Exo filtre :

$T = 1$; $\Omega_p = 0{,}25\pi$, $\Omega_s = 0{,}75$; $R_p = 7\,dB$, $A_s = 15\,dB$

1) l'ordre du filtre

$$w_p = 2\tan\left(\frac{\Omega_p}{2}\right) = 2\tan\left(\frac{0{,}25\pi}{2}\right) = 0{,}82$$

$$w_s = 2\tan\left(\frac{\Omega_s}{2}\right) = 2\tan\left(\frac{0{,}75\pi}{2}\right) = 4{,}82$$

$$N = \left\lceil \frac{\log\left[\left(10^{\frac{R_p}{10}} - 1\right) / \left(10^{\frac{A_s}{10}} - 1\right)\right]}{2\log\left(\frac{w_p}{w_s}\right)} \right\rceil = \lceil 0{,}57 \rceil = 1$$

entier sup

$$w_{cp} = \frac{w_p}{\left(10^{\frac{R_p}{10}} - 1\right)^{\frac{1}{2N}}} = \boxed{0{,}4}$$

$$w_{cs} = \frac{w_s}{\left(10^{\frac{A_s}{10}} - 1\right)^{\frac{1}{2N}}} = \boxed{0{,}87}$$

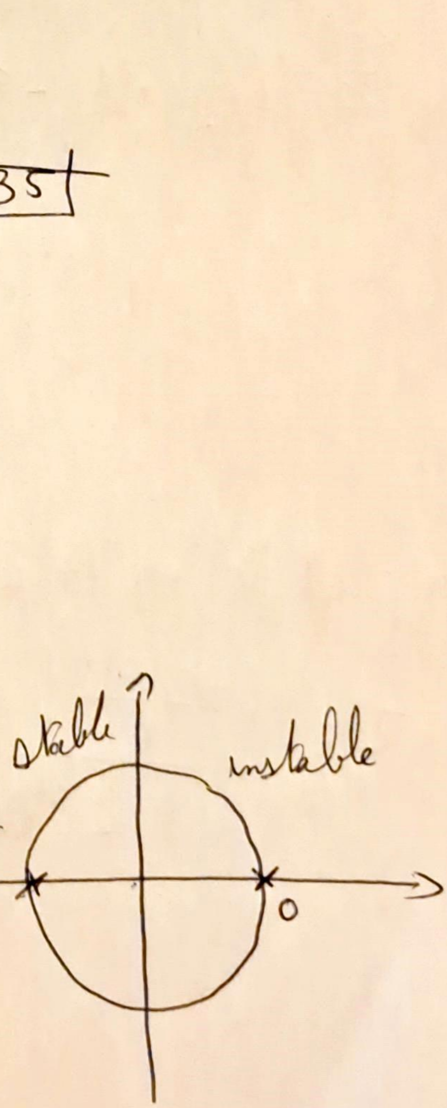
$$w_c = \frac{w_{cp} + w_{cs}}{2} = \frac{0{,}4 + 0{,}87}{2} = \boxed{0{,}635}$$

Les poles de $H(s)$:

N est impaire

$K = \{0, \ldots, 2N-1\}$ / $K \in [0, 1]$

$\theta = \frac{K}{N}\pi$ | pour $K = 0$: $\theta = 0$ | pour $K = 1$: $\theta = \pi$



calculer le pole

$$P_1 = W_c\, e^{+j\pi} = W_c\left[\cos(\pi) + j \overset{0}{\cancel{\sin(\pi)}}\right]$$

$$= 0{,}635 \times (-1)$$

$$\boxed{P_1 = -0{,}635}$$

la fonction de Butt— — . .

$$H(s) = \frac{(W_c)^n}{(S - P_1)} = \frac{(0,635)^1}{(S + 0,635)}$$

$$\boxed{P = -0,635}$$

2) calculer la position des poles dans le plan (z)

$$S = \frac{2}{\cancel{T}\,1}\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right)$$

$$\frac{S(1 + z^{-1})}{2} = (1 - z^{-1})$$

$$\frac{S}{2} + \frac{S z^{-1}}{2} - 1 + z^{-1} = 0$$

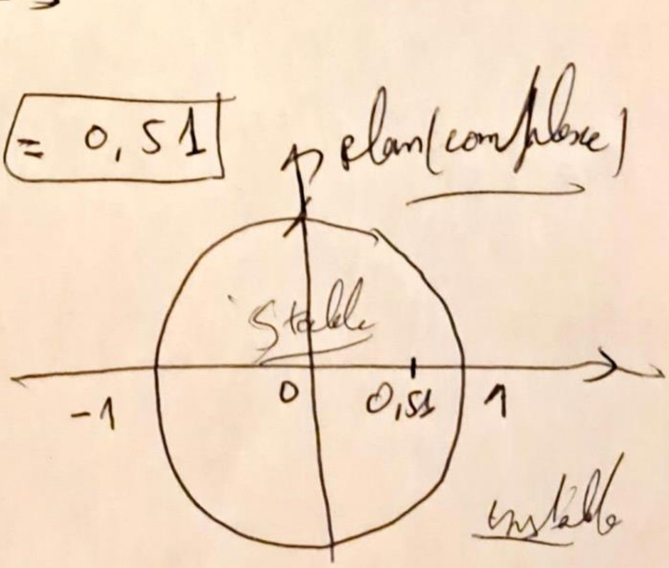
$$z^{-1}\left(\frac{S}{2} + 1\right) - 1 + \frac{S}{2} = 0$$

$$z^{-1} = \frac{1 - \frac{S}{2}}{\frac{S}{2} + 1}$$

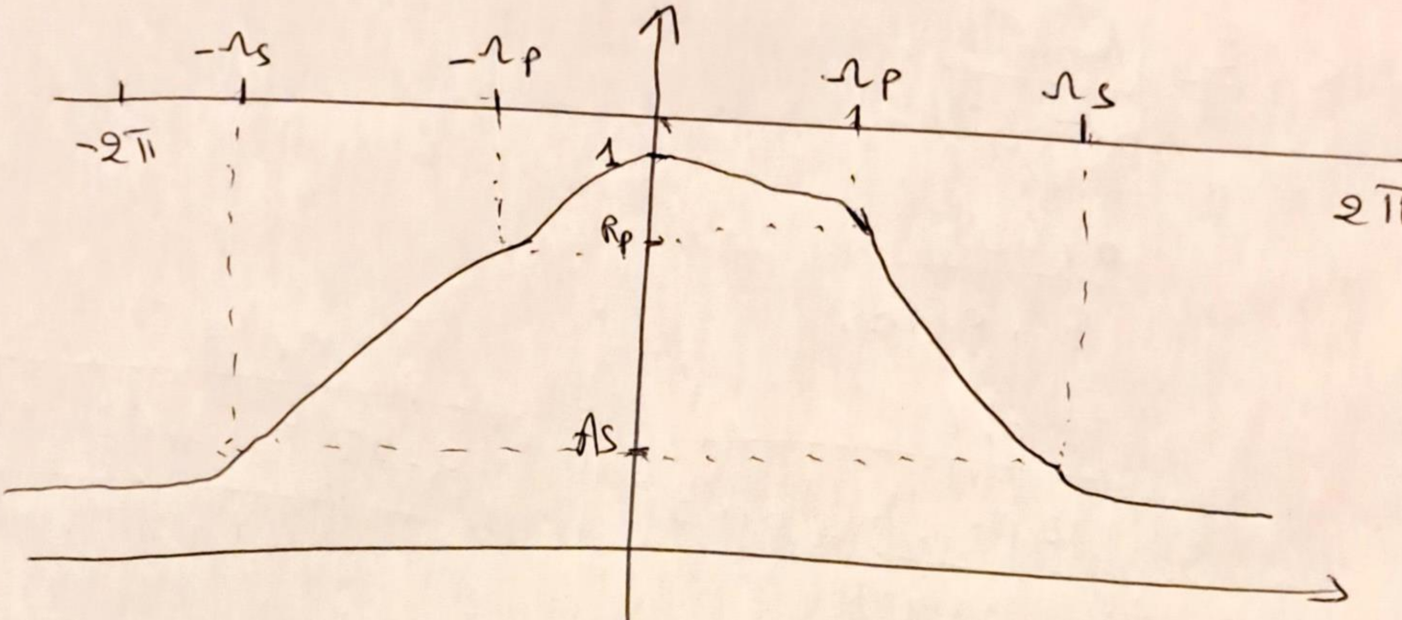
$$\boxed{Z = \frac{S/2 + 1}{1 - \frac{S}{2}}}$$

on pred p on la remplace dans s

$$Z = \frac{\left(\frac{-0,635}{2}\right) + 1}{1 + \left(\frac{0,635}{2}\right)} \boxed{= 0,51}$$



IIR



proposer une réalisation matérielle pour H(z)

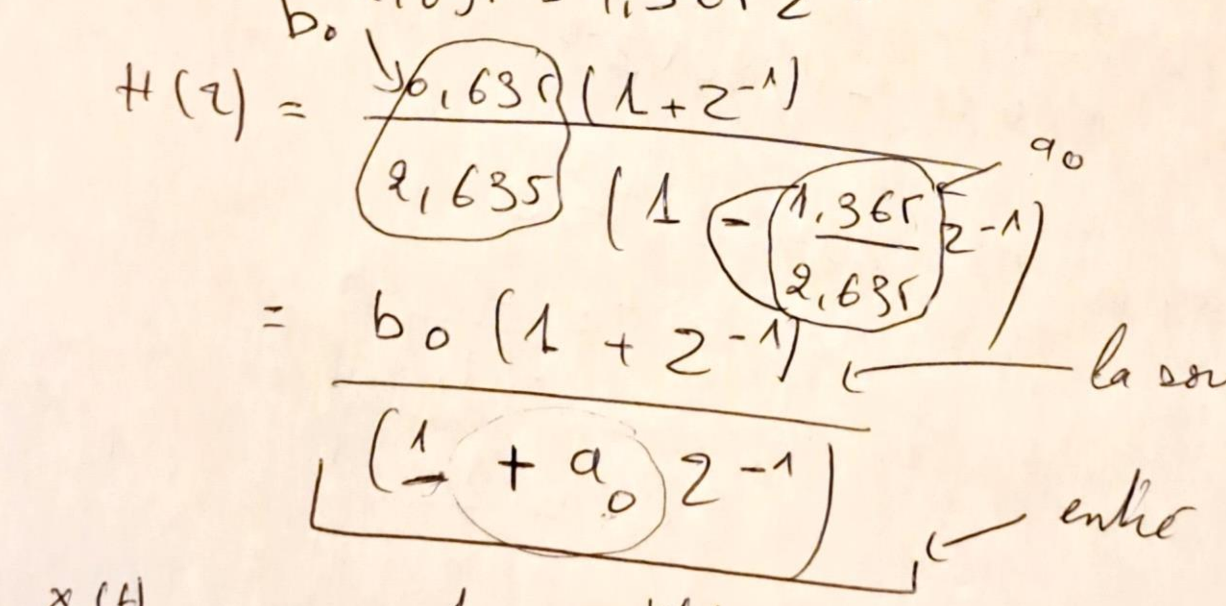
$$H(z) = \frac{0{,}635\,(1+z^{-1})}{2{,}635 - 1{,}365\,z^{-1}}$$

$$H(z) = \frac{\overset{b_0}{0{,}635}}{2{,}635}\;\frac{(1+z^{-1})}{\left(1 - \underset{a_0}{\frac{1{,}365}{2{,}635}}\,z^{-1}\right)}$$

$$= \frac{b_0\,(1+z^{-1})}{(1 + a_0\,z^{-1})}$$

la sortie

entrée



donner une relation pour $H(z)$

$$S = \frac{2}{T}\,\frac{(1-z^{-1})}{(1+z^{-1})}$$

$$H(s) = \frac{0,635}{s+0,635}$$

$$H(z) = \frac{0,635}{\dfrac{2(1-z^{-1})}{(1+z^{-1})} + 0,635} = \frac{0,635}{\dfrac{2-2z^{-1}+0,635(1+z^{-1})}{(1+z^{-1})}}$$

$$H(z) = \frac{0,635\,(1+z^{-1})}{2-2z^{-1}+0,635+0,635z^{-1}}$$

$$= \frac{0,635\,(1+z^{-1})}{2,635 - 1,365z^{-1}}$$

$$2,635 - 1,365z^{-1} = 0$$

$$z^{-1} = \frac{-2,635}{-1,365}$$

$$\boxed{Z = \frac{1,365}{2,635} = 0,51}$$

la fenetre :

1) As = 15 | As < Attenuation de rect

du comp on prend la fenetre rect

2) l'ordre du filtre on a :

$$\Omega_s - \Omega_p = \frac{1{,}8\,\pi}{M}$$

$$M = \frac{1{,}8\,\pi}{\boxed{\Omega_s - \Omega_p}} = \frac{1{,}8\,\pi}{0{,}75\pi - 0{,}25\pi} = \frac{1{,}8\,\cancel{\pi}}{\boxed{0{,}5\,\cancel{\pi}}} = \lceil 3{,}6 \rceil$$

Ω_c = fréquence de coupure

$$\boxed{M = 4}$$

$$\boxed{M - 1 = 3} \leftarrow$$ l'ordre du filtre

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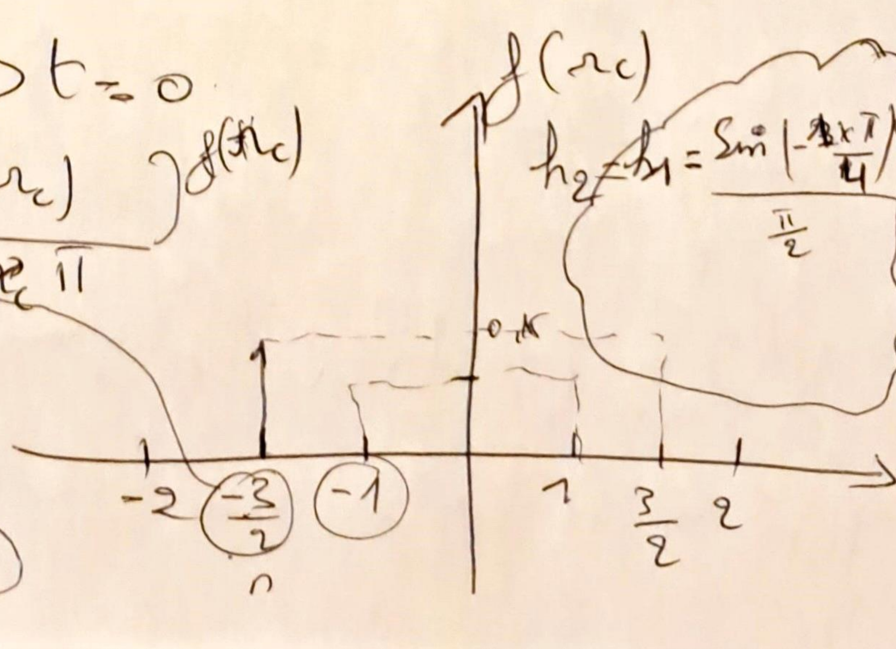
les coef du filtre :

l'intervale : $\left[-\frac{(M-1)}{2} ; \frac{(M-1)}{2}\right] = \left[-\frac{3}{2}, \frac{3}{2}\right]$

$$\begin{cases} 1 \Rightarrow t = 0 \\ \dfrac{\sin(n\Omega_c)}{n\,\cancel{\Omega_c}\,\pi} \end{cases} h(\Omega_c)$$

$$h_0 = h_3 = \frac{\sin\left(\frac{-3}{2} \times \frac{\pi}{2}\right)}{-\frac{3}{2}\pi} = \frac{\sin\left(-\frac{3\pi}{4}\right)}{-\frac{3\pi}{2}} = 0{,}15$$

$$h_2 = h_1 = \frac{\sin\left(-\frac{1 \times \pi}{4}\right)}{\frac{\pi}{2}}$$



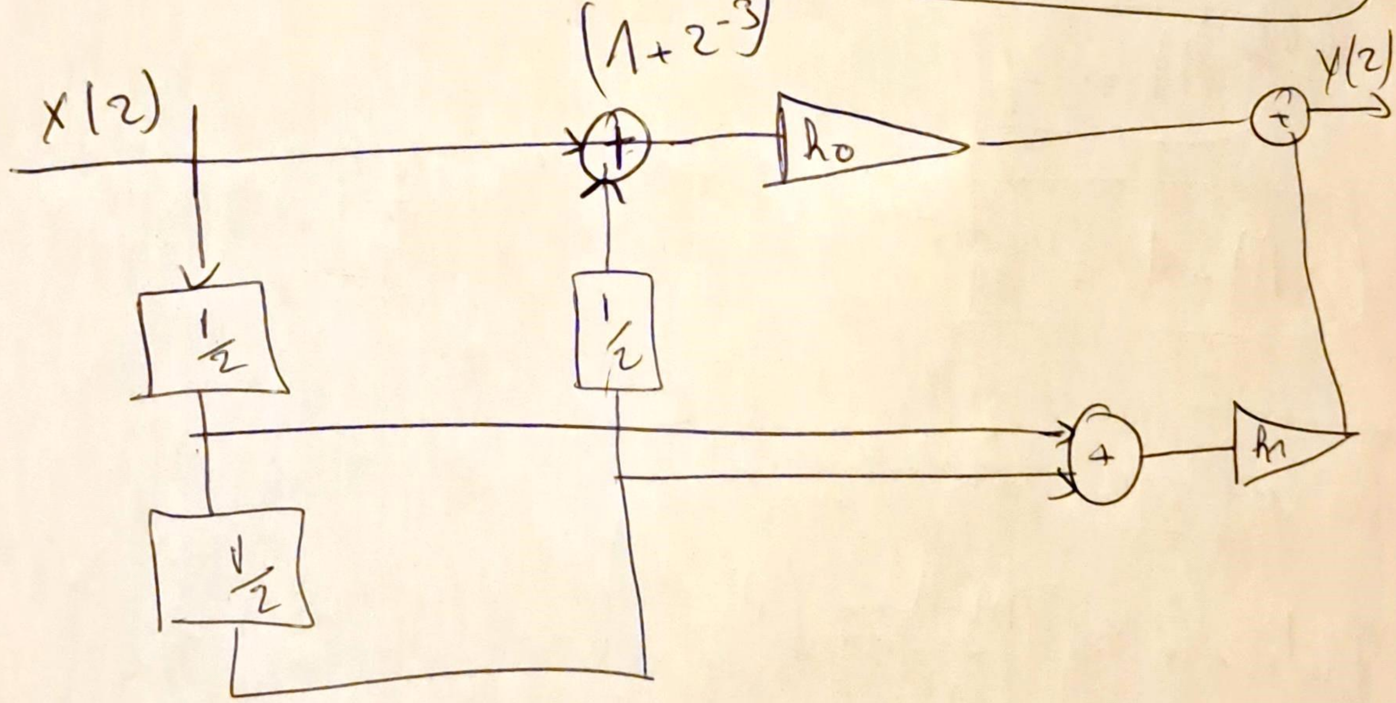
$$h_m = h_{(M-1)-m}$$

$$h_0 = h_3$$

$$h_1 = h_2$$

$$H[z] = h_0 + h_1(z^{-1}) + h_2(z^{-2}) + h_3 z^{-3}$$

$$\boxed{H[z] = h_0[1+z^{-3}] + h_1[z^{-1}+z^{-2}]}$$



Comparaison :

FIR : (3 retard, 3 add, 2 multi)

SIR : (1 retard, 2 ad, 3 multi)

FIR est plus coûteux que SIR mais plus performant