * x / 2447;
  d = x - 2447 * j / 80;
  x = j / 11;
  m = j + 2 - 12 * x;
  y = 100 * (n - 49) + i + x;
}

// converts integer (Julian day number) to day of week
string intToDay (int jd){
  return dayOfWeek[jd % 7];
}

int main (int argc, char **argv){
  int jd = dateToInt (3, 24, 2004);
  int m, d, y;
  intToDate (jd, m, d, y);
  string day = intToDay (jd);
```

```
    // expected output:
    //    2453089
    //    3/24/2004
    //    Wed
    cout << jd << endl
      << m << "/" << d << "/" << y << endl
      << day << endl;
}
```

7.4 Dates (Java)

```
// Example of using Java's built-in date calculation routines

import java.text.SimpleDateFormat;
import java.util.*;

public class Dates {
    public static void main(String[] args) {
        Scanner s = new Scanner(System.in);
        SimpleDateFormat sdf = new SimpleDateFormat("M/d/yyyy");
        while (true) {
            int n = s.nextInt();
            if (n == 0) break;
            GregorianCalendar c = new GregorianCalendar(n, Calendar.
                JANUARY, 1);
            while (c.get(Calendar.DAY_OF_WEEK) != Calendar.SATURDAY)
                c.add(Calendar.DAY_OF_YEAR, 1);
            for (int i = 0; i < 12; i++) {
                System.out.println(sdf.format(c.getTime()));
                while (c.get(Calendar.MONTH) == i) c.add(Calendar.
                    DAY_OF_YEAR, 7);
            }
        }
    }
}
```

7.5 Decimal output formatting (Java)

```
// examples for printing floating point numbers

import java.util.*;
import java.io.*;
import java.text.DecimalFormat;

public class DecFormat {
    public static void main(String[] args) {
        DecimalFormat fmt;

        // round to at most 2 digits, leave of digits if not needed
        fmt = new DecimalFormat("#.##");
        System.out.println(fmt.format(12345.6789)); // produces
            12345.68
        System.out.println(fmt.format(12345.0)); // produces 12345
        System.out.println(fmt.format(0.0)); // produces 0
        System.out.println(fmt.format(0.01)); // produces .1

        // round to precisely 2 digits
        fmt = new DecimalFormat("#.00");
        System.out.println(fmt.format(12345.6789)); // produces
            12345.68
        System.out.println(fmt.format(12345.0)); // produces 12345.00
        System.out.println(fmt.format(0.0)); // produces .00

        // round to precisely 2 digits, force leading zero
        fmt = new DecimalFormat("0.00");
        System.out.println(fmt.format(12345.6789)); // produces
            12345.68
        System.out.println(fmt.format(12345.0)); // produces 12345.00
        System.out.println(fmt.format(0.0)); // produces 0.00

        // round to precisely 2 digits, force leading zeros
        fmt = new DecimalFormat("000000000.00");
        System.out.println(fmt.format(12345.6789)); // produces
            000012345.68
        System.out.println(fmt.format(12345.0)); // produces
            000012345.00
        System.out.println(fmt.format(0.0)); // produces 000000000.00

        // force leading '+'
        fmt = new DecimalFormat("+0;-0");
        System.out.println(fmt.format(12345.6789)); // produces +12346
        System.out.println(fmt.format(-12345.6789)); // produces
            -12346
        System.out.println(fmt.format(0)); // produces +0

        // force leading positive/negative, pad to 2
        fmt = new DecimalFormat("positive 00;negative 0");
        System.out.println(fmt.format(1)); // produces "positive 01"
        System.out.println(fmt.format(-1)); // produces "negative 01"

        // qoute special chars (#)
        fmt = new DecimalFormat("text with '#' followed by #");
        System.out.println(fmt.format(12.34)); // produces "text with
            # followed by 12"

        // always show "."
        fmt = new DecimalFormat("#.#");
        fmt.setDecimalSeparatorAlwaysShown(true);
        System.out.println(fmt.format(12.34)); // produces "12.3"
        System.out.println(fmt.format(12)); // produces "12."
        System.out.println(fmt.format(0.34)); // produces "0.3"

        // different grouping distances:
        fmt = new DecimalFormat("#,####.###");
        System.out.println(fmt.format(123456789.123)); // produces
            "1,2345,6789.123"

        // scientific:
        fmt = new DecimalFormat("0.000E00");
        System.out.println(fmt.format(123456789.123)); // produces
            "1.235E08"
        System.out.println(fmt.format(-0.000234)); // produces "-2.34E
            -04"

        // using variable number of digits:
        fmt = new DecimalFormat("0");
        System.out.println(fmt.format(123.123)); // produces "123"
        fmt.setMinimumFractionDigits(8);
        System.out.println(fmt.format(123.123)); // produces
            "123.12300000"
        fmt.setMaximumFractionDigits(0);
        System.out.println(fmt.format(123.123)); // produces "123"

        // note: to pad with spaces, you need to do it yourself:
        // String out = fmt.format(...)
        // while (out.length() < targlength) out = " "+out;
    }
}
```

7.6 Regular expressions

```
// Code which demonstrates the use of Java's regular expression
    libraries.
// This is a solution for
```

```java
//
//   Loglan: a logical language
//   http://acm.uva.es/p/v1/134.html
//
// In this problem, we are given a regular language, whose rules can be
// inferred directly from the code.  For each sentence in the input, we must
// determine whether the sentence matches the regular expression or not.  The
// code consists of (1) building the regular expression (which is fairly
// complex) and (2) using the regex to match sentences.

import java.util.*;
import java.util.regex.*;

public class LogLan {

    public static String BuildRegex (){
        String space = " +";

        String A = "([aeiou])";
        String C = "([a-z&&[^aeiou]])";
        String MOD = "(g" + A + ")";
        String BA = "(b" + A + ")";
        String DA = "(d" + A + ")";
        String LA = "(l" + A + ")";
        String NAM = "([a-z]*" + C + ")";
        String PREDA = "(" + C + C + A + C + A + "|" + C + A + C + C + A + ")";

        String predstring = "(" + PREDA + "(" + space + PREDA + ")*)";
        String predname = "(" + LA + space + predstring + "|" + NAM + ")";
        String preds = "(" + predstring + "(" + space + A + space + predstring + ")*)";
        String predclaim = "(" + predname + space + BA + space + preds + "|" + DA + space +
            preds + ")";
        String verbpred = "(" + MOD + space + predstring + ")";
        String statement = "(" + predname + space + verbpred + space + predname + "|" +
            predname + space + verbpred + ")";
        String sentence = "(" + statement + "|" + predclaim + ")";

        return "^" + sentence + "$";
    }

    public static void main (String args[]){

        String regex = BuildRegex();
        Pattern pattern = Pattern.compile (regex);

        Scanner s = new Scanner(System.in);
        while (true) {

            // In this problem, each sentence consists of multiple lines, where the last
            // line is terminated by a period.  The code below reads lines until
            // encountering a line whose final character is a '.'.  Note the use of
            //
            //    s.length() to get length of string
            //    s.charAt() to extract characters from a Java string
            //    s.trim() to remove whitespace from the beginning and end of Java string
            //
            // Other useful String manipulation methods include
            //
            //    s.compareTo(t) < 0 if s < t, lexicographically
            //    s.indexOf("apple") returns index of first occurrence of "apple" in s
            //    s.lastIndexOf("apple") returns index of last occurrence of "apple" in s
            //    s.replace(c,d) replaces occurrences of character c with d
            //    s.startsWith("apple) returns (s.indexOf("apple") == 0)
            //    s.toLowerCase() / s.toUpperCase() returns a new lower/uppercased string
            //
            //    Integer.parseInt(s) converts s to an integer (32-bit)
            //    Long.parseLong(s) converts s to a long (64-bit)
            //    Double.parseDouble(s) converts s to a double

            String sentence = "";
            while (true){
                sentence = (sentence + " " + s.nextLine()).trim();
                if (sentence.equals("#")) return;
                if (sentence.charAt(sentence.length()-1) == '.') break;
            }

            // now, we remove the period, and match the regular expression

            String removed_period = sentence.substring(0, sentence.length()-1).trim();
            if (pattern.matcher (removed_period).find()){
                System.out.println ("Good");
            } else {
                System.out.println ("Bad!");
            }
        }
    }
}
```

8 Classical Problems

8.1 Illumination(Tarjan 2SAT)

```
/*
 tarjan + 2sat
You inherited a haunted house. Its floor plan is an n-by-n square grid
     with l lamps in fixed
locations and no interior walls. Each lamp can either illuminate its
    row or its column, but not both
simultaneously. The illumination of each lamp extends by r squares in
    both directions, so a lamp
unobstructed by an exterior wall of the house can illuminate as many
    as 2r + 1 squares.

n, r and l (1 < n, r, l < 1,000).
ri and ci (1 < ri, ci < n), indicating that there is a lamp in row ri
    and column ci.

YES if it is possible to illuminate all lamps as stated above
*/

#include <cstdio>
#include <vector>
#include <utility>
#include <algorithm>
#include <cstring>
#include <iostream>
#include <stack>

using namespace std;
typedef pair<int, int> pii;

const int MAXN = 10005;
vector<int> g[MAXN];
int d[MAXN], low[MAXN], scc[MAXN];
bool stacked[MAXN];
int ticks, current_scc;
stack<int> s;

void tarjan(int u) {
        d[u] = low[u] = ticks++;
        s.push(u);
        stacked[u] = true;
        const vector<int> &out = g[u];
        for (int k = 0, m = (int)out.size(); k < m; k++) {
                const int &v = out[k];
                if (d[v] == -1) {
                        tarjan(v);
                        low[u] = min(low[u], low[v]);
                } else if (stacked[v]) {
                        low[u] = min(low[u], low[v]);
                }
        }
        if (d[u] == low[u]) {
                int v;
                do {
                        v = s.top();
                        s.pop();
                        stacked[v] = false;
                        scc[v] = current_scc;
                } while (u != v);
                current_scc++;
        }
}

int n, r, l;
vector<pii> lamps;

int main() {
        cin >> n >> r >> l;
        for (int i = 1; i <= l; i++) {
                int x, y;
                cin >> x >> y;
                lamps.push_back(make_pair(x, y));
        }

        memset(d, -1, sizeof(d));

        for (int i = 0; i < l; i++) {
                int mr = lamps[i].first;
                for (int j = i + 1; j < l; j++) {
                        if (mr == lamps[j].first && abs(lamps[i].
                            second - lamps[j].second) <= 2 * r) {
                                g[i].push_back(j + l);
                                g[j].push_back(i + l);
                        }
                }
        }

        for (int i = 0; i < l; i++) {
                int mc = lamps[i].second;
                for (int j = i + 1; j < l; j++) {
                        if (mc == lamps[j].second && abs(lamps[i].
                            first - lamps[j].first) <= 2 * r) {
                                g[i + l].push_back(j);
                                g[j + l].push_back(i);
                        }
                }
        }

        for (int i = 0; i < l * 2; i++) {
                if (d[i] == -1) tarjan(i);
        }

        for (int i = 0; i < l; i++) {
                if (scc[i] == scc[i + l]) {
                        cout << "0" << endl;
                        return 0;
                }
        }

        cout << "1" << endl;
        return 0;
}
```

8.2 BuggyRobot (Dijkstra + state hash)

```
// go from S to G with minimal amount of edition to instruction L, R,
    U, and D

#include <string>
#include <vector>
#include <queue>
#include <iostream>
using namespace std ;
int moveind[128] ;
int moves[4] ;
const int INF = 1000000000 ;
int main() {
   int N, M ;
   cin >> N >> M ;
   string lin ;
   for (int i=0; i<M+2; i++)
      lin.push_back('#') ;
   for (int i=0; i<N; i++) {
      string inp ;
```

```
    cin >> inp ;
    lin.append(inp) ;
    lin.push_back('#') ;
}
for (int i=0; i<M+2; i++)
    lin.push_back('#') ;
int st = 0 ;
int end = 0 ;
for (int i=0; i<lin.size(); i++)
    if (lin[i] == 'S')
        st = i ;
    else if (lin[i] == 'G')
        end = i ;
string cmd ;
cin >> cmd ;
moveind['L'] = -1 ;
moveind['R'] = 1 ;
moveind['U'] = -M-1 ;
moveind['D'] = M+1 ;
moves[0] = -1 ;
moves[1] = 1 ;
moves[2] = -M-1 ;
moves[3] = M+1 ;
int atmod = lin.size() ;
vector<int> cost(lin.size()*(cmd.size()+1), INF) ;
vector<int> thislev ;
thislev.push_back(st) ;
cost[st] = 0 ;
int finished = -1 ;
for (int d=0; finished < 0; d++) {
    // first, all the additional nodes we can reach for free
    for (int i=0; i<thislev.size(); i++) {
        int at = thislev[i] ;
        int sqat = at % atmod ;
        int cmdat = at / atmod ;
        if (cmdat < cmd.size()) {
            int nsq = sqat + moveind[cmd[cmdat]] ;
            if (nsq == end)
                finished = d ;
            if (lin[nsq] == '#')
                nsq = sqat ;
            int st2 = nsq + (cmdat + 1) * atmod ;
            if (cost[st2] > d) {
                cost[st2] = d ;
                thislev.push_back(st2) ;
            }
        }
    }
    // now advance by additional moves; these cost
    vector<int> nextlev ;
    for (int i=0; finished < 0 && i<thislev.size(); i++) {
        int at = thislev[i] ;
        int sqat = at % atmod ;
        for (int mv=0; mv<4; mv++) {
            int nsq = sqat + moves[mv] ;
            if (nsq == end)
                finished = d + 1 ;
            int nat = at + moves[mv] ;
            if (lin[nsq] != '#' && cost[nat] > d + 1) {
                cost[nat] = d + 1 ;
                nextlev.push_back(nat) ;
            }
        }
    }
    swap(thislev, nextlev) ;
    nextlev.clear() ;
    if (thislev.size() == 0)
        break ;
}
    cout << finished << endl ;
}
```

8.3 Paint (dp + binary search backtrace)

```
//https://open.kattis.com/problems/paint
// the smallest number of slats that go unpainted with an optimal
    selection of painters
// dp with binary search backtrace
#include <bits/stdc++.h>

using namespace std;

typedef long long ll;
typedef pair<ll, ll> pll;
#define pb push_back
#define mp make_pair

ll n, k;

ll find_idx(vector<pll> &slabs, ll maxlen) {
    ll low = 0, high = k-1, mid;
    if (slabs[0].second > maxlen) return -1;
    while (low < high) {
        if (high == low+1) {
            if (slabs[high].second <= maxlen) return high;
            else return low;
        }
        mid = (low + high) / 2;
        if (slabs[mid].second > maxlen) high = mid;
        else low = mid;
    }
    return mid;
}

int main() {
    cin >> n >> k;
    vector<pll> slabs;
    for (ll i=0; i<k; i++) {
        ll a, b; cin >> a >> b;
        slabs.pb(mp(a, b));
    }

    sort(slabs.begin(), slabs.end(), [](pll &p1, pll &p2){return (p1.
        second < p2.second) || (p1.second == p2.second && p1.first <
        p2.first);});

    ll dp[k], cnt[k];
    dp[0] = slabs[0].second - slabs[0].first + 1;
    cnt[0] = 1;
    for (ll i=1; i<k; i++) {
        ll val1 = dp[i-1];
        ll idx = find_idx(slabs, slabs[i].first-1);
        ll val2 = idx < 0 ? 0 : dp[idx];
        val2 += slabs[i].second - slabs[i].first + 1;
        if (val1 > val2) {
            dp[i] = val1;
            cnt[i] = cnt[i-1];
        } else if (val1 < val2) {
            dp[i] = val2;
            cnt[i] = idx < 0 ? 1 : cnt[idx] + 1;
        } else {
            dp[i] = val1;
            cnt[i] = max(cnt[i-1], (idx < 0 ? 1 : cnt[idx] + 1));
        }
    }

    cout << n - dp[k-1] << endl;
```

```
}
```

8.4 Rainbow (dfs + mark)

```
// https://open.kattis.com/problems/rainbowroads
/*
You are given a tree with n nodes (stations), conveniently numbered
    from 1 to n.
Each edge in this tree has one of n colors.
A path in this tree is called a rainbow if all adjacent edges in the
    path have different colors.
Also, a node is called good if every simple path with that node as one
    of its endpoints is a rainbow path.
(A simple path is a path that does not repeat any vertex or edge.)

Find all the good nodes in the given tree.
*/
#include <iostream>
#include <map>
#include <vector>
#include <cstdio>

using namespace std;

map<int, vector<int> > adj[50005];
int last[50005];
bool locked[50005];

void mark(int src, int dst) {
  if (locked[dst]) return;
  if (last[dst] == 0) {
    last[dst] = src;
    for (auto entry : adj[dst]) {
      for (auto next : entry.second) {
        if (next != src) mark(dst, next);
      }
    }
  } else if (last[dst] != src) {
    mark(dst, last[dst]);
  }
}

int main() {
  int n;
  scanf("%d", &n);
  for (int i = 1; i < n; i++) {
    int a, b, c;
    scanf("%d %d %d", &a, &b, &c);
    adj[a][c].push_back(b);
    adj[b][c].push_back(a);
  }

  for (int i = 1; i <= n; i++) {
    if (locked[i]) continue;
    for (auto entry : adj[i]) {
      if (entry.second.size() > 1) {
        for (auto next : entry.second) mark(i, next);
      }
    }
  }

  int count = 0;
  for (int i = 1; i <= n; i++) {
    if (last[i] == 0) count++;
  }
  cout << count << endl;
  for (int i = 1; i <= n; i++) {
    if (last[i] == 0) cout << i << endl;
```

```
  }
}
```

8.5 Security badge (dfs + memorization)

```
//https://open.kattis.com/problems/securitybadge
// N, L, and B, denoting the number of rooms, of locks, and of badge
    numbers
// S and D noting the starting and destination rooms that we are
    interested in
// a b x y indicating that a lock permits passage from room a to room
    b (but not from b to a) for badges numbered from x to y, inclusive

#include <iostream>
#include <vector>
#include <set>

using namespace std;

struct door {
  int dst, lo, hi;
  door(int dst, int lo, int hi) : dst(dst), lo(lo), hi(hi) {}
};

typedef vector<door> VD;

VD adj[1024];
int s, t;

bool visited[1024];

void dfs(int cur, int id) {
  if (visited[cur]) return;
  visited[cur] = true;
  for (auto edge : adj[cur]) {
    if (id >= edge.lo && id <= edge.hi) dfs(edge.dst, id);
  }
}

bool accessible(int id) {
  for (int i = 0 ; i < 1024; i++) visited[i] = false;
  dfs(s, id);
  return visited[t];
}

int main() {
  int n, m, k;
  cin >> n >> m >> k;
  cin >> s >> t;
  set<int> boundaries;
  for (int i = 1; i <= m; i++) {
    int src, dst, lo, hi;
    cin >> src >> dst >> lo >> hi;
    adj[src].push_back(door(dst, lo, hi));
    boundaries.insert(lo);
    boundaries.insert(hi + 1);
  }
  int last = 0;
  bool lastGood = false;
  int ans = 0;
  for (auto id : boundaries) {
    if (lastGood) ans += id - last;
    lastGood = accessible(id);
    last = id;
  }
  cout << ans << endl;
}
```

8.6 Radio (string hashing)

```
// find smallest possible substring S'+S'+ +S' = S cabcabca abc
#include <iostream>
#include <string>
#include <vector>

using namespace std;

typedef uint64_t ull;

ull prime = (ull) (1e9+7);

ull hashstr(string s) {
    ull hash = 0;
    ull expo = 1;
    for (int k=0; k<s.length(); k++) {
        hash = (hash + (ull)(s[k] - 'a'+1) * expo) % prime;
        expo = (expo * 32) % prime;
    }
    return hash;
}

int main() {
    int n; cin >> n;
    string s; cin >> s;

    vector<ull> lhash(n), rhash(n), expos(n);
    ull hash = 0;
    ull expo = 1;
    for (int k=0; k<n; k++) {
        hash = (hash + (ull)(s[k] - 'a'+1) * expo) % prime;
        lhash[k] = hash;
        expos[k] = expo;
        expo = (expo * 32) % prime;
    }
    hash = 0;
    for (int k=n-1; k>=0; k--) {
        hash = (hash * 32 + (ull)(s[k] - 'a'+1)) % prime;
        rhash[k] = hash;
    }
    int l;
    for (l=1; l<n; l++) if (rhash[l] == lhash[n-l-1]) break;
    cout << l << endl;
}
```

8.7 Hanoi (recusive)

```
//https://open.kattis.com/problems/thathanoi
// num of step to complete hanoi state otherwise output no
#include <iostream>
#include <cmath>
using namespace std;

const int MAX = 50;
int loc[MAX+1];

bool count(int start, int dest, int work, int disk, long long moves,
     long long& ans)
{
    if (disk == 0)
        return true;
    else if (loc[disk] == dest) {
        if (!count(work, dest, start, disk-1, moves/2, ans))
            return false;
        ans += moves;
        return true;
    }
    else if (loc[disk] == start) {
        if (!count(start, work, dest, disk-1, moves/2, ans))
            return false;
        return true;
    }
    else
        return false;
}

int main()
{
    int n=0;

    long long moves = 1;
    bool valid = true;
    for(int i=0; i<3; i++) {
        int m;
        cin >> m;
        n += m;
        int prev = MAX+1;
        for(int j=0; j<m; j++) {
            int disk;
            cin >> disk;
            loc[disk] = i;
            moves *=2;
            if (disk > prev)
                valid = false;
            prev = disk;
        }
    }
    long long ans = 0;
    if (!valid || !count(0, 2, 1, n, moves/2, ans))      // moves = 2^
        n
        cout << "No" << endl;
    else
        cout << moves-1-ans << endl;
}
```

8.8 basesum (number theory)

```
/**
 *  given n, a, and b, find the smallest m>n such that the sum of the
     digits of m in
base a is the same as the sum of digits of m in base b.
 */

inline ll ceil_div(ll a, ll b) { return b ? ((a/b) + ((a%b) != 0)) :
    INF; }
inline ll add(ll a, ll b) { return (a >= INF - b) ? INF : (a + b); }
inline ll mul(ll a, ll b) { return b ? (a >= ceil_div(INF,b) ? INF : a
    *b) : 0; }

ll iters;

void convert(ll N, int BASE, int ans[MAXD]) {
  memset(ans,0,sizeof(ans[0])*MAXD); iters += MAXD;
  int i = 0;
  while(N) ans[i++] = (N%BASE), N /= BASE, ++iters;
  assert(i<=MAXD);
}

int tmp3[MAXD];
string convert_to_string(ll N, int BASE) {
  convert(N,BASE,tmp3);
  string ans;

  bool non_zero = false;
```

```cpp
  FORB(i,MAXD-1,0) {
    ++iters;
    if (tmp3[i] != 0) non_zero = true;
    if (non_zero) ans += (tmp3[i] < 10 ? '0' + tmp3[i] : 'A' + tmp3[i]
        - 10);
  }

  if (ans.empty()) return "0";
  return ans;
}

// find the next number > N with sum K (in given BASE)
int tmp[MAXD];
ll next(ll N, int BASE, int K) {
  convert(N,BASE,tmp);

  int sum = 0;
  FOR(i,MAXD) sum += tmp[i], ++iters;

  bool found = false;
  FOR(i,MAXD) {
    ++iters;
    if (tmp[i] < BASE-1 && sum + 1 <= K && sum - tmp[i] + (BASE-1)*(i
        +1) >= K) {
      // we can (and should) bump up here
      K -= sum + 1;
      tmp[i]++;

      FOR(j,MAXD) {
        ++iters;
        assert(K>=0);
        if (!K) break;
        assert(j <= i);
        assert(tmp[j] <= BASE-1);
        int add = min(K, BASE-1-tmp[j]);
        tmp[j] += add;
        K -= add;
      }

      found = true;
      break;
    } else {
      // continue step
      sum -= tmp[i];
      tmp[i] = 0;
    }
  }

  if (!found) return INF;

  ll ans = 0;
  FORB(i,MAXD-1,0) {
    ++iters;
    ans = mul(ans, BASE);
    ans = add(ans, tmp[i]);
  }

  return ans;
}

int tmp2[MAXD];
ll digit_sum(ll N, int BASE) {
  convert(N,BASE,tmp2);
  int ans = 0;
  FOR(i,MAXD) ans += tmp2[i];
  return ans;
}

ll points[2*MAXS];
ll nxtA[MAXS];
ll nxtB[MAXS];

int main() {
  ll N; int A,B;
  cin >> N >> A >> B;
  N++;

  iters = 0;

  while(true) {
    ll x = digit_sum(N,A);
    ll y = digit_sum(N,B);

    if (x==y) break;

    ++iters;

    // find all points where the ranges change
    FOR(v,MAXS) {
      if (nxtA[v] > N) continue;
      nxtA[v] = next(N,A,v);
      assert(nxtA[v] > N);
    }

    FOR(v,MAXS) {
      if (nxtB[v] > N) continue;
      nxtB[v] = next(N,B,v);
      assert(nxtB[v] > N);
    }

    int K = 0;
    FOR(v,MAXS) {
      if (nxtA[v] >= INF) continue;
      points[K++] = nxtA[v];
    }

    FOR(v,MAXS) {
      if (nxtB[v] >= INF) continue;
      points[K++] = nxtB[v];
    }

    sort(points, points+K);
    K = unique(points,points+K) - points;

    // find the next point where the ranges overlap
    ll low_x = x, high_x = x;
    ll low_y = y, high_y = y;

    ll p;
    FOR(z,K) {
      p = points[z];
      ll x = digit_sum(p,A);
      low_x = min(low_x, x);
      high_x = max(high_x, x);

      ll y = digit_sum(p,B);
      low_y = min(low_y, y);
      high_y = max(high_y, y);

      if (high_x >= low_y && low_x <= high_y) {
        // they now overlap
        N = p;
        break;
      }
    }
  }

  cout << N << /*" " << convert_to_string(N,A) << " " <<
      convert_to_string(N,B) <<*/ endl;
}
```

8.9 Vin Diagram (flood)

```
/*
7 7
AXXXX..
X...X..
X.XXXXX
X.X.X.X
XXXXX.X
..X...X
..XXXXB

A B    A intersect B
5 5          1

*/
#include <iostream>
#include <vector>
using namespace std;

const int MAXS = 1000;

vector<vector<int>> DIR = {{1, 0}, {-1, 0} ,{0, 1} , {0, -1}};

char graph[MAXS+2][MAXS+2];

inline bool isIntersection(int r, int c){
    return (graph[r][c] == 'X'
        && graph[r+1][c] == 'X'
        && graph[r-1][c] == 'X'
        && graph[r][c+1] == 'X'
        && graph[r][c-1] == 'X');
}

int flood(int r, int c, char mark, int nrows, int ncols){
    if (graph[r][c] != '.') return 0;
    graph[r][c] = mark;
    int count = 1;
    if (r > 0)
        count += flood(r-1, c, mark, nrows, ncols);
    if (r < nrows-1)
        count += flood(r+1, c, mark, nrows, ncols);
    if (c > 0)
        count += flood(r, c-1, mark, nrows, ncols);
    if (c < ncols-1)
        count += flood(r, c+1, mark, nrows, ncols);
    return count;
}

void findstart(int&r, int& c, char mark, int n, int m){
    int x, y;
    for(int i=1; i<=n; i++) for(int j=1; j<=m; j++) {
        if (graph[i][j] != mark) continue;
        for (auto &txy: DIR) {
            x = txy[0]; y = txy[1];
            if (graph[i+x][j+y] == '.' && graph[i-x][j-y] == '*') {
                r = i+x;
                c = j+y;
                return;
            }
        }
    }
}

int main(){
    int n, m;
    cin >> n >> m;

    int ax, ay, bx, by;
    int edges = 0;

    // extra side helper
    for (int j=0; j<m+2; j++)
        graph[0][j] = graph[n+1][j] = '.';
    for (int i=1; i<=n; i++)
        graph[i][0] = graph[i][m+1] = '.';

    for (int i=1; i<=n; i++) for (int j=1; j<=m; j++){
        cin >> graph[i][j];
        if (graph[i][j] == 'A'){ // mark 'A' coordinate
            ax = i; ay = j;
        }else if (graph[i][j] == 'B'){ // mark 'B' coordinate
            bx = i; by = j;
        }
        if (graph[i][j] != '.') edges++;
    }

        // set intersections
    for (int i=1; i<=n; i++) for (int j=1; j<=m; j++)
        if (isIntersection(i,j)) graph[i][j] = 'I';

    // fill edge connected to 'A'
    int r=ax, c=ay;
    bool done = false;
    int x, y;
    while(!done) {
        graph[r][c] = 'A';
        done = true;
        for (auto &txy: DIR) {
            x = txy[0]; y = txy[1];
            if (graph[r+x][c+y] == 'X' || graph[r+x][c+y] == 'I'){
                if (graph[r+x][c+y] == 'I') {
                    graph[r+x][c+y] = 'O';
                    r = r + 2*x;
                    c = c + 2*y;
                }else{
                    r = r + x;
                    c = c + y;
                }
                done = false;
                break;
            }
        }
    }

        // set remaining edge to 'B'
    for (int i=1; i<=n; i++) for (int j=1; j<=m; j++)
        if (graph[i][j] == 'X') graph[i][j] = 'B';

        // fill exterior '*' from (0,0)
    int exterior = flood(0, 0, '*', n+2, m+2);
    exterior -= 2*(n+m+2);

        // fill A area
    findstart(r, c, 'A', n, m);
    int anum = flood(r, c, 'a', n+2, m+2);
    // fill B area
    findstart(r, c, 'B', n, m);
    int bnum = flood(r, c, 'b', n+2, m+2);
    int abnum = n*m - edges - exterior - anum - bnum;

        cout << anum << ' ' << bnum << ' ' << abnum << endl;
    return 0;
}
```

8.10 substring (suffix)

```
#include <vector>
#include <algorithm>
#include <iostream>
#include <string>

using namespace std;
```

```
struct suffix
{
    int index;
    int rank[2];

    bool operator<(const suffix &other) const
    {
        if (rank[0] < other.rank[0])
            return true;
        if (rank[0] > other.rank[0])
            return false;
        return rank[1] < other.rank[1];
    }
};

void buildSuffixArray(string &txt, int n, vector<int> &sufarray)
{
    std::vector<suffix> suffixes;
    suffixes.resize(n);

    for (int i = 0; i < n; i++)
    {
        suffixes[i].index = i;
        suffixes[i].rank[0] = txt[i] - 'a';
        suffixes[i].rank[1] = ((i+1) < n)? (txt[i + 1] - 'a'): -1;
    }

    std::sort(suffixes.begin(), suffixes.end());
    std::vector<int> ind;
    ind.resize(n);

    for (int k = 4; k < 2*n; k = k*2)
    {
        int rank = 0;
        int prev_rank = suffixes[0].rank[0];
        suffixes[0].rank[0] = rank;
        ind[suffixes[0].index] = 0;

        for (int i = 1; i < n; i++)
        {
            if (suffixes[i].rank[0] == prev_rank &&
                suffixes[i].rank[1] == suffixes[i-1].rank[1])
            {
                prev_rank = suffixes[i].rank[0];
                suffixes[i].rank[0] = rank;
            }
            else
            {
                prev_rank = suffixes[i].rank[0];
                suffixes[i].rank[0] = ++rank;
            }
            ind[suffixes[i].index] = i;
        }

        for (int i = 0; i < n; i++)
        {
            int nextindex = suffixes[i].index + k/2;
            suffixes[i].rank[1] = (nextindex < n)?
                suffixes[ind[nextindex]].rank[0]: -1;
        }

        std::sort(suffixes.begin(), suffixes.end());
    }

    sufarray.clear();
    for (int i = 0; i < n; i++)
        sufarray.push_back(suffixes[i].index);
}

void kasai(string &txt, vector<int> &suffixArr, vector<int> &result)
{
    int n = suffixArr.size();
    result.resize(n, 0);
    vector<int> invSuff(n, 0);

    for (int i=0; i < n; i++)
        invSuff[suffixArr[i]] = i;

    int k = 0;

    for (int i=0; i<n; i++)
    {
        if (invSuff[i] == n-1)
        {
            k = 0;
            continue;
        }

        int j = suffixArr[invSuff[i]+1];

        while (i+k<n && j+k<n && txt[i+k]==txt[j+k])
            k++;

        result[invSuff[i]] = k;
        if (k>0)
            k--;
    }
}

int main()
{
    std::string s;
    std::cin >> s;
    std::vector<int> sufarray;
    buildSuffixArray(s, s.length(), sufarray);
    std::vector<int> lcp;
    kasai(s, sufarray, lcp);
    int maxlen = -1;
    int maxstr = -1;
    for (int i = 0; i < lcp.size(); i++)
    {
        if (lcp[i] > maxlen)
        {
            maxlen = lcp[i];
            maxstr = i;
        }
    }
    for (int j = 0; j < maxlen; j++)
    {
        std::cout << s[sufarray[maxstr] + j];
    }
    std::cout << std::endl;
}
```

8.11 average manhattan (comptational geometry)

```
/*
Within a zone:  Z = (l^2+3lr+r^2)w^3/15

Cx(i) = x0 + (l+2r)/(3(l+r))w

*/
#include <iostream>
#include <algorithm>
#include <iomanip>
#include <cstdlib>
#include <vector>
#include <math.h>
#include <tuple>
using namespace std ;
```

```
using ll = long long ;
using t3 = tuple<double, double, double> ;
int main() {
   cout << setprecision(15) ;
   int N{0} ;
   cin >> N ;
   vector<ll> xs(N), ys(N) ;
   for (int i=0; i<N; i++)
      cin >> xs[i] >> ys[i] ;
   double r {} ;
   for (int outer=0; outer<2; outer++) {
      int hix0 = min_element(xs.begin(), xs.end()) - xs.begin() ;
      int lox0 { hix0 } ;
      auto ht=[&](ll x, int p0, int p1) -> double {
         if (x == xs[p0])
            return ys[p0] ;
         if (x == xs[p1])
            return ys[p1] ;
         return ys[p0]+(x-xs[p0])*(ys[p1]-ys[p0])/(double)(xs[p1]-xs[
             p0]) ;
      } ;
      vector<t3> zones ;
      while (1) {
         int hix1 { (hix0 + 1) % N } ;
         int lox1 { (lox0 + N - 1) % N } ;
         ll x0 = max(xs[hix0], xs[lox0]) ;
         if (x0 == xs[hix1]) {
            hix0 = hix1 ;
            continue ;
         }
         if (x0 == xs[lox1]) {
            lox0 = lox1 ;
            continue ;
         }
         ll x1 { min(xs[hix1], xs[lox1]) } ;
         if (x1 < x0)
            break ;
         if (x1 == x0)
            throw "Failed while building zones" ;
         double lft { ht(x0, hix0, hix1) - ht(x0, lox0, lox1) } ;
         double rgt { ht(x1, hix0, hix1) - ht(x1, lox0, lox1) } ;
         zones.push_back(make_tuple(lft, rgt, (double)(x1-x0))) ;
//  cout << "Adding tuple xs " << x0 << " " << x1 << " heights " <<
    lft << " " << rgt << endl ;
         if (x1 == xs[hix1])
            hix0 = hix1 ;
         if (x1 == xs[lox1])
            lox0 = lox1 ;
      }
      double s {} ;
      double cxa {} ;
      double sa {} ;
      double a2 {} ;
      double x0 {} ;
      for (int i=0; i<zones.size(); i++) {
         t3 &z = zones[i] ;
         double lft { get<0>(z) }, rgt { get<1>(z) }, w { get<2>(z) }
             ;
         s += (lft*lft+3*lft*rgt+rgt*rgt)*w*w*w/15 ;
         double ta { (lft+rgt)*w/2 } ;
         double cx { x0 + (lft+2*rgt)/(3*(lft+rgt))*w } ;
         if (i)
            s += 2 * (cx - cxa / sa) * sa * ta ;
         a2 += ta*(2*sa+ta) ;
         sa += ta ;
         cxa += cx * ta ;
         x0 += w ;
      }
      r += s / a2 ;
      swap(xs, ys) ;
      reverse(xs.begin(), xs.end()) ;
      reverse(ys.begin(), ys.end()) ;
   }
   cout << r << endl ;
}
```