Exo 25

B_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
x_4	15	16	17	1	0	0	0	440
x_5	15	18	20	0	1	0	0	415
x_6	6	17	19	0	0	1	0	500
x_7	5	3	16	0	0	0	1	175
	14	37	35	0	0	0	0	*

$\uparrow$
ℓ

$440/16 = 27,5$

$415/18 \approx 23,06 \leftarrow k$

$500/17 \approx 29,41$

$175/3 \approx 58,33$

① avec x_4, x_5, x_6, x_7 , donc $B_0 = \{4,5,6,7\}$ et $H_0 = \{1,2,3\}$

② $\min\left(\underset{i \in \{1,2,3\}}{\text{Argmax}} (14\ 37\ 35)\right) = 2$, donc x_2

③ $\underset{i \in \{4,5,6,7\}}{\text{Argmin}} \left(\frac{440}{16}, \frac{415}{18}, \frac{500}{17}, \frac{175}{3}\right) = 5$, donc x_5

④ $'x_2' = 'x_5'/18 = \left(\frac{15}{18}, 1, \frac{20}{18}, 0, \frac{1}{18}, 0, 0 \mid \frac{415}{18}\right)$

$= \left(5/6, 1, 10/9, 0, 1/18, 0, 0 \mid \frac{415}{18}\right)$

$'x_4' = 'x_4' - 16\,'x_2'$

$= (15, 16, 17, 1, 0, 0, 0 \mid 440)$

$\quad - \left(\frac{40}{3}, 16, 160/9, 0, 8/9, 0, 0 \mid 8 \times 415/9\right)$

$= \left(\frac{5}{3}, 0, -7/9, 0, -8/9, 0, 0 \mid \frac{9 \times 440 - 8 \times 415}{9}\right)$

$= \left(5/3, 0, -7/9, 0, -8/9, 0, 0 \mid 640/9\right) = \left(\frac{5}{3}, 0, -\frac{7}{9}, 0, -\frac{8}{9}, 0, 0 \mid \frac{640}{9}\right)$

$'x_6' = 'x_6' - 17\,'x_2'$

$= (6, 17, 19, 0, 0, 1, 0 \mid 500)$

$\quad - \left(85/6, 17, 170/9, 0, 17/18, 0 \mid 17 \times 415/18\right)$

$= \left(-49/6, 0, 1/9, 0, 1/18, 0 \mid \frac{9000 - 17 \times 415}{18}\right) = \left(-\frac{49}{6}, 0, \frac{1}{9}, 0, \frac{1}{18}, 0 \mid \frac{1945}{18}\right)$

$'x_7' = 'x_7' - 3\,'x_2'$

$= (5, 3, 16, 0, 0, 0, 1 \mid 175)$

$\quad - \left(5/2, 3, 10/3, 0, 1/6, 0, 0 \mid 415/6\right)$

$= \left(5/2, 0, 38/3, 0, -1/6, 0, 1 \mid \frac{6 \times 175 - 415}{6}\right) = \left(\frac{5}{2}, 0, \frac{38}{3}, 0, -\frac{1}{6}, 0, 1 \mid \frac{635}{6}\right)$

$* = * + 37 \times \frac{415}{18} = * + \frac{15355}{18}$

$'c' = 'c' - 37\,'x_2'$

$= (14, 37, 35, 0, 0, 0, 0)$

$\quad - \left(\frac{37 \times 5}{6}, 37, 370/9, 0, 37/18, 0, 0\right)$

$= \left(\frac{6 \times 14 - 5 \times 37}{6}, 0, \frac{9 \times 35 - 370}{9}, 0, -\frac{37}{18}, 0, 0\right) = \left(-\frac{101}{6}, 0, -\frac{55}{9}, 0, -\frac{37}{18}, 0, 0\right)$

B_1	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
x_4	$5/3$	0	$-7/9$	1	$-8/9$	0	0	$640/9$
x_2	$5/6$	1	$10/9$	0	$1/18$	0	0	$415/18$
x_6	$-49/6$	0	$1/9$	0	$1/18$	1	0	$1945/18$
x_7	$5/2$	0	$38/3$	0	$-1/6$	0	1	$635/6$
	$-\dfrac{101}{6}$	0	$-55/9$	0	$-37/18$	0	0	$* + \dfrac{15355}{18}$

Comme $c \leq 0$ on en déduit que B_1 satisfait la CSO

(P) $\max (14 x_1 + 37 x_2 + 35 x_3 + *)$

S.C. $\begin{cases} 15 x_1 + 16 x_2 + 17 x_3 \leq 440 \\ 15 x_1 + 18 x_2 + 20 x_3 \leq 415 \\ 6 x_1 + 17 x_2 + 19 x_3 \leq 500 \\ 5 x_1 + 3 x_2 + 16 x_3 \leq 175 \\ x_1, x_2, x_3 \geq 0 \end{cases}$

solution optimale $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 415/18 \\ 0 \end{pmatrix}$

valeur optimale : $* + \dfrac{15355}{18}$