Swiss TPH

Lesson 6

How to solve mathematical models

Cameline Orlendo

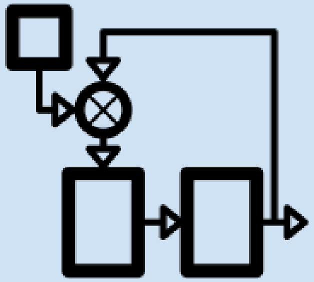
Learning outcomes

By the end of this lesson, you will be able to:

- **Construct** the basic SIS diagram; define compartments (S, I, S), transitions, and rates and build the differential equations step by step.
- **Define** equilibrium (disease-free and endemic).
- **Know how to calculat**e R0, and **how to define the threshold**

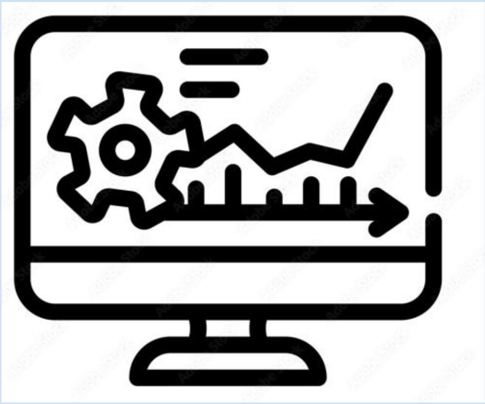
Recap: Modeling = a process where people from different worlds meet on a common ground

***biologist/epidemiologist/entomologists**:*
translate biological and population-level observations into flow diagrams

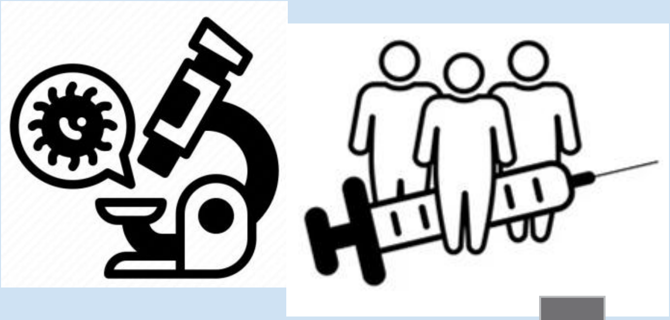


mathematician:
translates flow diagrams into equations

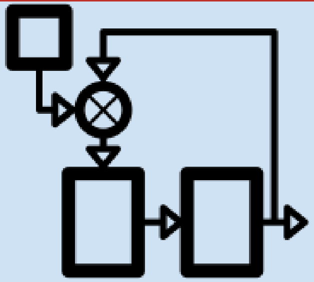
software engineer/computer science:
translates equations into computer code



Modeling = a process where people from different worlds meet on a common ground

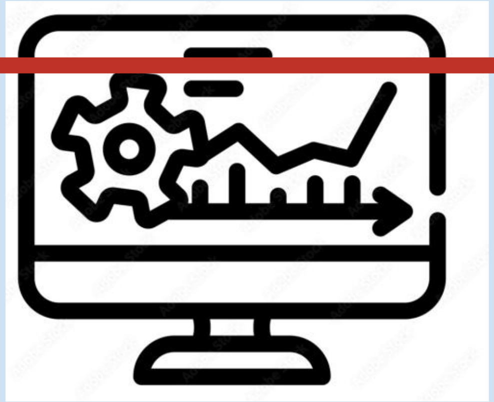


biologist/epidemiologist/entomologists: *translate biological and population-level observations into flow diagrams*

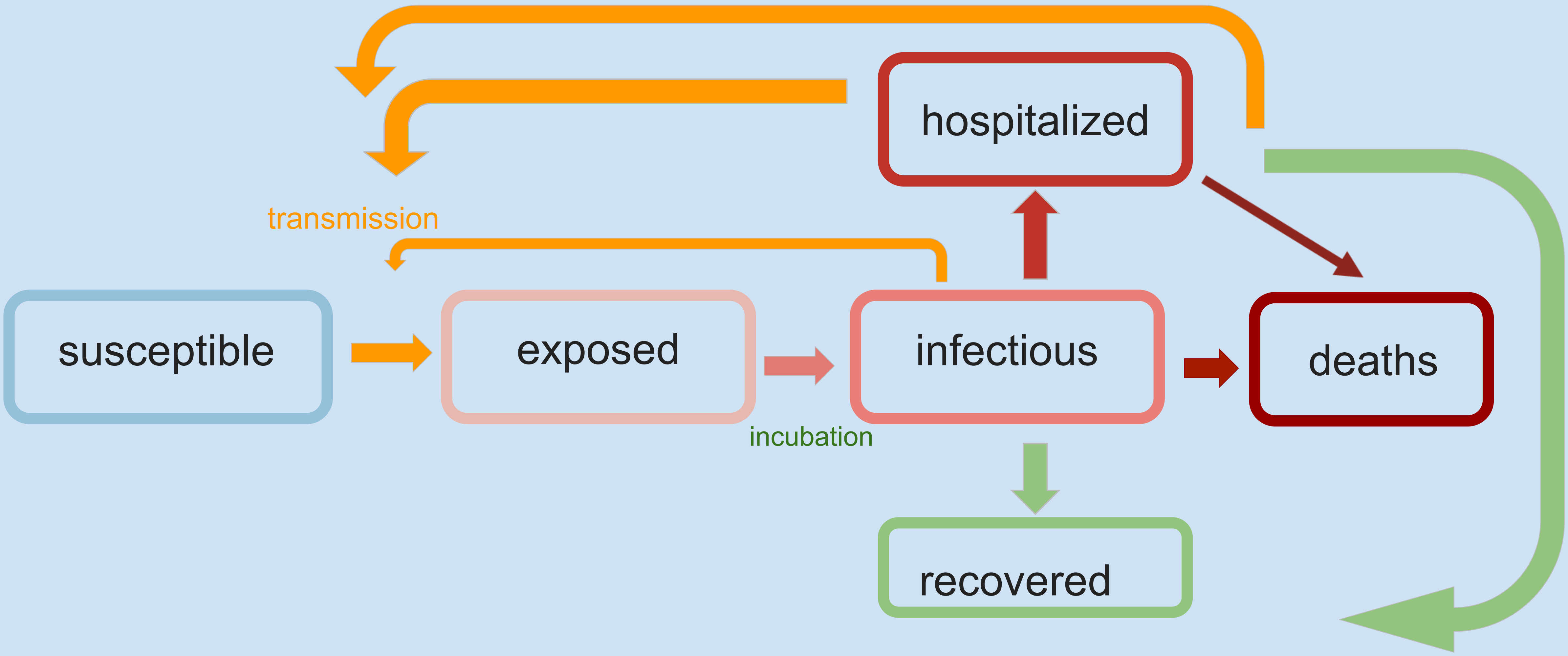


mathematician:
translates flow diagrams into equations

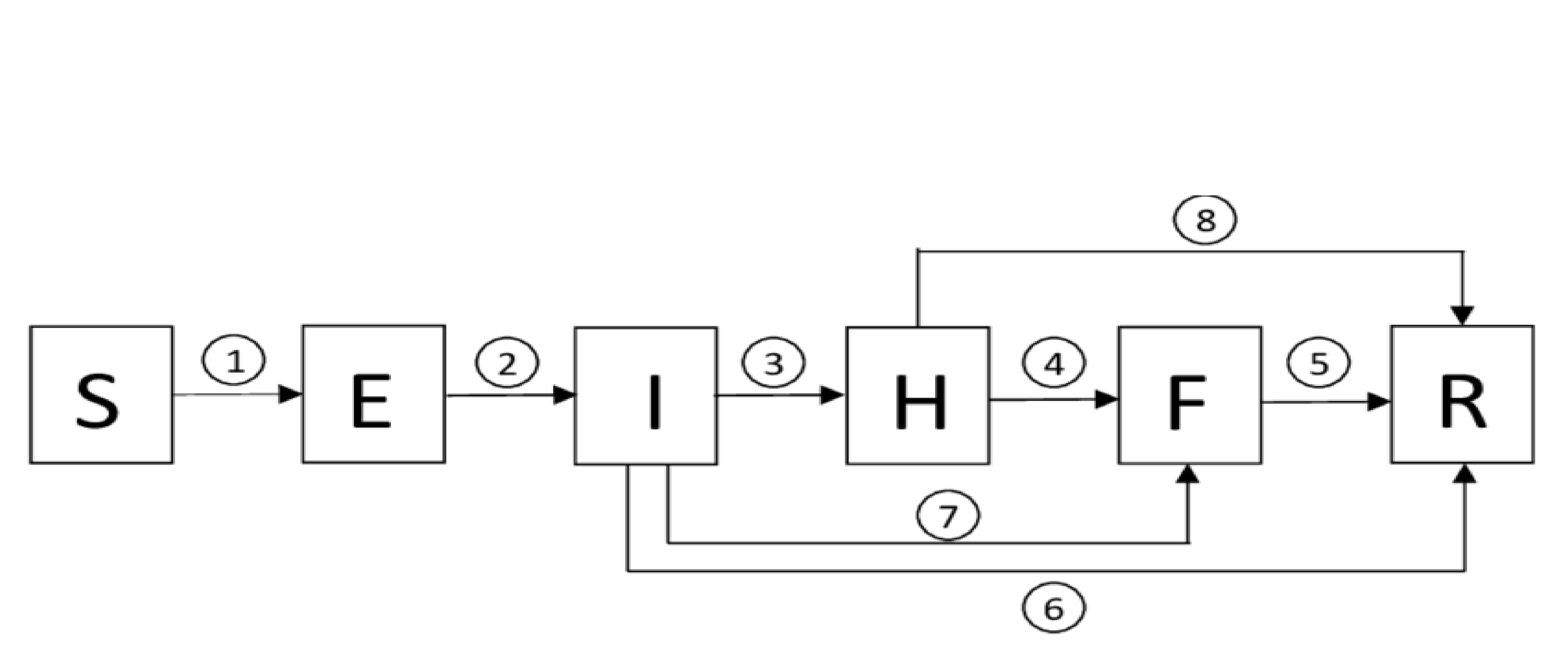
software engineer/computer science:
translates equations into computer code



Recap: Ebola paper

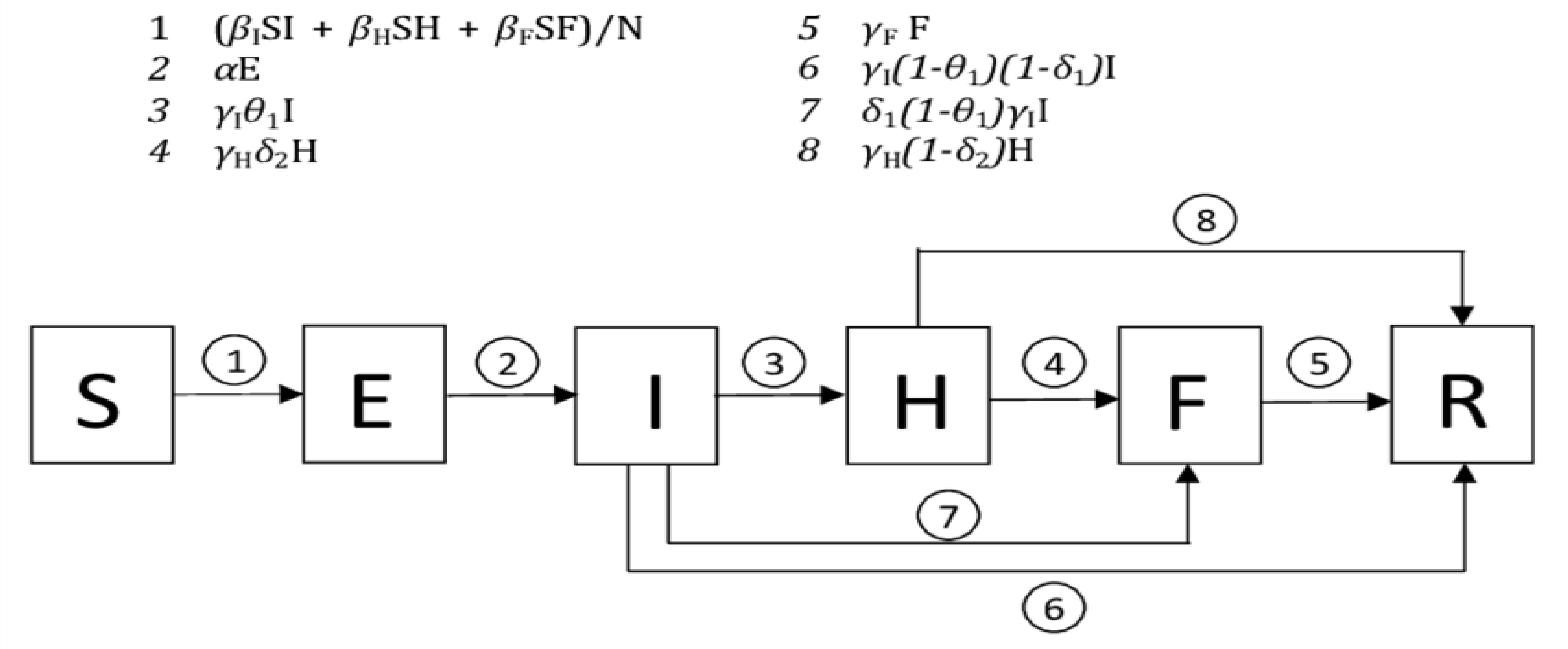


Recap: Ebola flow diagram



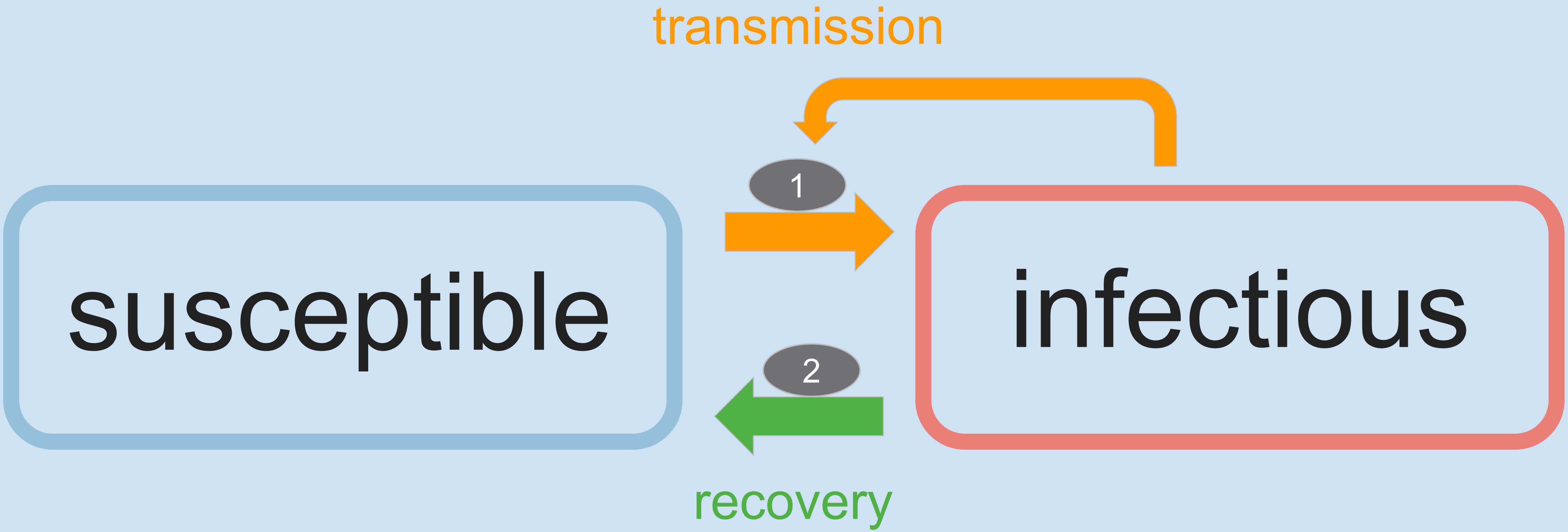
Hernandez-Suarez, C., & Lopez, O. M. (2022). https://doi.org/10.48550/arXiv.2208.11509

Recap: Ebola flow diagram



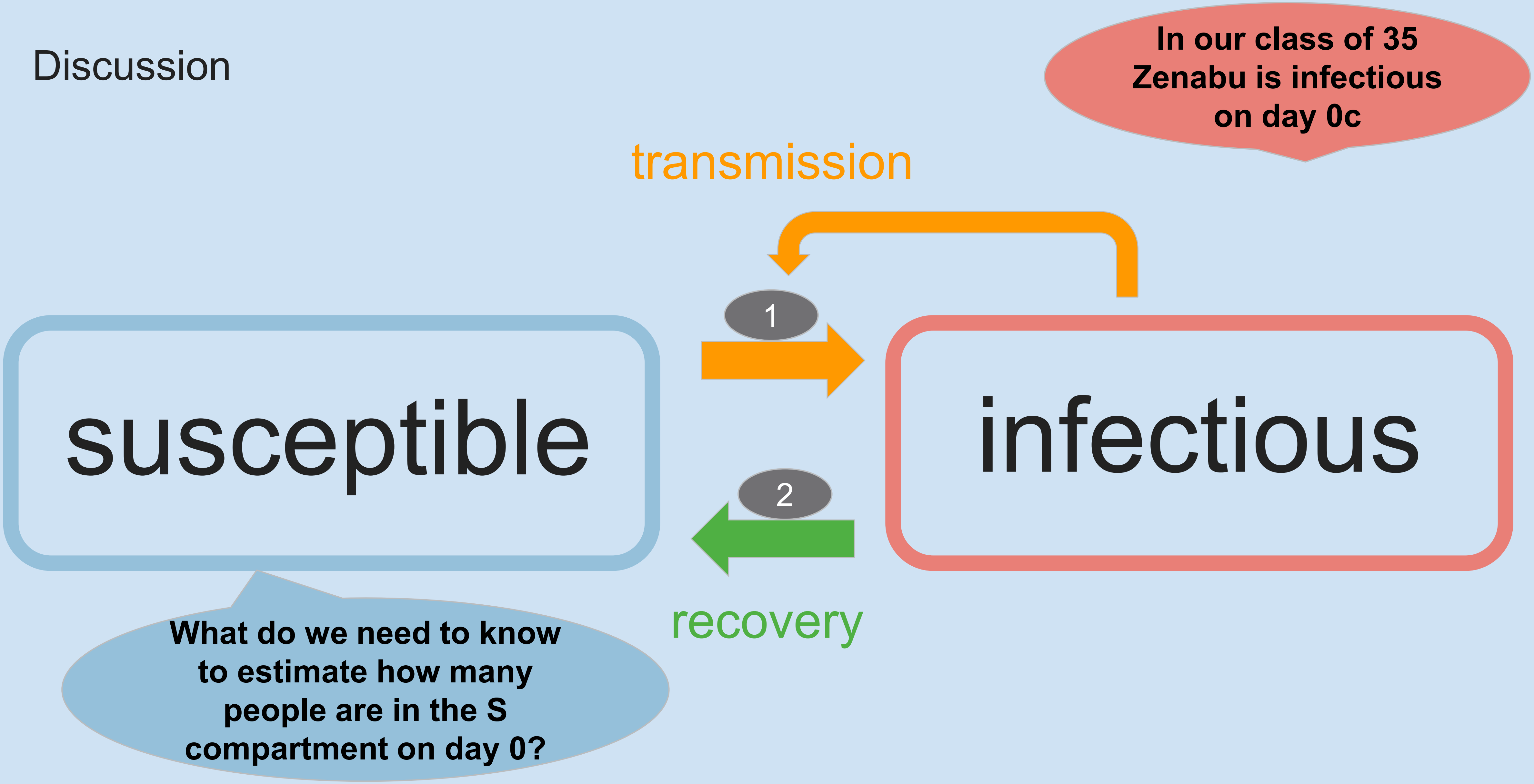
Hernandez-Suarez, C., & Lopez, O. M. (2022).
https://doi.org/10.48550/arXiv.2208.11509

Recap: Flow diagrams for disease transmission



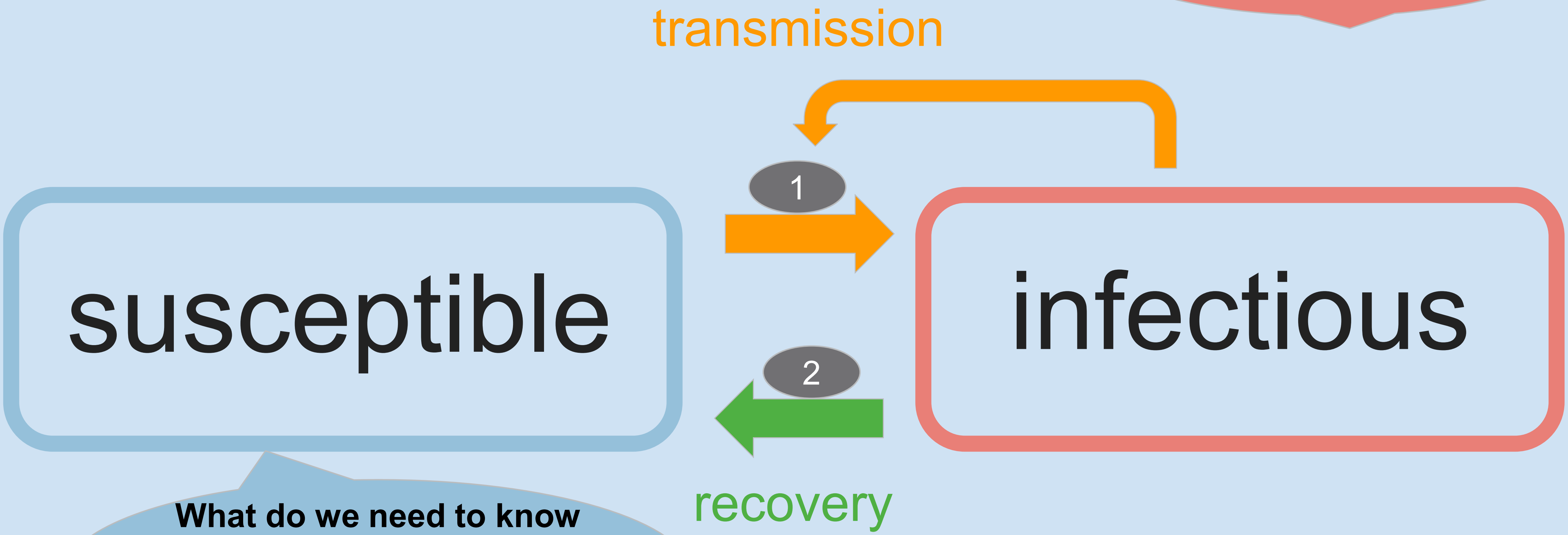
SIS model

Discussion


In our class of 35 Zenabu is infectious on day 0c
transmission
1
susceptible
infectious
2
recovery
What do we need to know to estimate how many people are in the S compartment on day 0?

Discussion

In our class of 35 Zenabu is infectious on day 0

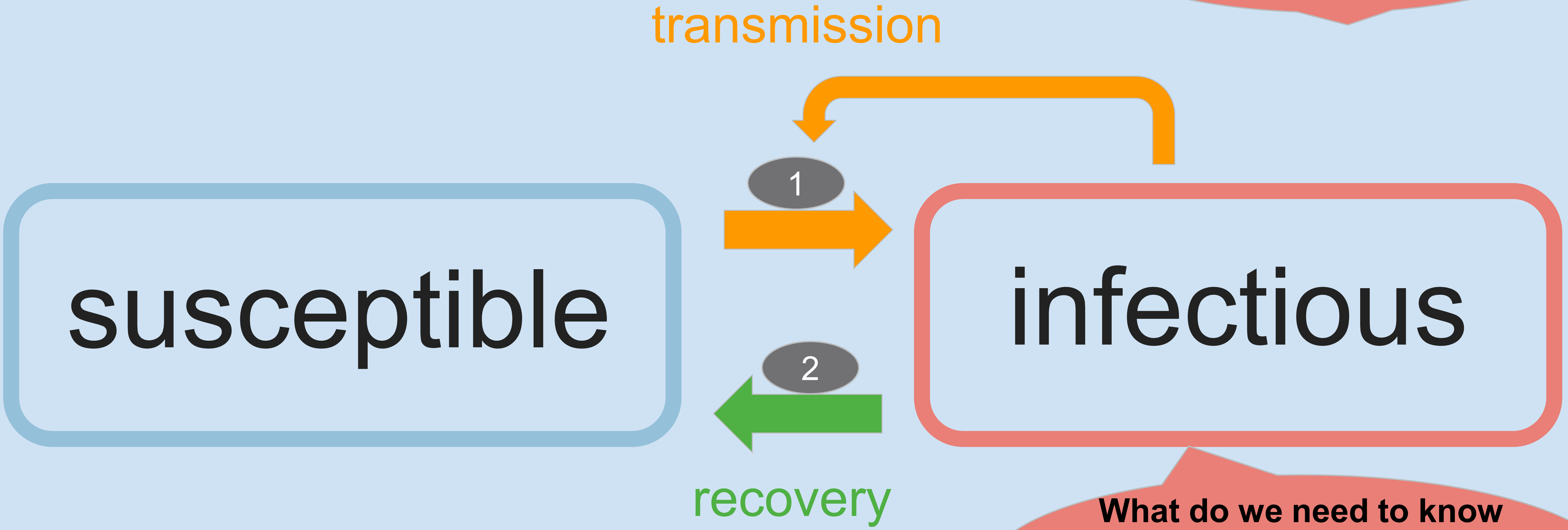


What do we need to know to estimate how many people are in the S compartment on day 0?

$$S(0) = 34$$

Discussion

In our class of 35 Zenabu is infectious on day 0

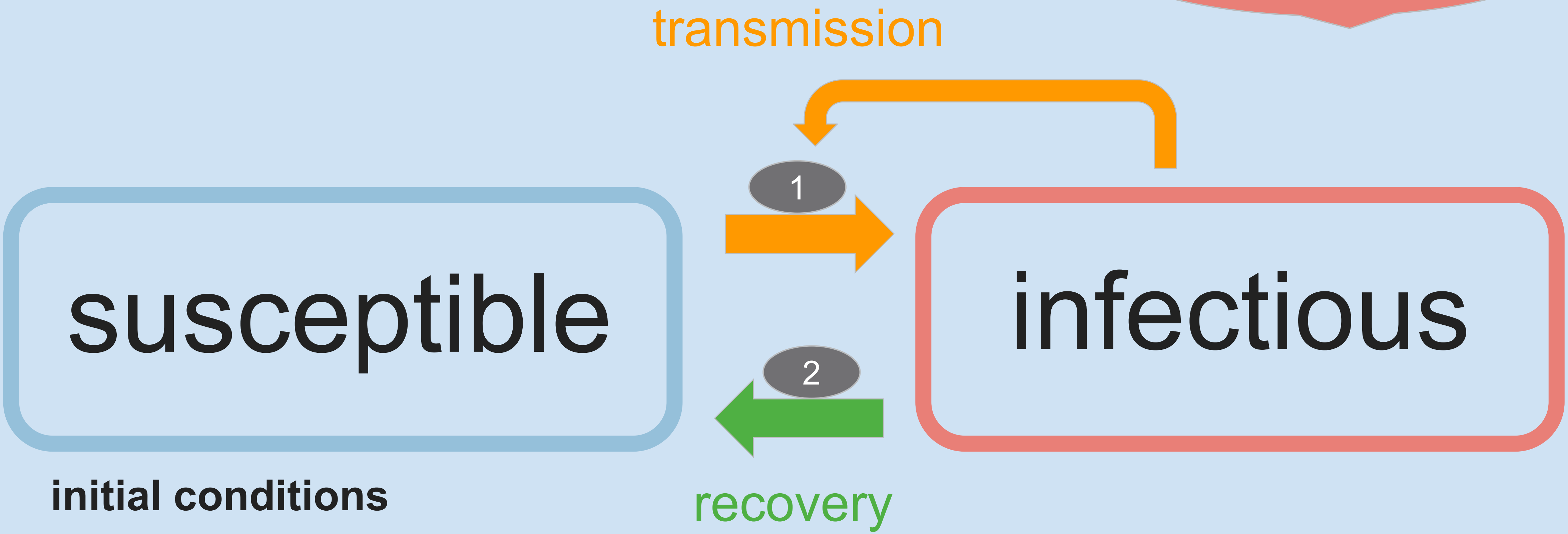


What do we need to know to estimate how many people are in the I compartment on day 0?

$$I(0) = 1$$

Discussion

In our class of 35 Zenabu is infectious on day 0

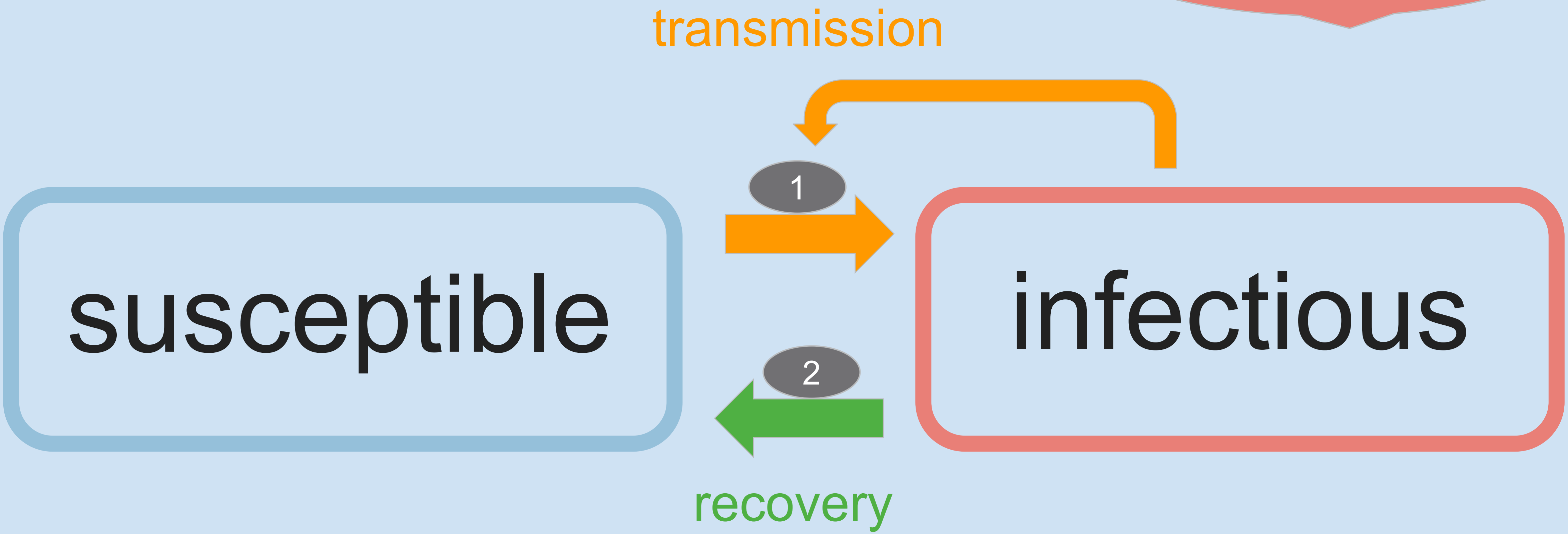


initial conditions

$$S(0) = 34,$$
$$I(0) = 1$$

Discussion

In our class of 35
Zenabu is infectious
on day 1



What do we need to know to estimate how many people are in the S and I compartments on day 1?

Discussion summary

In our class of 35 Zenabu is infectious on day 1

1. What do we need to know to estimate how many people are infectious on day 1?

How many new infections occur and how many recover.

1. Who can become newly infected? *Only those in the susceptible group.*
2. How does the number of infectious students from the previous day affect the risk for susceptibles today?

The more proportion of the infectious population there are, the higher the risk.

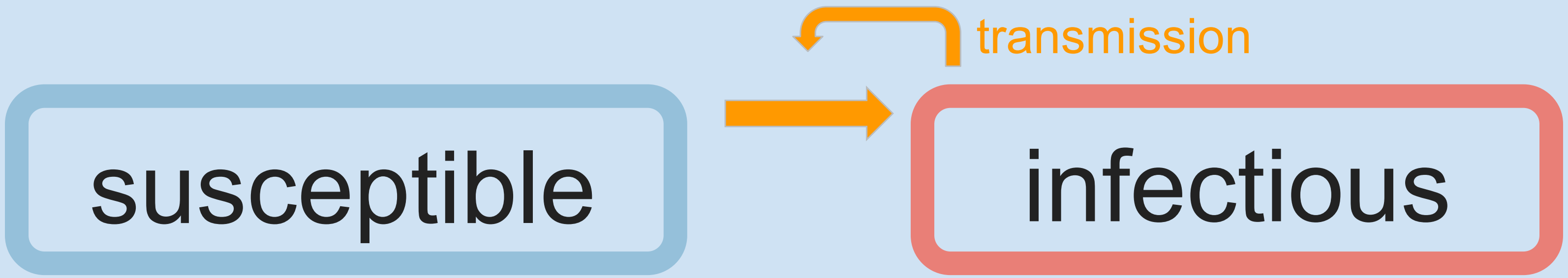
1. If a susceptible meets an infectious, do they always get infected?

(chance of transmission per contact).

1. And what about the students already infectious — do they all stay infectious tomorrow, or do some recover?

(chance of recovery each day)

Flow diagrams as differential equations



outflow **from** susceptible

$$S(t+1) = S(t) - \beta\frac{I(t)}{N}S(t)$$

inflow **into** infectious

$$I(t+1) = I(t) + \beta\frac{I(t)}{N}S(t)$$

assumption: all individuals are well mixed

: the population is closed, $S(t) + I(t) = N$

Flow diagrams as differential equations

assumption: all individuals are well mixed

: the population is $S(t) + I(t) = N$

closed,

outflow **from** susceptible

$$S(t+1) = S(t) - \beta \frac{I(t)}{N} S(t)$$

inflow **into** infectious

$$I(t+1) = I(t) + \beta \frac{I(t)}{N} S(t)$$

Transmission rate β

The rate at which a susceptible individual becomes infected per unit time, given contact with infectious individuals.

rate =expected number of events per time unit

Flow diagrams as differential equations

assumption: all individuals are well mixed

: the population is $S(t) + I(t) = N$

closed,

outflow **from** susceptible

$$S(t+1) = S(t) - \beta \frac{I(t)}{N} S(t)$$

inflow **into** infectious

$$I(t+1) = I(t) + \beta \frac{I(t)}{N} S(t)$$

Proportion infectious $\frac{I(t)}{N}$

The fraction of the population that is infectious at time *t.*

captures the "risk of exposure" for any susceptible individual.

Flow diagrams as differential equations

assumption: all individuals are well mixed
: the population is $S(t) + I(t) = N$
closed,

outflow **from** susceptible

$$S(t+1) = S(t) - \beta \frac{I(t)}{N} S(t)$$

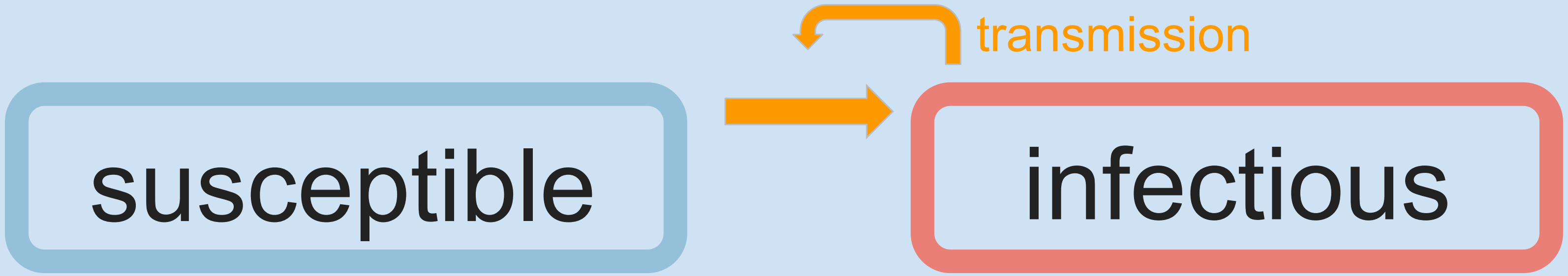
inflow **into** infectious

$$I(t+1) = I(t) + \beta \frac{I(t)}{N} S(t)$$

Susceptible population $S(t)$

How many susceptible people are present at time *t.*

Flow diagrams as differential equations



outflow from susceptible

$$S(t+1) = S(t) - \underbrace{\beta \frac{I(t)}{N} S(t)}_{\text{total new infections per unit time}}$$

inflow **into** infectious

$$I(t+1) = I(t) + \beta \frac{I(t)}{N} S(t)$$

assumption: all individuals are well mixed : the population is closed, $S(t) + I(t) = N$

Flow diagrams as differential equations

susceptible $\xleftarrow{\text{recovery}}$ infectious

inflow **into** susceptible

$$S(t+1) = S(t) + \gamma I(t)$$

outflow **from** infectious

$$I(t+1) = I(t) - \gamma I(t)$$

assumption: all individuals are well mixed

: the population is closed, $S(t) + I(t) = N$

Flow diagrams as differential equations

assumption: all individuals are well mixed

closed, : the population is $S(t) + I(t) = N$

inflow **into** susceptible

$$S(t+1) = S(t) + \gamma I(t)$$

outflow **from** infectious

$$I(t+1) = I(t) - \gamma I(t)$$

Recovery parameter γ

The rate at which infectious individuals recover per unit time.

Average infectious period: $\frac{1}{\gamma}$

Flow diagrams as differential equations

assumption: all individuals are well mixed

closed, : the population is $S(t) + I(t) = N$

inflow **into** susceptible

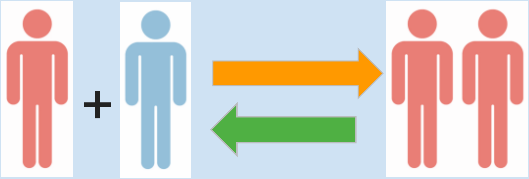
$$S(t+1) = S(t) + \underbrace{\gamma I(t)}$$

outflow **from** infectious

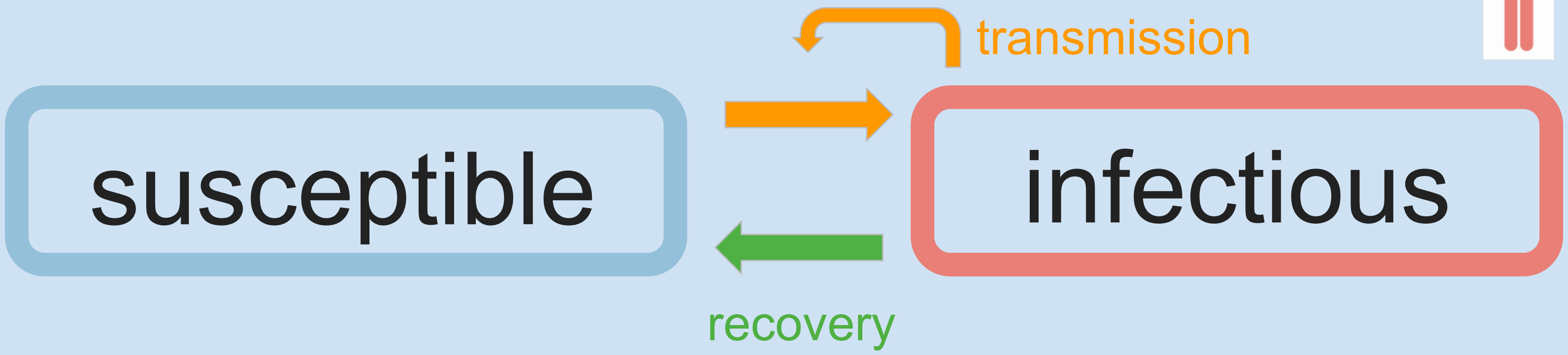
$$I(t+1) = I(t) - \gamma I(t)$$

Total number of people who recover in the population per unit time.

"coupled system"



Flow diagrams as differential equations

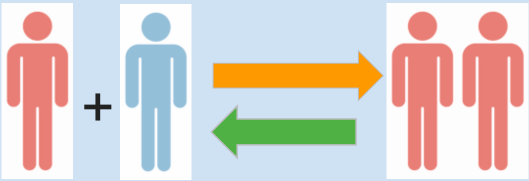


$$\frac{dS(t)}{dt} = -\beta \frac{I(t)}{N} S(t) + \gamma I$$

$$\frac{dI(t)}{dt} = \beta \frac{I(t)}{N} S(t) - \gamma I$$

$$\begin{aligned} S(0) &= S_0 \\ I(0) &= I_0 \\ S(t) + I(t) &= N \\ \Delta &\to 0 \end{aligned}$$

"coupled system"



Flow diagrams as differential equations

transmission

susceptible ⇄ infectious

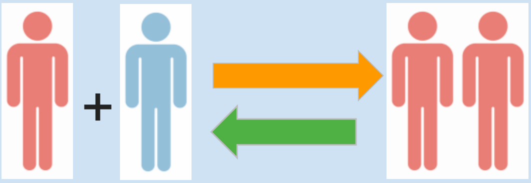
recovery

$$\frac{dS(t)}{dt} = -\beta\frac{I(t)}{N}S(t) + \gamma I$$

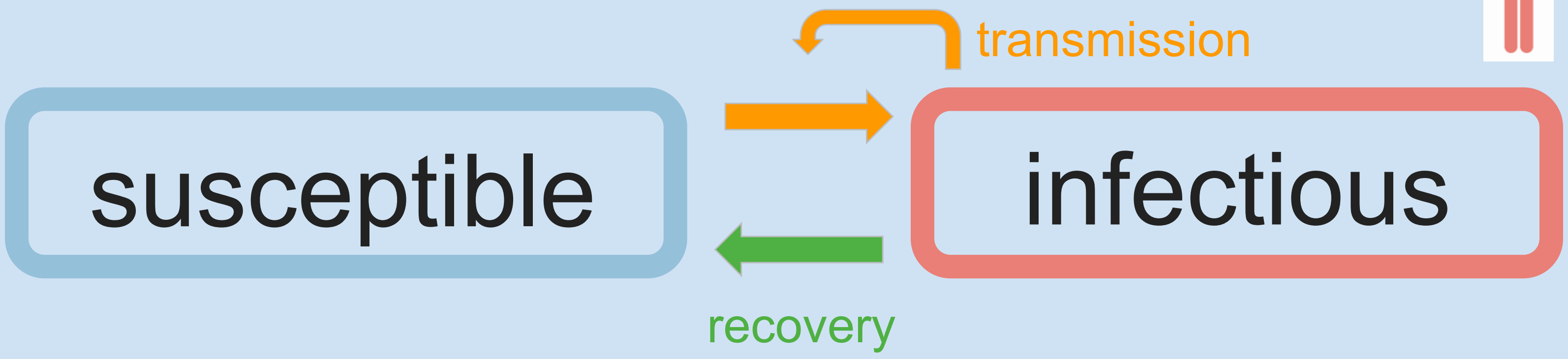
$$\frac{dI(t)}{dt} = \quad \frac{I(t)}{\quad} S(t) - \gamma I$$

$$S(0) = S_0$$
$$I(0) = I_0$$
$$S(t) + I(t) = N$$
$$\Delta \to 0$$

Rate of change of susceptible individuals over time *“coupled system”*



Flow diagrams as differential equations



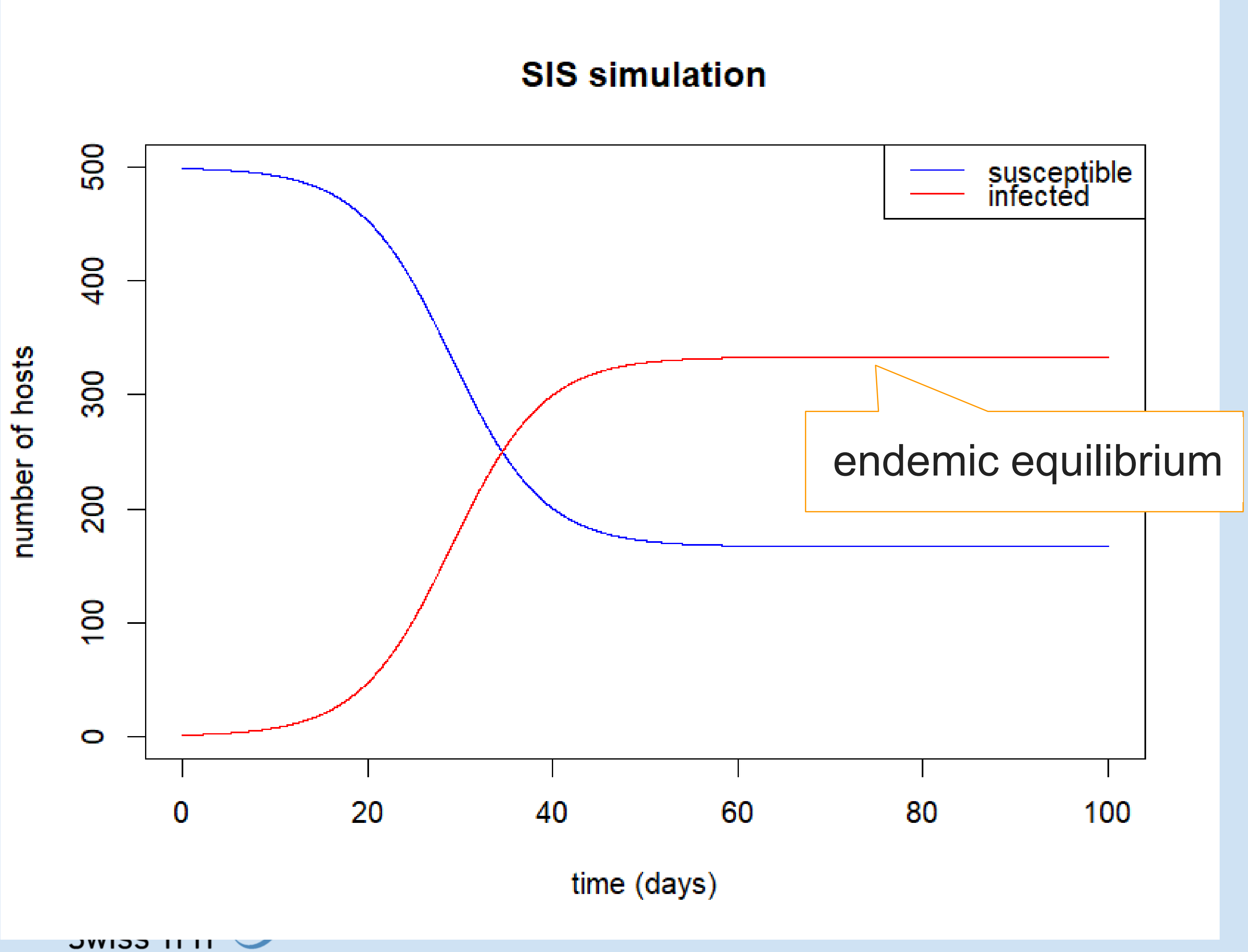
$$\frac{dS(t)}{dt} = -\beta \frac{I(t)}{N} S(t) + \gamma I$$

$$\frac{dI(t)}{dt} = \beta \frac{I(t)}{N} S(t) - \gamma I$$

$$S(0) = S_0$$
$$I(0) = I_0$$
$$S(t) + I(t) = N$$
$$\Delta \to 0$$

Rate of change of infectious individuals over time

Endemic and disease-free equilibrium for SIS system



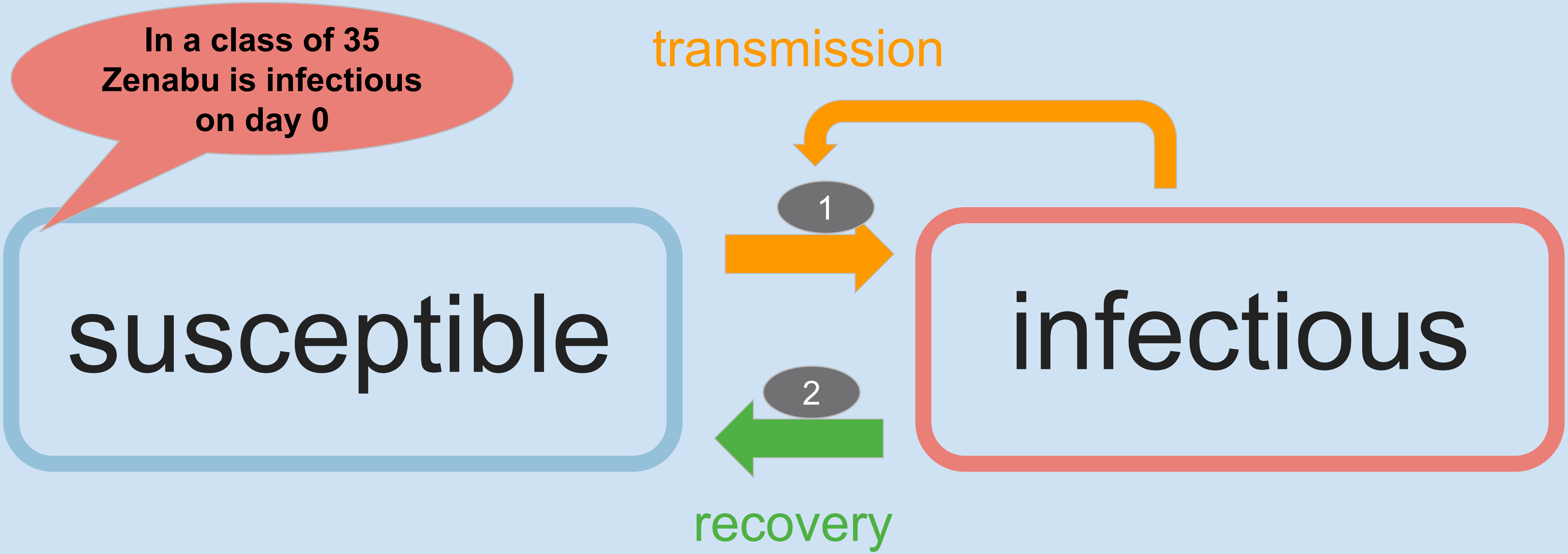
$$0 = -\beta \frac{I(t)}{N} S(t) + \gamma I$$

$$0 = \beta \frac{I(t)}{N} S(t) - \gamma I$$

$$\Leftrightarrow S^*(\infty) = \frac{\gamma}{\beta} N$$

$$I^*(\infty) = N - S^*(\infty) = N\left(1 - \frac{1}{\mathcal{R}_0}\right)$$

Practical: Infection OR No Infection



SIS model

Basic reproduction number

Introducing a single infection into a entirely susceptible population

$$\frac{dI}{dt} = \beta S \frac{I}{N} - \gamma I \qquad S \sim N, I(0) = I_0$$

$\beta - \gamma > 0$ exponential growth $\Leftrightarrow \beta > \gamma \Leftrightarrow \frac{\beta}{\gamma} > 1$

$\beta - \gamma < 0$ exponential growth $\Leftrightarrow \beta < \gamma \Leftrightarrow \frac{\beta}{\gamma} < 1$

"basic reproduction number" $\mathcal{R}_0 = \frac{\beta}{\gamma} \qquad I(\infty) = N\left(1 - \frac{1}{\mathcal{R}_0}\right)$

Acknowledgements

- Christian Selinger
- Emilie Pothin
- Billy Bauzille
- Zenabu Suboi
- Geoffrey Githinji

Swiss TPH

Asanteni sana

Cameline.Orlendo@swisstph.ch