Test 1

1.) $\int \frac{2}{3}\left(\sin 3x + \cos 3x\right) dx =$

$$= \frac{2}{3}\int\left(\sin 3x + \cos 3x\right) dx$$

$$= \frac{2}{3}\left(\int \sin 3x\, dx + \int \cos 3x\, dx\right)$$

$$= \frac{2}{3}\left(\frac{1}{3}\int \sin 3x\,(3dx) + \frac{1}{3}\int \cos 3x\,(3dx)\right)$$

$$= \frac{2}{3}\left(-\frac{1}{3}\cos 3x + \frac{1}{3}\sin 3x\right) + C$$

$$\boxed{= -\frac{2}{9}\cos 3x + \frac{2}{9}\sin 3x + C}$$

2. $\int \frac{dy}{16-9y^2} = \int \frac{dy}{4^2-(3y)^2} = \frac{1}{3}\int \frac{3dy}{4^2-(3y)^2} = \frac{1}{3}\left[\frac{1}{4}\text{ArcTan}\,\frac{3y}{4}\right] + C$

$$\boxed{\frac{1}{12}\text{ArcTan}\,\frac{3y}{4} + C}$$

3. $\int (x^3+5)\ln x \, dx$

$u = \ln x \quad du = 1/x$

$dv = x^3+5 \quad v = \frac{x^4}{4} + 5x$

$= \ln x \left(\frac{\cancel{x}^4}{4} + 5x\right) - \int \left(\frac{x^4}{4} + 5x\right) \frac{1}{x} \, dx$

$= \ln x \left(\frac{x^4}{4} + 5x\right) - \int \left(\frac{x^4}{4x} + \frac{5x}{x}\right) (dx)$

$= \ln x \left(\frac{x^4}{4} + 5x\right) - \int \left(\frac{x^3}{4} + 5\right) dx$

$= \ln x \left(\frac{x^4}{4} + 5x\right) - \frac{1}{4} \int x^3 dx + \int 5 dx$

$= \ln x \left(\frac{x^4}{4} + 5x\right) - \left(\frac{1}{16} x^4 + 5x\right) + C$

Test II

The item I've chosen is item number 2. $\int \frac{dy}{16+9y^2}$

The denominator of the integrant are both perfect square so I rewrote it to their square root squared. After that I see that the U which is 3y needs to have a derivative at the top which is 3dy so I put the 3 on the numerator and insert the reciprocal (1/3). Now that the integrant is satisfied I rewrote the fraction to the formula 1/a ArcTan u/a + C , finally I simplified it by multiplying the reciprocal (1/3) to the answer.