① $\begin{cases} \int \tan x \, dx = -\ln|\cos x| + C \\ \int \cot x \, dx = \ln|\sin x| + C \end{cases}$

② $\begin{cases} \int \sec x \, dx = \ln|\sec x + \tan x| + C \\ \int \csc x \, dx = \ln|\csc x - \cot x| + C \end{cases}$

③ $\begin{cases} \int \frac{1}{\sqrt{a^2-x^2}} dx = \arcsin\frac{x}{a} + C \\ \int \frac{1}{a^2+x^2} dx = \frac{1}{a}\arctan\frac{x}{a} + C \end{cases}$

④ $\begin{cases} \int \frac{1}{x^2-a^2} dx = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C \\ \int \frac{1}{a^2-x^2} dx = \frac{1}{2a}\ln\left|\frac{x+a}{x-a}\right| + C \end{cases}$

⑤ $\int \frac{1}{\sqrt{x^2 \pm a^2}} dx = \ln(x + \sqrt{x^2 \pm a^2}) + C$

$(\log_a x)' = \frac{1}{x\ln a}$

求导.

⑥ $\int a^x dx = \frac{a^x}{\ln a} + C$

$\Rightarrow \begin{cases} {a^x}' = \ln a \, a^x \\ a^x = e^{x\ln a} \sim x\ln a + 1. \end{cases}$

等价无穷小.

① $e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + o(x^n)$.

② $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+1})$

③ $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + o(x^{2n})$

④ $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \cdots + (-1)^n \frac{x^{n+1}}{n+1} + o(x^{n+1})$

⑤ $\frac{1}{1-x} = 1 + x + x^2 + \cdots + x^n + o(x^n)$

⑥ $(1+x)^m = 1 + mx + \frac{m(m-1)}{2!} x^2 + \cdots + \frac{m(m-1)\cdots(m-n+1)}{n!} x^n + o(x^n)$.

在x=0处展开

泰勒公式：$f(x) = f(x-x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \cdots + \frac{f^{(n+1)}(\xi)}{(n+1)!}(x-x_0)^{n+1}$

麦克劳林公式：$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^{(n+1)}(\xi)}{(n+1)!}x^{n+1}$.

高阶导数.

1). $[\sin(ax+b)]^{(n)} = a^n \sin(ax+b+\frac{n\pi}{2})$.

2). $(\cos(ax+b))^{(n)} = a^n \cos(ax+b+\frac{n\pi}{2})$.

3). $$\left(\frac{1}{ax+b}\right)^{(n)} = \frac{(-1)^n \cdot n! \, a^n}{(ax+b)^{n+1}}.$$

4). $(e^{ax+b})^{(n)} = a^n \cdot e^{ax+b}$.

$$\begin{cases} \tan x' = \sec^2 x \\ \cot x' = -\csc^2 x \end{cases} \qquad \begin{cases} a^{x\,\prime} = a^x \ln a \\ \log_a x' = \frac{1}{x \ln a} \end{cases}$$

$$\begin{cases} \sec' x = \sec x \tan x \\ \csc' x = -\csc x \cot x \end{cases}$$

$$\begin{cases} \sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} \\ \cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} \end{cases}$$

等价无穷小：

① $\sin x \sim x$,

② $\arcsin x \sim x$.

③ $\tan x \sim x$.

④ $\arctan x \sim x$.

⑤ $1-\cos x \sim \frac{1}{2}x^2 \sim \frac{1}{2}\sin^2 x$

⑥ $\sqrt[n]{1+x}-1 \sim \frac{1}{n}x$.

⑦ $e^x-1 \sim x$.

⑧ $\ln(1+x) \sim x$.

⑨ $\log_a(1+x) = \frac{x}{\ln a} \longrightarrow \frac{\ln(1+x)}{\ln a} \sim \frac{x}{\ln a}$.

⑩ $a^x-1 \sim x\ln a \Rightarrow e^{x\cdot\ln a}-1 \sim x\ln a$.

⑪ $(1+x)^{\mu}-1 \sim \mu x \Leftarrow (1+x)^{\frac{1}{n}}-1 \sim \frac{1}{n}x. \Leftarrow e^{\frac{1}{n}\ln(1+x)}-1 \sim \frac{1}{n}\ln(1+x) \sim \frac{1}{n}x.$

这里的μ是常数！！！