2008-2009 学年第二学期

微积分（数二）期末试题参考答案

一、填空题(每小题 3 分, 共 15 分)

1、$\frac{1}{3}|x|^3+C$，　　2、$\frac{1}{e}$　，　　3、$-\frac{1}{6}$

4、$\int_0^{\frac{\pi}{4}}d\theta\int_0^{\frac{\sin\theta}{\cos^2\theta}}f(r\cos\theta,r\sin\theta)rdr+\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}d\theta\int_0^{\frac{1}{\sin\theta}}f(r\cos\theta,r\sin\theta)rdr$

5、$\cos x(x+C)$

二、选择题(每小题 3 分, 共 15 分)

1—5：B C A C D

三、计算题(每题 8 分, 共 32 分)

1、解：$\int_1^2\frac{x^2-2}{x^3}e^xdx=\int_1^2\frac{e^x}{x}dx-\int_1^2\frac{2}{x^3}e^xdx=\int_1^2\frac{e^x}{x}dx+\int_1^2e^xd(\frac{1}{x^2})$

$$=\int_1^2\frac{e^x}{x}dx+\frac{1}{x^2}e^x\Big|_1^2-\int_1^2\frac{1}{x^2}e^xdx$$

$$=\int_1^2\frac{e^x}{x}dx+\frac{1}{x^2}e^x\Big|_1^2+\frac{1}{x}e^x\Big|_1^2-\int_1^2\frac{1}{x}e^xdx$$

$$=\frac{1}{4}e^2-e+\frac{1}{2}e^2-e=\frac{3}{4}e^2-2e$$

2、解：$\frac{\partial z}{\partial x}=2yf\cdot f'+2xg_1'+g_2'$

$$\frac{\partial^2 z}{\partial x\partial y}=2f\cdot f'+2yx(f')^2+2yxf\cdot f''+2xg_{12}''+g_{22}''$$

3、解：特征方程：$r^2-2r-3=0$，$r_1=3,r_2=-1$

对应齐次方程通解为：$Y = C_1 e^{-x} + C_2 e^{3x}$

设 $y_1^* = Axe^{-x}$ 为非齐次方程 $y'' - 2y' - 3y = e^{-x}$ 的特解.

设 $y_2^* = Bx + C$ 为非齐次方程 $y'' - 2y' - 3y = 3x + 1$ 的特解.

分别代入方程可得：

$$-Ae^{-x} - Ae^{-x} + Axe^{-x} - 2(Ae^{-x} - Axe^{-x}) - 3Axe^{-x} = e^{-x}$$

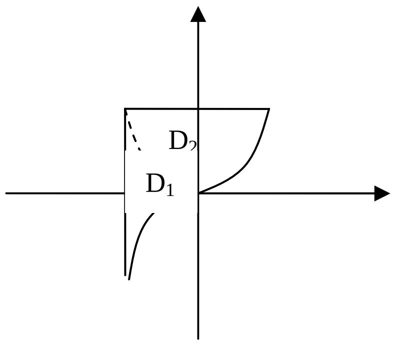
化简得： $-4A = 1$ ， $A = -\frac{1}{4}$ $\therefore y_1^* = -\frac{x}{4}e^{-x}$

同理有 $-2B - 3(Bx + C) = 3x + 1$

化简得 $-3Bx - 3C - 2B = 3x + 1$， $B = -1, C = \frac{1}{3}$ $\therefore y_2^* = -x + \frac{1}{3}$

所求非齐次方程通解为： $y = Y + y_1^* + y_2^* = C_1 e^{-x} + C_2 e^{3x} - \frac{x}{4}e^{-x} - x + \frac{1}{3}$

4、**解**：如图所示，把积分区域 D 分为 D_1, D_2 D_1, D_2 分别关于 x, y 轴对称，



$$\iint_D x[1 + yf(x^2 + y^2)]dxdy$$

$$= \iint_{D_1} x[1 + yf(x^2 + y^2)]dxdy + \iint_{D_2} x[1 + yf(x^2 + y^2)]dxdy$$

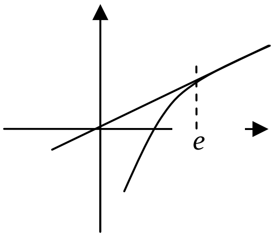
$$= \iint_{D_1} x[1 + yf(x^2 + y^2)]dxdy = \iint_{D_1} xdxdy = 2\int_{-1}^{0} dx \int_{0}^{-x^3} xdy = -\frac{2}{5}$$

四、解答题(每题 8 分, 共 24 分)

1、**解**：设切点为$(x_0,\ln x_0)$，则切线方程为 $y-\ln x_0=\frac{1}{x_0}(x-x_0)$

又过原点 $-\ln x_0=\frac{1}{x_0}(0-x_0)$，解得 $x_0=e$

切线方程 $y=\frac{1}{e}x$



$$V=V_{圆锥替}-V_1==\frac{1}{3}\pi e-\int_1^e \pi\ln^2 x dx==\frac{\pi}{3}(6-2e)$$

2、**解**： $\because\ \frac{\partial f}{\partial x}=axy^3+4y^2\cos x,\quad \frac{\partial f}{\partial y}=2+by\sin x-5x^2y^2$

$\therefore\ \frac{\partial^2 f}{\partial x\partial y}=3axy^2+8y\cos x=by\cos x-10xy^2=\frac{\partial^2 f}{\partial y\partial x}$

$\therefore\ a=-\frac{10}{3},\quad b=8$

3、**解**：设目标函数为 $d(x,y)=\frac{|2x+3y-6|}{\sqrt{2^2+3^2}}$，约束条件为 $x^2+4y^2=4$

构造拉格朗日函数

$$L(x,y)=d^2(x,y)-\lambda(x^2+4y^2-4)=\frac{1}{13}(2x+3y-6)^2-\lambda(x^2+4y^2-4)$$

$$\begin{cases} L_x=\frac{4}{13}(2x+3y-6)-2\lambda x=0 \\ L_y=\frac{6}{13}(2x+3y-6)-8\lambda y=0 \\ x^2+4y^2=4 \end{cases}$$

解得驻点 $\left(\frac{8}{5},\frac{3}{5}\right),\ \left(-\frac{8}{5},-\frac{3}{5}\right)$，由题意可得 $\left(-\frac{8}{5},-\frac{3}{5}\right)$为所求。

五、证明： $\because$ $$\frac{\partial u}{\partial x}=\varphi'(x+y)+\varphi'(x-y)+\psi(x+y)-\psi(x-y)$$

$$\frac{\partial^2 u}{\partial x^2}=\varphi''(x+y)+\varphi''(x-y)+\psi'(x+y)-\psi'(x-y)$$

$$\frac{\partial u}{\partial y}=\varphi'(x+y)-\varphi'(x-y)+\psi(x+y)+\psi(x-y)$$

$$\frac{\partial^2 u}{\partial y^2}=\varphi''(x+y)+\varphi''(x-y)+\psi'(x+y)-\psi'(x-y)$$

$$\therefore \quad \frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 u}{\partial y^2}$$

六、解：设雨滴下落的速度函数为$v(t)$

由牛顿第二定律 $$\frac{(50-t)}{1000}g-kv=\frac{(50-t)}{1000}v', \quad (t\in[0,50))$$

$$v'+\frac{1000k}{50-t}v=g$$

由一阶线性微分方程求解

$$v=e^{-\int\frac{1000k}{50-t}dt}[\int ge^{\int\frac{1000k}{50-t}dt}dt+C]=e^{1000k\ln(50-t)}[\int ge^{-1000k\ln(50-t)}dt+C]$$

$$=(50-t)^{1000k}[g\int(50-t)^{-1000k}dt+C]$$

$$=(50-t)^{1000k}[\frac{g}{k-1}(50-t)^{-1000k+1}+C]$$

$$=C(50-t)^{1000k}+\frac{g}{1000k-1}(50-t)$$

$v(0)=0$,代入上式得 $C=\dfrac{50^{1-1000k}g}{1-1000k}$

所以雨滴下落的速度函数为

$$v(t)=\frac{50^{1-1000k}g}{1-1000k}(50-t)^{1000k}+\frac{g}{1000k-1}(50-t), \quad t\in[0,50)$$