08 数 2 期末考试（A）卷答案

一、填空

1. $2\tan\frac{x}{2}+C$

2. $\cos x - x\sin x$

3. $\frac{1}{b-a}\int_a^b f(x)dx$

4. 充分

5. $\int_0^1 dy\int_{\sqrt{y}}^{2-y} f(x,y)dy$

二、选择题

1．D

2．A

3．D

4．C

5．B

三、计算

1．

$\int\frac{1}{1+\sin 2x}\mathrm{d}x$.

解 **解法一** $\int\frac{1}{1+\sin 2x}\mathrm{d}x=\int\frac{\mathrm{d}\tan x}{(1+\tan x)^2}=-\frac{1}{1+\tan x}+C$.

解法二
$$\int\frac{1}{1+\sin 2x}\mathrm{d}x=\int\frac{1-\sin 2x}{\cos^2 2x}\mathrm{d}x$$
$$=\frac{1}{2}\left(\tan 2x-\frac{1}{\cos 2x}\right)+C.$$

解法三
$$\int\frac{1}{1+\sin 2x}=\int\frac{1}{(\sin x+\cos x)^2}\mathrm{d}x$$
$$=\frac{1}{2}\int\frac{\mathrm{d}\left(x+\frac{\pi}{4}\right)}{\sin^2\left(x+\frac{\pi}{4}\right)}$$
$$=-\frac{1}{2}\cot\left(x+\frac{\pi}{4}\right)+C.$$

2．

解 令 $t=\sqrt{5-4x}$，则 $x=\frac{5-t^2}{4}$，$\mathrm{d}x=-\frac{1}{2}t\mathrm{d}t$，
$$\int_{-1}^{1}\frac{x}{\sqrt{5-4x}}\mathrm{d}x=\int_3^1\frac{5-t^2}{4t}\left(-\frac{1}{2}t\right)\mathrm{d}t=\int_3^1\frac{1}{8}(t^2-5)\mathrm{d}t$$
$$=\frac{1}{8}\left(\frac{1}{3}t^3-5t\right)\Big|_3^1=\frac{1}{6}.$$

3．

$$\text{解}\quad \frac{\partial z}{\partial x}=\frac{1}{1+\left(\frac{x+y}{1-xy}\right)^2}\cdot\left(\frac{x+y}{1-xy}\right)'_x$$

$$=\frac{1+y^2}{(1-xy)^2+(x-y)^2}=\frac{1}{1+x^2}.$$

$$\frac{\partial^2 z}{\partial x\partial y}=\frac{\partial}{\partial y}\left(\frac{1}{1+x^2}\right)=0.$$

4．

$$\text{解}\quad \iint_D 3x^2\sin^2 y\,\mathrm{d}x\,\mathrm{d}y=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin^2 y\,\mathrm{d}y\int_0^{\cos y}3x^2\,\mathrm{d}x$$

$$=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin^2 y\cos^3 y\,\mathrm{d}y$$

$$=2\int_0^{\frac{\pi}{2}}\sin^2 y(1-\sin^2 y)\,\mathrm{d}\sin y$$

$$=2\left[\frac{1}{3}\sin^3 y-\frac{1}{5}\sin^5 y\right]\Bigg|_0^{\frac{\pi}{2}}$$

$$=\frac{4}{15}.$$

四、解答题

1．0．

2．

$$\text{解}\quad \text{原式}=\int_0^{\pi}f(x)\sin x\,\mathrm{d}x+\int_0^{\pi}f''(x)\sin x\,\mathrm{d}x$$

$$=\int_0^{\pi}f(x)\,\mathrm{d}(-\cos x)+\int_0^{\pi}\sin x\,\mathrm{d}f'(x)$$

$$=-\cos x\,f(x)\Big|_0^{\pi}+\int_0^{\pi}\cos x\,f'(x)\,\mathrm{d}x$$

$$+\sin x\,f'(x)\Big|_0^{\pi}-\int_0^{\pi}f'(x)\cos x\,\mathrm{d}x$$

$$=f(\pi)+f(0)$$

3．

解　因为 $z=x\ln x+y^2\ln y$，

所以　$\frac{\partial z}{\partial x}=\ln x+1,\ \frac{\partial z}{\partial y}=2y\ln y+y.$

故　$\mathrm{d}z=(\ln x+1)\mathrm{d}x+(2y\ln y+y)\mathrm{d}y.$

五、证明题

1 .

证明 $\int_a^{a+T} f(x)\mathrm{d}x = \int_a^0 f(x)\mathrm{d}x + \int_0^T f(x)\mathrm{d}x + \int_T^{T+a} f(x)\mathrm{d}x$，在第三个积分中令 $x = T + t$，则

$$\int_T^{T+a} f(x)\mathrm{d}x = \int_0^a f(t)\mathrm{d}t = \int_0^a f(x)\mathrm{d}x = -\int_a^0 f(x)\mathrm{d}x.$$

故 $$\int_a^{a+T} f(x)\mathrm{d}x = \int_0^T f(x)\mathrm{d}x,$$

即积分与 a 的值无关.

2.

证明：原式＝$\lim\limits_{\substack{x\to 0\\ y=kx}} \frac{\sin(1-k)x}{x+kx} = \lim\limits_{\substack{x\to 0\\ y=kx}} \frac{(1-k)x}{x+kx} = \frac{1-k}{1+k}$.