

# Price Search and Consumption Inequality: Robust, Credible, and Valid Inference\*

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## Abstract

Using data on shopping expenditures from the Nielsen Homescan Dataset, I document that low-income consumers pay lower prices than high-income consumers and that both income groups have comparable distributions of food consumption levels. I investigate whether these facts are explained by heterogeneity in price search. I propose a model where consumers can pay lower prices by shopping more frequently. The impacts of shopping frequency on prices paid depend on the consumer shopping technology. I develop a revealed preference methodology to set identify the shopping technology in a computationally tractable fashion. My approach allows for nonparametric concave preferences, rich heterogeneity, and measurement error in prices. I show that the data are consistent with the model and that a doubling of shopping frequency decreases prices paid by about 19%. Additionally, I show that heterogeneity in price search mitigates between-group consumption inequality by almost 15% for a fraction of consumers but also creates within-group consumption inequality. My results show that price search explains part of the gap in prices paid between income groups and highlight the importance of price search for understanding consumption inequality.

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# 1 Introduction

The steady rise in income inequality has attracted much attention in the recent years. However, economics rather advocates to focus on consumption inequality as consumers are generally thought to derive utility from consumption. Also, consumption is a more reliable measure of lifetime resources since consumers seek to smooth it out. A notable difference between the trend in income inequality and consumption inequality could have serious implications for policy-making. Unfortunately, the empirical evidence documenting the evolution of consumption inequality has been mixed.<sup>1</sup> I contribute to this literature by quantifying how heterogeneity in price search impacts consumption inequality.

The first contribution of this paper is to provide a new revealed preference methodology to characterize a parsimonious model of price search.<sup>2</sup> The model shares key features with the price search literature such as the concavity of the utility function and the log-linearity of the shopping technology. However, to deal with misspecification issues I allow for nonparametric preferences and unrestricted unobserved heterogeneity. The second contribution of this paper is to relax the common homogeneity assumptions on the shopping technology which allows me to explore how it differs across several dimensions such as income, gender, and geographic location. The third contribution of this paper is to put forward an approach that allows one to statistically test structural models and achieve informative inference despite minimal assumptions on unobservables.

Price search describes the process whereby buyers actively seek to gauge the most favorable prices. Its importance has been recognized at least since the seminal paper of [Stigler \(1961\)](#) and has gained strong empirical support over the years.<sup>3</sup> In their influential paper, [Aguiar and Hurst \(2007\)](#) show that the drop in expenditures occurring around retirement is partly due to older households searching more intensively than younger ones. In another paper, [Griffith et al. \(2009\)](#) document savings from various shopping strategies and compare how those savings vary by income group. However, the impacts of search on prices paid by income group remains unknown. My paper addresses this gap by investigating how the shopping technology differs across income

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<sup>1</sup>For example, [Cutler and Katz \(1992\)](#) and [Aguiar and Bils \(2015\)](#) find that consumption inequality closely follows income inequality while [Slesnick \(1994\)](#) and [Krueger and Perri \(2006\)](#) find that consumption inequality has risen less significantly than income inequality. An overview of the literature is given by [Attanasio and Pistaferri \(2016\)](#).

<sup>2</sup>From a different angle, [Tipoe \(2021\)](#) provides a revealed preference characterization of a model with limited attention to prices à la [Gabaix \(2014\)](#).

<sup>3</sup>See, for example, [Sorensen \(2000\)](#), [Brown and Goolsbee \(2002\)](#), [Aguiar and Hurst \(2007\)](#), and [McKenzie et al. \(2011\)](#). For a general survey, see [Baye et al. \(2006\)](#).

groups.

My paper formalizes the empirical evidence documenting (i) the effects of price search on prices paid (Aguiar and Hurst, 2007), (ii) the use of price search as a mechanism to mitigate adverse income shocks (McKenzie et al., 2011; Nevo and Wong, 2019), and (iii) the wide heterogeneity in prices paid (Kaplan and Menzio, 2015; Kaplan et al., 2019; Hitsch et al., 2019). Additionally, by testing the main assumptions on which the price search literature relies, I provide a foundation for existing models of price search (Aguiar and Hurst, 2007; Pytka, 2017; Arslan et al., 2021), reinforce the plausibility of their empirical results, and improve existing estimates.

While instrumental variable is a standard tool to estimate the causal effect of a treatment on an outcome of interest, it requires the availability of an instrument that only affects the outcome of interest through its impact on the treatment. Unfortunately, it is often difficult to know a priori whether an instrument satisfy this exogeneity condition. Moreover, exogeneity is not always testable when the endogenous variable is continuous (Gunsilius, 2020).<sup>4</sup> My approach mitigates this issue by exploiting the structure of the model to obtain testable restrictions. In my application, I find that the exogeneity assumption is rejected in the model, thus precluding this path for estimation.

Absent exogenous variation in shopping intensity, I show that the model is set identified in the sense that many elasticities of price with respect to shopping intensity are consistent with the model given the data. This feature also holds more generally such as in the model of Aguilar and Hurst (2007). To address this problem, I provide a convenient characterization of the model that can be used to bound the elasticity. The implementation takes advantage of a revealed preference approach (Afriat, 1967; Diewert, 1973; Varian, 1982; Browning, 1989) and its extension to nonlinear budgets (Forges and Minelli, 2009). Accordingly, every consumer is treated separately such that no aggregation assumptions or restrictions on unobserved heterogeneity are imposed.

My empirical application uses the Nielsen Homescan Dataset which is a data set that tracks U.S. households' food purchases on each of their trips to a wide variety of retail outlets. I measure shopping intensity by number of shopping trips as it captures price variations across stores and price discounts found by frequently visiting stores. The panel structure of the data enables me to set identify the elasticity of price with respect to shopping intensity from individual

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<sup>4</sup>It is useful to note that overidentifying restrictions do not allow one to test instrument validity. See, for example, Parente and Silva (2012).

time-variation in shopping intensity. Under a mild assumption on residual factors impacting prices paid, I further show that the true impacts of search on prices paid are recovered. The validity of this assumption can be jointly tested along with the model.

In a validation study of the Nielsen Homescan Dataset, [Einav et al. \(2010\)](#) report severe measurement error in prices and provide information about its structure. The presence of measurement error requires special attention for two reasons. First, the model could be compatible with the true data but incompatible with the observed data, hence leading to the erroneous rejection of the model.<sup>5</sup> Second, measurement error can complicate empirical analyses by obscuring the true behavior of variables such as expenditures.<sup>6</sup> In turn, this can bias estimators in unpredictable ways. For example, measurement error may not be classical such that bias could arise even if it appears on the dependent variable in a standard regression setting. This is relevant because it implies that the estimation in [Aguiar and Hurst \(2007\)](#) could be biased.

The methodology developed by [Schennach \(2014\)](#) enables me to nonparametrically account for measurement error and statistically test the model. Furthermore, the extension of [Aguiar and Kashaev \(2018\)](#) allows me to impose the concavity of the utility function without increasing the dimensionality of the problem. I extend the applicability of their framework to allow for nonlinear budget sets and a mixture of parametric and nonparametric components in the model. This is achieved by using a rejection sampling algorithm in the implementation of the methodology.<sup>7</sup> This method has the advantage to be applicable in a broader set of models while remaining computationally tractable.

I find support for price search behavior in the Nielsen Homescan Dataset as the statistical test fails to reject the hypothesis that the model generated the data. Furthermore, I show that heterogeneity in price search only has a mild role in explaining the difference in average price paid between income groups. Rather, my results suggest that the difference in average price paid is driven by a difference in average good quality. This finding is consistent with [Broda et al. \(2009\)](#) who document that low-income consumers pay lower prices mainly through the purchase of less expensive goods within product categories.

I document that, on average, low-income consumers only shop slightly more

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<sup>5</sup>Measurement error has been shown to reverse conclusions about the validity of exponential discounting in single-individual households ([Aguiar and Kashaev, 2018](#)).

<sup>6</sup>See [Attanasio and Pistaferri \(2016\)](#) for an overview of how measurement error can cloud the evolution of consumption inequality.

<sup>7</sup>The rejection sampling algorithm exploits GPU parallel computing when applicable.

frequently than high-income consumers. Additionally, I show that low-income consumers have a slightly better average shopping technology compared to high-income consumers. That is, the average impact of search intensity on prices paid is stronger for low-income consumers. Since average food consumption levels are also comparable between income groups in my data, my results suggest that differences in price search and good quality are successful in reducing average food intake inequality.

Pulling all consumers together, the 95% confidence set on the average elasticity of average price with respect to shopping intensity states that doubling shopping intensity decreases the average price paid by about 19.1% to 19.5%. This effect is well-above the preferred estimate of 7.4% obtained by [Aguiar and Hurst \(2007\)](#) using income as an instrument.<sup>8</sup> Instead, the effect I find provides support for their estimate of 18.9% obtained using age as an instrument. Thus, my results show that the effects of shopping intensity on prices paid are almost three times larger than previously thought. Moreover, they rationalize the calibration of [Arslan et al. \(2021\)](#) used in a different model of price search.

In a counterfactual exercise, I show that a consumer in the lowest quintile of the shopping frequency distribution pay close to 15% more for a good than a consumer in the largest quintile. Assuming that savings from search are spent on the same good, this difference yields within-group consumption inequality of the same order. Furthermore, I show that price search has mixed impacts on between-group consumption inequality: I find that price search mitigates consumption inequality between low-income consumers with high shopping intensity and high-income consumers with low shopping intensity, while exacerbating it for low-income consumers with low shopping intensity and high-income consumers with high shopping intensity.

The rest of the paper is organized as follows. Section 2 introduces the model and provides testable restrictions. Section 3 describes the data set. Section 4 discusses the environment. Section 5 presents the statistical framework. Section 6 presents the empirical application. Section 7 concludes. The main proofs can be found in Appendix A5.

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<sup>8</sup>In addition to correcting the value of the elasticity, my confidence set is also much more precise than the 95% confidence set obtained with instrumental variables.

## 2 Description of the Model

This section introduces the notation, presents the model, characterizes the testable implications, and provides the procedure for recovering the identified set.

### 2.1 Notation

The scenario under consideration is that of households making purchases over a certain time window. Let  $\mathcal{N} := \{1, \dots, N\}$ ,  $\mathcal{L} := \{1, \dots, L\}$ , and  $\mathcal{T} := \{1, \dots, T\}$  denote the sets of households, commodities, and periods for which data are observable. For any household  $i \in \mathcal{N}$ , good  $l \in \mathcal{L}$  and time period  $t \in \mathcal{T}$ , denote prices by  $p_{i,l,t}$ , consumption by  $c_{i,l,t}$ , shopping intensity by  $a_{i,l,t}$ , and search productivity by  $\omega_{i,l,t}$ . I assume that  $P, C, A \subseteq \mathbb{R}_{++}^L$  and  $\Omega \subseteq \mathbb{R}^L$ , where  $P, C, A$  and  $\Omega$  correspond to the price, consumption, shopping intensity, and search productivity spaces, respectively. An observation on a given household  $i \in \mathcal{N}$  at time  $t \in \mathcal{T}$  is a triple  $(\mathbf{p}_{i,t}, \mathbf{c}_{i,t}, \mathbf{a}_{i,t}) \in P \times C \times A$ .<sup>9</sup> Accordingly, a data set is written as  $\{(\mathbf{p}_{i,t}, \mathbf{c}_{i,t}, \mathbf{a}_{i,t})\}_{t \in \mathcal{T}}$ .

### 2.2 The Consumer Problem

In every time period, the consumer is assumed to know her realization of search productivity and to choose consumption and shopping intensity accordingly. Formally, a consumer  $i \in \mathcal{N}$  behaves as if maximizing her lifetime utility subject to satisfying her budget constraints:

$$\max_{(\mathbf{c}_i, \mathbf{a}_i) \in C^T \times A^T} \sum_{t=1}^T \delta_i^{t-1} u_i(\mathbf{c}_{i,t}, \mathbf{a}_{i,t}) \quad (1)$$

subject to

$$\mathbf{p}_i(\mathbf{a}_{i,t}, \boldsymbol{\omega}_{i,t})' \mathbf{c}_{i,t} + s_{i,t+1} = y_{i,t} + s_{i,t},$$

where  $u_i : C \times A \rightarrow \mathbb{R}$  is the instantaneous utility function which is continuous, concave, strictly increasing in consumption, and decreasing in shopping intensity;  $\delta_i \in [\underline{\delta}, 1]$  is the discount factor, where  $\underline{\delta} \in (0, 1]$ ;  $s_{i,t+1}$  is savings in a risk-free asset;  $\mathbf{p}_i(\mathbf{a}, \boldsymbol{\omega}_{i,t})$  is the vector of continuously differentiable good-specific price functions  $p_{i,l} : A \times \Omega \rightarrow P$ ;  $y_{i,t} > 0$  is income; and  $\mathbf{p}_{i,t} := \mathbf{p}_i(\mathbf{a}_{i,t}, \boldsymbol{\omega}_{i,t})$ . The econometrician is assumed to only observe the data set  $x_i := \{(\mathbf{p}_{i,t}, \mathbf{c}_{i,t}, \mathbf{a}_{i,t})\}_{t \in \mathcal{T}}$ . As explained in the definition below, a data set that can be thought of as stemming from (1) is said to be rationalized by the model.

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<sup>9</sup>I use bold font to denote vectors and follow the convention that vectors are vector columns.

**Definition 1.** A data set  $x_i$  is rationalized by the model if there exist a utility function  $u_i(\cdot, \cdot)$ , a vector of price functions  $\mathbf{p}_i(\cdot, \cdot)$ , an income stream  $(y_{i,t})_{t \in \mathcal{T}}$ , a savings stream  $(s_{i,t+1})_{t \in \mathcal{T}}$ , and a discount factor  $\delta_i \in [\underline{\delta}, 1]$  such that  $(\mathbf{c}_{i,t}, \mathbf{a}_{i,t})_{t \in \mathcal{T}}$  solves the model (1).

The model has two distinctive features. First, the consumer gets utility from consumption and disutility from shopping intensity. The latter captures the opportunity cost of time such as foregone earnings and leisure. Second, the consumer can pay lower prices by shopping more intensively. The extent by which shopping intensity reduces prices paid depends on the consumer's ability to take advantage of sales and other deals such as coupons. The consumer problem boils down to finding the optimal trade-off between utility from consumption and disutility from shopping intensity.

This trade-off is illustrated in Figure 1 in the case where there is one good  $\mathcal{L} = \{1\}$  and one time period  $\mathcal{T} = \{1\}$ . The consumer has to choose a bundle that lies within her budget set  $\mathcal{B}_i := \{(\mathbf{c}, \mathbf{a}) \in C \times A : \mathbf{p}_i(\mathbf{a}, \boldsymbol{\omega}_i)' \mathbf{c} \leq y_i + s_i\}$ . This set is represented by the shaded area in Figure 1. The affordable bundle that maximizes the consumer utility is  $(c_i, a_i)$ . At this point, the indifference curve  $IC_i$  is tangent to the budget line, hence corresponding to the unique maximizer.<sup>10</sup>

In the remaining of the paper, I follow the price search literature and assume that the price functions are log-linear in shopping intensity.<sup>11</sup>

**Assumption 1.** For all  $l \in \mathcal{L}$ , the log price function is given by

$$\log(p_{i,l}(a_{i,l,t}, \omega_{i,l,t})) = \alpha_{i,l}^0 + \alpha_{i,l}^1 \log(a_{i,l,t}) - \omega_{i,l,t},$$

where  $\alpha_{i,l}^0 \in \mathbb{R}$  denotes the intercept and  $\alpha_{i,l}^1 \leq 0$  denotes the elasticity of price with respect to shopping intensity.

Assumption 1 implies that price functions are convex decreasing.<sup>12</sup> That is, prices paid decrease at a decreasing rate as shopping intensity or search productivity increases. This requirement captures decreasing marginal returns that arise due to the increasing difficulty of finding discounts surpassing the current

<sup>10</sup>In Appendix A1, I show that my model can be extended to home production and relates it to that of Aguiar and Hurst (2007).

<sup>11</sup>See, for example, Aguiar and Hurst (2007) and Arslan et al. (2021).

<sup>12</sup>A price function  $p : A \times \Omega \rightarrow \mathbb{R}_{++}$  is convex if  $p(a_s, \omega_s) \geq p(a_t, \omega_t) + \frac{\partial p(a_t, \omega_t)}{\partial a_t}(a_s - a_t) + \frac{\partial p(a_t, \omega_t)}{\partial \omega_t}(\omega_s - \omega_t)$  for all  $a_s, a_t \in A$  and all  $\omega_s, \omega_t \in \Omega$ . It is convex decreasing if it is convex and  $\frac{\partial p(a_t, \omega_t)}{\partial a_t}, \frac{\partial p(a_t, \omega_t)}{\partial \omega_t} \leq 0$ .

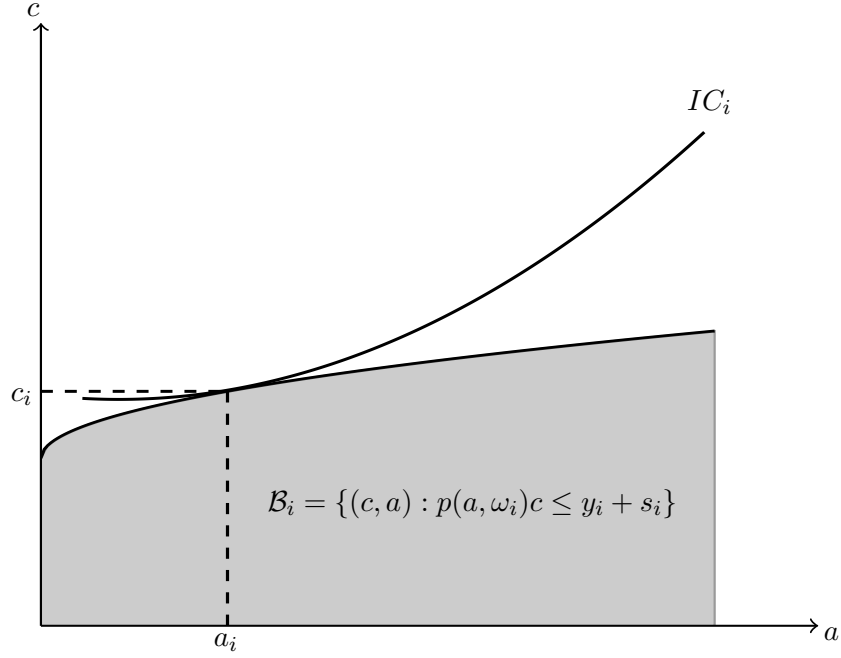


Figure 1: Optimal Choice with Price Search

best discount.<sup>13</sup> An example of budget set generated by a convex decreasing price function is illustrated in Figure 1 in the case of a single good. The figure shows that, while increasing shopping intensity allows a greater consumption level, the marginal benefit is decreasing in shopping intensity.

### 2.3 Identified Set

Given a data set  $x_i$ , I am interested in what can be learned about the elasticity  $\alpha_{i,l}^1$ . For any good  $l \in \mathcal{L}$ , the first-order conditions with respect to consumption and shopping intensity are

$$\delta^t \nabla_c u_i(\mathbf{c}_{i,t}, \mathbf{a}_{i,t})_l = \lambda_{i,t} p_{i,l,t} \quad (2)$$

$$\delta^t \nabla_a u_i(\mathbf{c}_{i,t}, \mathbf{a}_{i,t})_l = \lambda_{i,t} \alpha_{i,l}^1 a_{i,l,t}^{\alpha_{i,l}^1 - 1} e^{-(\omega_{i,l,t} - \alpha_{i,l}^0)} c_{i,l,t}. \quad (3)$$

Solving for the exponential function in equation (3) yields

$$e^{-(\omega_{i,l,t} - \alpha_{i,l}^0)} = \frac{\lambda_{i,t}^{-1} \delta^t \nabla_a u_i(\mathbf{c}_{i,t}, \mathbf{a}_{i,t})_l}{\alpha_{i,l}^1 a_{i,l,t}^{\alpha_{i,l}^1 - 1} c_{i,l,t}}.$$

<sup>13</sup>Stigler (1961) shows that the expected value of the minimum price is convex in search, therefore providing a theoretical motivation for this choice.



This equation can be substituted into the price function equation for good  $l$  to get rid of the intercept and search productivity. Further substituting  $\lambda_{i,t}^{-1}\delta^t$  by its expression from equation (2) then gives

$$\alpha_{i,l}^1(u_i) = \text{MRS}_{i,l} \cdot \frac{a_{i,l,t}}{c_{i,l,t}} \quad \forall l \in \mathcal{L}, \quad (4)$$

where  $\text{MRS}_{i,l} := \frac{\nabla_a u_i(\mathbf{c}_{i,t}, \mathbf{a}_{i,t})_l}{\nabla_c u_i(\mathbf{c}_{i,t}, \mathbf{a}_{i,t})_l} \leq 0$  denotes the marginal rate of substitution and highlights that the consumer would have to receive consumption to increase her shopping intensity.

Equation (4) shows that every preference that generates a distinct marginal rate of substitution induces a distinct shopping technology  $\alpha_{i,l}^1$ . The set of all elasticities that can be sustained by the model is captured by the identified set.

**Definition 2.** Under Assumption 1, the identified set is defined by

$$\mathcal{I}(x_i) := \{\alpha_i^1(u_i) : x_i \text{ is rationalized by the model, } u_i \in \mathcal{U}\},$$

where  $\mathcal{U}$  is the set of utility functions that are continuous, concave, strictly increasing in consumption, and decreasing in shopping intensity.

In other words, the identified set contains every possible elasticity of price with respect to shopping intensity such that data on consumption and shopping intensity can be thought of as maximizers of the consumer problem (1) for some well-behaved preferences.

## 2.4 Nonparametric Analysis of the Model

This subsection characterizes the testable implications of the model and provides a feasible procedure to recover the identified set. Let  $\odot$  denote the Hadamard product such that  $(\mathbf{v} \odot \mathbf{w})_j = v_j w_j$ . The following result characterizes the empirical content of the model.

**Theorem 1.** *Let  $x_i$  be a given data set. The statements (i) and (ii) below are related in the following ways: if (i) then (ii), and if the budget sets  $\{\mathcal{B}_{i,t}\}_{t \in \mathcal{T}}$  are convex then (ii) implies (i).*

- (i) *The data set is rationalized by the model, where the utility function  $u(\mathbf{c}_{i,t}, \mathbf{a}_{i,t})$  is continuous, concave, strictly increasing in  $\mathbf{c}_{i,t}$ , and decreasing in  $\mathbf{a}_{i,t}$ , and where the vector of price functions  $\mathbf{p}_i(\cdot, \cdot)$  satisfies Assumption 1.*

(ii) There exist numbers  $u_{i,t}$ ,  $\lambda_{i,t} > 0$ ,  $\alpha_{i,l}^0$ ,  $\alpha_{i,l}^1 \leq 0$ ,  $\omega_{i,l,t}$ , and a discount factor  $\delta_i \in [\underline{\delta}, 1]$ , such that for all  $s, t \in \mathcal{T}$ , the following system of inequalities is satisfied

$$\begin{aligned} u_{i,s} &\leq u_{i,t} + \lambda_{i,t} \delta_i^{-t} [\mathbf{p}'_{i,t}(\mathbf{c}_{i,s} - \mathbf{c}_{i,t}) + \boldsymbol{\rho}'_{i,t}(\mathbf{a}_{i,s} - \mathbf{a}_{i,t})] \\ 0 &< \mathbf{p}_{i,t} \\ 0 &\geq \boldsymbol{\rho}_{i,t}, \end{aligned}$$

$$\text{where } \rho_{i,l,t} := \alpha_{i,l}^1 a_{i,l,t}^{\alpha_{i,l}^1 - 1} e^{-(\omega_{i,l,t} - \alpha_{i,l}^0)} c_{i,l,t}.$$

The first set of inequalities in Theorem 1 (ii) represents the concavity of the utility function, where the numbers  $u_{i,t}$  and  $\lambda_{i,t} > 0$  can be thought of as utility numbers and marginal utilities of expenditure. The second and third sets of inequalities capture the assumptions that the utility function is strictly increasing in consumption and decreasing in shopping intensity, respectively. Theorem 1 states that any data set rationalized by the model must satisfy the inequalities in (ii). Moreover, any data set that satisfies these inequalities is rationalized by the model if the budget sets are convex.

Since the model is generally set identified, there may be many solutions to the inequalities in Theorem 1 (ii). These solutions are observationally equivalent in the sense that the data do not allow one to distinguish one from another. In particular, note that every solution to Theorem 1 (ii) sustains an elasticity  $\alpha_{i,l}^1$ . As a consequence, the set of solutions in Theorem 1 (ii) directly relates to the identified set.

**Corollary 1.** *If the inequalities in Theorem 1 (ii) are only necessary for the data to be rationalized by the model, then*

$$\mathcal{I}(x_i) \subset \{\boldsymbol{\alpha}_i^1 : x_i \text{ satisfies Theorem 1 (ii)}\}.$$

*If the budget sets  $\{\mathcal{B}_{i,t}\}_{t \in \mathcal{T}}$  are convex, then the inequalities in Theorem 1 (ii) are also sufficient for the data to be rationalized by the model and*

$$\mathcal{I}(x_i) = \{\boldsymbol{\alpha}_i^1 : x_i \text{ satisfies Theorem 1 (ii)}\}.$$

Corollary 1 states that conservative bounds on the identified set can always be recovered via Theorem 1. Moreover, these bounds are sharp whenever the budget sets are convex. Finally, note that given some value of  $(\lambda_{i,t})_{t \in \mathcal{T}}$  and  $\delta_i$ , the existence of a solution can be checked efficiently using linear programming.

**Remark.** Conditional on numbers  $(\lambda_{i,t})_{t \in \mathcal{T}} \in \mathbb{R}_{++}^L$  and a discount factor  $\delta_i \in [\underline{\delta}, 1]$ , the set of solutions  $\alpha_i^1$  in Theorem 1 is convex. As such, the identified set can be recovered by finding the smallest and largest value of  $\alpha_i^1$  for which a solution exists. If the budget sets are not convex, then the procedure returns conservative bounds on the identified set.

### 3 Data

This section presents the data set used in my empirical application and discusses its main source of measurement error.

#### 3.1 Sample Construction

For my empirical application, I use the Nielsen Homescan Dataset 2011 (henceforth referred to as the Homescan). This data set contains information on purchases made by a panel of U.S. households in a large variety of retail outlets. The data set is designed to be representative of the U.S. population based on a wide range of annually updated demographic characteristics including age, sex, race, education, and income.

Participating households are provided with a scanner device and instructed to record all of their purchases after each shopping trip. The scanner device first requires participants to specify the date and store associated with each trip. Then, they are prompted to enter the number of units bought. When an item is purchased at a store with point-of-sale data, the average weighted price of the item in that week and store is directly given to Nielsen and recorded as the price paid prior to any coupon. Otherwise, panelists enter the price paid prior to any deal or coupon using the scanner device. In either case, panelists record the amount saved from coupons. The final price paid is the recorded price paid minus coupon discounts.

The Homescan contains information on Universal Product Codes (UPC) belonging to one of 10 departments. In order to mitigate issues associated with stockpiling, I restrict my attention to the following four food departments: dry grocery, frozen foods, dairy, and packaged meat.<sup>14</sup> This selection leaves over a million distinct UPCs representing about 40% of all products in the Homescan. For each household, I also aggregate the data to monthly observations to further

<sup>14</sup>This choice implicitly assumes that food is weakly separable from other categories of goods. This assumption is empirically plausible (Cherchye et al., 2015), especially when the presence of measurement error is recognized (Fleissig and Whitney, 2008; Elger and Jones, 2008).

reduce stockpiling issues. The resulting UPC prices are calculated as the average UPC prices weighted by quantities purchased.

To obtain regular observations on each good, I aggregate UPCs to their department categories, yielding a total of four “goods”. The resulting aggregated prices are calculated as the average UPC prices weighted by quantities purchased. Even with this layer of aggregation, some households do not have purchases from each category of goods in every month. Since the model requires price observations in every time period, I discard those households from the analysis.

In addition to the above restriction, I only consider single households. This is motivated by recent evidence that exponential discounting may be rejected because of preference aggregation within a household.<sup>15</sup> Thus, including couple households could lead to the artificial rejection of the model.<sup>16</sup> Finally, I consider consumers that are at least 50 years old to exclude potential online shoppers. The inclusion of younger consumers has no significant impact on the results.

The data set focuses on households that satisfy the above criteria and who participated in the Homescan from April to September of the panel year 2011. The final sample contains 1645 consumers, 4 aggregated goods, and 6 monthly time periods. Additional details about the construction of the data set are provided in Appendix A2.

### 3.2 Sample Description

To assess whether my constructed sample is in line with the Homescan 2011, Table 1 compares demographics between the two samples.

Table 1: Summary Demographics

	N	Age	White (%)	Black (%)	Asian (%)	Other (%)	Male (%)	Female (%)	High school or less (%)
Sample	1645	65.45	86.14	11.43	0.55	1.88	34.04	65.96	27.90
Homescan 2011	62 092	57.45	83.77	9.38	3.17	3.99	74.99	90.41	46.21

*Notes:*  $N$  is the sample size. Age is calculated using the year of birth of the head household member. Since households may have many members in the Homescan 2011, I let the head household refer to the male in that case. The percentage of males and females includes both the head male and head female members in the household.

<sup>15</sup>As [Adams et al. \(2014\)](#) point out, inconsistencies may arise due to negotiation within a couple household. [Jackson and Yariv \(2015\)](#) further show that time inconsistent behavior will appear if individuals in a non-dictatorial household have different discount factors. By accounting for measurement error in survey data, [Aguiar and Kashaev \(2018\)](#) show that single households behave consistently with exponential discounting while couple households do not.

<sup>16</sup>This prediction is confirmed in my empirical application where I find that the model is rejected for couple households.

Table 1 shows that the average age, proportion of females, and education level are slightly higher in my sample than in the Homescan. The difference in age is natural given that my sample is restricted to consumers that are at least 50 years old. Despite the higher proportion of females and higher education level in my sample, the demographics are still fairly representative overall.

The fundamental assumption of the model is that consumers can decrease their prices paid by searching more intensively. To provide evidence that price search is an empirical feature of the data, Figure 2 displays how log prices vary with log number of shopping trips.

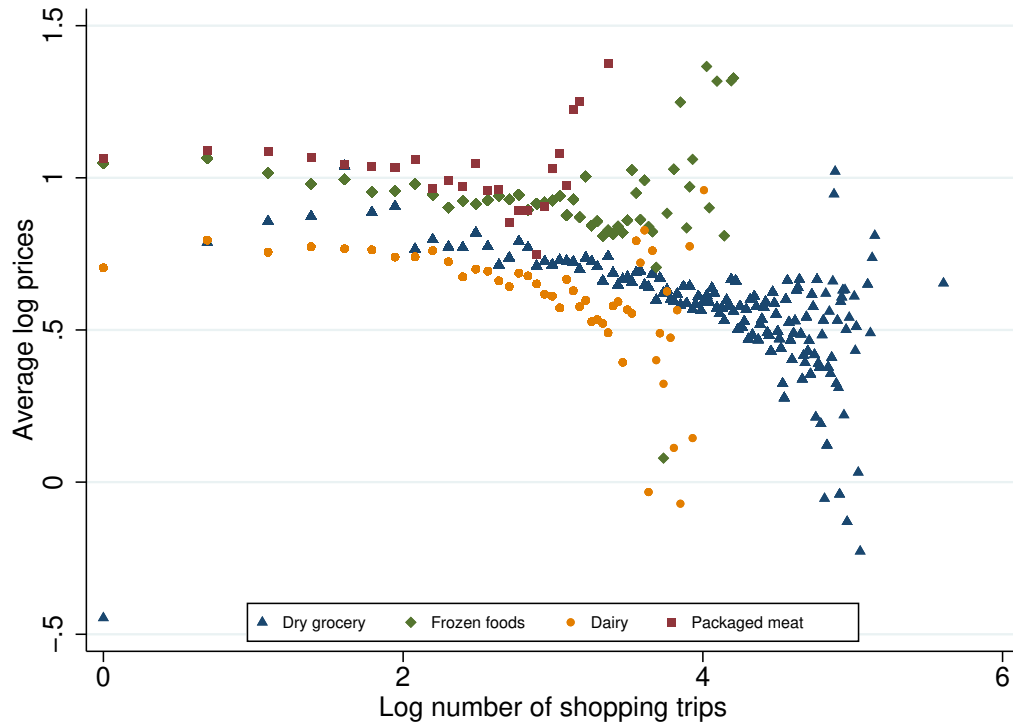


Figure 2: Average Log Price by Log Number of Shopping Trips

*Note:* The vertical axis reports the average log price, where the average is taken across consumers.

Consistent with the main hypothesis of the model, Figure 2 shows a negative relationship between prices paid and shopping intensity. That said, the true effect of shopping intensity on prices paid may be quite different from the one displayed in Figure 2. Consumers that go on many trips may do so because they do not find satisfactory discounts. This could explain the uptick in prices paid for larger values of shopping trips on frozen foods and packaged meat. Alternatively, those upticks could reflect the purchase of higher quality goods on

those shopping trips.

Next, Figure 3 compares differences in log prices paid between high- and low-income consumers for each category of goods.

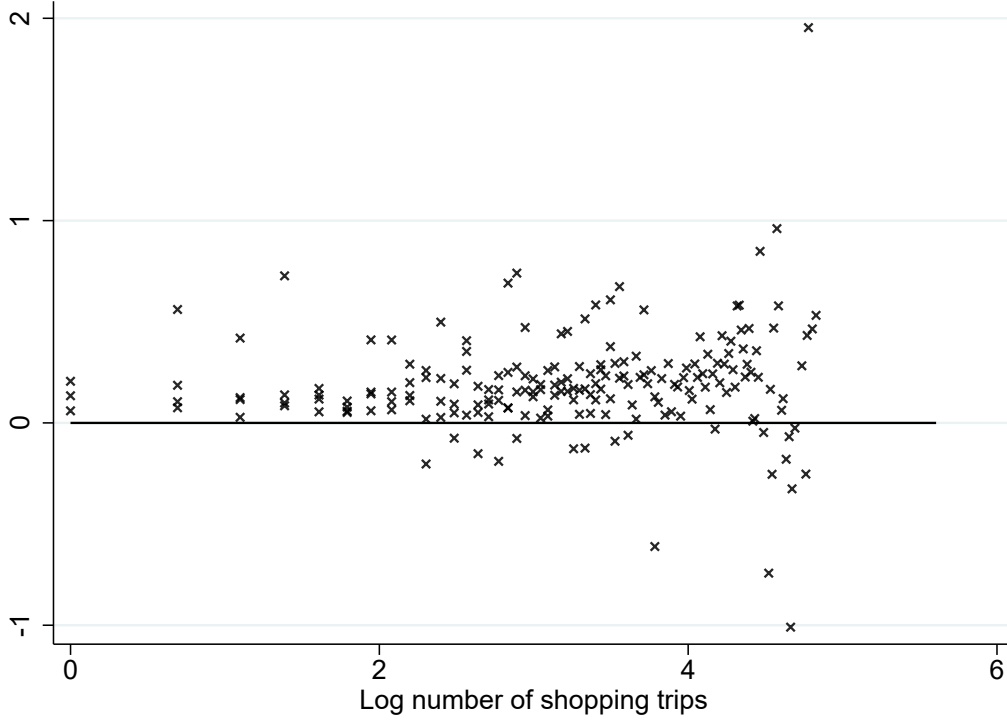


Figure 3: Difference in Average Log Price between Income Groups

*Notes:* The vertical axis reports the difference in average log price between high- and low-income consumers. The average is taken across consumers belonging to the same income group. High-income consumers are defined as those with an income greater than \$50,000. Low-income consumers are defined as those with an income lower than \$25,000.

The overwhelming trend shown in Figure 3 is that, conditional on shopping trips, high-income consumers pay higher prices relative to low-income consumers. Since this feature holds for almost every number of shopping trips, differences in prices paid cannot be attributed to differences in shopping intensity. A plausible reason for the difference could be that high-income consumers purchase goods of higher quality. Yet, a complementary explanation is that low-income consumers have a greater incentive to take advantage of sales and discounts to increase their consumption level. That is, low-income consumers could have a better shopping technology such that shopping intensity would be more effective in decreasing prices paid for them than for high-income consumers.

A possible approach to recover the true effects of shopping intensity on prices paid is to use instrumental variables. However, an issue with this approach is that

measurement error in prices could yield inconsistent estimators if measurement error is nonclassical or correlated with instruments. More generally, instruments could also fail the exogeneity assumption. In my empirical application, I show that such concerns about instrument validity are justified.

### 3.3 Measurement Error

The data collection process employed by Nielsen may induce measurement error for three reasons. First, conditional on a shopping trip, entry mistakes may arise as panelists self-report their purchases. Second, when a consumer purchases a UPC at a store that provides Nielsen with point-of-sale data, the price reported (before coupons) is the weighted average price during that week in that particular store. Thus, the reported price will be different from the price paid if the store changed the price during the week. Third, some consumers have loyalty cards whose discounts are not incorporated into the final price paid.

In a validation study of the Homescan 2004, [Einav et al. \(2010\)](#) use transactions from a large retailer in order to document the extent of measurement error. Consistent with the above observations, they find that price is the variable most severely hit by measurement error. Specifically, they find that around 50% of prices are accurately recorded. In contrast, around 90% of UPCs are accurately recorded by panelists on average. This number increases to 99% conditional on the quantity being equal to one. Accordingly, I focus exclusively on measurement error in prices in my application.

Since prices are mismeasured, observed prices  $(\mathbf{p}_{i,t})_{t \in \mathcal{T}}$  are different from true prices paid by the consumer  $(\mathbf{p}_{i,t}^*)_{t \in \mathcal{T}}$ . Let the difference between their logarithms define measurement error:  $\mathbf{m}_{i,t} := \log(\mathbf{p}_{i,t}) - \log(\mathbf{p}_{i,t}^*)$  for all  $t \in \mathcal{T}$ .<sup>17</sup> Using price data from a large retailer, [Einav et al. \(2010\)](#) show that the difference between observed and true log prices is centered around zero in the Homescan 2004. Formally, one cannot reject that the difference in sample means of log prices is zero at the 95% confidence level.

As Nielsen’s method of data collection has not changed since their study, I take their finding as support for mean zero measurement error in log prices in the Homescan 2011.

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<sup>17</sup>This definition makes no assumption on the way measurement error arises. For example, measurement error could be additive or multiplicative and be correlated across goods or time periods.

**Assumption 2.** *For all  $l \in \mathcal{L}$  and  $t \in \mathcal{T}$ , the following moment condition holds:*

$$\mathbb{E}[\log(p_{l,t})] = \mathbb{E}[\log(p_{l,t}^*)].$$

Assumption 2 says that expected observed log prices and true log prices are the same for each good and time period. Together, they yield a total of  $L \cdot T$  moments on measurement error.

## 4 Environment

In this section, I discuss existing assumptions and impose additional structure on the model in preparation of the empirical application.

Conditional on the log-linear specification implied by Assumption 1, price functions are otherwise free to vary across goods and consumers. This heterogeneity is important as goods may not be subject to the same discounts and consumers may not have access to the same set of stores.<sup>18</sup> Furthermore, note that the price function for any good  $l \in \mathcal{L}$  only depends on the shopping intensity on that good. This precludes complementarities that may naturally arise, for instance, if two goods are in a same aisle in a store. Since goods are aggregated to coarse categories in the data set, this issue should be largely mitigated.

Consistent with the ability of the consumer to transfer income across time, I restrict the marginal utility of expenditure to be constant over time.

**Assumption 3.** *For all  $t \in \mathcal{T}$ , the marginal utility of expenditure is constant and such that  $\lambda_{i,t} = 1$ .*

Assumption 3 requires that the marginal utility of expenditure be invariant to changes in income. This quasilinearity assumption is justified in my empirical application as the data span only a period of 6 months for which unexpected changes in income should be negligible.<sup>19</sup> Moreover, food tends to be income inelastic such that changes in income should have mild impacts on preferences.<sup>20</sup>

The next assumption imposes the average search productivity to be time-invariant and ensures that price search is refutable. To see why, note that Assumption 1 implies that the true average price paid must decrease whenever

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<sup>18</sup>Hence, even if store chains apply a nearly-uniform pricing rule (DellaVigna and Gentzkow, 2019), the shopping technology may still differ across consumers.

<sup>19</sup>Quasilinearity is also used by Echenique et al. (2011) and Allen and Rehbeck (2020) in a similar scanner data set on food expenditures.

<sup>20</sup>See, for example, Anderson and Blundell (1984), Erdil (2006), and Selvanathan and Selvanathan (2006).



shopping intensity increases if there is no change in the average search productivity. Therefore, inconsistencies with price search arise whenever this relationship is violated in the data.<sup>21</sup>

**Assumption 4.** *For all  $t \in \mathcal{T}$ ,  $\mathbb{E}[\bar{\omega}_t] = 0$ , where  $\bar{\omega}_{i,t} := L^{-1} \sum_{l=1}^L \omega_{i,l,t}$  denotes the average search productivity across goods.*

Assumption 4 allows time-varying search productivity for specific goods as long as the overall search productivity remains constant. Permitting search productivity for a particular good to change over time is important in my application because of the coarse aggregation of the data. Indeed, since a consumer may purchase different baskets of goods in different time periods, prices may vary due to variations in the composition of the baskets of goods. As such, Assumption 4 can be interpreted as requiring the average quality of the baskets of goods purchased to be time-invariant across consumers.

Aside from the above restriction, Assumption 4 is quite general as it does not presume anything about the underlying stochastic process of search productivity. Conditional on the aggregate average search productivity being time-invariant, it allows individual-specific search productivity to vary arbitrarily with both observables and unobservables. In particular, it includes Markovian processes often assumed in the production function literature.<sup>22</sup>

Given the log-linearity of the price functions, Assumption 4 implies that average log true prices should be around the mean, conditional on shopping trips. In accordance with this prediction, Figure 4 shows that the distribution of observed average log prices is centered around its mean. I report the unconditional distribution for expositional purposes; similar shapes are obtained for the conditional ones.<sup>23</sup>

It is useful to note that, under Assumption 2, measurement error balances out such that the observed mean of average log price asymptotically coincides with the true mean of average log price. As such, the observed mean of average log price shown in Figure 4 can be thought of as the true mean of average log price under Assumption 2.

Lastly, I bound the support of the elasticity of price with respect to shopping intensity to gain further identification power.

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<sup>21</sup>See Appendix A3 for analytical power results.

<sup>22</sup>See, for example, Olley and Pakes (1996), Levinsohn and Petrin (2003), Akerberg et al. (2015), and Gandhi et al. (2020).

<sup>23</sup>Consistent with Assumption 1, distributions conditioned on larger values of shopping trips tend to be centered around lower values of log prices.

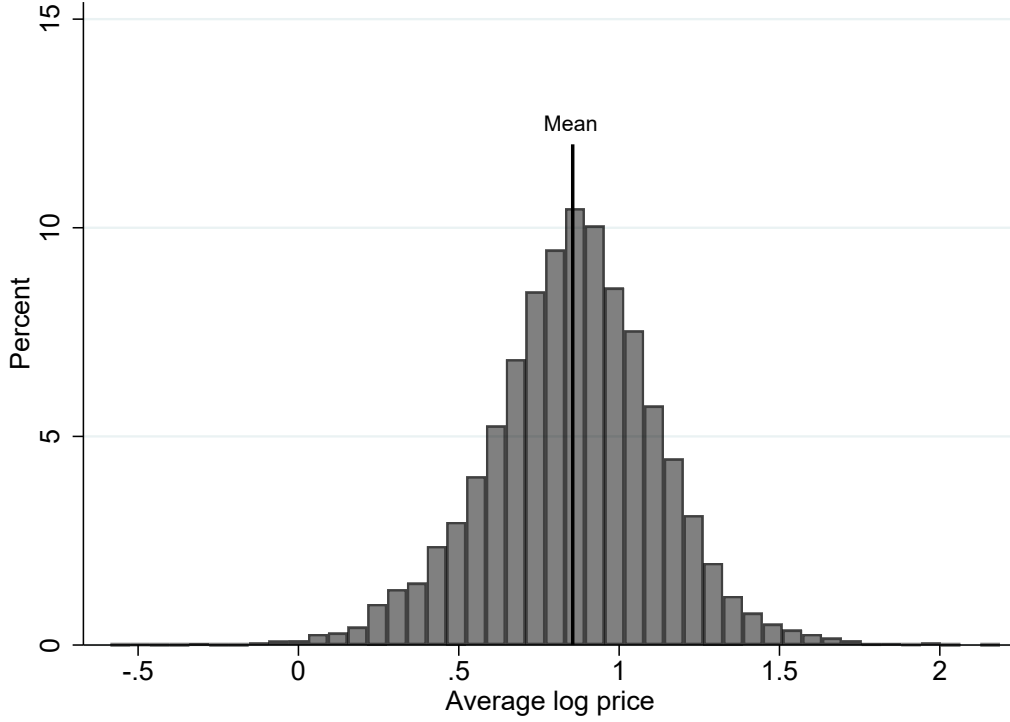


Figure 4: Distribution of Average Log Prices

**Assumption 5.** For all  $l \in \mathcal{L}$ ,  $\alpha_{i,l}^1 \in [-1, 0]$ .

Assumption 5 constrains the elasticity of price with respect to shopping intensity to be in  $[-1, 0]$  for every good  $l \in \mathcal{L}$ . In comparison, [Aguiar and Hurst \(2007\)](#) obtain a point-estimate of  $-0.074$  for the elasticity of average price with respect to shopping intensity using the Homescan 1993-1995.<sup>24</sup> As such, Assumption 5 should give enough flexibility for the needs of the data.

**Remark.** *Measurement error implicitly accommodates various shocks that may occur outside the model. For example, changes in prices induced by supply shocks would be absorbed by the moments on measurement error provided they satisfy Assumption 2. Likewise, exogenous shocks can be absorbed by search productivity provided they satisfy Assumption 4. Accordingly, the model is robust to a variety of perturbations.*

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<sup>24</sup>Their estimate is obtained using an instrumental variable approach and is for a single aggregated good.

## 5 Statistical Framework

In this section, I extend the deterministic framework presented in the paper to a statistical one susceptible to testing and inference. To this end, I exploit the Entropic Latent Variable Integration via Simulation (ELVIS) methodology of Schennach (2014) and the refinement of Aguiar and Kashaev (2018). I then show that inference on the true expected elasticity can be obtained in the model.

### 5.1 Characterization of the Model via Moment Functions

From now on, consumer data sets should be viewed as independent and identically distributed draws from some distribution. Let  $\mathcal{X} := P \times C \times A$  and  $\mathcal{E}|\mathcal{X}$  be the support of the latent random variables conditional on  $\mathcal{X}$ . Moreover, let  $x_i \in \mathcal{X}$  denote the observed random data and  $e_i \in \mathcal{E}|\mathcal{X}$  denote the latent random variables  $(u_{i,t}, \delta_i, \alpha_i, \rho_{i,t}, \omega_{i,t}, \mathbf{m}_{i,t})_{t \in \mathcal{T}}$ .

To make the model amenable to statistical testing, I first express it through a set of moments. Therefore, for all  $l \in \mathcal{L}$  and  $s, t \in \mathcal{T}$ , I write the model defined by Assumptions 1-5 with the following moment functions:

$$\begin{aligned} g_{i,s,t}^u(x_i, e_i) &:= \mathbb{1} \left( u_{i,s} - u_{i,t} - \delta_i^{-t} [\mathbf{p}_{i,t}^{*l}(\mathbf{c}_{i,s} - \mathbf{c}_{i,t}) - \rho_{i,t}'(\mathbf{a}_{i,s} - \mathbf{a}_{i,t})] \leq 0 \right) - 1, \\ g_{i,l,t}^p(x_i, e_i) &:= \mathbb{1} \left( \log(p_{i,l,t}^*) - (\alpha_{i,l}^0 + \alpha_{i,l}^1 \log(a_{i,l,t}) - \omega_{i,l,t}) = 0 \right) - 1, \\ g_{i,l,t}^m(x_i, e_i) &:= \log(p_{i,l,t}) - \log(p_{i,l,t}^*), \\ g_{i,l,t}^\omega(x_i, e_i) &:= \bar{\omega}_{i,t}, \end{aligned}$$

where the first set of functions characterizes the concavity of the utility function, the second the log-linearity of the price functions, the third measurement error, and the last search productivity. The latent random variables satisfy their support constraints:  $\delta_i \in [\delta, 1]$ ,  $-\alpha_i^1 \in [0, 1]$ ,  $\rho_{i,t} \leq 0$  and  $\mathbf{p}_{i,t}^* > 0$ , where  $\rho_{i,t}$  further satisfies

$$\rho_{i,t} := \frac{\partial \mathbf{p}_i(\mathbf{a}_{i,t}, \omega_{i,t})}{\partial \mathbf{a}_{i,t}} \odot \mathbf{c}_{i,t}.$$

This equality constraint implies that  $\rho_{i,t}$  is completely determined by the data and latent variables  $(u_{i,t}, \delta_i, \alpha_i, \omega_{i,t}, \mathbf{m}_{i,t})_{t \in \mathcal{T}}$ .

Every consumer has a total of  $T^2 + L \cdot T + L \cdot T + T$  moment functions, written as  $\mathbf{g}_i(x_i, e_i) := (\mathbf{g}_i^u(x_i, e_i)', \mathbf{g}_i^p(x_i, e_i)', \mathbf{g}_i^m(x_i, e_i)', \mathbf{g}_i^\omega(x_i, e_i)')'$  for short. Arbitrary combinations of these sets of functions are denoted with their superscripts bundled together. For example,  $\mathbf{g}_i^{m,\omega}(x_i, e_i)$  is the set of functions on measurement error and search productivity. Note that the moment functions

$\mathbf{g}_i(x_i, e_i)$  depend on unobservables. As such, the latent variables have to be drawn from some distribution for the moment functions to be evaluated.

Since Assumptions 2 and 4 only impose centering conditions on the expected measurement error and the expected average search productivity, the latent variables are essentially unrestricted for any particular consumer. Therefore, empirical content is gained by averaging the moment functions  $\mathbf{g}_i(x_i, e_i)$  across consumers.<sup>25</sup> The distribution of the latent variables that satisfy individual-specific moment functions  $\mathbf{g}_i^{u,p}(x_i, e_i)$  may then only do so when the expectation of  $\mathbf{g}_i^{m,\omega}(x_i, e_i)$  deviates from zero.

## 5.2 Statistical Price Search Rationalizability

Let  $\mathcal{M}_{\mathcal{X}}$ ,  $\mathcal{M}_{\mathcal{E},\mathcal{X}}$ , and  $\mathcal{M}_{\mathcal{E}|\mathcal{X}}$  denote the set of all probability measures defined over  $\mathcal{X}$ ,  $(\mathcal{E}, \mathcal{X})$ , and  $\mathcal{E}|\mathcal{X}$ , respectively. Moreover, let  $\mathbb{E}_{\mu \times \pi}[\mathbf{g}_i(x_i, e_i)] := \int_{\mathcal{X}} \int_{\mathcal{E}|\mathcal{X}} \mathbf{g}_i(x_i, e_i) d\mu d\pi$ , where  $\mu \in \mathcal{M}_{\mathcal{E}|\mathcal{X}}$  and  $\pi \in \mathcal{M}_{\mathcal{X}}$ . The moment functions previously defined allow me to define the statistical rationalizability of a data set with respect to price search.<sup>26</sup>

**Definition 3.** Under Assumptions 1-5, a random data set  $x := \{x_i\}_{i=1}^N$  is price search rationalizable (PS-rationalizable) if

$$\inf_{\mu \in \mathcal{M}_{\mathcal{E}|\mathcal{X}}} \|\mathbb{E}_{\mu \times \pi_0}[\mathbf{g}_i(x_i, e_i)]\| = 0,$$

where  $\pi_0 \in \mathcal{M}_{\mathcal{X}}$  is the observed distribution of  $x$ .

That is, the data are PS-rationalizable if there exists a distribution of the latent random variables conditional on the data such that the expected moment functions are satisfied. In practice, searching over the set of all conditional distributions represents a daunting task. Fortunately, the following result shows that the problem can be greatly simplified without loss of generality.<sup>27</sup>

**Theorem 2.** *The following are equivalent:*

(i) *A random data set  $x$  is PS-rationalizable.*

(ii)  $\min_{\gamma \in \mathbb{R}^{L \cdot T + T}} \|\mathbb{E}_{\pi_0}[\tilde{\mathbf{h}}_i(x_i; \gamma)]\| = 0,$

<sup>25</sup>I show that the model defined by Assumptions 1-5 is refutable in Appendix A3.

<sup>26</sup>This definition follows the notion of identified set in Schennach (2014).

<sup>27</sup>See Aguiar and Kashaev (2018) for the weak technical assumptions required for this result to hold.

where

$$\tilde{\mathbf{h}}_i(x_i; \gamma) := \frac{\int_{e_i \in \mathcal{E}|\mathcal{X}} \mathbf{g}_i^{m,\omega}(x_i, e_i) \exp(\gamma' \mathbf{g}_i^{m,\omega}(x_i, e_i)) \mathbb{1}(\mathbf{g}_i^{u,p}(x_i, e_i) = 0) d\eta(e_i|x_i)}{\int_{e_i \in \mathcal{E}|\mathcal{X}} \exp(\gamma' \mathbf{g}_i^{m,\omega}(x_i, e_i)) \mathbb{1}(\mathbf{g}_i^{u,p}(x_i, e_i) = 0) d\eta(e_i|x_i)},$$

and where  $\eta(\cdot|x_i)$  is an arbitrary user-specified distribution function supported on  $\mathcal{E}|\mathcal{X}$  such that  $\mathbb{E}_{\pi_0}[\log(\mathbb{E}_{\eta}[\exp(\gamma' \mathbf{g}_i^{m,\omega}(x_i, e_i))|x_i])]$  exists and is twice continuously differentiable in  $\gamma$  for all  $\gamma \in \mathbb{R}^{L \cdot T + T}$ .

*Proof.* See Theorem 2.1 in Schennach (2014) and Theorem 4 in Aguiar and Kashaev (2018).  $\square$

In words, Theorem 2 (ii) averages out the unobservables in  $\mathbf{g}_i(x_i, e_i)$  according to some conditional distribution.<sup>28</sup> The particularity of  $\eta(\cdot|x_i)$  is to preserve the set of values that the objective function can take before the latent variables have been averaged out. As such, any minimum achieved under  $\mu$  can also be achieved under  $\eta(\cdot|x_i)$ . The dimensionality of the problem is then further reduced by noting that the concavity of the utility function and the log-linearity of the price functions are only restricting the conditional support of the unobservables. Thus, one can draw from the conditional distribution  $\tilde{\eta}(\cdot|x_i) := \mathbb{1}(\mathbf{g}_i^{u,p}(x_i, \cdot) = 0) \eta(\cdot|x_i)$  rather than leaving the moment functions  $\mathbf{g}_i^{u,p}(x_i, \cdot)$  in the optimization problem.

In most applications, the distribution  $\tilde{\eta}(\cdot|x_i)$  may be taken to be proportional to a normal distribution:

$$d\tilde{\eta}(\cdot|x_i) \propto \exp(-\|\mathbf{g}_i^{m,\omega}(x_i, e_i)\|^2),$$

where the value of the mean and variance are inconsequential for the validity of the result. To draw from this distribution, the first step is to obtain latent variables that satisfy the moment functions  $\mathbf{g}_i^{u,p}(x_i, e_i)$  and can be achieved by rejection sampling. Then, a standard Metropolis-Hastings algorithm can be used to draw from the distribution.<sup>29</sup>

### 5.3 Statistical Testing

The notion of PS-rationalizability together with Theorem 2 provides a feasible way of checking whether the data are consistent with the model. To statistically

<sup>28</sup>Schennach (2014) shows the existence of an admissible conditional distribution  $\eta(\cdot|x_i)$  and gives a generic construction for it.

<sup>29</sup>Additional details about the implementation are given in Appendix A4.

test the PS-rationalizability of a data set, let

$$\hat{\mathbf{h}}(\gamma) := \frac{1}{N} \sum_{i=1}^N \tilde{\mathbf{h}}_i(x_i, \gamma)$$

and

$$\hat{\mathbf{\Omega}}(\gamma) := \frac{1}{N} \sum_{i=1}^N \tilde{\mathbf{h}}_i(x_i, \gamma) \tilde{\mathbf{h}}_i(x_i, \gamma)' - \hat{\mathbf{h}}(\gamma) \hat{\mathbf{h}}(\gamma)'$$

denote the sample analogues of  $\tilde{\mathbf{h}}$  and its variance, respectively. Furthermore, let  $\hat{\mathbf{\Omega}}^-$  denote the generalized inverse of the matrix  $\hat{\mathbf{\Omega}}$ . Schennach (2014) shows that the test statistic

$$\text{TS}_N := N \inf_{\gamma \in \mathbb{R}^{L \cdot T + T}} \hat{\mathbf{h}}(\gamma)' \hat{\mathbf{\Omega}}^-(\gamma) \hat{\mathbf{h}}(\gamma)$$

is stochastically bounded by a  $\chi^2$  distribution with  $d_m := L \cdot T + T$  degrees of freedom ( $\chi_{d_m}^2$ ).<sup>30</sup> As such, the PS-rationalizability of a data set can be checked by comparing the value of the test statistic against the critical value of the chi-square distribution with  $d_m$  degrees of freedom.

#### 5.4 Inference

Conditional on the data being consistent with PS-rationalizability, the next step is to make inferences on parameters of interest. First, the next result shows that inference on the true average elasticity of average price is possible in the model.

**Proposition 1.** *The average expected elasticity of average price with respect to shopping intensity is given by*

$$\frac{1}{L} \sum_{l=1}^L \mathbb{E} \left[ \frac{\partial \overline{\log(p_t^*)}}{\partial \log(a_{l,t})} \right] = \frac{1}{L} \mathbb{E} [\bar{\alpha}^1],$$

where  $\overline{\log(p_t^*)}$  is the true average log price across goods and  $\bar{\alpha}^1$  is the average elasticity of price with respect to shopping intensity.

Proposition 1 states that the average expected effect of an increase in shopping intensity on the average price can be recovered from data on prices paid and search intensity. The reason why it can be achieved in the model is that

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<sup>30</sup> Aguiar and Kashaev (2018) further show that the test has an asymptotic power equal to one.

Assumption 4 restricts the expected average search productivity to be time-invariant. Therefore, any variation in the expected average price must be caused exclusively by variations in shopping intensity.

In the statistical framework previously outlined, inference can be made by adding a moment on a parameter of interest. Proposition 1 suggests to choose the following moment:

$$L^{-1}\mathbb{E}[\bar{\alpha}^1] = \theta_0,$$

where  $\bar{\alpha}^1$  is the average elasticity of price with respect to shopping intensity and  $\theta_0 \in [-0.25, 0]$  is the average expected elasticity of average price with respect to shopping intensity.<sup>31</sup> As before, this condition can be encapsulated in a moment function:

$$g_i^\alpha(x_i, e_i) := L^{-1}\bar{\alpha}_i^1 - \theta_0.$$

A conservative 95% confidence set on  $\theta_0$  can be obtained by inverting the test statistic:

$$\{\theta_0 \in \Theta : TS_N(\theta_0) \leq \chi_{d_m+1, 0.95}^2\},$$

where  $TS_N(\theta_0)$  is the test statistic at a fixed value of  $\theta_0$ .

## 6 Empirical Application

In this section, I check whether the data are PS-rationalizable, set estimate the shopping technology across multiple demographics, relate price search to consumption inequality, and test the validity of instrumental variable within the model.

### 6.1 Price Search

In what follows, I set  $\underline{\delta} = 0.95$  such that the support of the discount factor is  $[0.95, 1]$ . By applying the above methodology to my sample, I find that PS-rationalizability is not rejected by the data at the 95% confidence level. More precisely, I obtain a test statistic of 36.38, which is below the chi-square critical value of 43.77.<sup>32</sup> Since the model is not rejected by the data, I can invert the statistical test to obtain a 95% confidence set on the average expected elasticity

<sup>31</sup>The support of  $\theta_0$  is given by  $L^{-1}$  times the support of the average elasticity of price with respect to shopping intensity  $\bar{\alpha}_i = [-1, 0]$ .

<sup>32</sup>In contrast, I find that PS-rationalizability is rejected by the data in couple households as the test statistic is 392.56.

of average price with respect to shopping intensity. The results for all consumers, low-income consumers, and high-income consumers are reported in Figure 5.

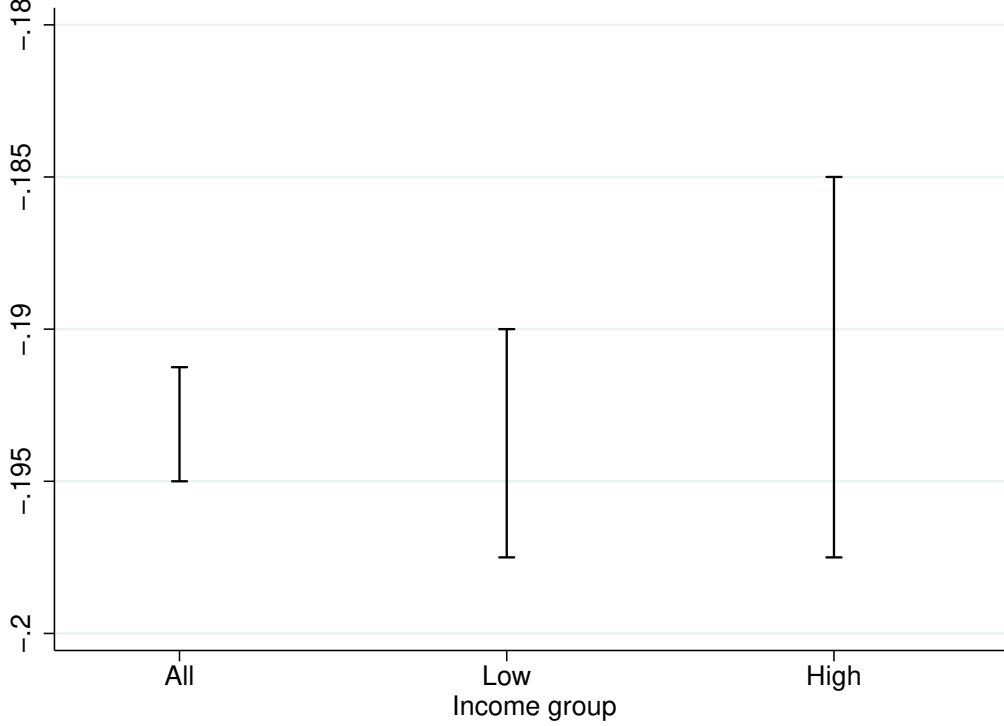


Figure 5: Average Expected Elasticity of Average Price with Respect to Shopping Intensity ( $\mathbb{E}[\bar{\alpha}]$ )

*Notes:* The vertical axis represents the average expected elasticity of average price with respect to shopping intensity. Low-income consumers are defined as those with an income lower than \$25,000. High-income consumers are defined as those with an income greater than \$50,000. The sample size is 1645 for all, 660 for low-income consumers, and 348 for high-income consumers.

In this figure, we see that the average expected elasticity of average price with respect to shopping intensity is about  $-0.1925$ . That is, doubling shopping intensity decreases the average price paid by about 19.25%. Figure 5 also shows that low-income consumers have a slightly better shopping technology compared to high-income consumers. However, this difference is insufficient to fully explain the large gap in prices paid documented in Figure 3. More generally, low-income consumers could tend to pay lower prices because of a higher shopping frequency. To investigate this prospect, Figure 6 displays the distribution of log number of shopping trips by income group.

Overall, Figure 6 shows that the distribution of shopping intensity is almost identical between income groups. Consumers earning less than \$25,000 a year are only making 1.65% more shopping trips per month than consumers earning





Figure 6: Log Number of Shopping Trips Density by Department Category

*Notes:* Low-income consumers are defined as those with an income lower than \$25,000. High-income consumers are defined as those with an income greater than \$50,000. The sample size is 660 for low-income consumers and 348 for high-income consumers.

more than \$50,000 a year. Combined with my finding that income groups have a similar shopping technology, it follows that heterogeneity in price search only explains a small portion of the difference in average price paid between low- and high-income consumers. Consequently, the gap in average price paid is likely due to a difference in average good quality.<sup>33</sup>

Lastly, to investigate whether the shopping technology differs across other dimensions than income, I set estimate the shopping technology by gender, education, geographic location, and occupation. These categories are defined as follows: gender separates the sample between males and females; education between consumers that did not graduate college and those that graduated college; geographic location between consumers in rural states and those in urban states; and occupation between workers and retirees.<sup>34</sup>

<sup>33</sup>Strictly speaking, the difference in average price paid is mainly due to time-invariant factors. In my application, quality is the main candidate time-invariant factor affecting the difference in average price paid between low- and high-income consumers.

<sup>34</sup>A rural state is defined as one where the proportion of the urban population is below that of the U.S. population, and an urban state as one where the proportion of the urban population

For every demographic, the 95% confidence set is  $[-0.1975, -0.19]$ , with the exception of workers where the confidence set is  $[-0.195, -0.19]$ .<sup>35</sup> Since the model allows for sizable heterogeneity, the confidence sets on the average expected elasticity of average price with respect to shopping intensity reflect inherent homogeneity in the average shopping technology rather than induced homogeneity through ex ante restrictions.

## 6.2 Consumption Inequality

Since low-income consumers tend to pay lower prices, they may be able to achieve a similar consumption level to high-income consumers despite their lower income. To assess the extent of consumption inequality in the data, Figure 7 compares the distribution of log consumption between low- and high-income consumers.



Figure 7: Log Consumption Density by Department Category

*Notes:* Consumption represents the number of UPCs purchased. Low-income consumers are defined as those with an income lower than \$25,000. High-income consumers are defined as those with an income greater than \$50,000. The sample size is 660 for low-income consumers and 348 for high-income consumers.

is above that of the U.S. population.

<sup>35</sup>The 95% confidence set is identical across those demographics at a precision level of 0.0025. Therefore, any difference in the confidence set must be by less than 0.005.

Figure 7 shows that the distribution of consumption is close to identical across income groups. On average, low-income consumers only have a 0.43% lower consumption level per month compared to high-income consumers.

To evaluate the impacts of heterogeneity in price search on consumption inequality, I compare how much more a consumer at the 20th quantile of the shopping intensity distribution would pay compared to one at the 80th quantile. I shall refer to the former type of consumers as infrequent shoppers and the latter type as frequent shoppers. The results are presented in Figure 8 and are obtained by using the shopping technologies estimated in Figure 5 and the observed distribution of number of shopping trips.

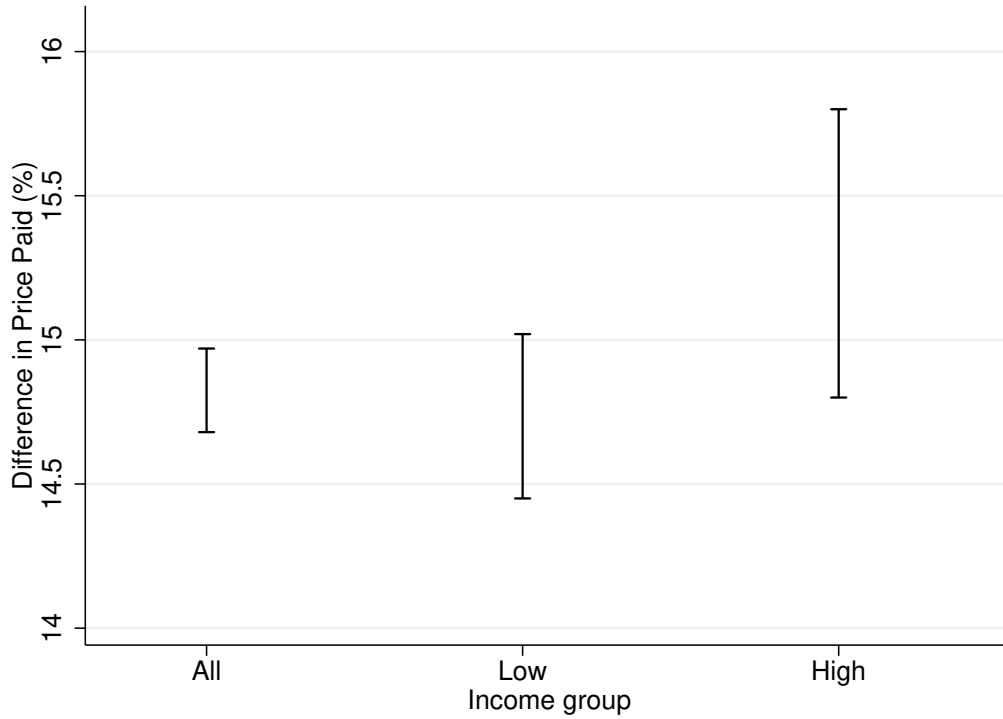


Figure 8: Difference in Price Paid due to Heterogeneity in Price Search

*Notes:* The vertical axis represents how much more consumers at the 20th quantile of the distribution of number of shopping trips pay compared to those at the 80th quantile.

Figure 8 shows that infrequent shoppers pay almost 15% more on the goods they purchase than frequent shoppers. Assuming that expenditure is fixed and that savings from search is spent on those same goods, the consumption level of frequent shoppers is 15% larger than that of infrequent shoppers. Figure 8 also shows that consumption inequality is larger among high-income consumers than among low-income consumers. Finally, Figure 8 shows that search has

two opposite effects on between-group consumption inequality. On one hand, price search mitigates consumption inequality between low-income consumers with high shopping frequency and high-income consumers with low shopping frequency. On the other hand, price search exacerbates consumption inequality between low-income consumers with low shopping frequency and high-income consumers with high shopping frequency.

### 6.3 Instrumental Variable

The statistical framework allows me to impose additional moments to test the validity of an instrument within the structure of the model. For conciseness, I consider the validity of income as it is the main instrument used in [Aguiar and Hurst \(2007\)](#). Income is a valid instrument if the exogeneity assumption  $\mathbb{E}[y_t \omega_{l,t}] = 0$  holds for all  $l \in \mathcal{L}$  and  $t \in \mathcal{T}$ . By imposing those moments at  $t = 1$ , this additional restriction brings the total number of moments to 34 and yields a test statistic of 80.12.<sup>36</sup> Since this value is above the chi-square critical value of 48.60, instrumental variable validity is rejected in my data set.<sup>37</sup> Moreover, income would also need to satisfy  $\mathbb{E}[y_t m_{l,t}] = 0$  for all  $l \in \mathcal{L}$  and  $t \in \mathcal{T}$  to be valid if measurement error in prices is additive. The model is rejected with a test statistic of 113.69 when further imposing this set of moments at  $t = 1$ .

### 6.4 Discussion

Price search allows consumers to pay prices that depend on their own shopping intensity. The resulting heterogeneity in prices paid means that expenditure may give an erroneous account of consumption and, hence, consumption inequality. In a calibrated model of price search, [Arslan et al. \(2021\)](#) show that consumption inequality is significantly smaller than expenditure inequality in the Homescan 2004. Consistent with their finding, [Figure 3](#) and [Figure 7](#) display disparity in prices paid despite similar consumption levels between income groups. Since the difference in expenditures is attributable to lower prices paid by low-income consumers, an analysis based on expenditures would overstate actual consumption inequality in the data.

The conclusion that consumption inequality is negligible between income groups appears somewhat counterintuitive, especially given the overwhelming

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<sup>36</sup>Imposing instrument validity on additional time periods would further increase the test statistic.

<sup>37</sup>This result does not directly translate into the rejection of income in [Aguiar and Hurst \(2007\)](#) because the aggregation of their data set is different from mine. Also, the rejection could be due to the incompatibility between the exogeneity restriction and the model restrictions.

evidence that consumption inequality has increased over time such as in [Aguiar and Bils \(2015\)](#).<sup>38</sup> This finding can be reconciled by two observations. First, [Aguiar and Bils \(2015\)](#) measure the change in consumption inequality by comparing relative expenditures on luxury versus necessity goods. Since my data set only contains food categories (necessities), the rise in consumption inequality can be explained by changes in expenditures on luxury goods. Second, consumption inequality is typically measured from data on expenditures, thus ignoring differences in prices paid between income groups. As discussed previously, this can lead to an inflated measure of consumption inequality.

My application shows that price search has a strong impact on average prices paid. However, the set estimates in Figure 5 show that low-income consumers have only a slightly better average shopping technology compared to high-income consumers. Thus, most of the price differentials between income groups displayed in Figure 3 are due to differences in good quality. Using data from the National Health and Nutrition Examination Survey, [Wang et al. \(2014\)](#) document an increase in diet quality inequality between socioeconomic groups.<sup>39</sup> To the extent that lower quality goods are also less healthy, consumption inequality between income groups likely takes the form of diet quality inequality.

## 7 Conclusion

This paper proposes a flexible model to estimate the impacts of price search on consumption inequality. To guarantee the robustness of my inferences, I do not parameterize preferences and allow for unrestricted unobserved heterogeneity. Additionally, I show that the true average elasticity of average price with respect to shopping intensity can be recovered. To ensure the validity of the model, I go a step further and statistically test its assumptions through the use of a recent econometric framework. This approach allows me to make credible inferences without relying on the availability of instrumental variables.

The model I propose is shown to be consistent with the Nielsen Homescan data. Using aggregated data on food expenditures, I obtain a robust confidence set on the effect of an average increase in shopping intensity on the average price paid. I find that a doubling of shopping frequency decreases average prices paid

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<sup>38</sup>See [Attanasio and Pistaferri \(2016\)](#) for a review of the literature investigating the evolution of consumption inequality.

<sup>39</sup>Individuals with low socioeconomic status are defined as those with less than 12 years of education and an income below 130% of the poverty line. Individuals with high socioeconomic status are defined as those with more than 12 years of education and an income above 350% above the poverty line.

by about 19%. Moreover, I show that heterogeneity in price search has significant but mixed implications on consumption inequality. Price search reduces consumption inequality between low-income consumers with high shopping frequency and high-income consumers with low shopping frequency but exacerbates it between low-income consumers with low shopping frequency and high-income consumers with high shopping frequency.

The methodology put forward in this paper may be useful in other settings. I model the shopping technology as a function that takes shopping intensity as an input and returns price paid as an output. In this light, the estimation of the shopping technology is related to the estimation of production functions such as in [Gandhi et al. \(2020\)](#).<sup>40</sup> In a firm setting, my centering condition on the error term amounts to assuming that the average productivity across firms is constant over time. This allows firm-specific productivity shocks to follow essentially unrestricted processes. Thus, my methodology could be used to identify the production function without appealing to Markovian processes.

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<sup>40</sup>See also [Levinsohn and Petrin \(2003\)](#) for an earlier treatment on the estimation of gross output production functions and [Olley and Pakes \(1996\)](#) and [Akerberg et al. \(2015\)](#) for the related problem of estimating value-added production functions.

## Appendix

### A1: Relationship with Models of Household Production

Although the focus of this paper is on the price function, the framework of the model is consistent with one of household production similar in spirit to that of [Becker \(1965\)](#). As an illustration, I extend my model to one of household production and shows that it has close ties with that of [Aguiar and Hurst \(2007\)](#). For ease of comparison, I consider the static version of my model.

Suppose that, in addition to spending time shopping, the household can spend time in home production denoted by  $h \in \mathbb{R}_{++}$ . By using that time input along with market goods, the household can produce some homemade good  $K$  by using its (concave) home production function  $f(h, \mathbf{c})$ .<sup>41</sup> The household's problem therefore becomes

$$\begin{aligned} \max_{(\mathbf{c}, \mathbf{a}, K, h) \in C \times A \times \mathbb{R}_{++}^2} \quad & u(\mathbf{a}, K, h) \quad \text{s.t.} \quad \mathbf{p}(\mathbf{a}, \boldsymbol{\omega}_t)' \mathbf{c} = y_t \\ & f(\mathbf{c}, h) = K. \end{aligned}$$

One can get rid of the second constraint by substituting it for  $K$  in the utility function, yielding

$$\max_{(\mathbf{c}, \mathbf{a}, h) \in C \times A \times \mathbb{R}_{++}} u(\mathbf{a}, f(\mathbf{c}, h), h) \quad \text{s.t.} \quad \mathbf{p}(\mathbf{a}, \boldsymbol{\omega}_t)' \mathbf{c} = y_t.$$

Assuming the opportunity cost of time is additively separable, linear, and identical for the shopper and the home producer, the problem boils down to

$$\max_{(\mathbf{c}, \mathbf{a}, h) \in C \times A \times \mathbb{R}_{++}} u(f(\mathbf{c}, h)) + \boldsymbol{\mu}_t' \mathbf{a} + \mu_t h \quad \text{s.t.} \quad \mathbf{p}(\mathbf{a}, \boldsymbol{\omega}_t)' \mathbf{c} = y_t,$$

where  $\mu_t$  denotes the disutility from the time spent on either activity. Since both  $u(\cdot)$  and  $f(\cdot, \cdot)$  are unobservable concave functions, this maximization problem is observationally equivalent to

$$\max_{(\mathbf{c}, \mathbf{a}, h) \in C \times A \times \mathbb{R}_{++}} f(\mathbf{c}, h) + \boldsymbol{\mu}_t' \mathbf{a} + \mu_t h \quad \text{s.t.} \quad \mathbf{p}(\mathbf{a}, \boldsymbol{\omega}_t)' \mathbf{c} = y_t,$$

and we have thereby recovered a model with the same implications to that of

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<sup>41</sup>One can think of market goods as comestible such as eggs, sugar and pecans. By spending  $h$  unit of time cooking, the household can transform these “raw goods” into a pecan pie, the final good consumed by the household.

Aguiar and Hurst (2007).<sup>42</sup> To see why, assume the solution is interior and take the first-order conditions:

$$\begin{aligned}\frac{\partial f}{\partial c} &= \lambda_t \mathbf{p}(\mathbf{a}, \boldsymbol{\omega}_t) \\ \boldsymbol{\mu} &= \lambda_t \frac{\partial \mathbf{p}(\mathbf{a}, \boldsymbol{\omega}_t)}{\partial \mathbf{a}} \odot \mathbf{c} \\ \mu &= -\frac{\partial f}{\partial h}.\end{aligned}$$

It follows that the marginal rate of transformation (MRT) between time and goods in shopping equals the MRT in home production:

$$\frac{\partial f}{\partial h} / \frac{\partial f}{\partial c_l} = -\frac{\frac{\partial p_l(a_l, \omega_{l,t})}{\partial a_l} \cdot c_l}{p_l(a_l, \omega_{l,t})} \quad \forall l \in \mathcal{L}.$$

This derivation shows that the household production version of my model naturally extends that of Aguilar and Hurst (2007). Conditional on knowing the price function, this last equation can be used to identify the home production function, a point that was cleverly exploited by Aguilar and Hurst (2007) in a parametric setting. Note that using exponential discounting as defined in (1) would yield  $\lambda_t \delta^{-t}$  instead of  $\lambda_t$  in the first-order conditions and leave the MRT unchanged.

## A2: Sample Construction

The Homescan contains information on purchases made by U.S. households in a wide variety of retail outlets. After every trip to a retail outlet, information about the trip is recorded by the panelist via a scanner device. Each trip may have one or many UPC purchases. In total, there are 66,321,848 purchases in the panel year 2011. Among them, 43,432,246 pertain to the departments of dry grocery, frozen foods, dairy and packaged meat. Since some purchases in the panel year are outside of the calendar year 2011, I remove them from the sample. This operation drops 751,479 purchases, leaving a total of 42,680,767 purchases.

For each household-month, I average UPC prices across trips. Precisely, for any household  $i \in \mathcal{N}$  and month  $t \in \mathcal{T}$ , the weighted average price for a given

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<sup>42</sup>Despite that the two maximization problems are observationally equivalent, eliminating the utility function changes the interpretation of the model.



UPC is given by

$$\bar{p}_{i,UPC,t} = \frac{\sum_{trips_i \in t} p_{i,UPC,trips_i} c_{i,UPC,trips_i}}{\sum_{trips_i \in t} c_{i,UPC,trips_i}},$$

where  $trips_i$  denotes a trip of household  $i$ . This aggregation is only computed for UPCs that are purchased by a given household in a given month.

The Homescan has a total of 4,510,908 distinct UPCs, with 1,633,850 that belong to the four departments considered: dry grocery, frozen foods, dairy, and packaged meat. To keep the analysis tractable and mitigate stockpiling issues, I aggregate UPCs to their department categories. For each household-month, the weighted average price for a given department  $l \in \mathcal{L}$  is given by

$$p_{i,l,t} = \frac{\sum_{UPC \in l} \bar{p}_{i,UPC,t} c_{i,UPC,t}}{\sum_{UPC \in l} c_{i,UPC,t}}.$$

Furthermore, I only keep data from April to September. The main reason for limiting the number of goods and time periods is to control the computational burden. Since the number of parameters to solve for in the model is given by  $L \cdot T + T$ , the nonlinear optimization problem becomes quickly intractable when either  $L$  or  $T$  increases.

As the methodology requires the data to be strictly positive, I drop households that do not meet this requirement for any aggregated good and month. These conditions bring down the number of households from 62,092 to 16,025. Further limiting the sample to single households that are at least 50 years old decreases the number of households to 1668. Finally, I drop households that have zero prices paid, thus decreasing the sample size to 1645.<sup>43</sup>

I restrict the sample to single households to avoid the false rejection of the model. As [Adams et al. \(2014\)](#) point out, inconsistencies may arise due to negotiation within a couple household. [Jackson and Yariv \(2015\)](#) further show that time inconsistent behavior will appear if individuals in a non-dictatorial household have different discount factors. By accounting for measurement error in survey data, [Aguilar and Kashaev \(2018\)](#) show that single households behave consistently with exponential discounting while couple households do not.

### A3: Power Analysis

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<sup>43</sup>Zero prices may arise because of “free-good” promotions or if the household enters a price equal to zero and no historical information regarding a valid price for the UPC is available.

In this section, I show that price search and utility maximization are both refutable under Assumptions 1-5. I then provide empirical evidence that these additional restrictions are not necessary for the model to be rejected by the data.

### Convexity of the Log-linear Shopping Technology

Let the price function for any good  $l \in \mathcal{L}$  have the log-linear shopping technology specified by Assumption 1:

$$\log(p_{l,t}(a_{l,t}, \omega_{l,t})) = \alpha_l^0 + \alpha_l^1 \log(a_{l,t}) - \omega_{l,t}.$$

It is easy to see that, for any  $l \in \mathcal{L}$ , the Hessian of the log price function is

$$H(a_{l,t}, \omega_{l,t}) = \begin{bmatrix} -\frac{\alpha_l^1}{a_{l,t}^2} & 0 \\ 0 & 0 \end{bmatrix}.$$

The principal minors are  $D_1 = -\frac{\alpha_l^1}{a_{l,t}^2} \geq 0$ ,  $D_2 = 0$ , and  $D_3 = 0$ . Accordingly, the log price functions are convex and, therefore, the price functions logarithmically convex.<sup>44</sup>

### Falsifiability of Price Search

Suppose that Assumptions 1-5 are satisfied and let  $\mathcal{L} = \{1, 2\}$ ,  $\mathcal{T} = \{1, 2\}$ . Almost surely, let observed prices be such that  $\mathbf{p}_1 = [1, 2]'$ ,  $\mathbf{p}_2 = [3, 4]'$ , shopping intensity be such that  $\mathbf{a}_1 = [1, 2]'$ ,  $\mathbf{a}_2 = [2, 3]'$ , and consumption be such that  $\mathbf{c}_t > 0$  for  $t = 1, 2$ .

Convexity of the log price functions implies that for all  $l \in \mathcal{L}$  and  $s, t \in \mathcal{T}$ , we have

$$\log\left(\frac{p(a_{l,s}, \omega_{l,s})}{p(a_{l,t}, \omega_{l,t})}\right) \geq \frac{\nabla_a p(a_{l,t}, \omega_{l,t})}{p(a_{l,t}, \omega_{l,t})}(a_{l,s} - a_{l,t}) + \frac{\nabla_\omega p(a_{l,t}, \omega_{l,t})}{p(a_{l,t}, \omega_{l,t})}(\omega_{l,s} - \omega_{l,t}).^{45}$$

The above expression can be written more concisely as

$$\log\left(\frac{p_{l,s}^*}{p_{l,t}^*}\right) \geq \frac{\rho_{l,t}}{p_{l,t}^* c_{l,t}}(a_{l,s} - a_{l,t}) - (\omega_{l,s} - \omega_{l,t}) \quad \forall s, t \in \mathcal{T}.$$

<sup>44</sup>A function  $f$  is logarithmically convex if the composition of the logarithm with  $f$  is itself a convex function.

<sup>45</sup>Note that this expression is well-defined since prices are strictly positive.

Summing up these inequalities for each good  $l \in \mathcal{L}$  and dividing by  $L$  gives

$$\frac{1}{L} \sum_{l=1}^L \log \left( \frac{p_{l,s}^*}{p_{l,t}^*} \right) \geq \frac{1}{L} \sum_{l=1}^L \frac{\rho_{l,t}}{p_{l,t}^* c_{l,t}} (a_{l,s} - a_{l,t}) - (\bar{\omega}_s - \bar{\omega}_t) \quad \forall s, t \in \mathcal{T},$$

where  $\bar{\omega}_t := \frac{1}{L} \sum_{l=1}^L \omega_{l,t}$  for all  $t \in \mathcal{T}$ . Taking the expectation for  $s = 1, t = 2$  and using the assumptions that  $\mathbb{E}[\log(\mathbf{p}_t)] = \mathbb{E}[\log(\mathbf{p}_t^*)]$  and  $\mathbb{E}[\bar{\omega}_t] = 0$  for all  $t \in \mathcal{T}$ , we get

$$0 > \frac{1}{L} \sum_{l=1}^L (\mathbb{E}[\log(p_{l,1})] - \mathbb{E}[\log(p_{l,2})]) \geq -\frac{1}{L} \sum_{l=1}^L \mathbb{E} \left[ \frac{\rho_{l,2}}{p_{l,2}^* c_{l,2}} \right]. \quad (5)$$

Noting that the random variables on the right-hand side are always negative, it follows that the negative of their expectations are positive:  $-\mathbb{E} \left[ \frac{\rho_{l,2}}{p_{l,2}^* c_{l,2}} \right] \geq 0$  for all  $l \in \mathcal{L}$ . Clearly, inequality (5) yields a contradiction. In other words, the data are inconsistent with the model provided the price functions are log-linear.

### Falsifiability of Utility Maximization

Suppose that Assumptions 1-5 are satisfied and let  $\mathcal{L} = \{1, 2\}$ ,  $\mathcal{T} = \{1, 2\}$ . Almost surely, let observed prices be such that  $\mathbf{p}_1 = [1, 2]'$ ,  $\mathbf{p}_2 = [3, 4]'$ , shopping intensity be such that  $\mathbf{a}_1 = [2, 3]'$ ,  $\mathbf{a}_2 = [1, 2]'$ , and consumption be such that  $\mathbf{c}_1 = [1, 1]'$ ,  $\mathbf{c}_2 = [2, 2]'$ . Furthermore, suppose that the discount factor is such that  $\delta = 1$  almost surely.

Concavity of the utility function implies that for all  $s, t \in \mathcal{T}$

$$u(\mathbf{c}_s, \mathbf{a}_s) - u(\mathbf{c}_t, \mathbf{a}_t) \leq \nabla_{\mathbf{c}} u(\mathbf{c}_t, \mathbf{a}_t)'(\mathbf{c}_s - \mathbf{c}_t) + \nabla_{\mathbf{a}} u(\mathbf{c}_t, \mathbf{a}_t)'(\mathbf{a}_s - \mathbf{a}_t).$$

Summing up these inequalities for  $s = 1, t = 2$  and  $s = 2, t = 1$ , we can obtain

$$0 \leq [(\mathbf{p}_2^* - \mathbf{p}_1^*)'(\mathbf{c}_1 - \mathbf{c}_2) + (\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1)'(\mathbf{a}_1 - \mathbf{a}_2)],$$

For concavity to be analytically refuted, it is clear that Assumption 2 needs to

be changed to  $\mathbb{E}[\mathbf{p}_t] = \mathbb{E}[\mathbf{p}_t^*]$  for all  $t \in \mathcal{T}$ . Taking the expectation then yields

$$\begin{aligned}
0 &\leq (\mathbb{E}[\mathbf{p}_2] - \mathbb{E}[\mathbf{p}_1])'(\mathbf{c}_1 - \mathbf{c}_2) + (\mathbb{E}[\mathbf{p}_2] - \mathbb{E}[\mathbf{p}_1])'(\mathbf{a}_1 - \mathbf{a}_2) \\
&= -4 + \sum_{l=1}^L (\mathbb{E}[\rho_{l,2}] - \mathbb{E}[\rho_{l,1}]) \\
&\leq -4 - \sum_{l=1}^L \mathbb{E}[\rho_{l,1}] \\
&\leq -4 + \frac{1}{2} + \frac{2}{3} \\
&< 0,
\end{aligned}$$

where the first equality substituted the expected value of true prices for their expected observed values, the second inequality used the assumption that  $\rho_t \leq 0$  for all  $t \in \mathcal{T}$ , and the third inequality exploited the fact that  $\alpha_l \in [-1, 0]$  for all  $l \in \mathcal{L}$ . Indeed, the latter allows us to obtain the support of  $\rho_1$  since  $\rho_{l,1} = \alpha_{l,1} \cdot \frac{p_{l,1}c_{l,1}}{a_{l,1}}$  for all  $l \in \mathcal{L}$ . Picking  $\alpha_{l,1} = -1$  yields the third inequality.

Clearly, the previous inequalities yield a contradiction. As such, utility maximization can be rejected by the data under Assumptions 1-5 provided  $\mathbb{E}[\mathbf{p}_t] = \mathbb{E}[\mathbf{p}_t^*]$  for all  $t \in \mathcal{T}$  (instead of Assumption 2) and  $\delta = 1$  almost surely.

### Falsifiability of the Model: Empirical Evidence

I have shown analytically that the model defined by Assumptions 1-5 can be rejected by the data with only two time periods if either (1) the price functions are log-linear, or (2) the discount factor equals one almost surely and measurement error satisfies  $\mathbb{E}[\mathbf{p}_t] = \mathbb{E}[\mathbf{p}_t^*]$  for all  $t \in \mathcal{T}$ .

To complement the above analysis, I now provide empirical evidence that the model can be rejected by the data under Assumptions 2-5 if the price functions are convex decreasing.<sup>46</sup> This corresponds to the fully nonparametric version of the model. To this end, I consider a data set where  $\mathbf{p}_1 = [1, 2]'$ ,  $\mathbf{p}_2 = [3, 4]'$ ,  $\mathbf{a}_1 = [1, 2]'$ ,  $\mathbf{a}_2 = [2, 3]'$ , and  $\mathbf{c}_1 = [1, 4]'$ ,  $\mathbf{c}_2 = [3, 2]'$ . I let the sample size be 500 where, for simplicity, every consumer is assumed to have the same data set.

The results derived previously do not allow me to conclude that the model has any empirical content without the log-linearity of the price functions. Nevertheless, an application of the methodology to the constructed data set yields a test statistic of 476.98, well-above the chi-square critical value of 12.59.

<sup>46</sup>Formally, a function  $f : \mathbb{R}^L \rightarrow \mathbb{R}$  is convex if and only if  $f(\mathbf{x}) \geq f(\mathbf{y}) + \nabla'_{\mathbf{y}} f(\mathbf{y})(\mathbf{x} - \mathbf{y})$  for all  $\mathbf{x}, \mathbf{y} \in \mathbb{R}^L$ . It is convex decreasing if it is convex and  $\nabla f(\mathbf{y}) \leq 0$  for all  $\mathbf{y}$ .

## A4: Implementation

In this section, I provide a pseudo-algorithm of the ELVIS approach proposed by Schennach (2014) specialized to my model. Furthermore, I provide pseudo-algorithms for the construction of the conditional distribution  $\tilde{\eta}$  and the integration of the latent variables.

### Pseudo-Code

#### Step 1

- Fix the number of goods  $L$  and the number of time periods  $T$ .
- Fix the data set  $x = (x_i)_{i=1}^N$ , where  $x_i = (\mathbf{p}_{i,t}, \mathbf{c}_{i,t}, \mathbf{a}_{i,t})_{t \in T}$ .
- Fix the moments defining the model:  $\mathbf{g}_i^u, \mathbf{g}_i^p, \mathbf{g}_i^m, \mathbf{g}^\omega$ .
- Fix the support of the structural parameters:  $\delta_i \in [\underline{\delta}, 1]$  and  $\alpha_i^1 \in [-1, 0]$ .
- Fix the conditional distribution of the latent variables  $\tilde{\eta}$ .

#### Step 2

for  $i = 1 : N$

- Integrate the latent variables under  $\tilde{\eta}(\cdot|x_i)$  to obtain  $\tilde{\mathbf{h}}_i(x_i, \gamma)$ .

end

- Compute  $\hat{\mathbf{h}}(\gamma) = \frac{1}{N} \sum_{i=1}^N \tilde{\mathbf{h}}_i(x_i, \gamma)$ .
- Compute  $\hat{\mathbf{\Omega}}(\gamma) = \frac{1}{N} \sum_{i=1}^N \tilde{\mathbf{h}}_i(x_i, \gamma) \tilde{\mathbf{h}}_i(x_i, \gamma)' - \hat{\mathbf{h}}(\gamma) \hat{\mathbf{h}}(\gamma)'$ .
- Compute the objective function:  $\text{ObjFct}(\gamma) = N \hat{\mathbf{h}}(\gamma)' \hat{\mathbf{\Omega}}(\gamma) - \hat{\mathbf{h}}(\gamma)$ .

#### Step 3

- Compute  $\text{TS}_N = \min_{\gamma} \text{ObjFct}(\gamma)$ .

#### Step 1 (Construction of $\tilde{\eta}$ )

The distribution  $\tilde{\eta}$  can be taken to be proportional to a normal distribution:

$$d\tilde{\eta}(\cdot|x_i) \propto \exp(-\|\mathbf{g}_i^{m,\omega}(x_i, e_i)\|^2),$$

where  $\mathbf{g}_i^{m,\omega}$  is the set of moments on measurement error and search productivity. The following pseudo-code details how to construct the conditional distribution by using rejection sampling and applying Metropolis-Hastings on each passing draw. I draw true prices instead of measurement error as it ensures true prices to be strictly positive. Let  $R > 0$ .

**while**  $r \leq R$

- Draw candidate latent variables  $e_i^c = (\delta_i, \mathbf{p}_{i,t}^*, \boldsymbol{\alpha}_i^1, \boldsymbol{\omega}_{i,t})_{t \in T}$  such that their support constraints are satisfied.
- Given  $x_i$  and  $e_i^c$ , check whether the model is satisfied by using Theorem 1. If the model is not satisfied, go a step back.
- Draw  $\zeta$  from  $U[0, 1]$
- If  $-\left(\|\mathbf{g}_i^{m,\omega}(x_i, e_i^c)\|^2 - \|\mathbf{g}_i^{m,\omega}(x_i, e_i^{r-1})\|^2\right) > \log(\zeta)$ , set  $e_i^r$  to  $e_i^c$ . Else, set  $e_i^r$  to  $e_i^{r-1}$ .
- Set  $r = r + 1$

**end**

## Step 2 (Latent Variable Integration)

- Fix  $x_i$ ,  $\tilde{\eta}$ , and  $\gamma$ .
- Set  $\tilde{\mathbf{h}}_i(x_i, \gamma) = 0$

**while**  $r \leq R$

- Draw  $e_i^c$  proportional to  $\tilde{\eta}(\cdot|x_i)$ .
- Draw  $\zeta$  from  $U[0, 1]$
- If  $[\mathbf{g}_i^{m,\omega}(x_i, e_i^c) - \mathbf{g}_i^{m,\omega}(x_i, e_i^{r-1})]' \gamma > \log(\zeta)$ , set  $e_i^r$  to  $e_i^c$ . Else, set  $e_i^r$  to  $e_i^{r-1}$ .
- Compute  $\tilde{\mathbf{h}}_i(x_i, \gamma) = \tilde{\mathbf{h}}_i(x_i, \gamma) + \mathbf{g}_i^{m,\omega}(x_i, e_i^r)/R$
- Set  $r = r + 1$

end

## A5: Proofs

To reduce the notational burden, I remove the subscript  $i$  from the variables in the rest of this section.

### Proof of Theorem 1

(i)  $\implies$  (ii)

Suppose the data have been generated by (1) where the utility function is continuous, concave, strictly increasing in consumption and decreasing in shopping intensity. Then, the first-order conditions of the consumer's problem are given by

$$\begin{aligned}\nabla_c u(\mathbf{c}_t, \mathbf{a}_t) &= \lambda_t \delta^{-t} \mathbf{p}_t, \\ \nabla_a u(\mathbf{c}_t, \mathbf{a}_t) &= \lambda_t \delta^{-t} \frac{\partial \mathbf{p}(\mathbf{a}_t, \boldsymbol{\omega}_t)}{\partial \mathbf{a}_t} \odot \mathbf{c}_t.\end{aligned}$$

Since the utility function is assumed strictly increasing in consumption, it must be that  $\nabla_c u(\mathbf{c}, \mathbf{a}) = \lambda_t \delta^{-t} \mathbf{p}_t > 0$ . Accordingly, it follows that  $\mathbf{p}_t > 0$ . Likewise, the assumption that the utility function is decreasing in shopping intensity entails  $\nabla_a u(\mathbf{c}_t, \mathbf{a}_t) = \lambda_t \delta^{-t} \frac{\partial \mathbf{p}(\mathbf{a}_t, \boldsymbol{\omega}_t)}{\partial \mathbf{a}_t} \odot \mathbf{c}_t \leq 0$  and, hence,  $\frac{\partial \mathbf{p}(\mathbf{a}_t, \boldsymbol{\omega}_t)}{\partial \mathbf{a}_t} \odot \mathbf{c}_t \leq 0$ . Finally, concavity of the utility function implies

$$u(\mathbf{c}_s, \mathbf{a}_s) - u(\mathbf{c}_t, \mathbf{a}_t) \leq \nabla_c u(\mathbf{c}_t, \mathbf{a}_t)'(\mathbf{c}_s - \mathbf{c}_t) + \nabla_a u(\mathbf{c}_t, \mathbf{a}_t)'(\mathbf{a}_s - \mathbf{a}_t) \quad \forall s, t \in \mathcal{T}.$$

Combining the first-order conditions with concavity of the utility function and letting  $u_t := u(\mathbf{c}_t, \mathbf{a}_t)$  for all  $t \in \mathcal{T}$  yields

$$u_s - u_t \leq \lambda_t \delta^{-t} \left[ \mathbf{p}_t'(\mathbf{c}_s - \mathbf{c}_t) + \left( \frac{\partial \mathbf{p}(\mathbf{a}_t, \boldsymbol{\omega}_t)}{\partial \mathbf{a}_t} \odot \mathbf{c}_t \right)' (\mathbf{a}_s - \mathbf{a}_t) \right] \quad \forall s, t \in \mathcal{T},$$

where  $\frac{\partial p_l(a_{l,t}, \omega_{l,t})}{\partial a_{l,t}} = \alpha_l^1 a_{l,t}^{\alpha_l^1 - 1} e^{-(\omega_{l,t} - \alpha_l^0)}$  for all  $l \in \mathcal{L}$  due to Assumption 1.

(ii)  $\implies$  (i)

Starting from the first set of inequalities in Theorem 1 (ii), note that

$$\boldsymbol{\rho}_t = \frac{\partial \mathbf{p}(\mathbf{a}_t, \boldsymbol{\omega}_t)}{\partial \mathbf{a}_t} \odot \mathbf{c}_t,$$

such that

$$u_s \leq u_t + \lambda_t \delta^{-t} \left[ \mathbf{p}'_t(\mathbf{c}_s - \mathbf{c}_t) + \left( \frac{\partial \mathbf{p}(\mathbf{a}_t, \boldsymbol{\omega}_t)}{\partial \mathbf{a}_t} \odot \mathbf{c}_t \right)' (\mathbf{a}_s - \mathbf{a}_t) \right] \quad \forall s, t \in \mathcal{T}.$$

Fix some  $t \in \mathcal{T}$  and let  $t_1 := t$ . Consider any sequence of finite indices  $\tau = \{t_i\}_{i=1}^m$ ,  $m \geq 2$ ,  $t_i \in \mathcal{T}$ . Let  $\mathcal{I}$  be the set of all such indices and define

$$u(\mathbf{c}, \mathbf{a}) = \min_{\tau \in \mathcal{I}} \left\{ \lambda_{t_m} \delta^{-t_m} \left[ \mathbf{p}'_{t_m}(\mathbf{c} - \mathbf{c}_{t_m}) + \left( \frac{\partial \mathbf{p}(\mathbf{a}_{t_m}, \boldsymbol{\omega}_{t_m})}{\partial \mathbf{a}_{t_m}} \odot \mathbf{c}_{t_m} \right)' (\mathbf{a} - \mathbf{a}_{t_m}) \right] \right. \\ \left. + \sum_{i=1}^{m-1} \lambda_{t_i} \delta^{-t_i} \left[ \mathbf{p}'_{t_i}(\mathbf{c}_{t_{i+1}} - \mathbf{c}_{t_i}) + \left( \frac{\partial \mathbf{p}(\mathbf{a}_{t_i}, \boldsymbol{\omega}_{t_i})}{\partial \mathbf{a}_{t_i}} \odot \mathbf{c}_{t_i} \right)' (\mathbf{a}_{t_{i+1}} - \mathbf{a}_{t_i}) \right] \right\}.$$

This function is the pointwise minimum of a collection of linear functions in  $(\mathbf{c}, \mathbf{a})$ . As such,  $u(\mathbf{c}, \mathbf{a})$  is concave and continuous. Moreover, the second set of inequalities in Theorem 1 (ii) guarantees that the utility function is strictly increasing in consumption. Likewise, the third set of inequalities implies that it is decreasing in shopping intensity. Finally, note that  $\alpha_l^0$ ,  $\alpha_l^1$ , and  $(\omega_{l,t})_{t \in \mathcal{T}}$  directly identify the log-linear price function for good  $l \in \mathcal{L}$ .

If the budget sets  $\{\mathcal{B}_t\}_{t=1}^T$  are convex, then the first-order conditions of the model are necessary and sufficient for a maximum. Therefore, I am left to show that  $\lambda_t \delta^{-t} \mathbf{p}_t \in \nabla_{\mathbf{c}} u(\mathbf{c}_t, \mathbf{a}_t)$  and  $\lambda_t \delta^{-t} \frac{\partial \mathbf{p}(\mathbf{a}_t, \boldsymbol{\omega}_t)}{\partial \mathbf{a}_t} \odot \mathbf{c}_t \in \nabla_{\mathbf{a}} u(\mathbf{c}_t, \mathbf{a}_t)$  for all  $t \in \mathcal{T}$ .

Let  $\epsilon > 0$ ,  $t \in \mathcal{T}$ , and note that by definition of  $u(\cdot, \cdot)$ , there is some sequence of indices  $\tau \in \mathcal{I}$  such that

$$u(\mathbf{c}_t, \mathbf{a}_t) + \epsilon > \lambda_{t_m} \delta^{-t_m} \left[ \mathbf{p}'_{t_m}(\mathbf{c}_t - \mathbf{c}_{t_m}) + \left( \frac{\partial \mathbf{p}(\mathbf{a}_{t_m}, \boldsymbol{\omega}_{t_m})}{\partial \mathbf{a}_{t_m}} \odot \mathbf{c}_{t_m} \right)' (\mathbf{a}_t - \mathbf{a}_{t_m}) \right] \\ + \sum_{i=1}^{m-1} \lambda_{t_i} \delta^{-t_i} \left[ \mathbf{p}'_{t_i}(\mathbf{c}_{t_{i+1}} - \mathbf{c}_{t_i}) + \left( \frac{\partial \mathbf{p}(\mathbf{a}_{t_i}, \boldsymbol{\omega}_{t_i})}{\partial \mathbf{a}_{t_i}} \odot \mathbf{c}_{t_i} \right)' (\mathbf{a}_{t_{i+1}} - \mathbf{a}_{t_i}) \right] \\ \geq u(\mathbf{c}_t, \mathbf{a}_t).$$

Add any bundle  $(\mathbf{c}, \mathbf{a}) \in C \times A$  to the sequence and use the definition of



$u(\cdot, \cdot)$  once again to obtain

$$\begin{aligned}
& \lambda_{t_m} \delta^{-t_m} \left[ \mathbf{p}'_{t_m}(\mathbf{c}_t - \mathbf{c}_{t_m}) + \left( \frac{\partial \mathbf{p}(\mathbf{a}_{t_m}, \boldsymbol{\omega}_{t_m})}{\partial \mathbf{a}_{t_m}} \odot \mathbf{c}_{t_m} \right)' (\mathbf{a}_t - \mathbf{a}_{t_m}) \right] \\
& + \sum_{i=1}^{m-1} \lambda_{t_i} \delta^{-t_i} \left[ \mathbf{p}'_{t_i}(\mathbf{c}_{t_{i+1}} - \mathbf{c}_{t_i}) + \left( \frac{\partial \mathbf{p}(\mathbf{a}_{t_i}, \boldsymbol{\omega}_{t_i})}{\partial \mathbf{a}_{t_i}} \odot \mathbf{c}_{t_i} \right)' (\mathbf{a}_{t_{i+1}} - \mathbf{a}_{t_i}) \right] \\
& + \lambda_t \delta^{-t} \left[ \mathbf{p}'_t(\mathbf{c} - \mathbf{c}_t) + \left( \frac{\partial \mathbf{p}(\mathbf{a}_t, \boldsymbol{\omega}_t)}{\partial \mathbf{a}_t} \odot \mathbf{c}_t \right)' (\mathbf{a} - \mathbf{a}_t) \right] \\
& \geq u(\mathbf{c}, \mathbf{a}).
\end{aligned}$$

Hence,

$$u(\mathbf{c}_t, \mathbf{a}_t) + \epsilon + \lambda_t \delta^{-t} \left[ \mathbf{p}'_t(\mathbf{c} - \mathbf{c}_t) + \left( \frac{\partial \mathbf{p}(\mathbf{a}_t, \boldsymbol{\omega}_t)}{\partial \mathbf{a}_t} \odot \mathbf{c}_t \right)' (\mathbf{a} - \mathbf{a}_t) \right] > u(\mathbf{c}, \mathbf{a}).$$

Since  $\epsilon > 0$ ,  $t \in \mathcal{T}$  and  $(\mathbf{c}, \mathbf{a})$  were arbitrary, we get

$$u(\mathbf{c}_t, \mathbf{a}_t) + \lambda_t \delta^{-t} \left[ \mathbf{p}'_t(\mathbf{c} - \mathbf{c}_t) + \left( \frac{\partial \mathbf{p}(\mathbf{a}_t, \boldsymbol{\omega}_t)}{\partial \mathbf{a}_t} \odot \mathbf{c}_t \right)' (\mathbf{a} - \mathbf{a}_t) \right] \geq u(\mathbf{c}, \mathbf{a}).$$

This corresponds to the definition of concavity and, therefore, it must be that  $\lambda_t \delta^{-t} \mathbf{p}_t$  and  $\lambda_t \delta^{-t} \frac{\partial \mathbf{p}(\mathbf{a}_t, \boldsymbol{\omega}_t)}{\partial \mathbf{a}_t} \odot \mathbf{c}_t$  are supergradients of  $u(\mathbf{c}_t, \mathbf{a}_t)$ . Next, I show that we can construct a utility function that guarantees the solution to exist.

Let  $\Gamma := \max_{l \in \mathcal{L}, t \in \mathcal{T}} \{a_{l,t}\}$ . For every  $l \in \mathcal{L}$ , let  $h_l(\cdot)$  be a continuously differentiable function satisfying  $h_l(0) = 0$ ,  $h'_l(x) > 0$ ,  $h''_l(x) \geq 0$  for  $x \in \mathbb{R}_+$  and  $\lim_{x \rightarrow \infty} h'_l(x) = \infty$ .<sup>47</sup> To see that there exists a utility function such that a solution exists, define  $\hat{u}(\mathbf{c}, \mathbf{a}) := u(\mathbf{c}, \mathbf{a}) - \sum_{l=1}^L h_l(\max\{0, a_l - \Gamma\})$ . As before, this function is concave, continuous, strictly increasing in consumption, and decreasing in shopping intensity. Furthermore, note that  $\hat{u}(\mathbf{c}, \mathbf{a}) \leq u(\mathbf{c}, \mathbf{a})$  for all  $(\mathbf{c}, \mathbf{a}) \in C \times A$  and  $\hat{u}(\mathbf{c}_t, \mathbf{a}_t) = u(\mathbf{c}_t, \mathbf{a}_t)$  for all  $t \in \mathcal{T}$ . Thus,  $(\mathbf{c}_t, \mathbf{a}_t)_{t \in \mathcal{T}}$  is still a solution to the consumer problem. Finally, note that  $\hat{u}(\mathbf{c}, \mathbf{a}) \rightarrow -\infty$  whenever  $\mathbf{a} \rightarrow \infty$  along some dimension. This follows from the piecewise linearity of  $u(\cdot, \cdot)$  and the assumption that  $\lim_{x \rightarrow \infty} h'_l(x) = \infty$ .

### Proof of Proposition 1

Assumption 1 states that the price function for any good  $l \in \mathcal{L}$  is given by:

$$\log(p_{l,t}^*) = \alpha_l^0 + \alpha_l^1 \log(a_{l,t}) - \omega_{l,t}.$$

<sup>47</sup>This construction is analogous to that of Deb et al. (2018).

Due to measurement error in prices, we only get to make inference from

$$\log(p_{l,t}) = \alpha_l^0 + \alpha_l^1 \log(a_{l,t}) - \omega_{l,t}.$$

Summing this equation across goods and dividing by  $L$  yields

$$\overline{\log(p_{l,t})} = \frac{1}{L} \sum_{l=1}^L [\alpha_l^0 + \alpha_l^1 \log(a_{l,t})] - \bar{\omega}_{l,t},$$

where  $\overline{\log(p_{l,t})}$  denotes the average log price paid and  $\bar{\omega}_{l,t}$  denotes the average search productivity. Further taking the expectation simplifies the equation to

$$\mathbb{E} [\overline{\log(p_{l,t})}] = \frac{1}{L} \sum_{l=1}^L (\mathbb{E} [\alpha_l^0] + \mathbb{E} [\alpha_l^1 \log(a_{l,t})]),$$

where Assumption 4 was used to eliminate the expected average search productivity. By Assumption 2, the above can be written as

$$\mathbb{E} [\overline{\log(p_{l,t}^*)}] = \frac{1}{L} \sum_{l=1}^L (\mathbb{E} [\alpha_l^0] + \mathbb{E} [\alpha_l^1 \log(a_{l,t})]).$$

Taking the derivative with respect to  $\log(a_{l,t})$  and invoking Leibniz integration rule, one gets

$$\mathbb{E} \left[ \frac{\partial \log(p_{l,t}^*)}{\partial \log(a_{l,t})} \right] = \frac{1}{L} \mathbb{E} [\alpha_l^1].$$

Finally, summing this equation for each good  $l \in \mathcal{L}$  and dividing by  $L$  gives

$$\frac{1}{L} \sum_{l=1}^L \mathbb{E} \left[ \frac{\partial \log(p_{l,t}^*)}{\partial \log(a_{l,t})} \right] = \frac{1}{L} \mathbb{E} [\bar{\alpha}^1],$$

where  $\bar{\alpha}^1 := \frac{1}{L} \sum_{l=1}^L \alpha_l^1$  is the average shopping technology across goods.

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