① 计算 $\int \frac{x\ln x}{(x^2-1)^2}dx$.

析：思考：$\left[\frac{1}{(x^2-1)}\right]' = \frac{-1}{(x^2-1)^2}\cdot 2x$.

$$原式 = \frac{1}{2}\int \ln x \, d\left(\frac{1}{x^2-1}\right) = \frac{1}{2}\ln x\cdot(x^2-1)^{-1} - \frac{1}{2}\int \frac{1}{x(x+1)(x-1)}dx.$$

$$\frac{A}{x}+\frac{B}{x+1}+\frac{C}{x-1} = \frac{A(x^2-1)+B(x^2-x)+C(x^2+x)}{x(x+1)(x-1)} = \frac{1}{x(x+1)(x-1)}. \Rightarrow \begin{cases} A=-1 \\ B=\frac{1}{2} \\ C=\frac{1}{2}. \end{cases}$$

$$故原式 = \frac{1}{2}\ln x\cdot(x^2-1)^{-1} - \frac{1}{4}\int\left(\frac{1}{x+1}+\frac{1}{x-1}-\frac{2}{x}\right)dx$$
$$= \frac{1}{2}\ln x\,(x^2-1)^{-1} - \frac{1}{4}\ln|x^2-1| + \frac{1}{2}\ln|x| + C.$$

小结：不定积分的运用.

② 计算 $\int_0^2 x^2\sqrt{2x-x^2}\,dx$.

$$原式 = \int_0^2 x^2\sqrt{1-(x-1)^2}\,dx. \qquad 令\ t=x-1.$$

奇函数．舍去.

$$原式 = \int_{-1}^{1}(t+1)^2\sqrt{1-t^2}\,dt = \int_{-1}^{1}(t^2+1)\sqrt{1-t^2}\,dt + \underline{\int_{-1}^{1}2t\sqrt{1-t^2}\,dt}.$$

令 $t=\sin\theta$. $dt=\cos\theta\, d\theta$.

$$原式 = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(1+\sin^2\theta)\cos^2\theta\, d\theta = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1-\sin^4\theta\, d\theta = \pi - 2\left(\frac{3}{4}\times\frac{1}{2}\times\frac{\pi}{2}\right) = \frac{5}{8}\pi$$

定积分奇偶性运用；关于平方换元；三角函数计算公式.

③ 设$g(x)$二阶可导且$g(0)=1$，$g'(0)=2$，$g''(0)=1$. 设$f(x)=\begin{cases}\dfrac{g(x)-e^{2x}}{x}, & x\neq 0.\\ 0, & x=0.\end{cases}$

求$f'(0)$，并讨论$f'(x)$在$x=0$处的连续性. 不能使用洛必达!!!

$$f'(0)=\lim_{x\to 0}\frac{f(x)-f(0)}{x}=\lim_{x\to 0}\frac{g(x)-e^{2x}}{x^2}\overset{\frac{0}{0}}{=}\lim_{x\to 0}\frac{g'(x)-2e^{2x}}{2x}=\frac{1}{2}\lim_{x\to 0}\frac{g'(x)-2}{x}+\frac{1}{2}\lim_{x\to 0}\frac{2-2e^{2x}}{x}$$

导数的定义.

$$=\frac{1}{2}g''(0)+\frac{1}{2}\lim_{x\to 0}(-4e^{2x})$$

$$=-\frac{3}{2}.$$

由 $f'(x)=\begin{cases}\dfrac{x[g'(x)-2e^{2x}]-[g(x)-e^{2x}]}{x^2}, & x\neq 0.\\ -\frac{3}{2}, & x=0.\end{cases}$

$$\lim_{x\to 0}f'(x)=\lim_{x\to 0}\frac{g'(x)-2e^{2x}}{x}-\lim_{x\to 0}\frac{g(x)-e^{2x}}{x^2}=\underbrace{\lim_{x\to 0}\frac{g'(x)-2}{x}}_{f'(0)}+\underbrace{\lim_{x\to 0}\frac{2-2e^{2x}}{x}}_{(-4)}-\underbrace{\lim_{x\to 0}\frac{g(x)-e^{2x}}{x^2}}_{g'(0)}$$

$=\frac{3}{2}$

$= -2.$

小结：$g(x)$二阶可导 $\Leftrightarrow$ $g(x)$二阶导数存在 $\Leftrightarrow$ $g'(x)$可导且$g(x)$连续.

注意区分!!!