

## Credit Suisse Case Study

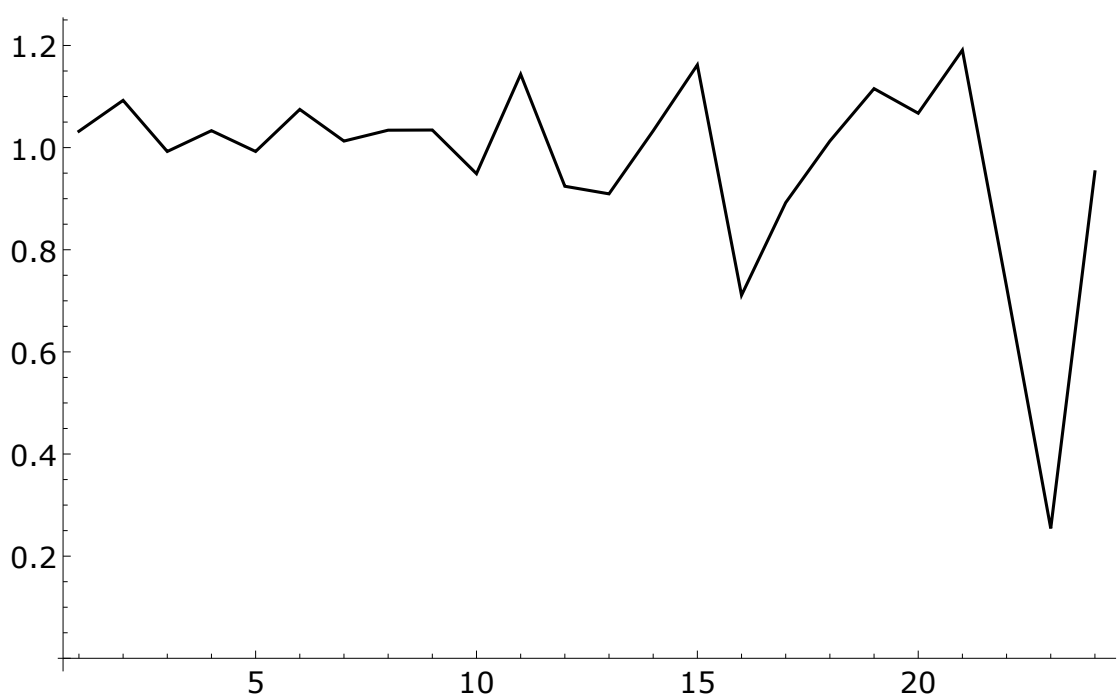
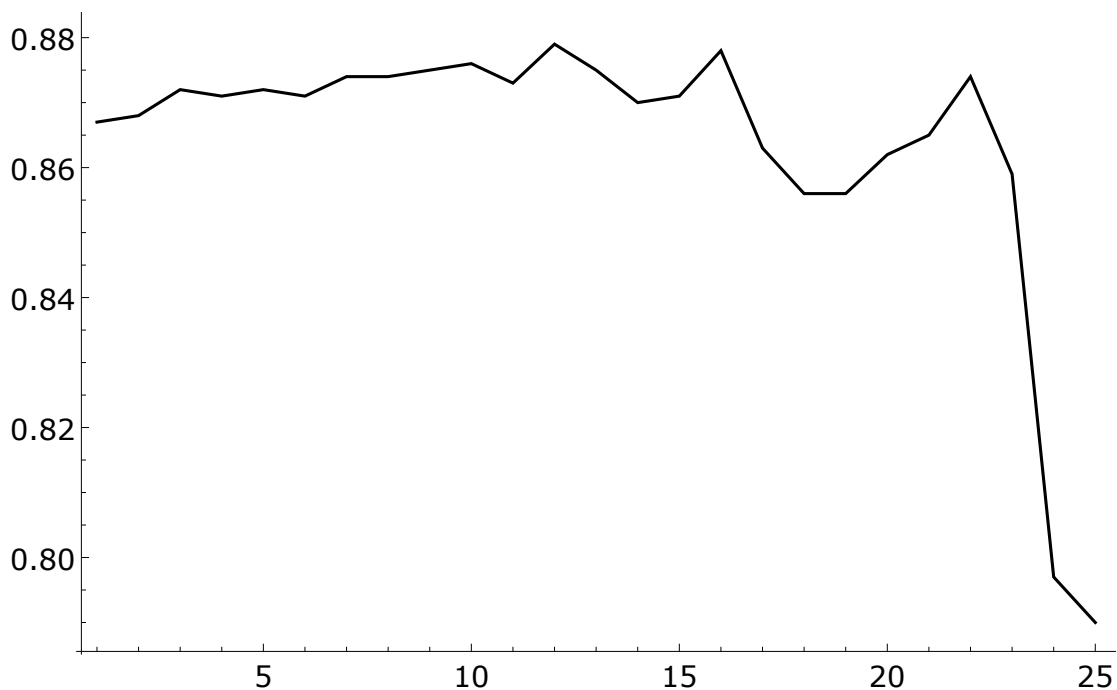
### CET-I ratio parameter estimation

```
DataB = Import["Wolfram Mathematica/Data/Credit_Suisse_Data_17-23.xlsx",  
              [导入] [偏导]  
              {"Data", 2, Table[i, {i, 2, 26}], 4}];  
              [表格]  
DataJ = Differences[Tan[Pi * ((1 - 2 * DataB) / 2)] + Cot[Pi * (1 - DataB[[1]])]];  
      [差分] [正切] [圆周率] [余切] [圆周率]  
(* solvency shocks *)  
UnitRootTest[DataJ] (* stationarity check *)  
[单位根检验]  
% < 0.05  
0.00073546  
True  
  
DataH = DataJ - Quantile[SmoothKernelDistribution[DataJ], 0];  
      [分位数] [平滑核分布]  
(* positive transformation *)  
nH = Length[DataH]  
      [长度]  
24
```

```

dp = ListLinePlot[1 - DataB, PlotStyle -> Black, AxesStyle -> Black,
  [绘制点集的线条] [绘图样式] [黑色] [坐标轴样式] [黑色]
  TicksStyle -> Directive["Label", 15], ImageSize -> Large,
  [刻度样式] [指令] [标签] [图像尺寸] [大]
  PlotRange -> Full]
[绘制范围] [全范围]
dp1 = ListLinePlot[DataH, PlotStyle -> Black, AxesStyle -> Black,
  [绘制点集的线条] [绘图样式] [黑色] [坐标轴样式] [黑色]
  TicksStyle -> Directive["Label", 15], ImageSize -> Large, PlotRange -> Full]
[指令] [标签] [图像尺寸] [大] [绘制范围] [全范围]

```



```

(* Kernel density estimation *)
kernelH = SmoothKernelDistribution[DataH];
           [平滑核分布]
fH[x_] := PDF[kernelH, x]; (* kernel density *)
           [概率密度函数]

(* Model PDFs continuous part *)
f[λ_, α_, β_, t_, x_] :=
  Exp[(-λ) * t] * (λ * β^α * t * x^(α - 1) * (Exp[(-β) * x] / (α - 1)!) *
    [指数形式] [指数形式]
    HypergeometricPFQ[{}, Table[1 + i / α, {i, 1, α}],
    [广义超几何函数] [表格]
    λ * t * (β * (x / α))^α]; (* general PDF *)
                                [概率密度函数]
f0[λ_, β_, t_, x_] := Exp[(-λ) * t] * (Exp[(-β) * x] * Sqrt[λ * β * (t / x)] *
    [指数形式] [指数形式] [平方根]
    BesselI[1, 2 * Sqrt[λ * β * t * x]]); (* relaxed PDF *)
    [平方根] [概率密度函数]

T = 1/4; (* quarterly frequency *)
relaxedest =
  Maximize[{Sum[Log[f0[λ, β, T, DataH[[i]]]], {i, 1, nH}], λ > 0, β > 0},
    [求和] [对数]
    {λ, β}]; (* MLE (relaxed) *)
λhat0 = λ /. relaxedest[[2]];
βhat0 = β /. relaxedest[[2]];
Print[{λhat0, βhat0}]
[打印]
{147.835, 37.9971}

λhat0 > 15 (* if false use a different method for reestimation *)
True

genest = For[α = 1, α <= 5, α++,
  [For循环]
  {est[α] = Maximize[{Sum[Log[f[λ, α, β, T, DataH[[i]]]], {i, 1, nH}],
    [最大点值] [求和] [对数]
    λ > 0, β > 0}, {λ, β}],
  Print[est[α]]}] (* MLE *)
[打印]
{1.94434, {λ → 147.835, β → 37.9971}}
{2.51877, {λ → 116.83, β → 60.0564}}
{2.79354, {λ → 106.477, β → 82.1014}}
{2.95453, {λ → 101.293, β → 104.139}}
{3.06027, {λ → 98.1804, β → 126.174}}

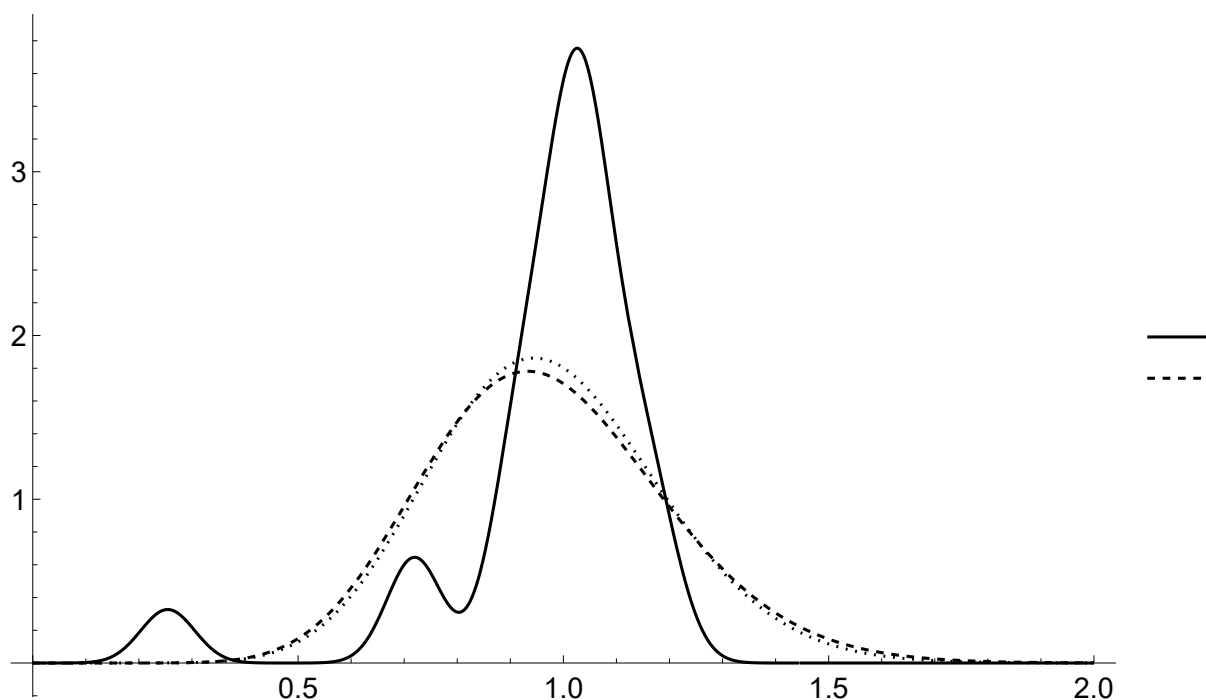
```

```

 $\alpha$ hat =
  Sum[If[est[ $\alpha$ ][[1]] == Max[Table[est[a][[1]], {a, 1, 5}]],  $\alpha$ , 0], { $\alpha$ , 1, 5}];
 $\lambda$ hat = Sum[If[est[ $\alpha$ ][[1]] == Max[Table[est[a][[1]], {a, 1, 5}]],
   $\lambda$  /. est[ $\alpha$ ][[2]], 0], { $\alpha$ , 1, 5}];
 $\beta$ hat = Sum[If[est[ $\alpha$ ][[1]] == Max[Table[est[a][[1]], {a, 1, 5}]],
   $\beta$  /. est[ $\alpha$ ][[2]], 0], { $\alpha$ , 1, 5}];
Print[{ $\lambda$ hat,  $\alpha$ hat,  $\beta$ hat}]
{98.1804, 5, 126.174}

p1 = Plot[{fH[x], f0[ $\lambda$ hat0,  $\beta$ hat0, T, x], f[ $\lambda$ hat,  $\alpha$ hat,  $\beta$ hat, T, x]},
  {x, 0, 2}, PlotStyle -> {Black, Dashed, Dotted},
  PlotTheme -> "Monochrome", TicksStyle -> Directive["Label", 15],
  PlotLegends -> {"Fitted", "Fitted ( $\alpha = 1$ )"}, ImageSize -> Large, PlotRange -> Full]

```



## Stock price parameter estimation

```

Rets = Import["Wolfram Mathematica/Data/Credit_Suisse_Data_17-23.xlsx",
  {"Data", 1, Table[i, {i, 2, 1583}], 7}];

```

```
nR = Length[Rets]
```

```
|长度
```

```
1582
```

```
dp2 = ListLinePlot[Rets, PlotStyle -> Gray, AxesStyle -> Black,
```

```
|绘制点集的线条
```

```
|绘图样式
```

```
|灰色
```

```
|坐标轴样式
```

```
|黑色
```

```
TicksStyle -> Directive["Label", 15], ImageSize -> Large, PlotRange -> Full]
```

```
|刻度样式
```

```
|指令
```

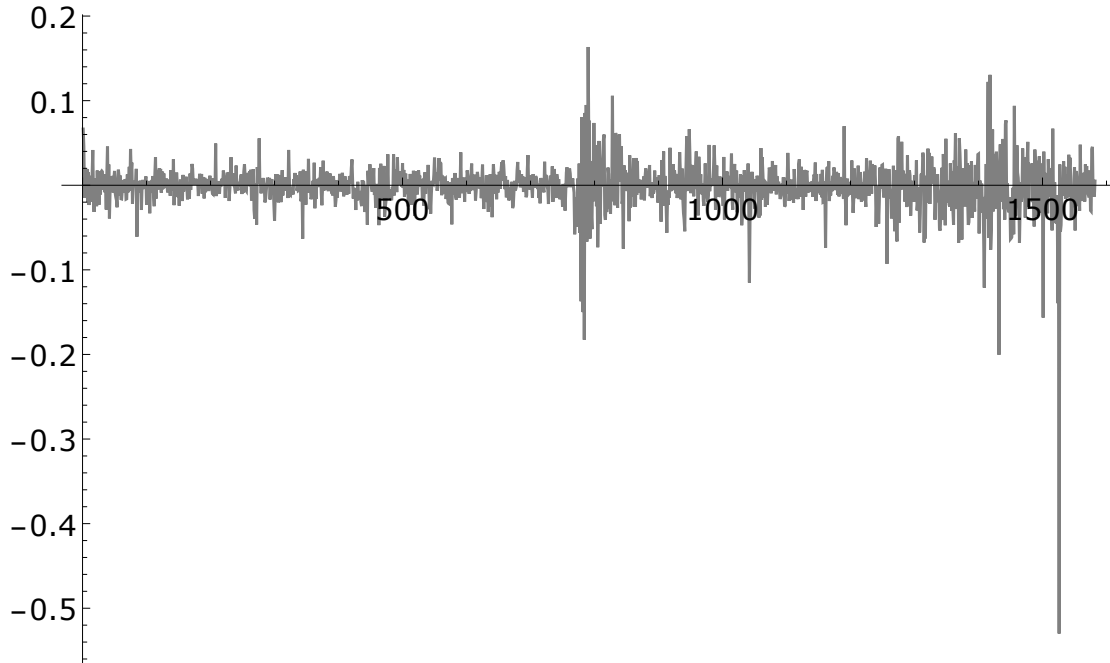
```
|标签
```

```
|图像尺寸
```

```
|大
```

```
|绘制范围
```

```
|全范围
```



```
UnitRootTest[Rets] (* stationarity check *)
```

```
|单位根检验
```

```
% < 0.05
```

```
 $7.9505 \times 10^{-19}$ 
```

```
True
```

```

(* Kernel density estimation *)
kernelX = SmoothKernelDistribution[Rets];
    [平滑核分布]
fRets[x_] := PDF[kernelX, x]; (* kernel density *)
    [概率密度函数]
(* Model PDFs (at least daily freq.) *)
fX[λ1_, α_, β_, μ_, σ_, λ2_, μV_, σV_, η_, Δ_, x_] :=
Module[{v = μ + μV / Δ, u = Sqrt[σ^2 + σV^2 / Δ]},
    [平方根]
Ψ = (Δ^((α - 1) / 2) / Sqrt[2 * Pi * u^2]) * (β * (u / η))^α * Exp[-(x - v * Δ)^2 /
    [平方根] [圆周率] [指数形式]
    (2 * u^2 * Δ) + (η * (x - v * Δ) + β * u^2 * Δ)^2 / ((2 * η * u)^2 * Δ)] *
    ParabolicCylinderD[-α, (η * (x - v * Δ) + β * u^2 * Δ) / (η * u * Sqrt[Δ])];
    [抛物柱面函数] [平方根]
λ1 * λ2 * Δ^2 * Ψ + λ2 * Δ * ((1 - λ1 * Δ) / Sqrt[2 * Pi * u^2 * Δ]) *
    [平方根] [圆周率]
    Exp[-(x - v * Δ)^2 / (2 * u^2 * Δ)] + λ1 * Δ * (1 - λ2 * Δ) * Ψ +
    [指数形式]
    (1 - λ1 * Δ) * ((1 - λ2 * Δ) / Sqrt[2 * Pi * σ^2 * Δ]) *
    [平方根] [圆周率]
    Exp[-(x - μ * Δ)^2 / (2 * σ^2 * Δ)]];
    [指数形式]

Δt = 1 / 252; (* daily frequency *)
λ1hat = λhat;
genretest =
Maximize[{Sum[Log[fX[λ1hat, αhat, βhat, μ, σ, λ2, μV, σV, η, Δt, Rets[[i]]]],
    [最大点值] [求和] [对数]
    {i, 1, nR}], -1 < μ < 1, 0 < σ < 1,
    λ2 > 0, -1 < μV < 1, 0 < σV < 1, 0 < η}, {μ, σ, λ2, μV, σV, η}]
(* constrained MLE *)

{3744.67, {μ → -0.123329, σ → 0.202993,
    λ2 → 2.31387, μV → 0.067659, σV → 0.0179964, η → 1.79029}}

μhat = μ /. genretest[[2]];
σhat = σ /. genretest[[2]];
λ2hat = λ2 /. genretest[[2]];
μVhat = μV /. genretest[[2]];
σVhat = σV /. genretest[[2]];
ηhat = η /. genretest[[2]];
Print[{μhat, σhat, λ2hat, μVhat, σVhat, ηhat}]
[打印]

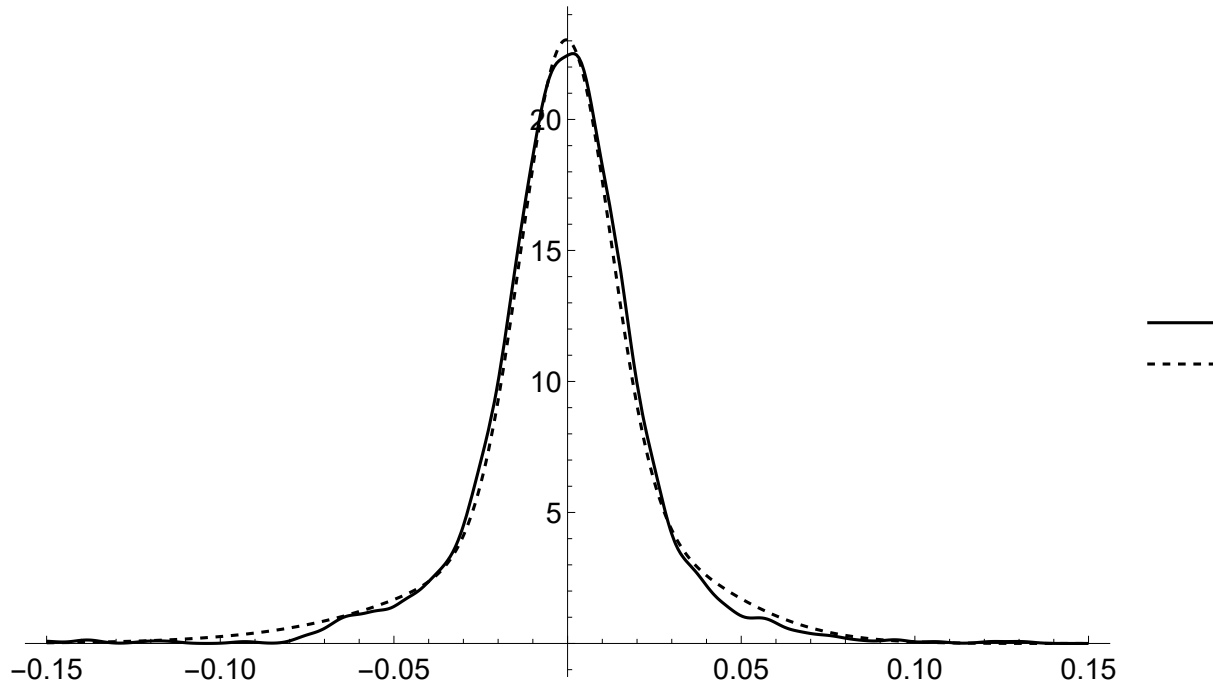
{-0.123329, 0.202993, 2.31387, 0.067659, 0.0179964, 1.79029}

```

```

p2 = Plot[{fRets[x], fX[λ1hat, αhat, βhat, μhat,
  绘图
    ohat, λ2hat, μVhat, σVhat, ηhat, Δt, x]}], {x, -0.15, 0.15},
PlotStyle -> {Black, Dashed}, PlotTheme -> "Monochrome",
  绘图样式      黑色      虚线      绘图主题
  TicksStyle -> Directive["Label", 15],
  刻度样式      指令      标签
PlotLegends -> {"Kernel", "Fitted"}, ImageSize -> Large, PlotRange -> Full]
  绘图的图例      图像尺寸      大      绘制范围      全范围
(* density comparison *)

```



CoCo valuation

```

(* Distributional formulas *)
(* whole PDF of Erlang-CPP *)
    概率密度函数
fh[λ_, α_, β_, t_, x_] :=
    Exp[(-λ) * t] * (DiracDelta[x] + λ * β^α * t * x^(α - 1) * (Exp[(-β) * x] / (α - 1)!) *
    指数形式      狄拉克δ函数      指数形式
    HypergeometricPFQ[{}, Table[1 + i / α, {i, 1, α}], λ * t * (β * (x / α))^α]);
    广义超几何函数      表格
(* continuous part of PDF *)
    概率密度函数
fc[λ_, α_, β_, t_, x_] := λ * β^α * t * x^(α - 1) * (Exp[(-λ) * t - β * x] / (α - 1)!) *
    指数形式
    HypergeometricPFQ[{}, Table[1 + i / α, {i, 1, α}],
    表格
    λ * t * (β * (x / α))^α];
(* time derivative of continuous part *)
tdfc[λ_, α_, β_, t_, x_] :=
    λ * β^α * x^(α - 1) * (Exp[(-λ) * t - β * x] / (α - 1)!) *
    指数形式
    (λ * t * ((β * x)^α / Pochhammer[1 + α, α]) *
    波赫汉默
    HypergeometricPFQ[{}, Table[2 + i / α, {i, 1, α}], λ * t * (β * (x / α))^α] -
    表格
    (λ * t - 1) * HypergeometricPFQ[{}, Table[1 + i / α, {i, 1, α}],
    广义超几何函数      表格
    λ * t * (β * (x / α))^α];
(* expectation of negative part of CCpp *)
I1[λ_, α_, β_, (t_)?NumericQ] := I1[λ, α, β, t] =
    数值量判定
    NIntegrate[(1 - β * (y / (λ * α * t))) * fc[λ, α, β, t, y], {y, 0, λ * α * (t / β)},
    数值积分
    AccuracyGoal -> 3];
    准确度目标
(* a rewarding probability *)
I2[λ_, α_, β_, (t_)?NumericQ, (x_)?NumericQ] := I2[λ, α, β, t, x] =
    数值量判定      数值量判定
    NIntegrate[tdfc[λ, α, β, t, y], {y, 0, x}, AccuracyGoal -> 3];
    数值积分      准确度目标
(* supremum probability of CCpp *)
SupPr[λ_, α_, β_, t_, x_] :=
    (1 - Exp[(-λ) * t] - NIntegrate[fc[λ, α, β, t, y], {y, 0, x + λ * α * (t / β)}]) +
    指数形式      数值积分
    NIntegrate[I1[λ, α, β, t - s] *
    数值积分
    ((β / (λ * α)) * (I2[λ, α, β, s, x + λ * α * (s / β)] - λ * Exp[(-λ) * s]) +
    指数形式
    fc[λ, α, β, s, x + λ * α * (s / β)]), {s, 0, t}, AccuracyGoal -> 3];
    准确度目标

```



```

(* Simplified distributional formulas:  $\alpha=1$  *)
fh0[λ_, β_, t_, x_] := Exp[(-λ) * t] * (DiracDelta[x] +
    [指数形式] [狄拉克δ函数]
    Exp[(-β) * x] * Sqrt[λ * β * (t / x)] * BesselI[1, 2 * Sqrt[λ * β * t * x]]);
    [平方根] [第一类修正贝塞... [平方根]
fc0[λ_, β_, t_, x_] := Exp[(-λ) * t - β * x] * Sqrt[λ * β * (t / x)] *
    [指数形式] [平方根]
    BesselI[1, 2 * Sqrt[λ * β * t * x]];
    [第一类修正贝塞... [平方根]
tdfc0[λ_, β_, t_, x_] := λ * β * Exp[(-λ) * t - β * x] *
    [指数形式]
    (Hypergeometric0F1Regularized[1, λ * β * t * x] -
    [正则化的合流超几何函数0F1]
    λ * t * Hypergeometric0F1Regularized[2, λ * β * t * x]);
    [正则化的合流超几何函数0F1]
I10[λ_, β_, (t_)?NumericQ] :=
    [数值量判定]
    I1[λ, β, t] = NIntegrate[(1 - β * (x / (λ * t))) * fc0[λ, β, t, x],
    [数值积分]
    {x, 0, λ * (t / β)}, AccuracyGoal -> 3];
    [准确度目标]
I20[λ_, β_, t_, (x_)?NumericQ] := I2[λ, β, t, x] =
    [数值量判定]
    NIntegrate[tdfc0[λ, β, t, y], {y, 0, x}, AccuracyGoal -> 3];
    [数值积分] [准确度目标]
SupPr0[λ_, β_, t_, x_] :=
    (1 - Exp[(-λ) * t] - NIntegrate[fc0[λ, β, t, y], {y, 0, x + λ * (t / β)}]) +
    [指数形式] [数值积分]
    NIntegrate[I10[λ, β, t - s] *
    [数值积分]
    ((β / λ) * (I20[λ, β, s, x + λ * (s / β)] - λ * Exp[(-λ) * s]) +
    [指数形式]
    fc0[λ, β, s, x + λ * (s / β)]), {s, 0, t},
    AccuracyGoal ->
    [准确度目标]
    3];

```

Market parameters (with artificial values)

```

W = 0.07; (* watermark *)
C0 = 0.11; (* current CET1 ratio *)
S0 = 15; (* current stock price *)
K = 10; (* principal *)
c = 6.75/2; (* semiannual coupons *)
w = 0.75; (* write-down porportion *)
r = 0.0168; (* risk-free rate *)
q = 0.02; (* dividend yield *)
T = 5; (* maturity *)
M = 2*T; (* coupon number *)
m = 0.03; (* net profit *)
l = 0.04; (* loan growth s.t. CET1 ratio equals watermark at infinity *)
n0 = 0.8; (* current one minus CET-1 ratio *)

(* converted trigger barrier *)
Jbar = Tan[Pi * ((1 - 2*W)/2)] + Cot[Pi * (1 - DataB[[1]])]
      [正切][圆周率] [余切][圆周率]
Jbar0 = Jbar;
2.22136

(* Govt. intervention-related functionals *)

```

```

E1[k1_, c1_, t_, u_] := Exp[k1^2 * (t / (2 * c1^2))] *
    Tanh[Sqrt[2 * c1^2 * t^2 * u]] / Sqrt[2 * c1^2 * t^2 * u] - 1) *
    Sqrt[Sech[Sqrt[2 * c1^2 * t^2]]];
(* continuous part expectation *)

ac[μV_, σV_, i_] :=
    Which[i == 1, 1 - (1/2) * Erfc[(μV + σV) / Sqrt[2 * σV^2]], i == 2,
    (1/2) * (Erfc[(μV + σV) / Sqrt[2 * σV^2]] - Erfc[(μV + 2 * σV) / Sqrt[2 * σV^2]]),
    i == 3,
    (1/2) * (Erfc[(μV + 2 * σV) / Sqrt[2 * σV^2]] -
    Erfc[(μV + 3 * σV) / Sqrt[2 * σV^2]]), i == 4,
    (1/2) * Erfc[(μV + 3 * σV) / Sqrt[2 * σV^2]], True, None];
E2[k2_, c2_, λ2_, λ30_, μV_, σV_, t_, u_] :=
    Exp[(-u) * λ30 * ((1 - Exp[(-k2) * T]) / k2) +
    (λ2 / k2) * Sum[ac[μV, σV, i] * Exp[(-i) * c2 * (u / k2)] * (ExpIntegralEi[
    i * u * (c2 / k2)] - ExpIntegralEi[i * u * c2 * (Exp[(-k2) * t] / k2))],
    {i, 1, 4}] - λ2 * t]; (* jump part expectation *)

```

Write-down CoCo

```

(* write-off *)
Timing[
    Wdz = K * Exp[(-r) * T] * (1 - SupPr[λhat, Ceiling[αhat], βhat, T, Jbar]) +
    Sum[c * Exp[(-r) * k * (T / M)] *
    (1 - SupPr[λhat, Ceiling[αhat], βhat, k * (T / M), Jbar]), {k, 1, M}]]
{2.96875, 41.2376}

```

```

(* write-
  down (quadrature rule applied for fast approximation with NN steps ) *)
NN = 4 * T;
κ1hat = 0.09;
ς1hat = 1.4;
κ2hat = 3.2;
ς2hat = 0.9;
λ30hat = 0; (* artificial values *)
Timing[WD =
  计算时间
  WDZ + (1 - w) * K * Sum[Exp[(-r) * (T / NN) * j] * E1[κ1hat, ς1hat, j * (T / NN), 1] *
    求和 指数形式
    E2[κ2hat, ς2hat, λ2hat, λ30hat, μVhat, σVhat, j * (T / NN), 1] *
    (SupPr[λ1hat, Ceiling[αhat], βhat, (j + 1) * (T / NN), Jbar] -
      向上取整
      SupPr[λ1hat, Ceiling[αhat], βhat, j * (T / NN), Jbar]), {j, 1, NN}]]
  {10.2188, 41.238}

```

Convertible AT1 bond pricing

```

(* quadrature rule applied for fast approximation with NN steps *)
ψ1[p_] := (1 + ηhat * (p / βhat)) ^ (-αhat) - 1;
ψ2[p_] := Exp[μVhat + σVhat^2 / 2] - 1; (* exponential correcting terms *)
  指数形式
Q[p_] := p * q + (1 - p) * r + (1 / 2) * p * (1 - p) * σhat^2 +
  λ1hat * (p * ψ1[1] - ψ1[p]) + λ2hat * (p * ψ2[1] - ψ2[p]);
(* power-adjusted dividend *)
CV[p_] :=
  WDZ + w * K * Sum[Exp[(-Q[p]) * (T / NN) * j] * E1[κ1hat + p * σhat * ς1hat, ς1hat,
    求和 指数形式
    j * (T / NN), 1] * E2[κ2hat, ς2hat, λ2hat * (1 + ψ2[p]),
    μVhat + p * σVhat^2, σVhat, j * (T / NN), 1] * (SupPr[λhat * (1 + ψ1[p]),
    Ceiling[αhat], βhat + ηhat * p, (j + 1) * (T / NN), Jbar] -
      向上取整
      SupPr[λhat * (1 + ψ1[p]), Ceiling[αhat], βhat + ηhat * p, j * (T / NN), Jbar]),
    向上取整
    {j, 1, NN}];

```

```
DiscretePlot[CV[p], {p, 0, 1, 0.1}, Joined -> True,
  离散图      连接点      真
  Filling -> None, ImageSize -> Large, PlotRange -> All]
  填补      无      图像尺寸      大      绘制范围      全部
```

