Math 105 Project

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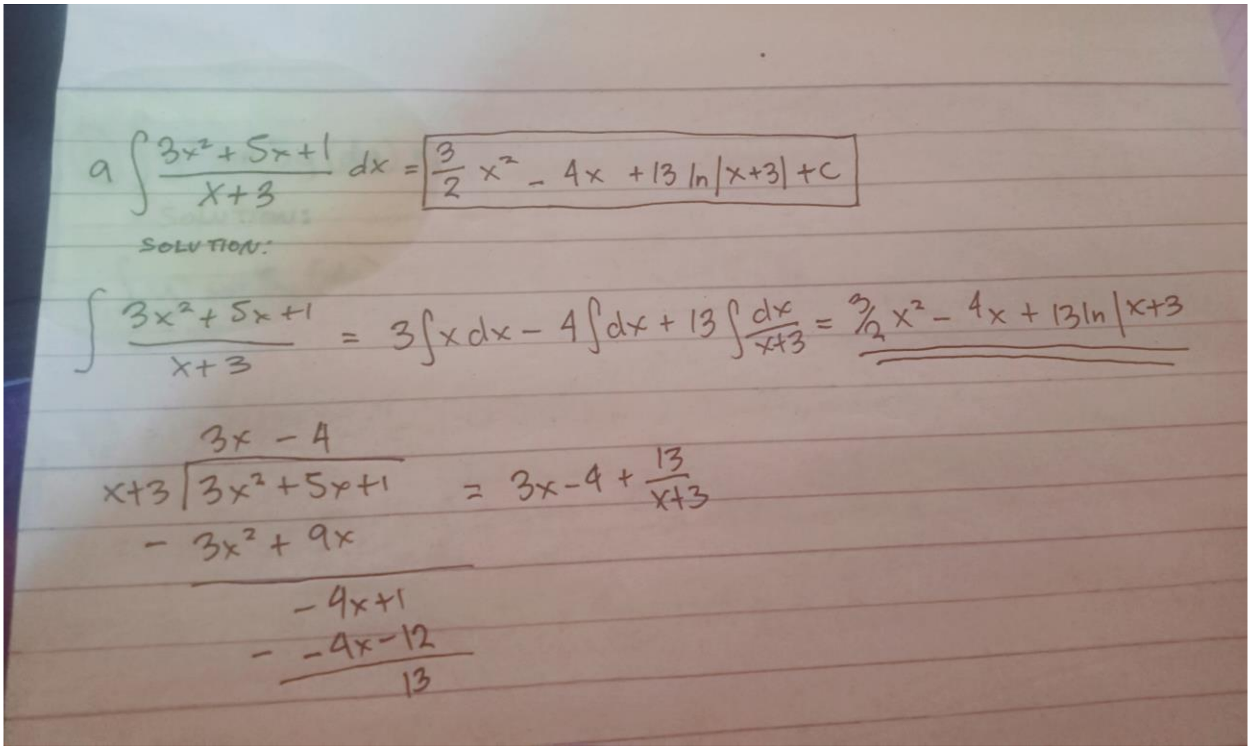
A.

Gottfried Wilhelm (von) Leibniz was a German polymath with great knowledge in mathematics, philosophy and science. He believes that all of human reasoning can be lessen into calculations of a sort, and that these calculations could resolve the dissimilarity of opinions. Born in July 1, 1646 and died on 14th of November 1716 at the age of 70. His contributions on many subjects or fields are scattered in a large number of journals which accounted to tens of thousands of letters and unpublished manuscripts. Furthermore, Leibniz is now regarded as the inventor and contributor of calculus, providing a clear set of rules for the solution in infinitesimal quantities.

The calculus ratiocinator a concept that is made by Leibniz which is similar to symbolic logic, is a way of making such calculations more practical. Leibniz wrote many letters in an attempt to establish calculus, but was remained unpublished until the selection that is edited by Carl Immanuel Gerhardt (1859) appeared. His skill and vast knowledge in mathematics can be seen in his notation for calculus, where he thought that symbols are significant for human understanding. However, Leibniz was accused of plagiarism by Isaac Newton but after careful examinations of their works it was concluded that both mathematicians arrived on their results independently with Leibniz starting with integration and Newton starting on differentiation.

B.

1.Indefinite Integral

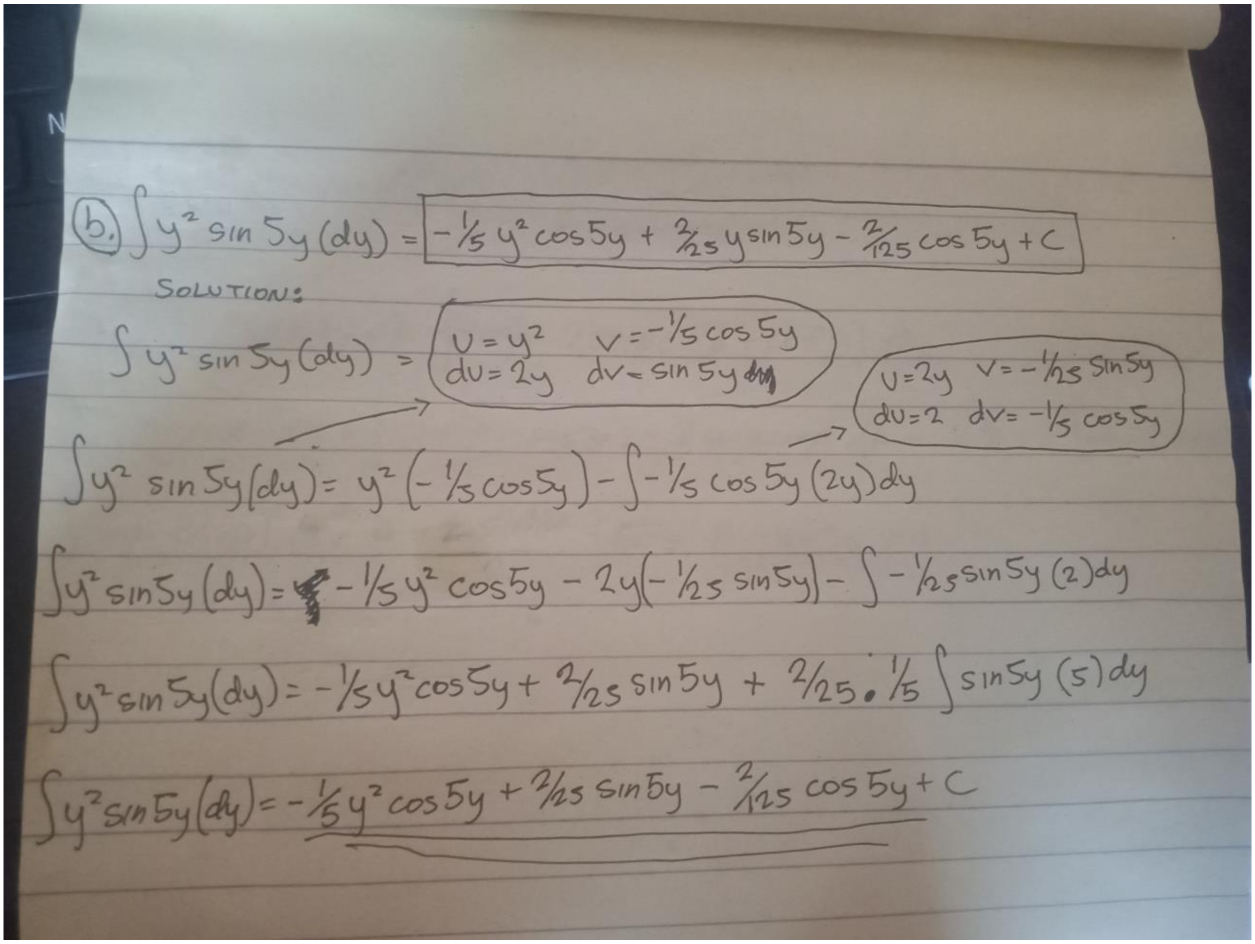


a $\int \frac{3x^2+5x+1}{x+3}\,dx = \boxed{\frac{3}{2}x^2 - 4x + 13\ln|x+3| + C}$

SOLUTION:

$$\int \frac{3x^2+5x+1}{x+3} = 3\int x\,dx - 4\int dx + 13\int \frac{dx}{x+3} = \underline{\underline{3/2\,x^2 - 4x + 13\ln|x+3}}$$

$$\begin{array}{r} 3x-4 \\ x+3\,\overline{)\,3x^2+5x+1} \\ -\ 3x^2+9x \\ \hline -4x+1 \\ -\ -4x-12 \\ \hline 13 \end{array} = 3x-4+\frac{13}{x+3}$$



ⓑ $\int y^2 \sin 5y\,(dy) = \boxed{-\frac{1}{5}y^2\cos 5y + \frac{2}{25}y\sin 5y - \frac{2}{125}\cos 5y + C}$

SOLUTION:

$\int y^2 \sin 5y\,(dy) =$

$u = y^2 \quad v = -\frac{1}{5}\cos 5y$

$du = 2y \quad dv = \sin 5y$

$u = 2y \quad v = -\frac{1}{25}\sin 5y$

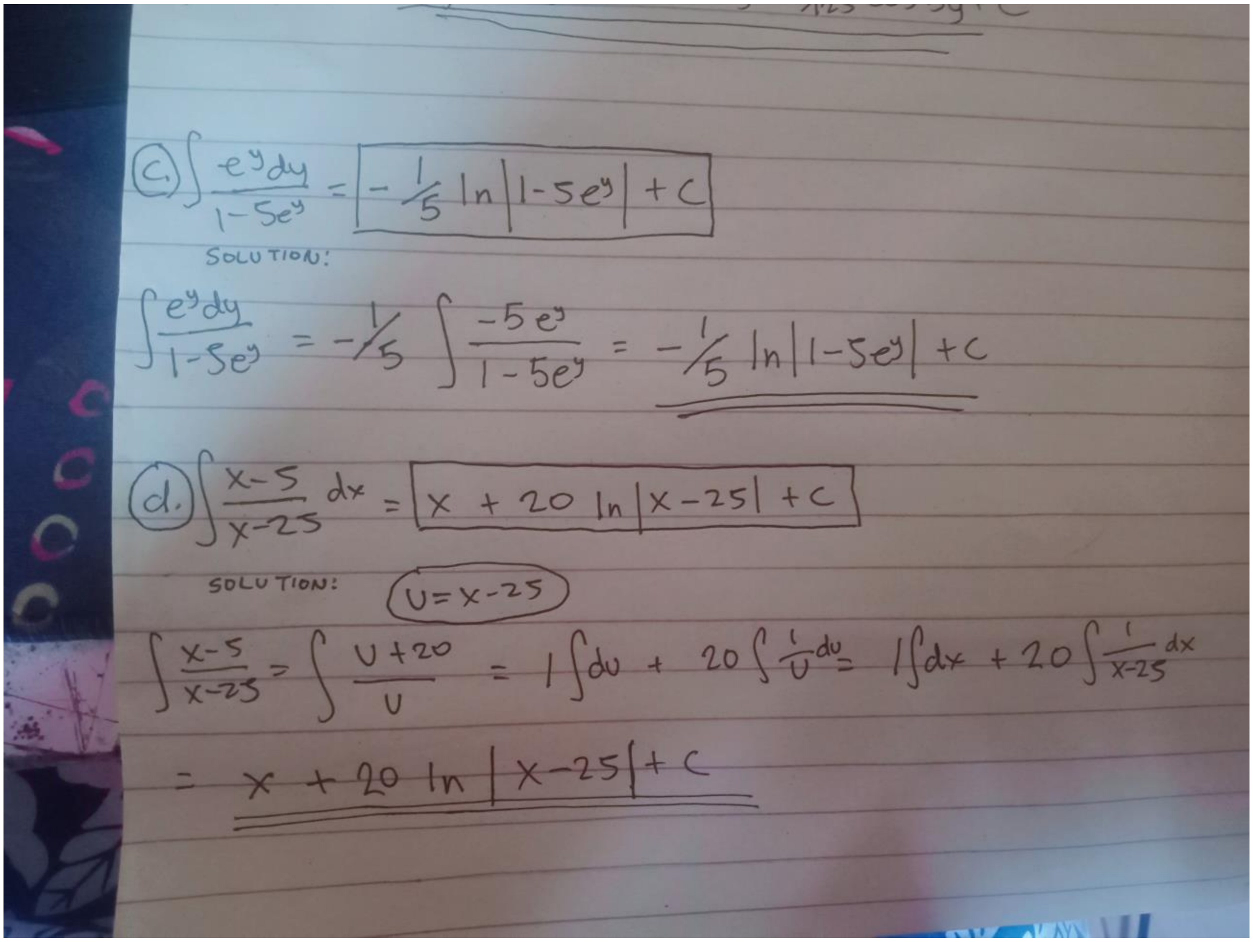
$du = 2 \quad dv = -\frac{1}{5}\cos 5y$

$$\int y^2 \sin 5y\,(dy) = y^2\left(-\frac{1}{5}\cos 5y\right) - \int -\frac{1}{5}\cos 5y\,(2y)\,dy$$

$$\int y^2 \sin 5y\,(dy) = -\frac{1}{5}y^2\cos 5y - 2y\left(-\frac{1}{25}\sin 5y\right) - \int -\frac{1}{25}\sin 5y\,(2)\,dy$$

$$\int y^2 \sin 5y\,(dy) = -\frac{1}{5}y^2\cos 5y + \frac{2}{25}\sin 5y + \frac{2}{25}\cdot\frac{1}{5}\int \sin 5y\,(5)\,dy$$

$$\int y^2 \sin 5y\,(dy) = \underline{\underline{-\frac{1}{5}y^2\cos 5y + \frac{2}{25}\sin 5y - \frac{2}{125}\cos 5y + C}}$$



(c.) $\int \frac{e^y dy}{1-5e^y} = \boxed{-\frac{1}{5}\ln|1-5e^y| + c}$

SOLUTION:

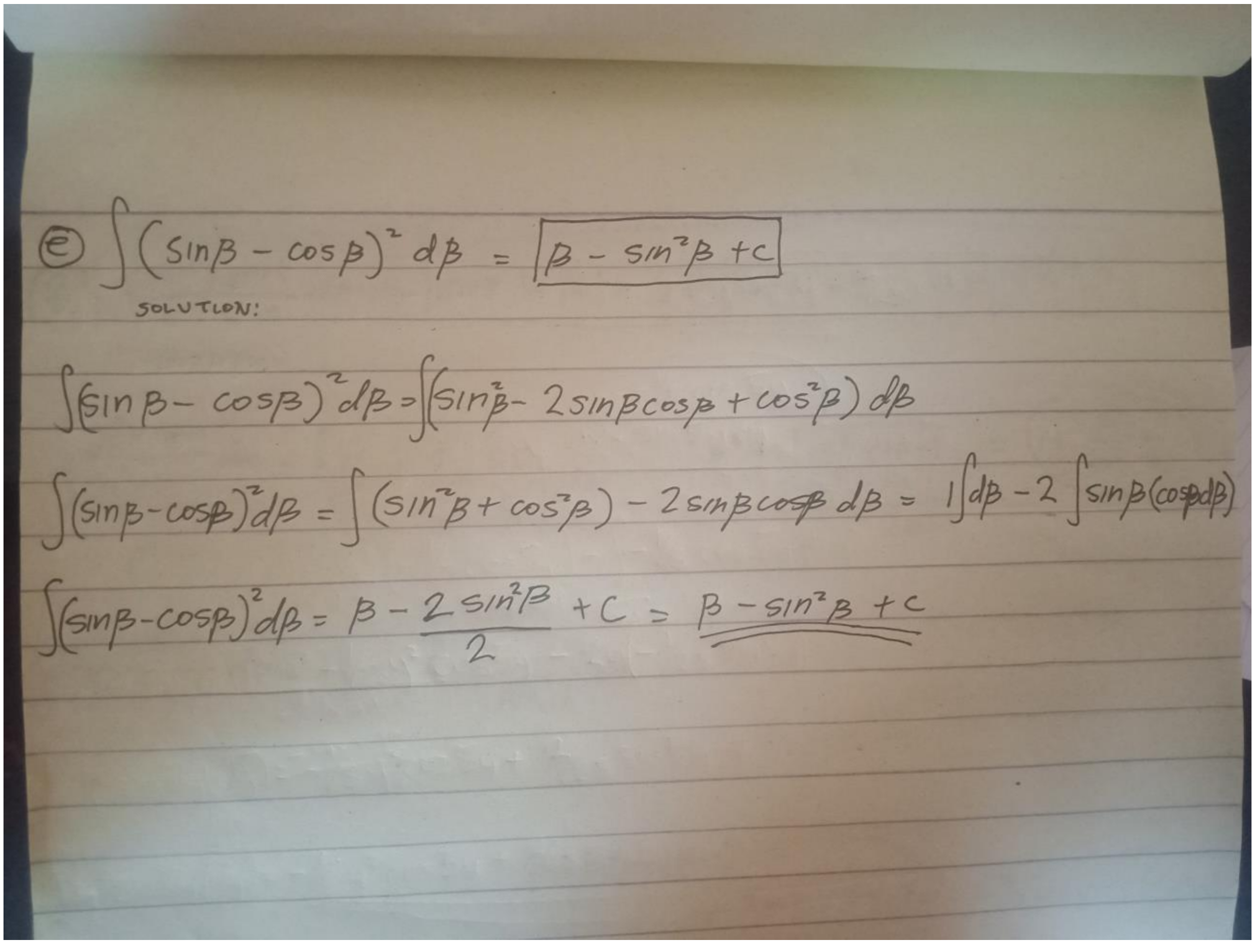
$$\int \frac{e^y dy}{1-5e^y} = -\frac{1}{5}\int \frac{-5e^y}{1-5e^y} = -\frac{1}{5}\ln|1-5e^y| + c$$

(d.) $\int \frac{x-5}{x-25}dx = \boxed{x + 20\ln|x-25| + c}$

SOLUTION: $U = x-25$

$$\int \frac{x-5}{x-25} = \int \frac{U+20}{U} = 1\int du + 20\int \frac{1}{U}du = 1\int dx + 20\int \frac{1}{x-25}dx$$

$$= x + 20\ln|x-25| + c$$



ⓔ $\int (\sin\beta - \cos\beta)^2 \, d\beta = \boxed{\beta - \sin^2\beta + c}$

SOLUTION:

$$\int (\sin\beta - \cos\beta)^2 \, d\beta = \int (\sin^2\beta - 2\sin\beta\cos\beta + \cos^2\beta) \, d\beta$$

$$\int (\sin\beta - \cos\beta)^2 \, d\beta = \int (\sin^2\beta + \cos^2\beta) - 2\sin\beta\cos\beta \, d\beta = 1\int d\beta - 2\int \sin\beta(\cos\beta \, d\beta)$$

$$\int (\sin\beta - \cos\beta)^2 \, d\beta = \beta - \frac{2\sin^2\beta}{2} + C = \underline{\underline{\beta - \sin^2\beta + c}}$$

(f.) $\int \frac{m^3+m-1}{m^3-m}\,dm = \boxed{m + \ln|m| + \tfrac{1}{2}\ln|2m-2| - \tfrac{3}{2}\ln|2m+2| + C}$

SOLUTION

$$\int \frac{m^3+m-1}{m^3-m}\,dm = \int\left(1 + \frac{2m-1}{m^3-m}\right)dm$$

$$\begin{array}{r} 1 \\ m^3-m\,\overline{\big)\,m^3+m-1} \\ \underline{m^3-m} \\ 2m-1 \end{array} = 1 + \frac{2m-1}{m^3-m}$$

$$\int \frac{m^3+m-1}{m^3-m}\,dm = 1\int dm + \int \frac{2m-1}{m^3-m}\,dm$$

$$1\int dm = m$$

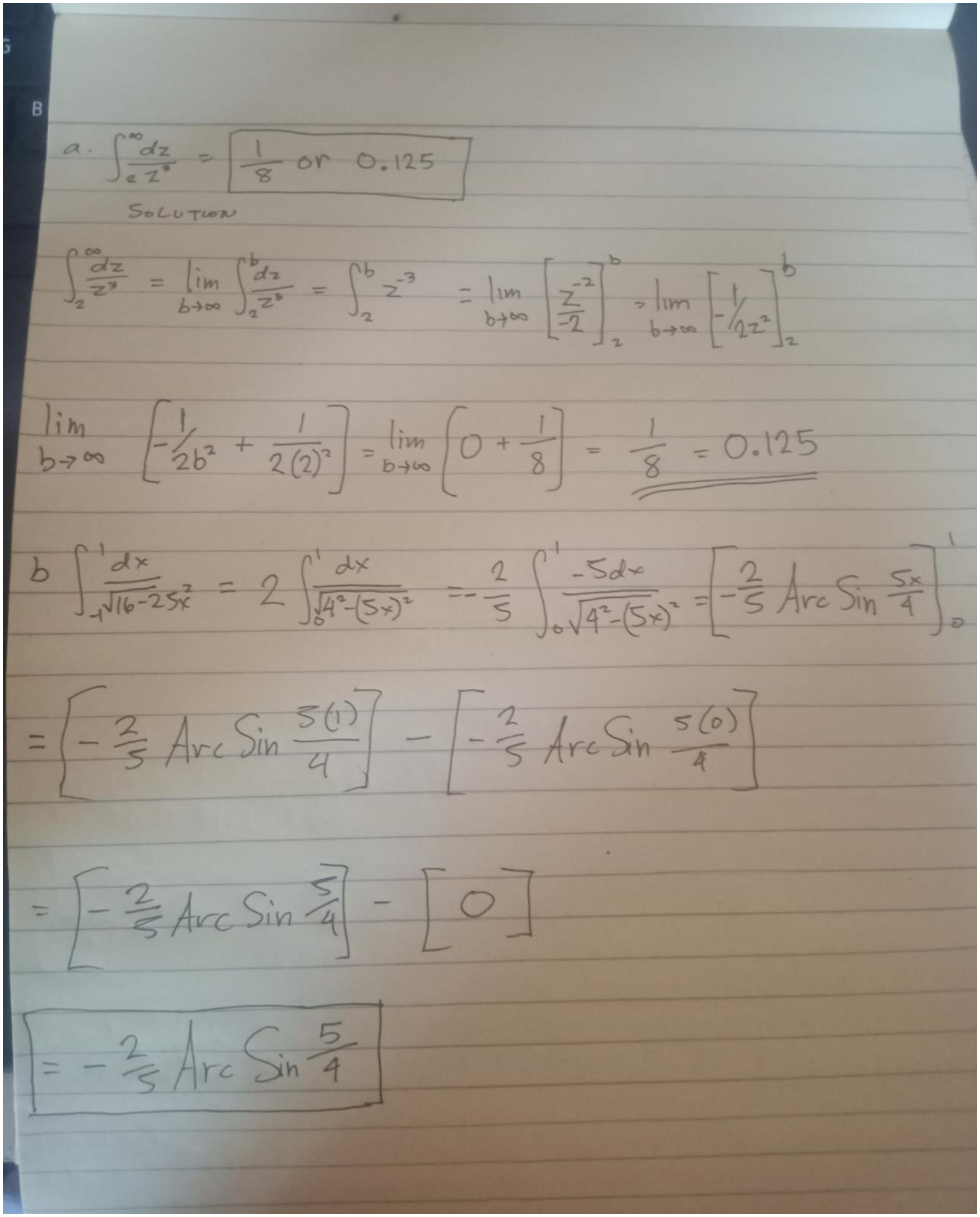
$$\int\left[\frac{2m-1}{m^3-m} = \frac{A}{m} + \frac{B}{m-1} + \frac{C}{m+1}\right]^{m(m-1)(m+1)} = 2m-1 = A(m^2-1) + B(m^2+m) + C(m^2-m)$$

$m=1$	$m=-1$	$m=0$
$2(1)-1 = A(1^2-1)+B(1^2+1)+C(1^2-1)$	$2(-1)-1 = A(-1^2-1)+B(-1^2-1)+C(-1^2+1)$	$2(0)-1 = A(0-1)+B(0)+C(0)$
$1 = A(0)+B(2)+C(0)$	$-3 = A(0)+B(0)+C(2)$	$-1 = -A$
$\frac{1}{2} = \frac{2B}{2}$	$\frac{-3}{2} = \frac{2C}{2}$	$\frac{-1}{-1} = \frac{-A}{-1}$
$\frac{1}{2} = B$	$-\frac{3}{2} = C$	$1 = A$

$$\int \frac{2m-1}{m^3-m} = \int \frac{1}{m}dm + \frac{1}{2}\int \frac{2dm}{2m-2} - \frac{3}{2}\int \frac{2dm}{2m-2} = \ln|m| + \frac{1}{2}\ln|2m-2| - \frac{3}{2}\ln|2m-2|$$

$$\int \frac{m^3+m-1}{m^3-m}\,dm = 1\int dm + \int \frac{2m-1}{m^3-m} = \underline{\underline{m + \ln|m| + \tfrac{1}{2}\ln|2m-2| - \tfrac{3}{2}\ln|2m-2| + C}}$$

2.Definite Integral



a. $\int_{2}^{\infty} \frac{dz}{z^3} = \boxed{\frac{1}{8} \text{ or } 0.125}$

SOLUTION

$$\int_{2}^{\infty} \frac{dz}{z^3} = \lim_{b\to\infty} \int_{2}^{b} \frac{dz}{z^3} = \int_{2}^{b} z^{-3} = \lim_{b\to\infty} \left[\frac{z^{-2}}{-2}\right]_{2}^{b} = \lim_{b\to\infty} \left[-\frac{1}{2z^2}\right]_{2}^{b}$$

$$\lim_{b\to\infty} \left[-\frac{1}{2b^2} + \frac{1}{2(2)^2}\right] = \lim_{b\to\infty} \left[0 + \frac{1}{8}\right] = \underline{\underline{\frac{1}{8} = 0.125}}$$

b $\int_{-1}^{1} \frac{dx}{\sqrt{16-25x^2}} = 2\int_{0}^{1} \frac{dx}{\sqrt{4^2-(5x)^2}} = -\frac{2}{5} \int_{0}^{1} \frac{-5dx}{\sqrt{4^2-(5x)^2}} = \left[-\frac{2}{5} \text{Arc Sin} \frac{5x}{4}\right]_{0}^{1}$

$$= \left[-\frac{2}{5} \text{Arc Sin} \frac{5(1)}{4}\right] - \left[-\frac{2}{5} \text{Arc Sin} \frac{5(0)}{4}\right]$$

$$= \left[-\frac{2}{5} \text{Arc Sin} \frac{5}{4}\right] - [0]$$

$$\boxed{= -\frac{2}{5} \text{Arc Sin} \frac{5}{4}}$$