AE 498 Planetary Defense, HW1

Problem (100 points)

The minimum orbit intersection distance (MOID) between two heliocentric Keplerian orbits is defined as the minimum Euclidean distance between the two orbits, independent of timing:

$$\mathrm{MOID} = \min_{f_1, f_2 \in [0, 2\pi)} \|\boldsymbol{r}_1(f_1) - \boldsymbol{r}_2(f_2)\|,$$

where $\boldsymbol{r}_{1,2}$ are the position vectors of celestial bodies 1 and 2 with respect to the center they orbit and $f_{1,2}$ are true anomalies (or eccentric/hyperbolic anomalies) for each orbit. Defining the squared distance

$$d^2(f_1, f_2) = \|\boldsymbol{r}_1(f_1) - \boldsymbol{r}_2(f_2)\|^2,$$

stationary points satisfy

$$\frac{\partial d^2}{\partial f_1} = 0 \quad \text{and} \quad \frac{\partial d^2}{\partial f_2} = 0,$$

where the MOID is the global minimum of the distance function over $(f_1, f_2) \in [0, 2\pi) \times [0, 2\pi)$.

Your task is to calculate the MOID on 01/01/2026 for the following cases (Keplerian Orbital Elements are provided in Table 1):

1. (30 points) The Earth and asteroid 99942 Apophis,
2. (30 points) The Earth and asteroid 2024 YR4,
3. (25 points) Asteroid 99942 Apophis and asteroid 2024 YR4,
4. (15 points) The Earth and 3I/ATLAS.

Please provide your answers in terms of both the minimum distance in km and astronomical units as well as where in terms of (f_1, f_2) the MOID was found! Feel free to use any algorithm / methodology you prefer, but please document your choice in your homework report! Please also upload any code you generated to GitHub and post a link to the code you've used in your write-up!
Good hunting!

$\mathcal{O}$	Earth	Apophis	2024 YR4	3I/ATLAS
a [km]	1.4765067E+08	1.3793939E+08	3.7680703E+08	-3.9552667E+07
e [ ]	9.1669995E-03	1.9097084E-01	6.6164147E-01	6.1469268E+00
i [deg]	4.2422693E-03	3.3356539E+00	3.4001497E+00	1.7512507E+02
ω [deg]	6.64375167E+01	1.2919949E+02	1.3429905E+02	1.2817255E+02
Ω [deg]	1.4760836E+01	2.0381969E+02	2.7147904E+02	3.2228906E+02

Table 1: Keplerian Orbital Elements for 01/01/2026 with respect to the Solar System Barycenter. Source: JPL Horizons