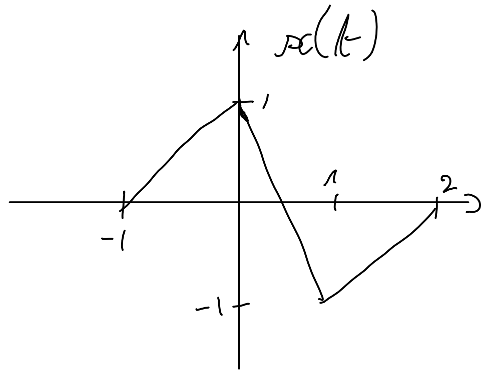
DM1

Thursday, September 22, 2022 5:10 PM

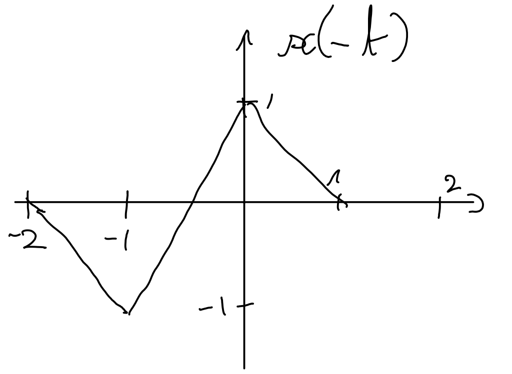
On défini $u(t) = \begin{cases} 1 & \text{si } t \geq 0 \\ 0 & \text{sinon} \end{cases}$

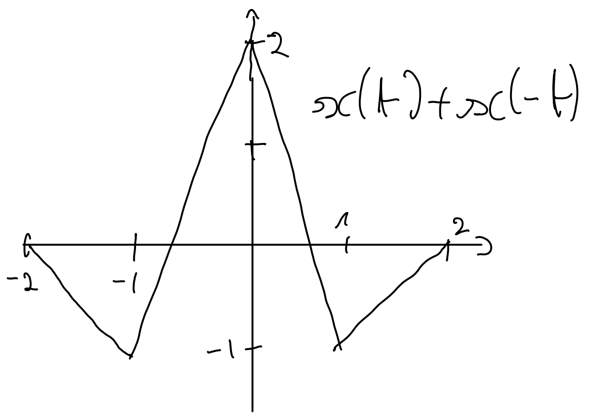
Exercice 1 :

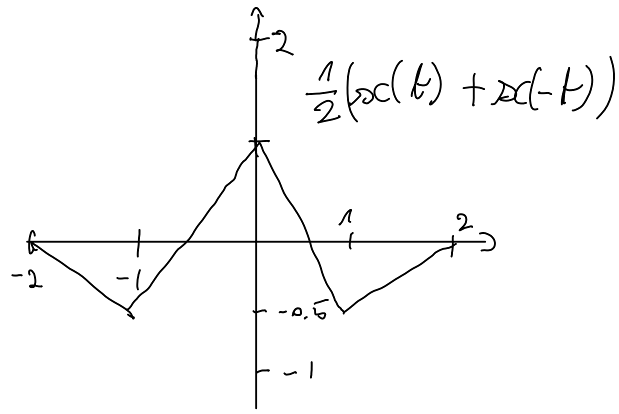
$$x(t) = (t+1)\big(u(t+1) - u(t)\big) + (-2t+1)\big(u(t) - u(t-1)\big) + (t-2)\big(u(t-1) - u(t-2)\big)$$



$$x(-t) = (-t+1)\big(u(-t+1) - u(-t)\big) + (2t+1)\big(u(-t) - u(-t-1)\big) + (-t-2)\big(u(-t-1) - u(-t-2)\big)$$

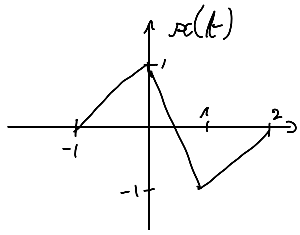


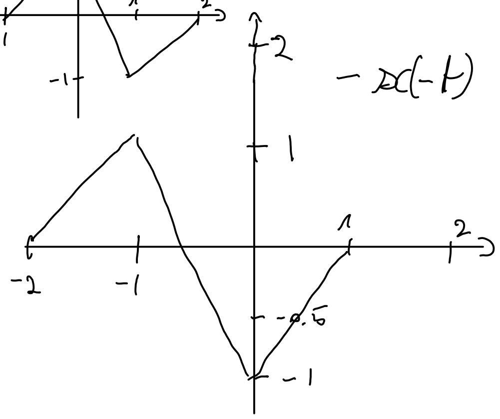


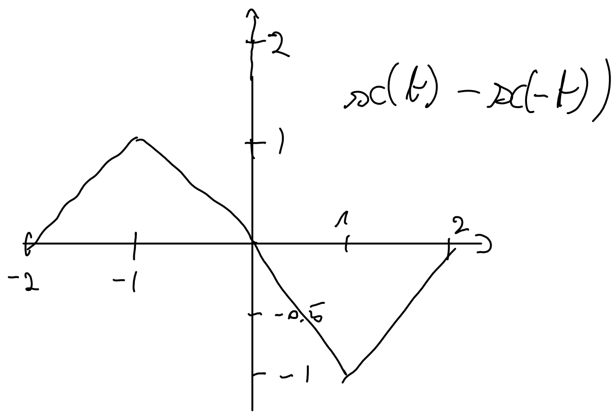


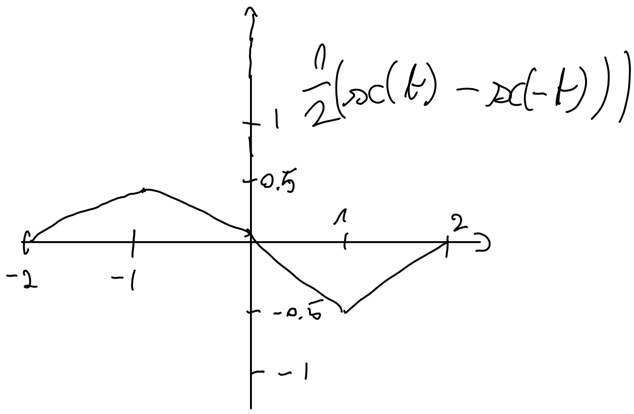
Composante paire de $x(t)$:

$$x_p(t) = \frac{1}{2}\big(x(t) + x(-t)\big)$$









Composante impaire de $x(t)$

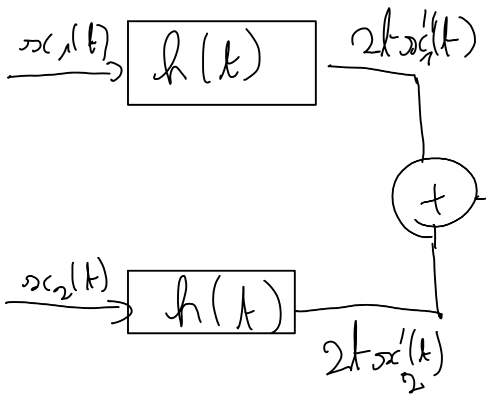
$$x_{i}(t) = \frac{1}{2}(x(t) - x(-t))$$

$$x(t) = x_i(t) + x_p(t)$$

Exercice 2 :

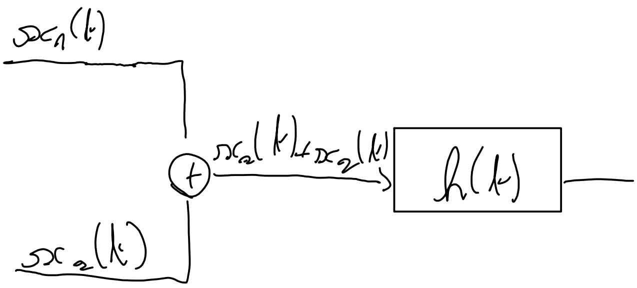
a) $y(t) = t^2 \frac{dx(t)}{dt} = 2t\, x'(t)$

a) Additivité



$$y_1(t) = 2t\, x_1'(t) + 2t\, x_2'(t)$$
$$= 2t\,(x_1'(t) + x_2'(t))$$

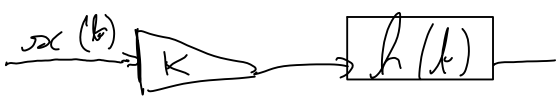
car $\frac{d(f(x)+g(x))}{dx} = f'(x) + g'(x)$



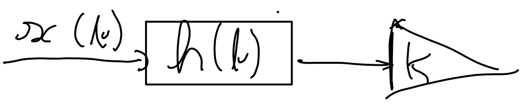
$y_2(t) = 2t\,(x_1'(t) + x_2'(t))$

$y_1(t) = y_2(t)$ le système est additif

Homogène



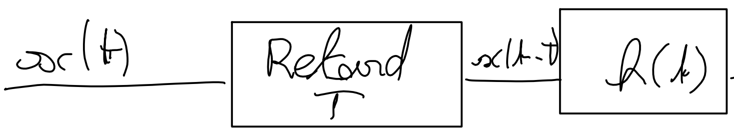
$y_1(t) = 2t\,(K x'(t))$



$y_2(t) = K \times 2t\, x'(t)$

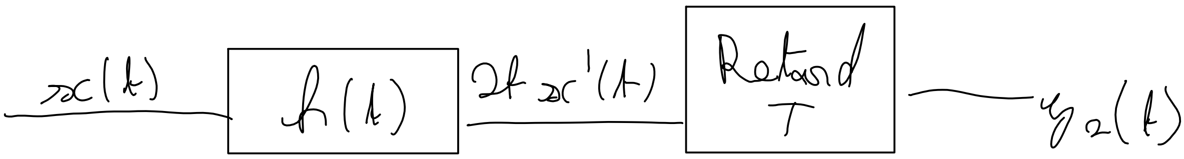
$y_1(t) = y_2(t)$ le système est homogène

invariant dans le temps



$y_1(t) = 2t\, x'(t-T)$

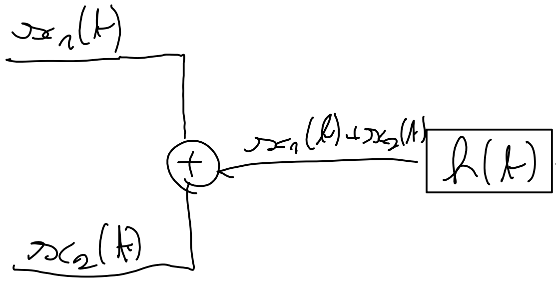
car $\dfrac{d(x(t-T))}{dt} = x'(t-T)$



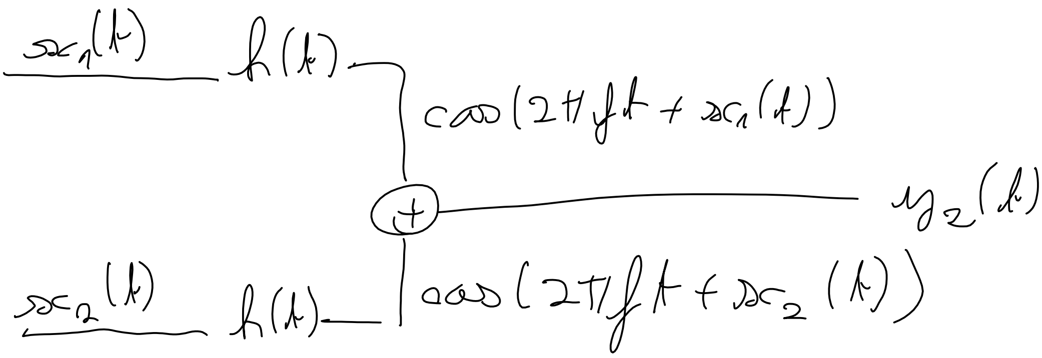
$y_2(t) = 2(t-T)\, x'(t-T) \neq y_1$ donc le système n'est pas invariant dans le temps

b) $y(t) = \cos(2\pi f t + x(t))$

Additivité :



$\rightarrow y_1(t) = \cos(2\pi f t + x_1(t) + x_2(t))$



$y_2(t) = \cos(2\pi f t + x_1(t)) + \cos(2\pi f t + x_2(t))$

$\cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right) \ldots$

$y_2(t) \neq y_1(t)$

Donc le système n'est pas additif

Homogène

Rapidement :
- $y_1(t) = K\cos(2\pi f t + x(t))$
- $y_2(t) = \cos(2\pi f t + K x(t))$
- $y_1(t) \neq y_2(t) \Rightarrow$ Système non Homogène

Invariance dans le temps

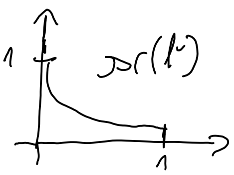
$$x(t) \to \boxed{\text{retard } T} \xrightarrow{x(t-T)} \boxed{h(t)} \to y_1(t) = \cos(2\pi f t + x(t-T))$$

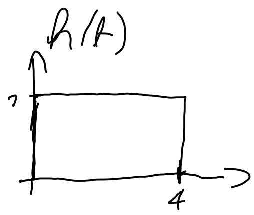
$$x(t) \to \boxed{h(t)} \longrightarrow \boxed{\text{retard } T} \to y_2(t) = \cos(2\pi f(t-T) + x(t-T))$$

$y_1(t) \neq y_2(t)$ donc le système n'est pas invariant dans le temps

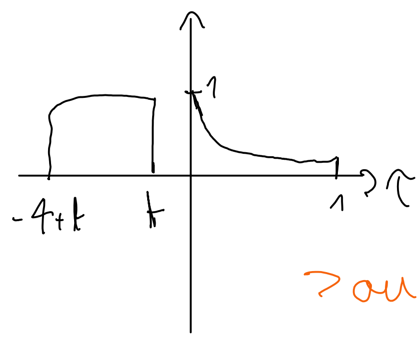
Exercice 3

a)
- $x(t) = e^{-t}u(t)$
- $h(t) = u(t) - u(t-4)$



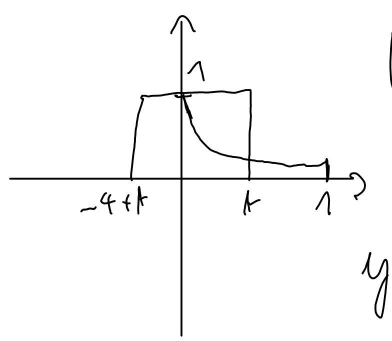


Les différents cas :



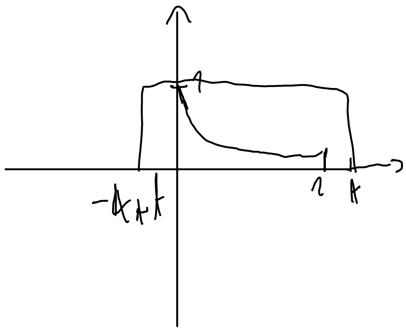
$t \leq 0,\ y(t) = 0$

$>$ ou $\geq$ $\rightarrow$ pas d'influence sur la valeur de $y(t)$, car c'est une $\int$



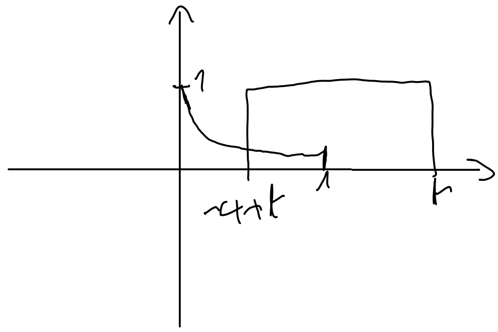
$\begin{cases} t \geq 0 \\ -4+t \leq 0 \end{cases} \Leftrightarrow \begin{cases} t \geq 0 \\ t < 4 \end{cases} \Leftrightarrow 0 < t < 4$

$$y(t) = \int_0^t e^{-\tau}\,d\tau = \left[-e^{-\tau}\right]_0^t = -e^{-t} + 1$$



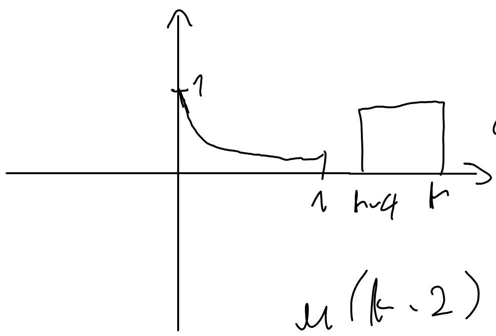
$\begin{cases} t \geq 1 \\ -4+t < 0 \end{cases} \iff \begin{cases} t > 1 \\ t < 4 \end{cases} \iff 1 < t < 4$

$$y(t) = \int_0^1 e^{-\tau} d\tau = -e^{-1} + 1$$



$\begin{cases} -4+t < 1 \\ -4+t > 0 \end{cases} \iff \begin{cases} t < 5 \\ t > 4 \end{cases} \iff 4 < t < 5$

$$y(t) = \int_{t-4}^1 e^{-\tau} d\tau = -e^{-1} + e^{-t+4}$$



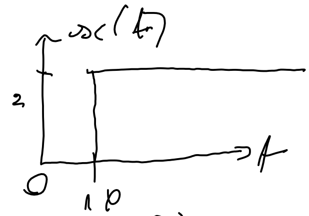
$t - 4 > 1$

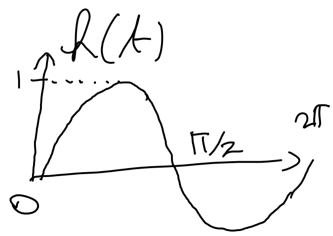
$\iff t > 4 \quad y(t) = 0$

$u(t.2)$

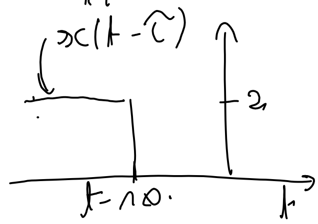
b) $x(t) = 2\, u(t-10)$

$h(t) = \sin(2t)\, u(t)$

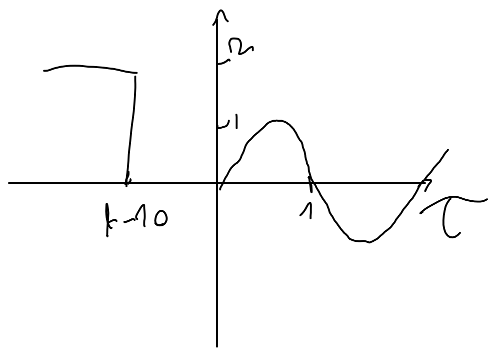




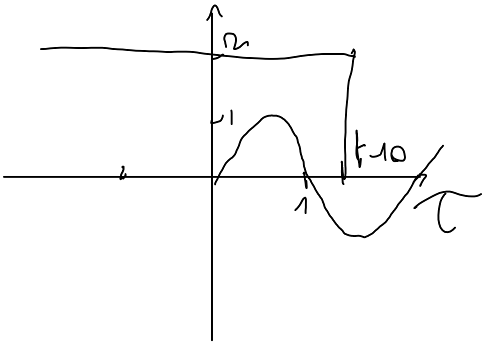
$$x(t-\tau) = \begin{cases} 2 \text{ si } t - \tau - 10 \geq 0 \\ \quad \iff -\tau \geq -t + 10 \\ \quad \iff \tau < t - 10 \\ 0 \text{ sinon} \end{cases}$$



<u>Les differents cas :</u>



$t - 10 \leq 0 \iff t \leq 10, \quad y(t) = 0$



$t-10 > 0 \iff t > 10$

$$y(t) = \int_0^{t-10} 2\sin(2\tau)\,d\tau$$

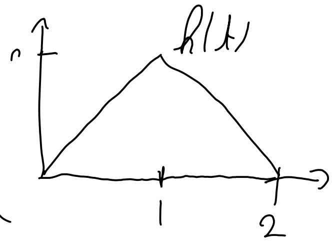
$$= -\cos(2\tau)\Big|_0^{t-10}$$

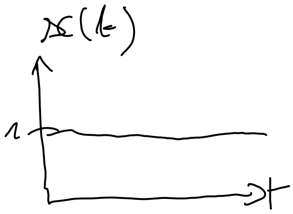
$$= 1 - \cos(2t-20)$$

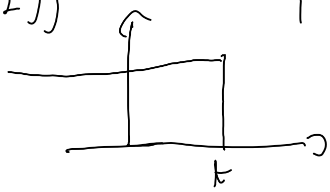
c) $x(t) = u(t)$

$h(t) = t\,(u(t) - u(t-1))$
$\quad + (-t+2)(u(t-1) - u(t-2))$

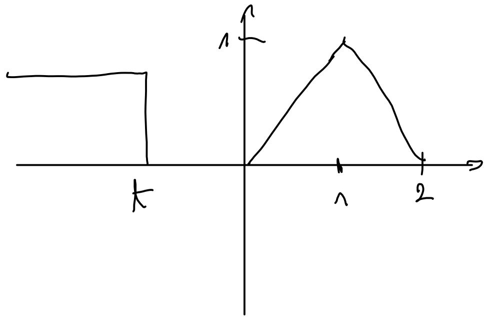
$x(t-\tau) = \begin{cases} 1 \text{ si } t-\tau \geq 0 \\ \iff \tau \leq t \end{cases}$



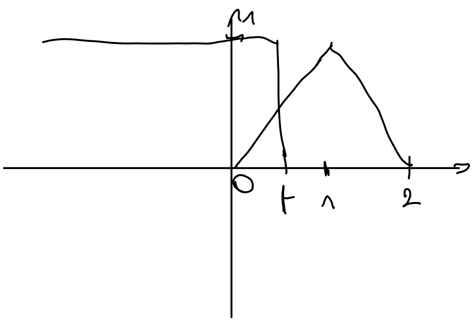




Les différents cas

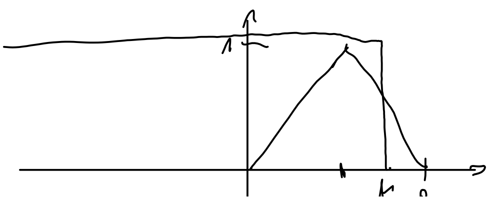


$t < 0, \quad y(t) = 0$



$0 < t < 1, \quad y(t) = \int_0^t \tau\,d\tau = \frac{1}{2}\tau^2\Big|_0^t$

$$= \frac{1}{2}t^2$$



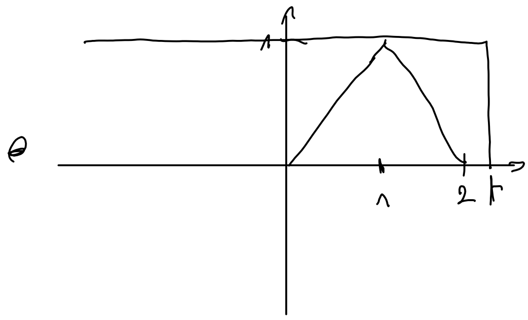
$1 < t < 2$

$$y(t) = \int_0^1 \tau\,d\tau + \int_1^t 2-\tau\,d\tau$$

$$= \frac{1}{2} + \left[2\tau - \frac{1}{2}\tau^2\right]_1^t$$

$$= \frac{1}{2} + 2t - \frac{1}{2}t^2 - 2 + \frac{1}{2}$$

$$= -\frac{1}{2}t^2 + 2t - 1$$



$t > 2$,

$$y(t) = \int_0^1 \tau\, d\tau + \int_1^2 2-\tau\, d\tau$$

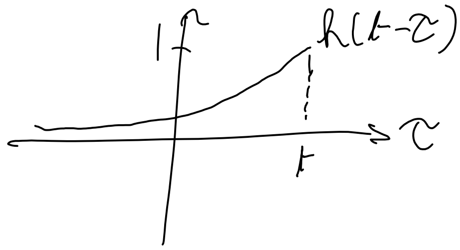
$$= \frac{1}{2} + \left[2\tau - \frac{1}{2}\tau^2\right]_1^2$$

$$= \frac{1}{2} + 4 - 2 - 2 + \frac{1}{2} = 1$$

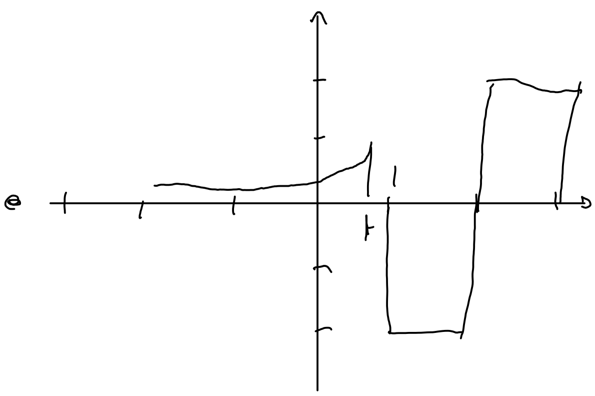
d) $x(t) = -2(u(t-1) - u(t-2)) + 2(u(t-2) - u(t-3))$

$h(t) = e^{-t}u(t)$

$$h(t-\tau) = \begin{cases} e^{-\tau} & \text{si } \tau \leq t \\ 0 & \text{sinon} \end{cases}$$



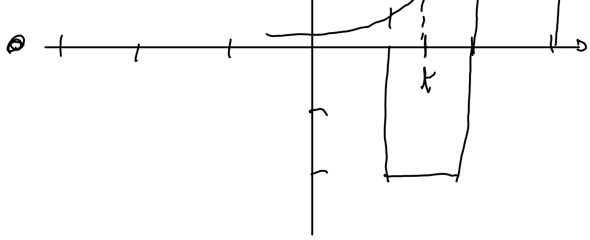
Les different cas :



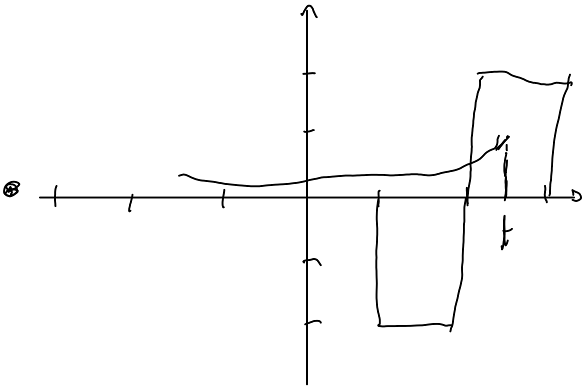
$t < 1$, $y(t) = 0$

$1 < t < 2$

$$y(t) = \int^t -2e^{-\tau} d\tau$$

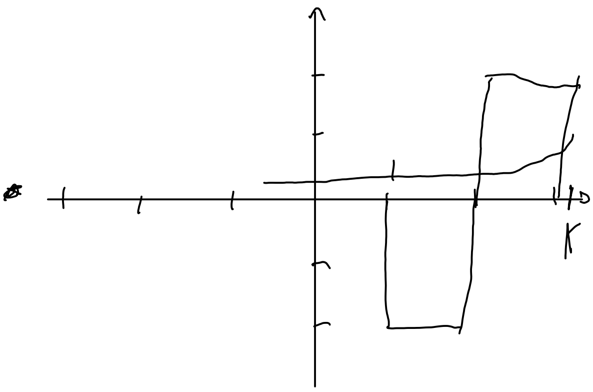


$$= -2\left[-e^{-\tau}\right]_1^t$$
$$= -2(-e^{-t} + e^{-1})$$



$2 < t < 3$

$$y(t) = \int_1^2 -2e^{-\tau}\,d\tau + \int_2^t 2e^{-\tau}\,d\tau$$
$$= 2\left(\int_2^t e^{-\tau}\,d\tau - \int_1^2 e^{-\tau}\,d\tau\right)$$
$$= 2\left(-e^{-\tau}\Big|_2^t + e^{-\tau}\Big|_1^2\right)$$
$$= 2(-e^{-t} + e^{-2} + e^{-2} - e^{-1})$$
$$= 2(2e^{-2} - e^{-1} - e^{-t})$$



$t > 3$

$$y(t) = \int_1^2 -2e^{-\tau}\,d\tau + \int_2^3 2e^{-\tau}\,d\tau$$
$$= 2\left(-e^{-\tau}\Big|_2^3 + e^{-\tau}\Big|_1^2\right)$$
$$= 2(-e^{-3} + e^{-2} + e^{-2} - e^{-1})$$
$$= 2(2e^{-2} - e^{-3} - e^{-1})$$

Exercice 4 :

```
t = 0:0.001:10
```

```
t = 1×10001
         0    0.0010    0.0020    0.0030    0.0040    0.0050    0.0060    0.0070 ···
```

```
w = exp(-t)
```

```
w = 1×10001
    1.0000    0.9990    0.9980    0.9970    0.9960    0.9950    0.9940    0.9930 ···
```

```
x = t.*exp(-t)
```

```
x = 1×10001
         0    0.0010    0.0020    0.0030    0.0040    0.0050    0.0060    0.0070 ···
```

```
y = exp(-t)+t.*exp(-t)
```

```
y = 1×10001
    1.0000    1.0000    1.0000    1.0000    1.0000    1.0000    1.0000    1.0000 ···
```

```
plot(t,w,t,x,t,y)
xlabel('t')
legend('w(t) = exp(-t)', 'x(t) = t*exp(-t)', 'y(t) = exp(-t) + t*exp(-t)')
```