SSE (Least Squares) Fitting: Step-by-Step Instructions

1. Define the model and the initial parameters/states

- Keep your SIS–SI ODE exactly as before (humans S_h, I_h; mosquitoes S_m, I_m).
- Add `Ch` (cumulative human infections) so you can compute weekly incidence from week-to-week increments.
- Introduce a transmission scaler `alpha_scale` so `alpha ← alpha_base × alpha_scale`.

2. Simulate to get the model's weekly incidence

- Integrate daily over 52 weeks from day 1
- At days `0,7,14,…,364`, compute `weekly_incidence_t = Ch(day_t) − Ch(day_{t−1})` for `t = 1,…,52`.
- Predicted weekly cases: `X_hat_t(θ) = ρ_scale · I_t(alpha_scale)`, with `θ = {alpha_scale, ρ_scale}`.

3. Define the least-squares objective (weekly counts recommended)

- Sum of Squared Errors (SSE): $SSE(\theta) = \Sigma_{t=1}^{52} [X_{obs}(t) - X_{hat}_t(\theta)]^2$
- Root Mean Squared Error (RMSE): $RMSE(\theta) = sqrt{ SSE(\theta) / 52 }$
- Minimize SSE or RMSE; they yield the same minimizer.

Note: Fit on cumulative weekly totals. Use the function diff.

4. Set sensible starting values

- Simulate once with `alpha_scale = 1` to obtain $I_t(1)$.
- Level match: $\rho_0 = (\Sigma_t X_{obs}(t)) / (\Sigma_t I_t(1))$
- Optimize on the **log-scale** to keep parameters positive: $\vartheta = (\log alpha\_scale, \log \rho\_scale).$

5. Find the parameters that minimize SSE/RMSE

- Use `optim` (e.g., method = `"BFGS"`) or `nlminb` on log-parameters.
- Optionally apply simple bounds (on natural scales): `alpha_scale ∈ [0.05, 20], ρ_scale ∈ [1e−3, 100]`.

6. Extract the parameters and summarize

- Back-transform: $alpha\_scale\_hat = exp(\hat{\vartheta}1),\ \rho\_scale\_hat = exp(\hat{\vartheta}2).$
- Compute and report the minimized RMSE (or SSE).

7. Visualize the fit

- Overlay observed weekly cases `X_obs(t)` with the fitted mean `X_hat_t = ρ_scale_hat · I_t(alpha_scale_hat)` as a line.
- (Optional) Plot cumulative curves as a secondary check.