Analysis of Variance (ANOVA)

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Agenda

- 0. Lesson overview
- 1. Two-Sample *t*-Tests
- 2. One-Way ANOVA
- 3. ANOVA with Data from a Randomized Block Design
- 4. ANOVA Post Hoc Tests
- 5. Two-Way ANOVA with Interactions
- 6. Summary

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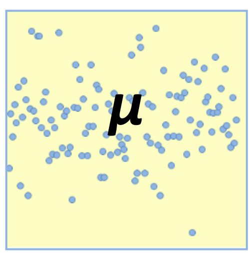
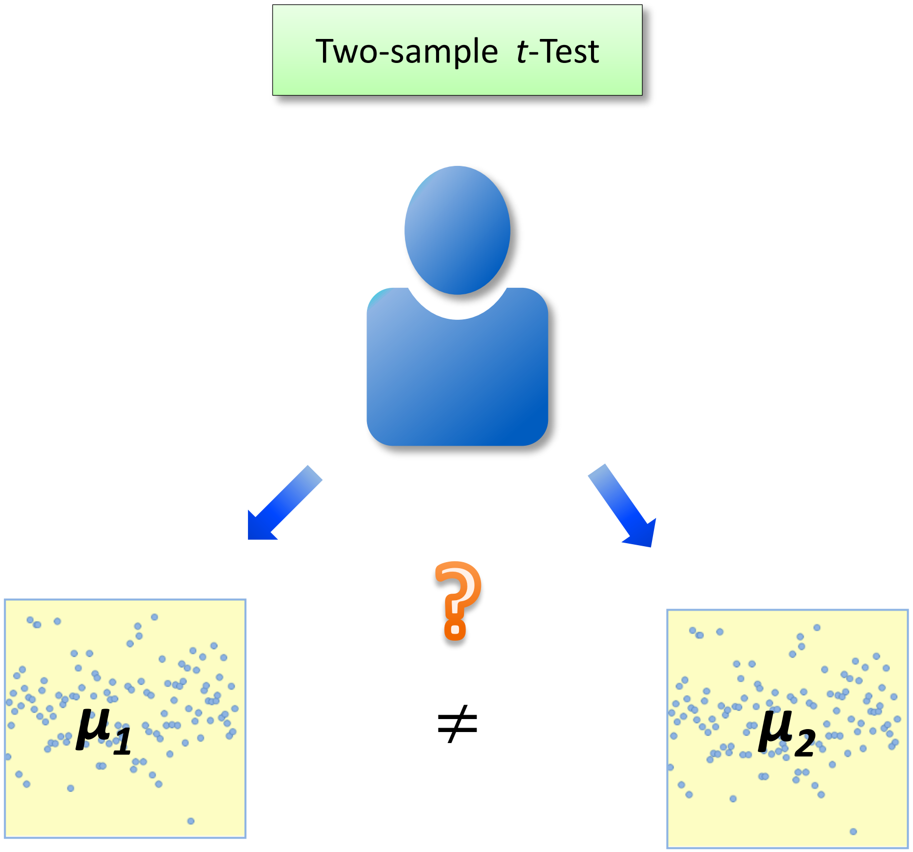
Lesson overview

One sample t-Test

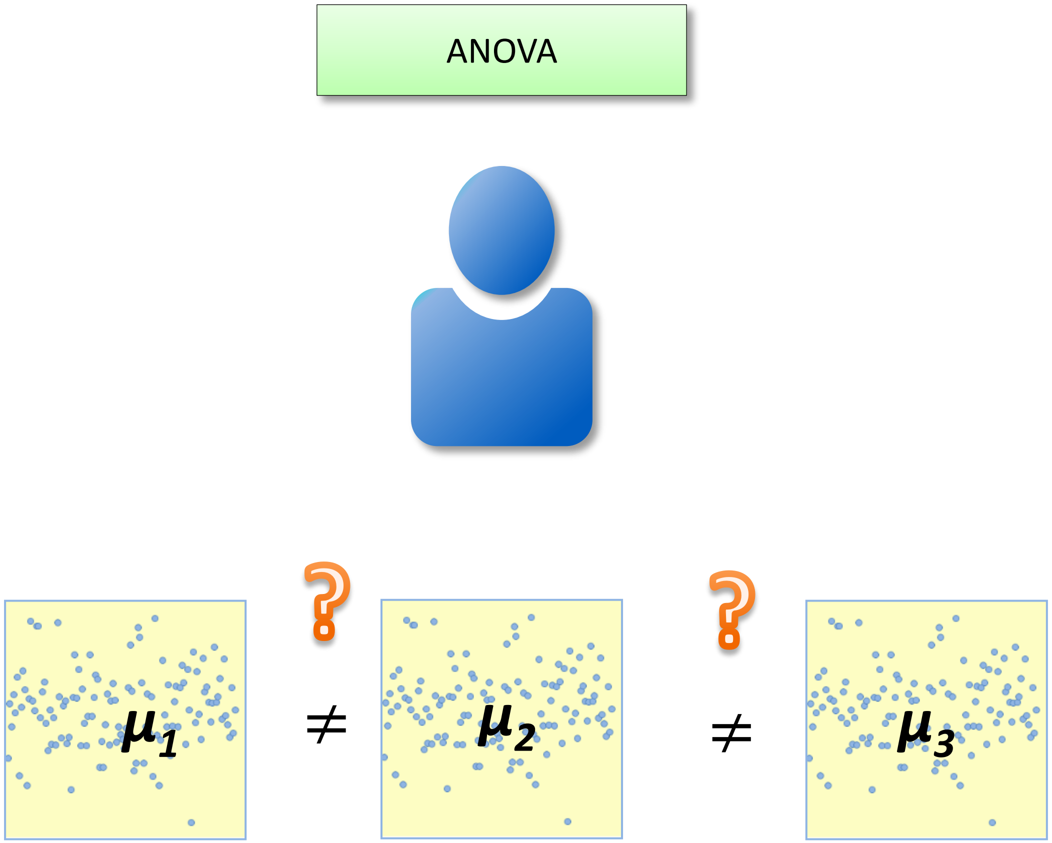


μ_0

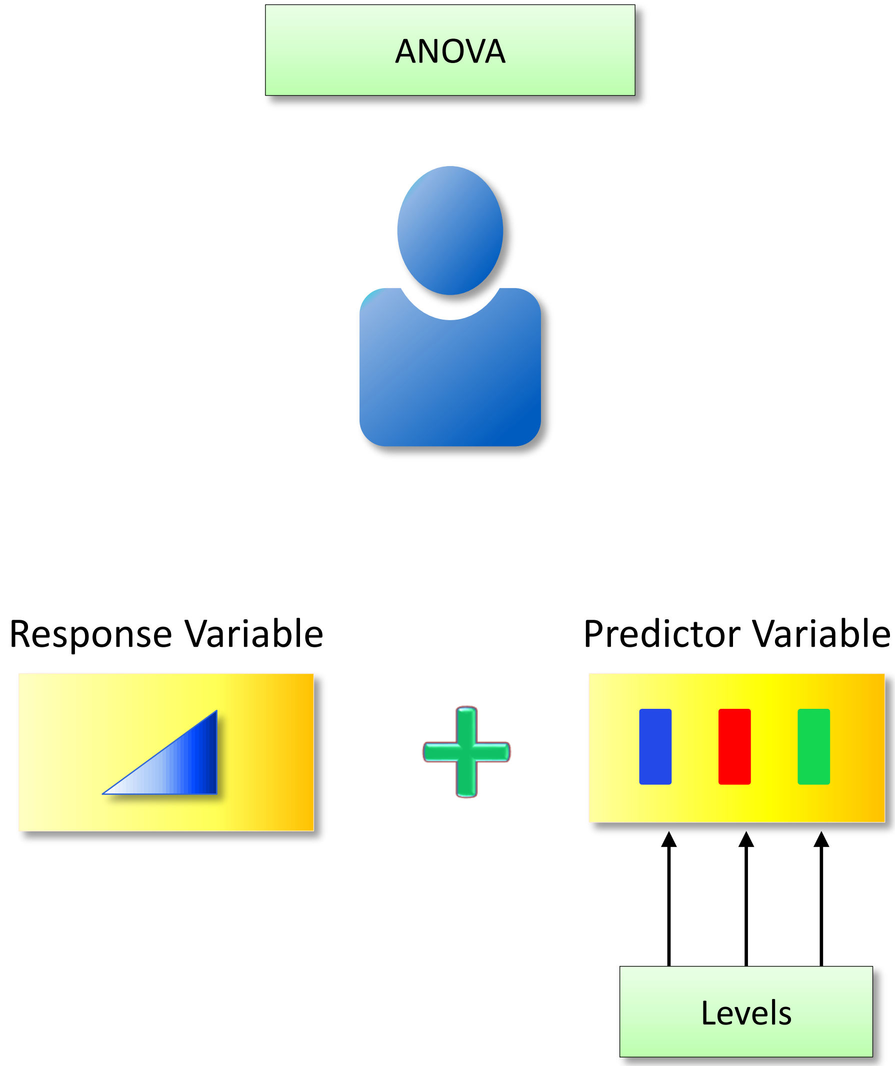
Lesson overview



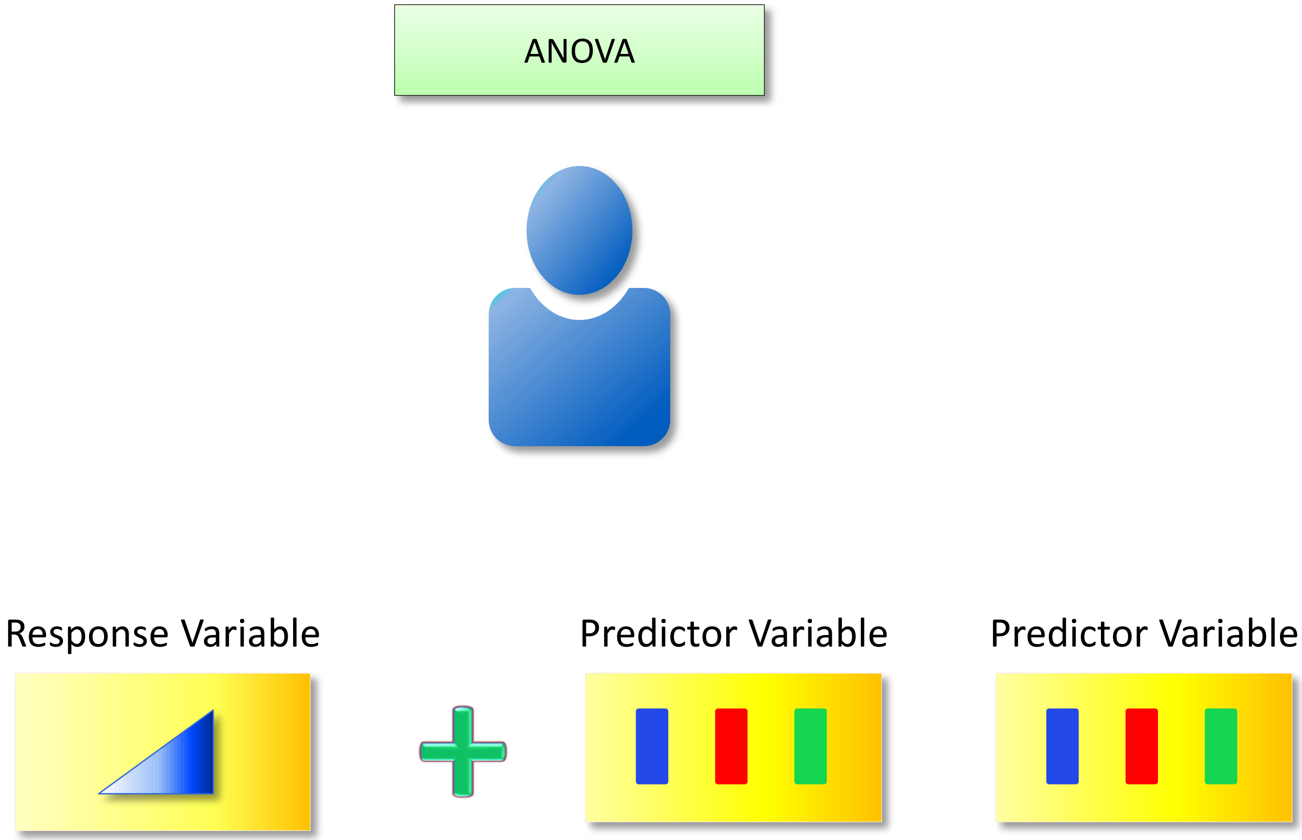
Lesson overview



Lesson overview



Lesson overview



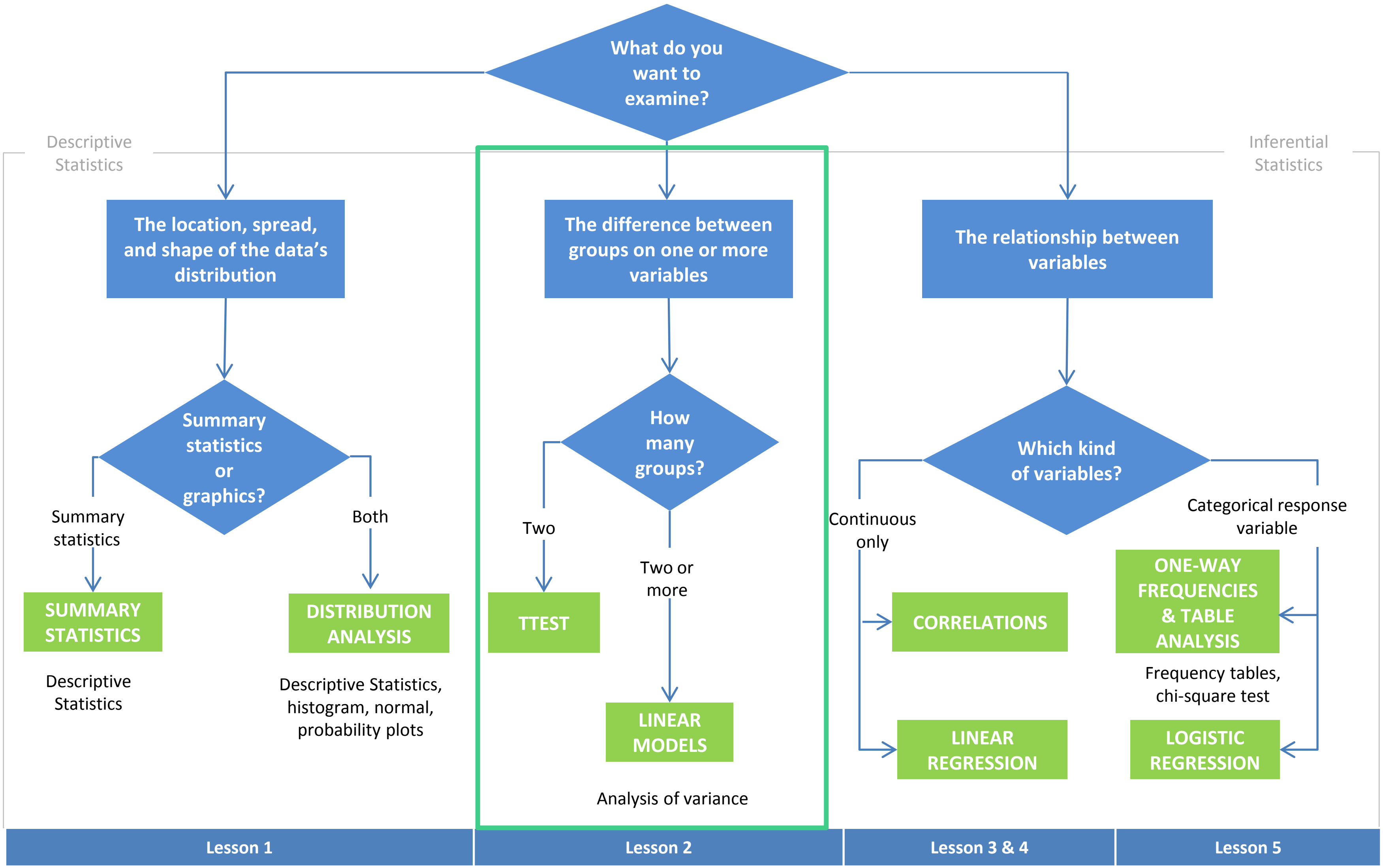
Lesson overview

One sample *t*-Test

Two-sample *t*-Test

ANOVA

Lesson overview



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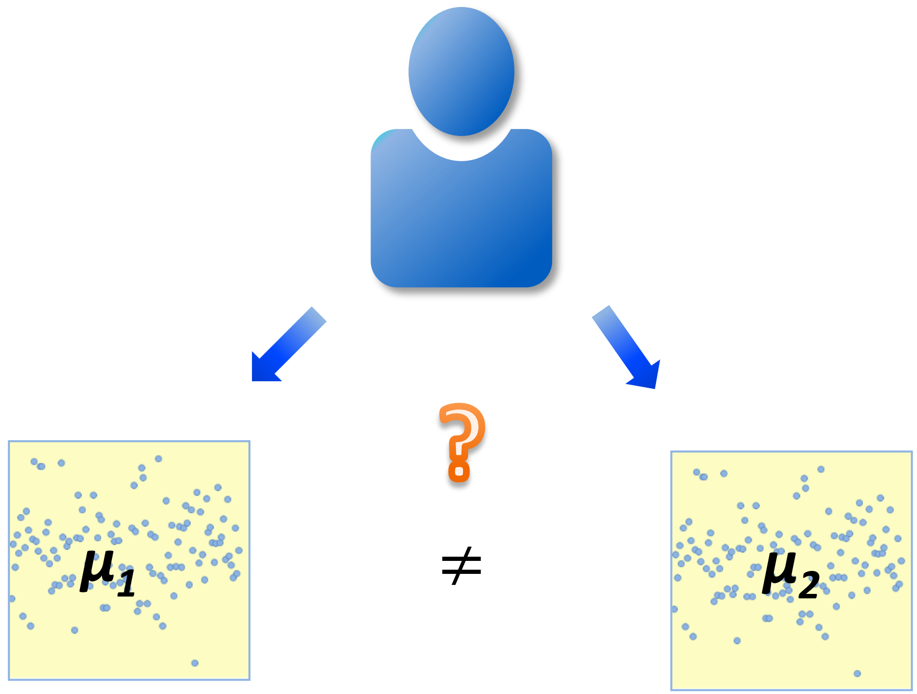
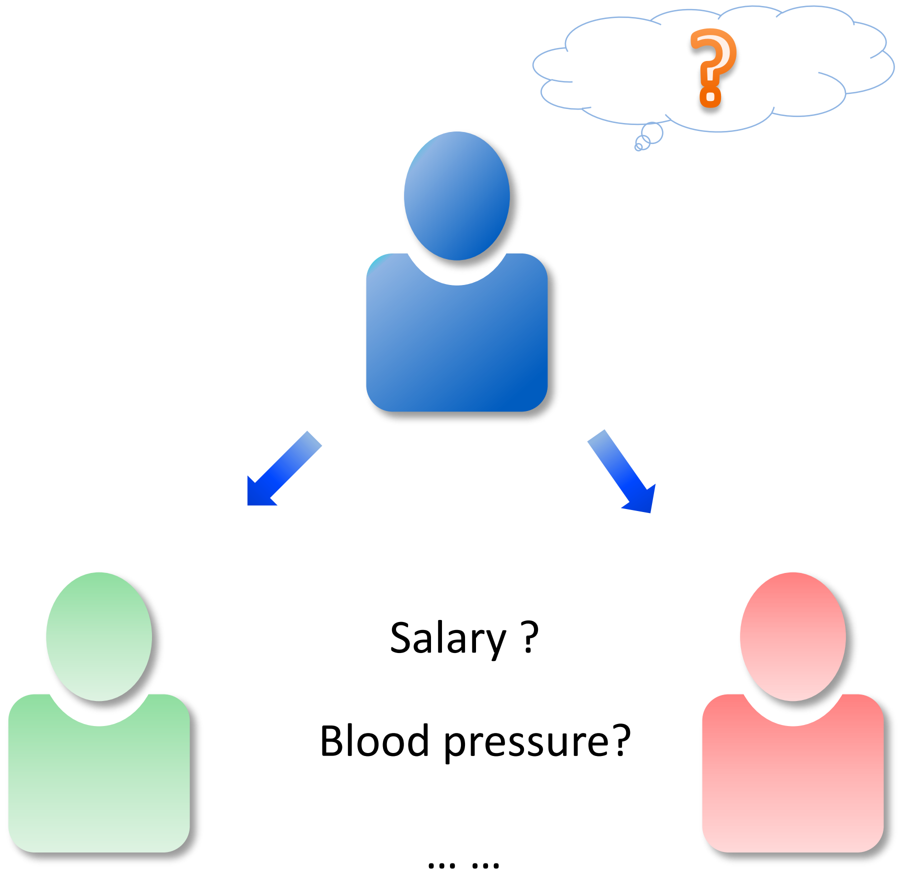
The Two-Sample *t*-Test: Introduction



The Two-Sample *t*-Test: Introduction



The Two-Sample *t*-Test: Introduction

In this topic, we will learn to do the following:

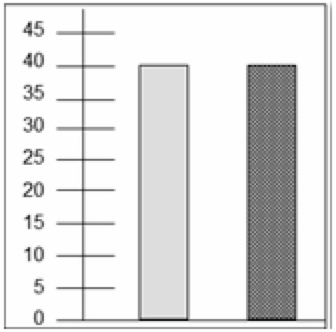
- Analyze differences between two population means using the *t*-Test task
- Verify the assumption of and perform a two-sample *t*-Test
- perform a one-sided *t*-Test

The Two-Sample *t*-Test: Introduction

Two-Sample *t*-Test

H_0: $\boldsymbol{\mu_1 = \mu_2}$

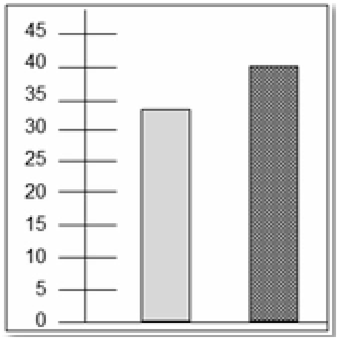
H_0: $\boldsymbol{\mu_1 - \mu_2} = 0$



Two-Sample *t*-Test

H_a: $\boldsymbol{\mu_1 \neq \mu_2}$

H_a: $\boldsymbol{\mu_1 - \mu_2} \neq 0$



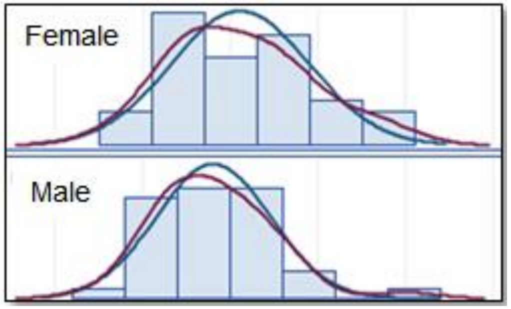
Two-Sample t-Tests: Assumptions

Before we start the analysis, examine the data to verify that the statistical assumption are valid:

- Independent observations:

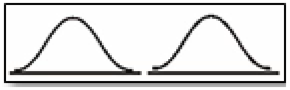
  No information provided by other observations

  No impact between the observations
- Normally distributed data for each group



- Equal variances for each group.

$$\sigma_1{}^2 = \sigma_2{}^2$$



Two-Sample t-Tests: *F*-Test for Equality of Variance

F-Statistic

$$F = \frac{\max({s_1}^2, {s_2}^2)}{min({s_1}^2, {s_2}^2)}$$

H_0: ${\sigma_1}^2 = {\sigma_2}^2$

$F \cong 1$

H_1: ${\sigma_1}^2 \neq {\sigma_2}^2$

F = large value

When the null hypothesis is true, what value will the *F*-Statistic be close to?

Two-Sample t-Tests:
Examining the Equal Variance *t*-Test and *p*-Values

?

Method	Variances	DF	t Value	Pr > |t|
Pooled	Equal	6.0	7.40	0.0003
Satterthwaite	Unequal	5.8	7.40	0.0004

F-Test for Equal Variance

$H_0\!: \sigma_1{}^2 = \sigma_2{}^2$

Equality of Variances				
Method	Num DF	Den DF	F Value	Pr > F
Folded F	3	3	1.51	0.7446

$0.7446 > 0.05$

Two-Sample t-Tests:
Examining the Equal Variance *t*-Test and *p*-Values

$\sigma_1^2 = \sigma_2^2$

Method	Variances	DF	t Value	Pr > |t|
Pooled	Equal	6.0	7.40	0.0003
Satterthwaite	Unequal	5.8	7.40	0.0004

Equality of Variances				
Method	Num DF	Den DF	F Value	Pr > F
Folded F	3	3	1.51	0.7446

0.0003 < 0.05

H_0: $\mu_1 - \mu_2 = 0$

H_a: $\mu_1 - \mu_2 \neq 0$

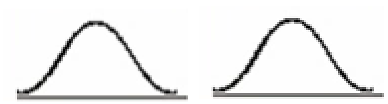
Two-Sample t-Tests:
Examining the Unequal Variance *t*-Test and *p*-Values

?

Method	Variances	DF	t Value	Pr > |t|
Pooled	Equal	13.0	-1.78	0.0979
Satterthwaite	Unequal	11.1	-2.45	0.0320

F-Test for Equal Variance

$H_0\text{: } \sigma_1{}^2 = \sigma_2{}^2$



Equality of Variances				
Method	Num DF	Den DF	F Value	Pr > F
Folded F	9	4	15.28	0.0185

0.0185 < 0.05

Two-Sample t-Tests:
Examining the Unequal Variance *t*-Test and *p*-Values

$\sigma_1^{\,2} \neq \sigma_2^{\,2}$

Method	Variances	DF	t Value	Pr > |t|
Pooled	Equal	13.0	-1.78	0.0979
Satterthwaite	Unequal	11.1	-2.45	0.0320

Equality of Variances				
Method	Num DF	Den DF	F Value	Pr > F
Folded F	9	4	15.28	0.0185

$0.0320 < 0.05$

H_0: $\boldsymbol{\mu_1 - \mu_2} = 0$

H_a: $\boldsymbol{\mu_1 - \mu_2} \neq 0$

Two-Sample t-Tests: *Demo*

Scenario: compare two group's means, girl's and boy's SAT scores

Identify the data

TestScores

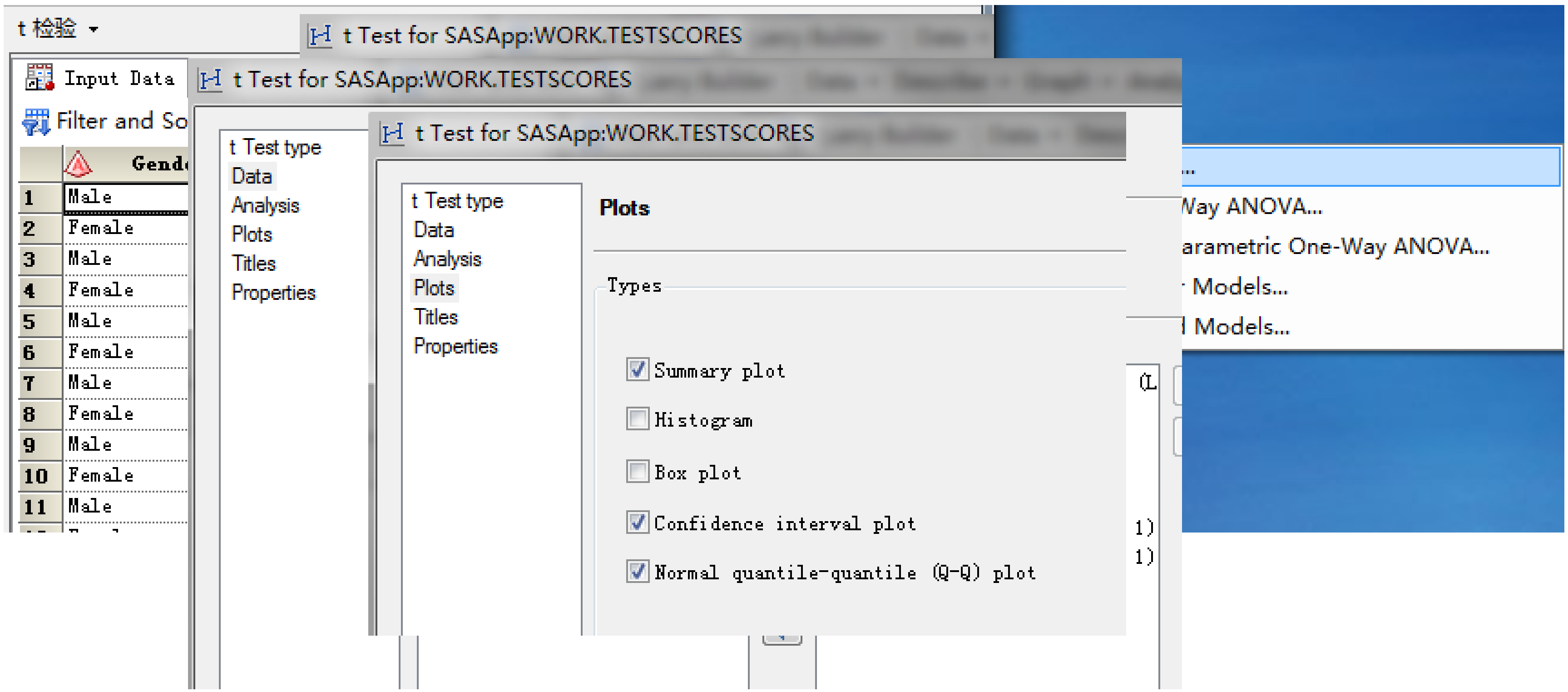
Gender	SATScore	IDNumber
Male	1170	61469897
Female	1090	33081197
Male	1240	68137597
Female	1000	37070397
Male	1210	64608797
Female	970	60714297
Male	1020	16907997
Female	1490	9589297
Male	1200	93891897
Female	1260	85859397

Classification variable?

Continuous variable to analyze?

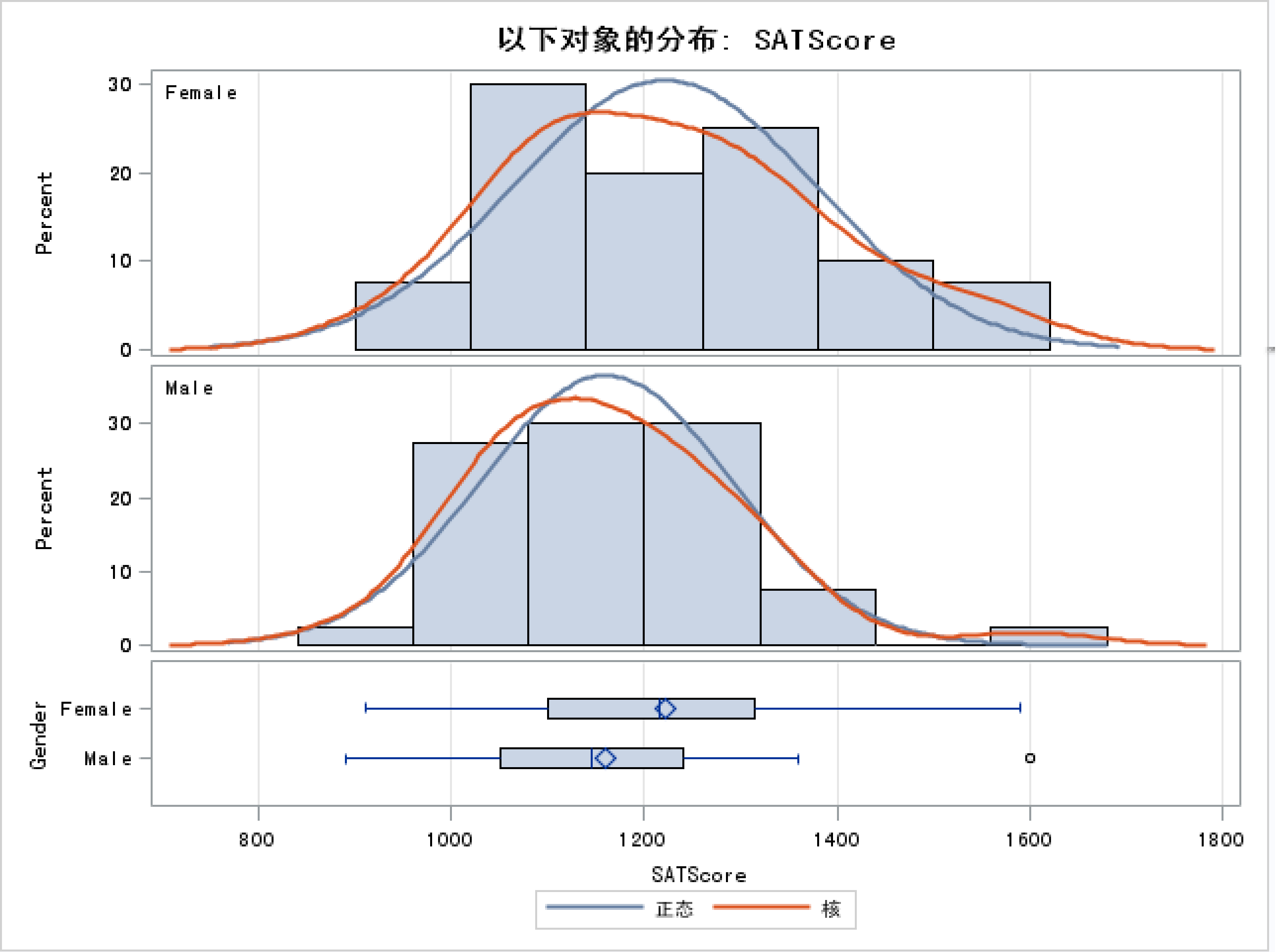
Two-Sample t-Tests: *Demo*

Analyze > ANOVA > t Test



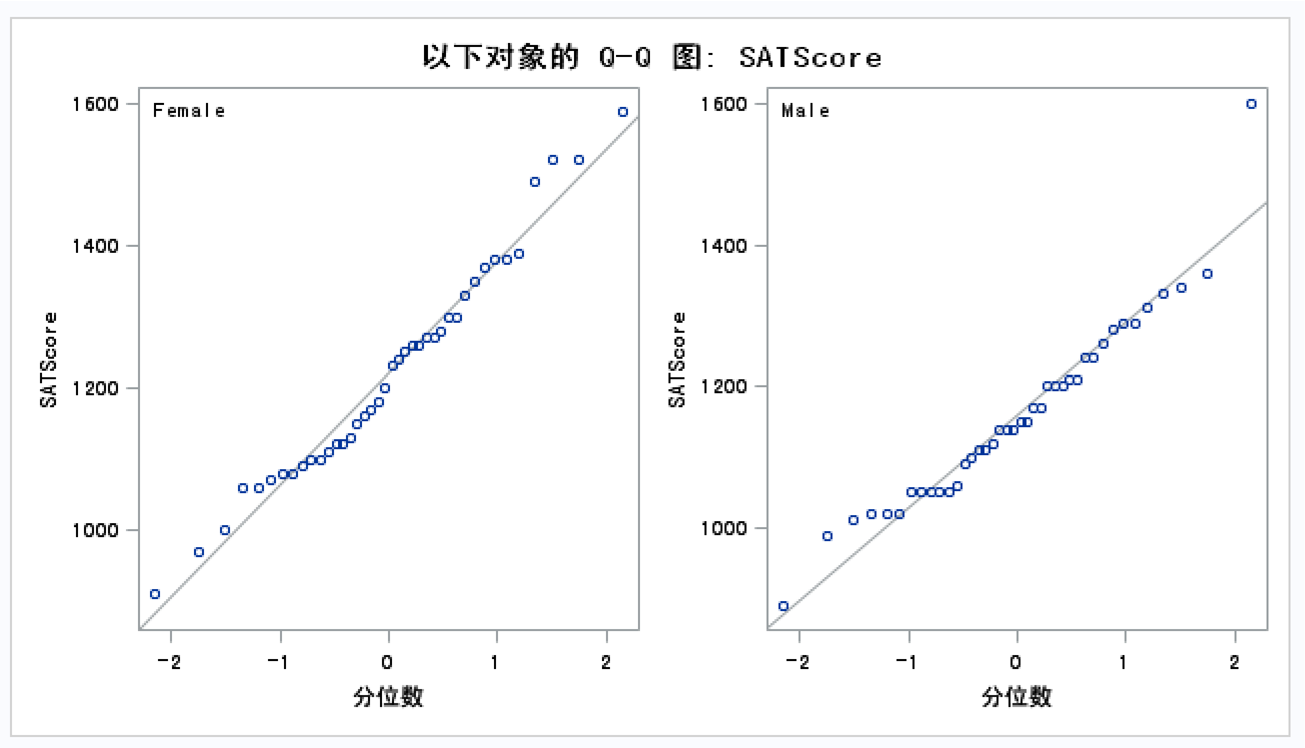
Two-Sample t-Tests: *Demo*

Result interpreting: Normal? ✓



Two-Sample t-Tests: *Demo*

Result interpreting Normal?



Two-Sample t-Tests: *Demo*

Result interpreting

t Test

The TTEST Procedure

Variable: SATScore

Gender	N	均值	标准差	标准误差	最小值	最大值
Female	40	1221.0	157.4	24.8864	910.0	1590.0
Male	40	1160.3	130.9	20.7008	890.0	1600.0
Diff (1-2)		60.7500	144.8	32.3706		

Gender	方法	均值	95% CL 均值		标准差	95% CL 标准差	
Female		1221.0	1170.7	1271.3	157.4	128.9	202.1
Male		1160.3	1118.4	1202.1	130.9	107.2	168.1
Diff (1-2)	Pooled	60.7500	-3.6950	125.2	144.8	125.2	171.7
Diff (1-2)	Satterthwaite	60.7500	-3.7286	125.2			

H_0: $\boldsymbol{\mu_1} - \boldsymbol{\mu_2} = 0$

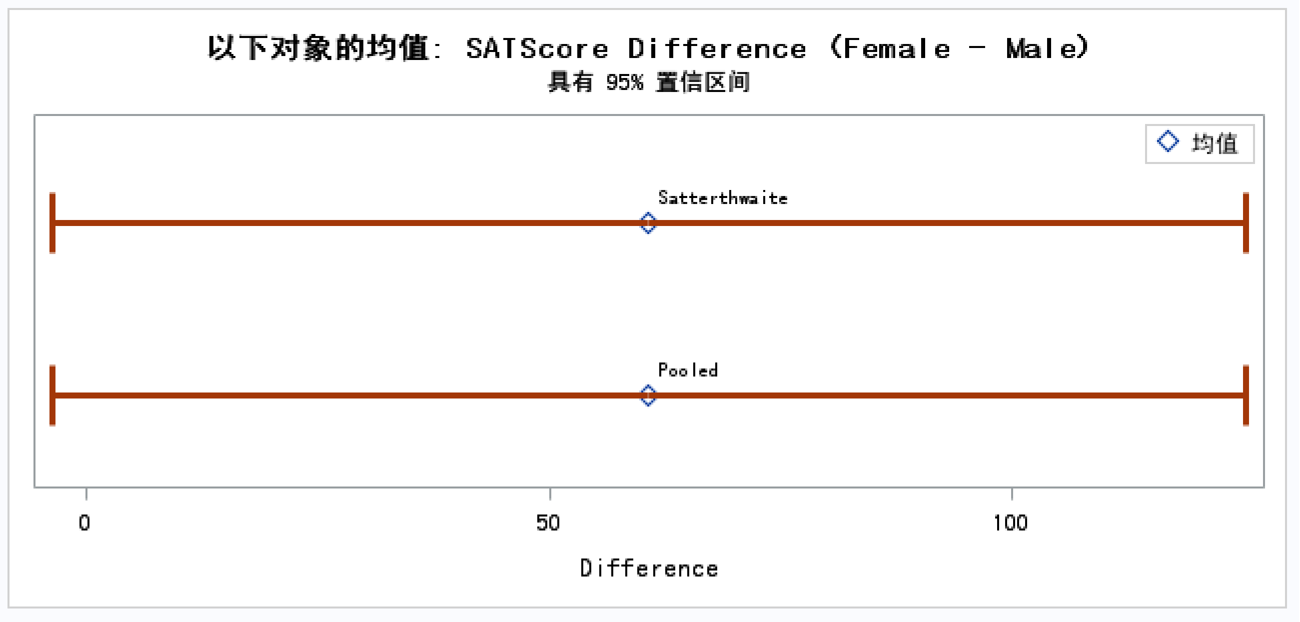
方法	方差	自由度	t 值	Pr > |t|
Pooled	Equal	78	1.88	0.0643
Satterthwaite	Unequal	75.497	1.88	0.0644

H_0: $\sigma_1^2 = \sigma_2^2$

方差等价				
方法	分子自由度	分母自由度	F 值	Pr > F
Folded F	39	39	1.45	0.2545

Two-Sample t-Tests: *Demo*

Result interpreting



The confidence interval for the mean difference (-3.6950, 125.2) includes 0. this implies that you cannot say with 95% confidence that the difference between boys and girls is not zero. Therefore, it also implies that the p-value is greater than 0.05.

Two-Sample t-Tests: One-Sided Tests

Two-sample *t*-Test → H$_a$: $\mu_1 \neq \mu_2$

H$_a$: $\mu_1 > \mu_2$

New drug only have positive effect

H$_a$: $\mu_1 - \mu_2 < 0$

One-Sided Test

Two-Sample t-Tests: One-Sided Tests

One-Sided Test

H_0: $\boldsymbol{\mu_1} \leq \boldsymbol{\mu_2}$

One direction

One-Sided Test

H_0: $\boldsymbol{\mu_1} \geq \boldsymbol{\mu_2}$

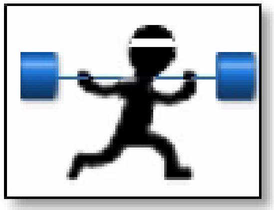
One direction

Two-Sample t-Tests: One-Sided Tests

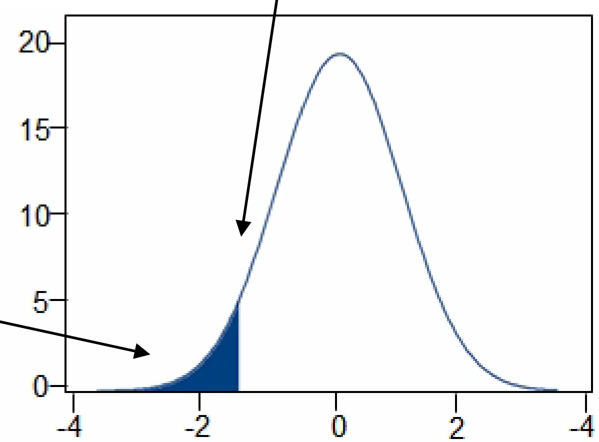
One-Sided Test

H_0: $\boldsymbol{\mu_1} > \boldsymbol{\mu_2}$

H_a: $\boldsymbol{\mu_1} < \boldsymbol{\mu_2}$



Critical region



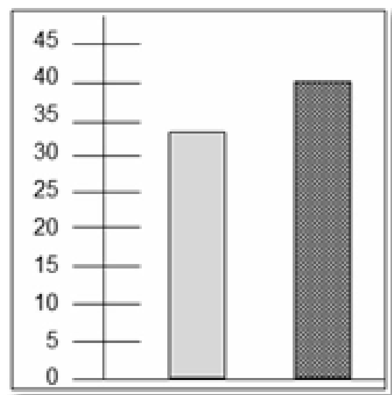
Two-Sample t-Tests: One-Sided Tests

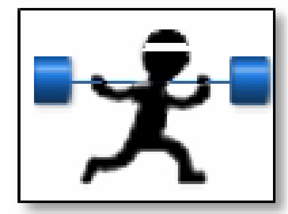
What is Power?

One-Sided Test

H_0: $\mu_1 > \mu_2$

H_a: $\mu_1 < \mu_2$



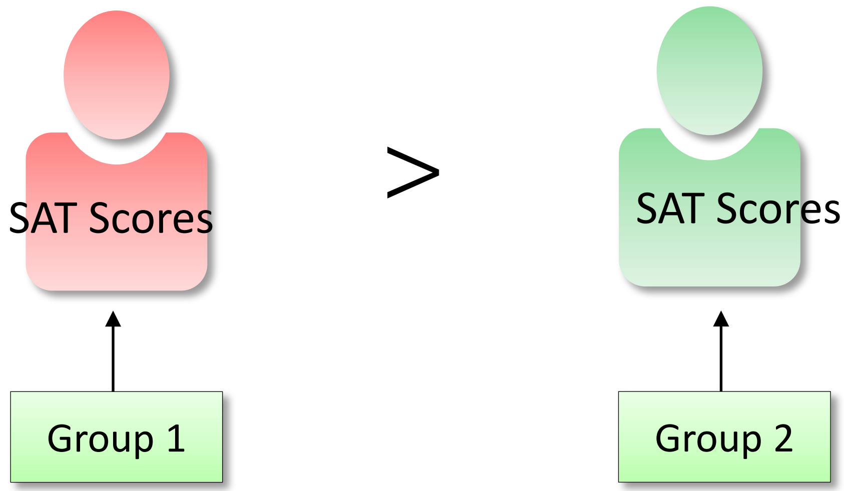


Power is the probability when your test will reject the null hypothesis when the hypothesis is false.

Or the probability when you detect a difference when the difference actually exists.

Two-Sample t-Tests: One-Sided Tests/Scenario

one-sided upper-tailed t-test

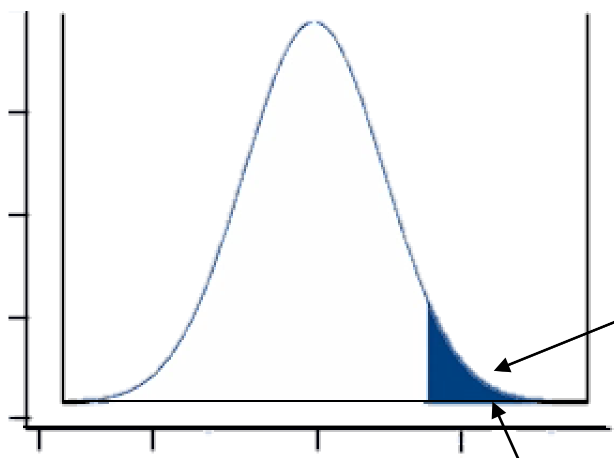


H_0: $\boldsymbol{\mu_1} - \boldsymbol{\mu_2} \leq 0$

H_a: $\boldsymbol{\mu_1} - \boldsymbol{\mu_2} > 0$

Two-Sample t-Tests: One-Sided Tests/Scenario

one-sided upper-tailed t-test



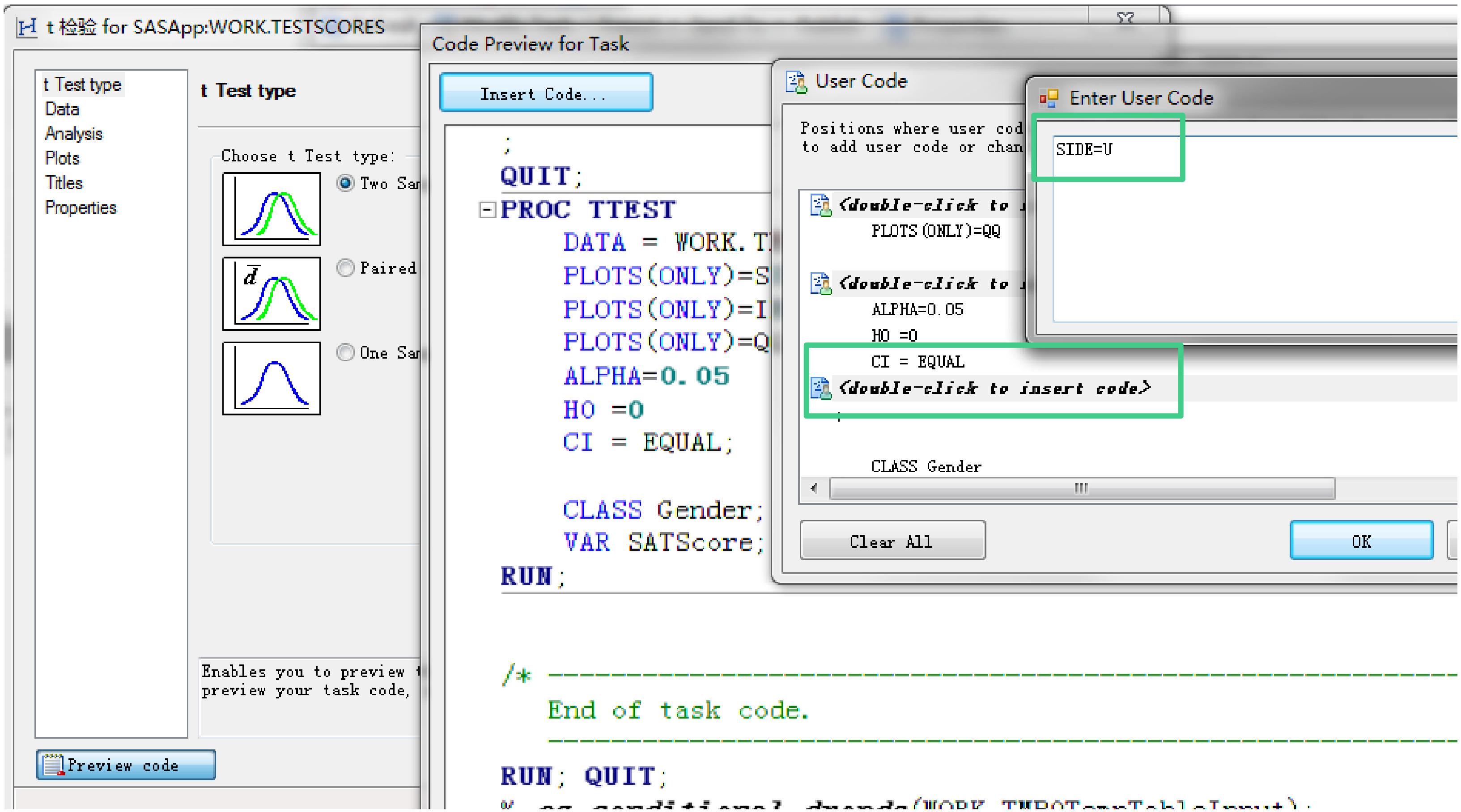
t statistic

H$_0$: $\boldsymbol{\mu_1} - \boldsymbol{\mu_2} \leq 0$

H$_a$: $\boldsymbol{\mu_1} - \boldsymbol{\mu_2} > 0$

Two-Sample t-Tests: One-Sided Tests/Scenario

Analyze > ANOVA > t Test > Two sample > Preview Code> Insert code

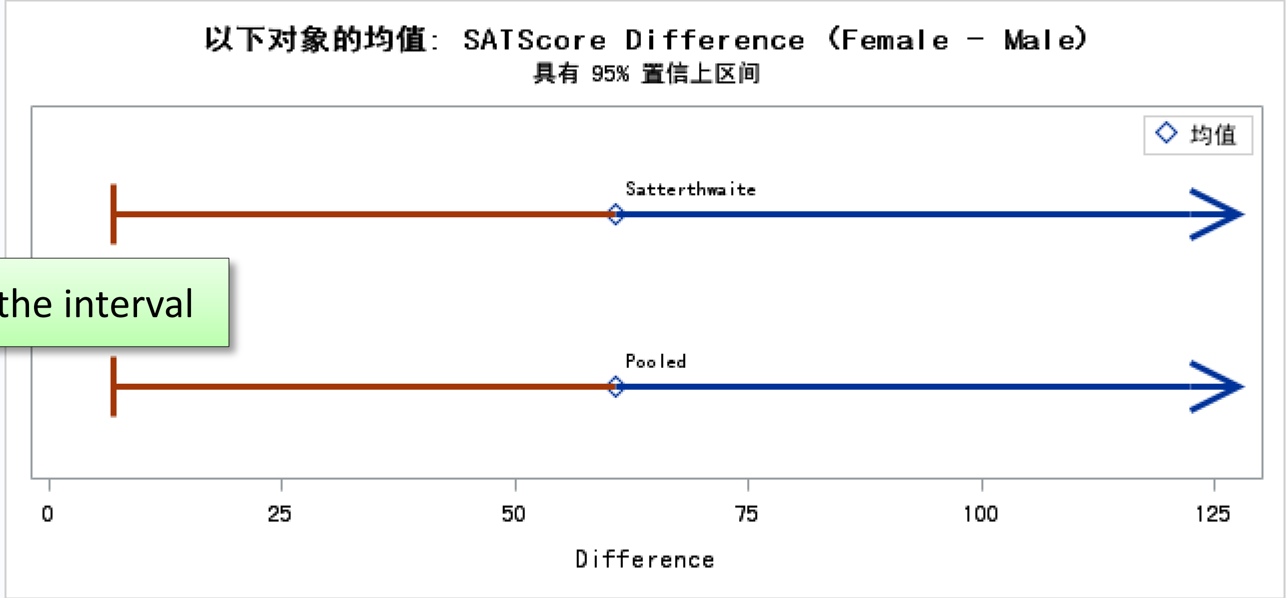


Two-Sample t-Tests: One-Sided Tests/Scenario

Gender	方法	均值	95% CL 均值		标准差	95% CL 标准差	
Female		1221.0	1170.7	1271.3	157.4	128.9	202.1
Male		1160.3	1118.4	1202.1	130.9	107.2	168.1
Diff (1-2)	Pooled	60.7500	6.8651	正无穷大	144.8	125.2	171.7
Diff (1-2)	Satterthwaite	60.7500	6.8436	正无穷大			

方法	方差	自由度	t 值	Pr > t
Pooled	Equal	78	1.88	0.0321
Satterthwaite	Unequal	75.497	1.88	0.0322

0.0321< 0.05



0 is not in the interval

Two-Sample t-Tests

Question 1.

What justifies the choice of a one-sided test versus a two-sided test

a) The need for more statistical power
b) Theoretical and subject-matter considerations
c) The non-significance of a two-sided test
d) The need for an unbiased test statistic

Answer: b

Two-Sample t-Tests

Question 2.

A professor suspects her class is performing below the department average of 73%. She decides to test this claim. Which of the following is the correct alternative hypothesis?

a) $\mu < 0.73$
b) $\mu > 0.73$
c) $\mu \neq 0.73$

Answer: a

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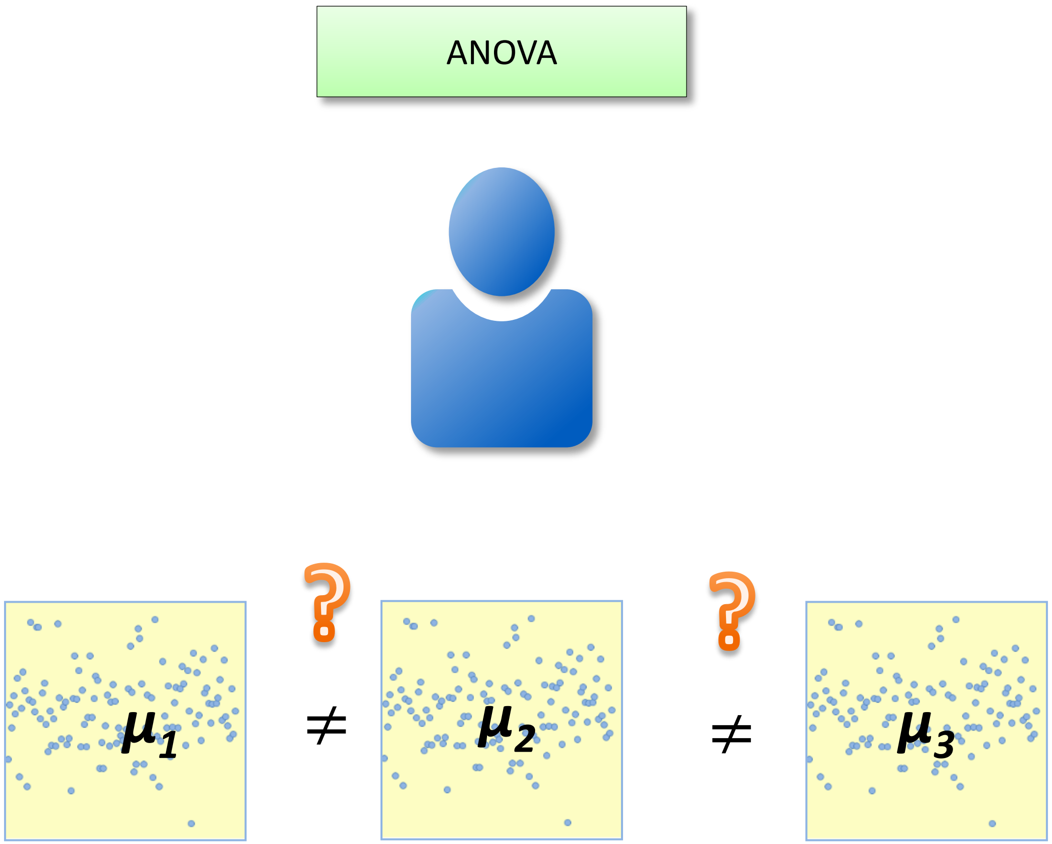
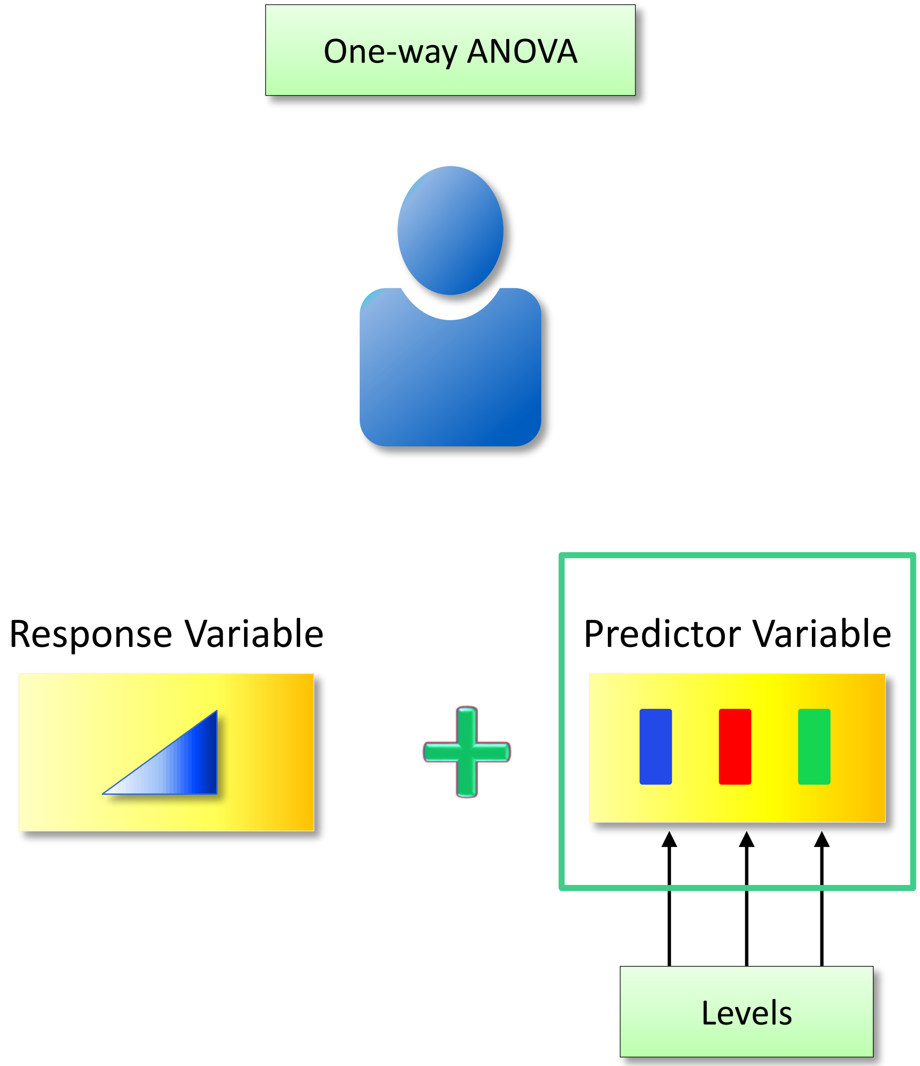
One-Way ANOVA: Introduction



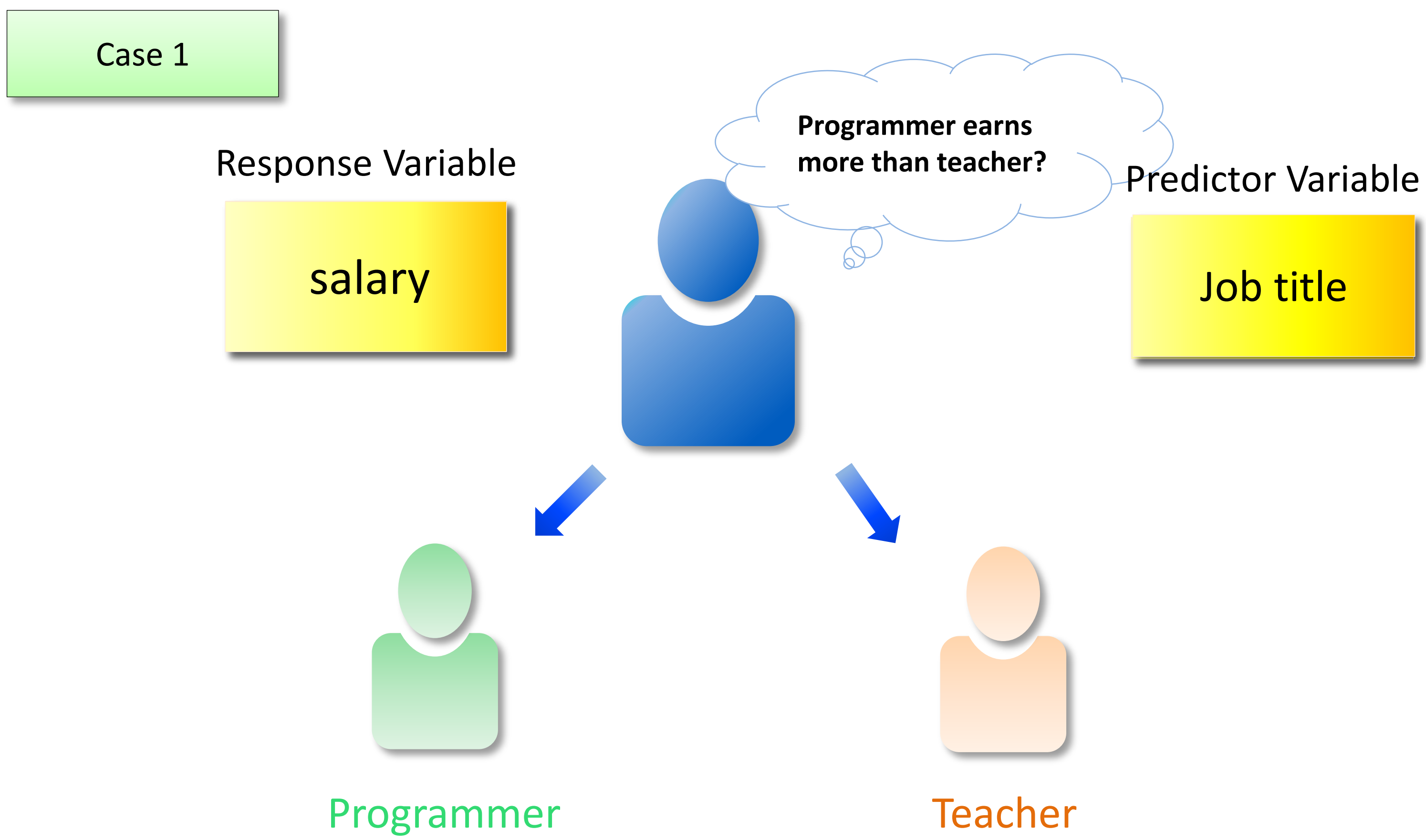
One-Way ANOVA: Objective

- Analyze difference between population means using the Linear Models task
- Verify the assumption of analysis variance.

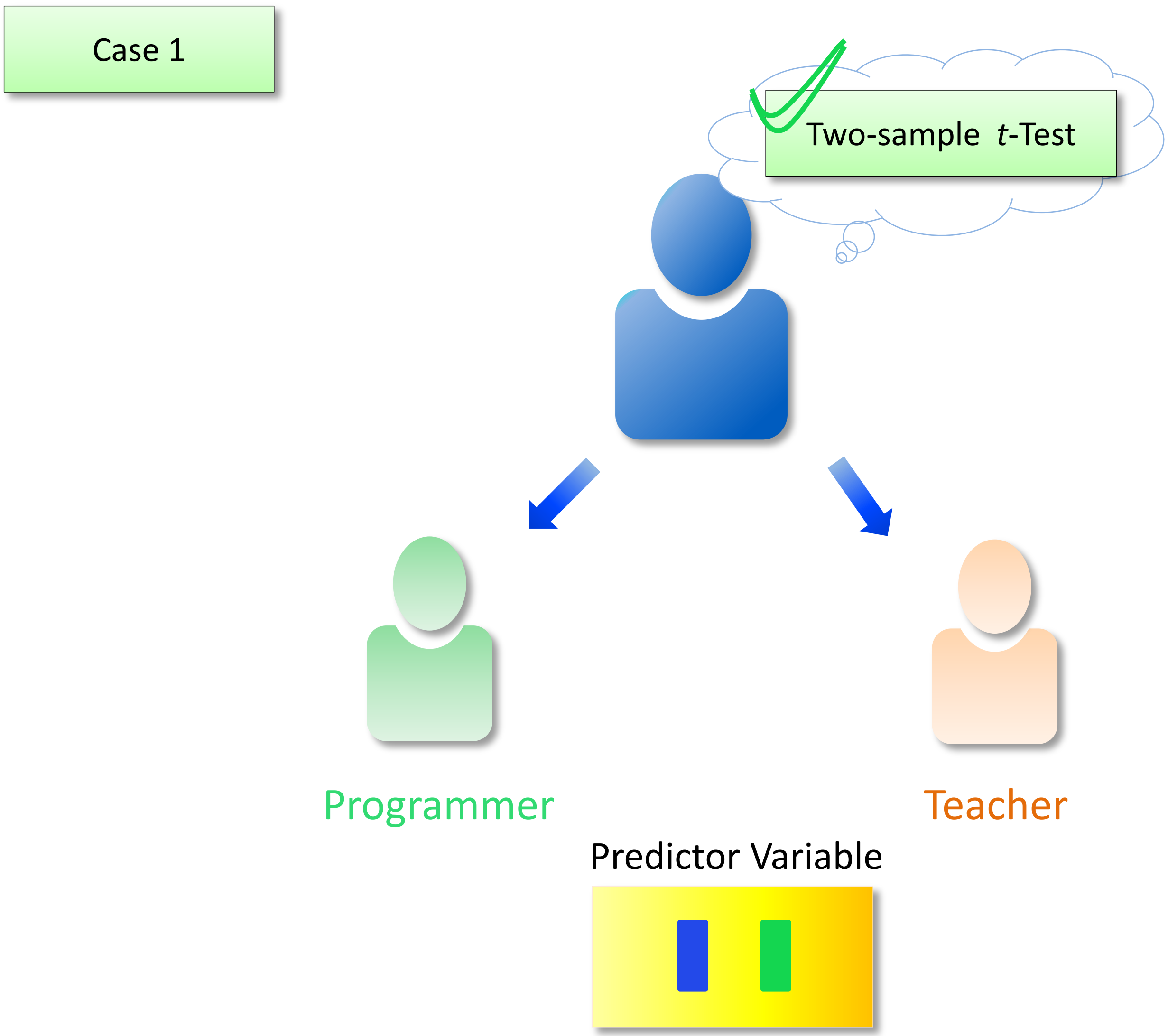
One-Way ANOVA: ANOVA overview



One-Way ANOVA: ANOVA overview



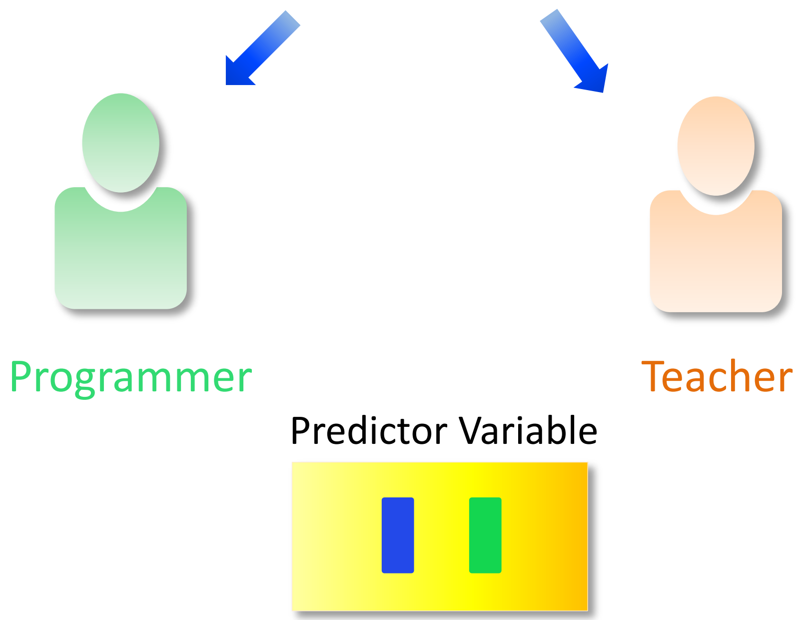
One-Way ANOVA: ANOVA overview



One-Way ANOVA: ANOVA overview

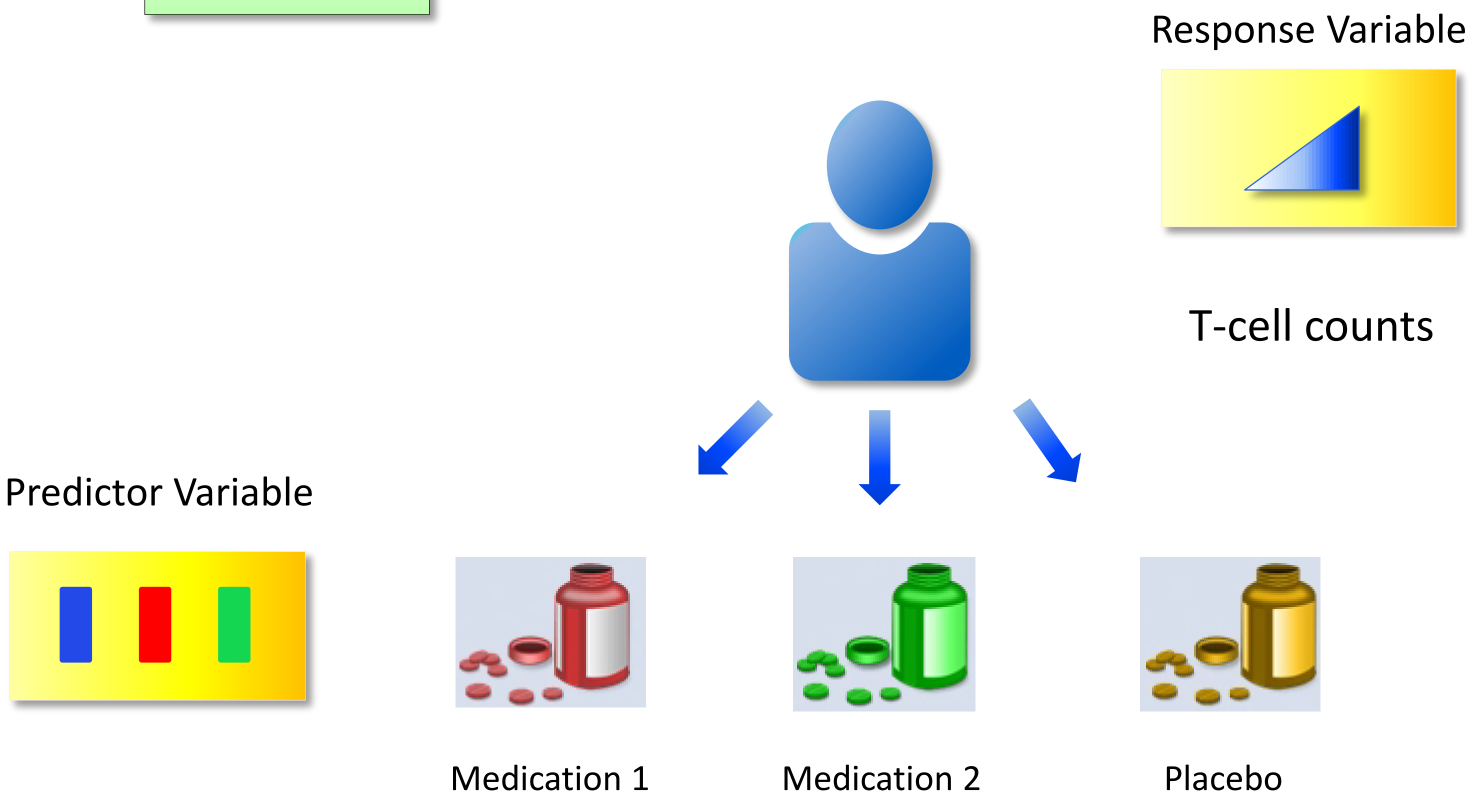
Case 1

F statistic = *t* statistic^2



One-Way ANOVA: ANOVA overview

Case 2



One-Way ANOVA: ANOVA overview

Case 2

Two-sample *t*-Test

Medication 1

Medication 2

Medication 2

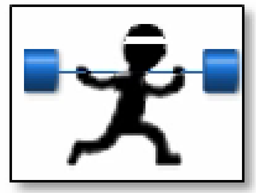
Placebo

Placebo

Medication 1

One-Way ANOVA: ANOVA overview

Case 2



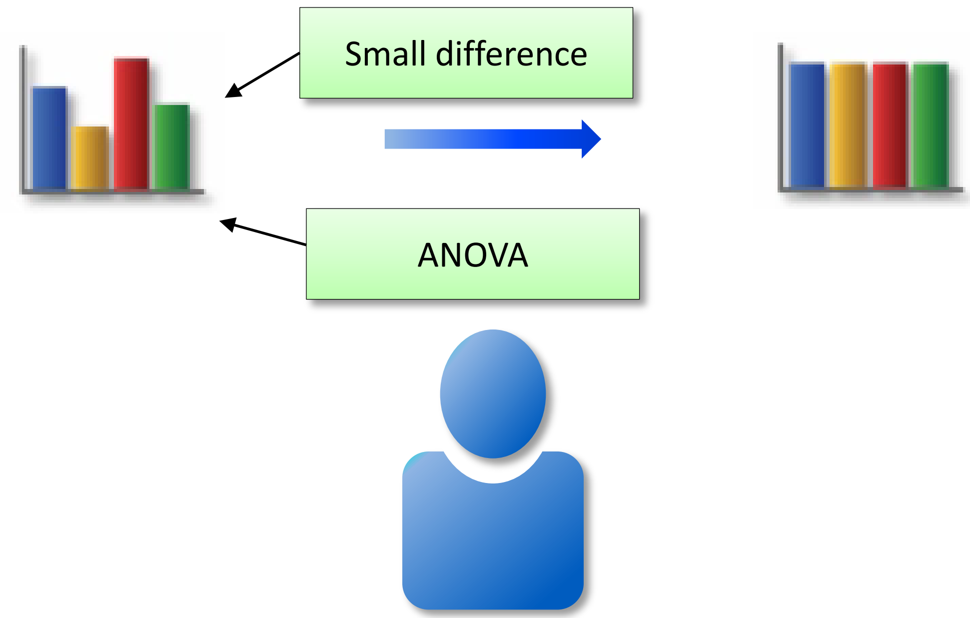
Medication 1

One-way ANOVA

Placebo

Medication 2

One-Way ANOVA: The ANOVA Hypothesis



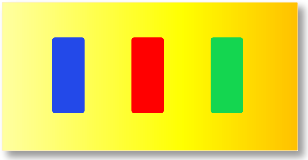
$$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$$

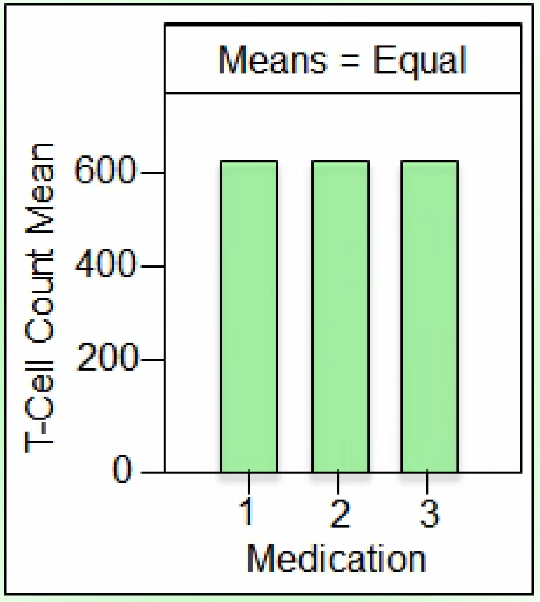
One-Way ANOVA: The ANOVA Hypothesis

ANOVA

H_0: $\mu_1 = \mu_2 = \mu_3$

Predictor Variable



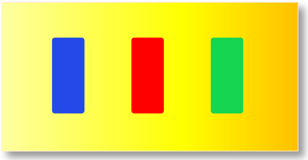


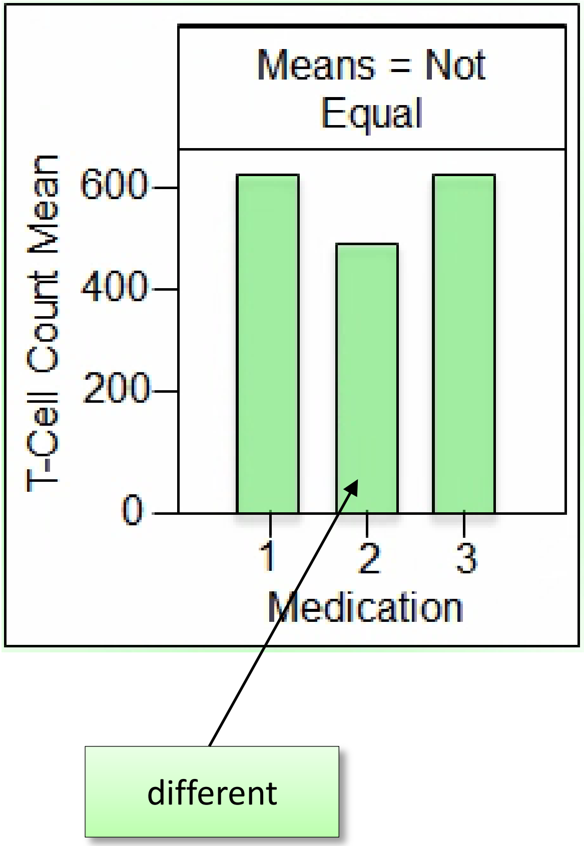
One-Way ANOVA: The ANOVA Hypothesis

ANOVA

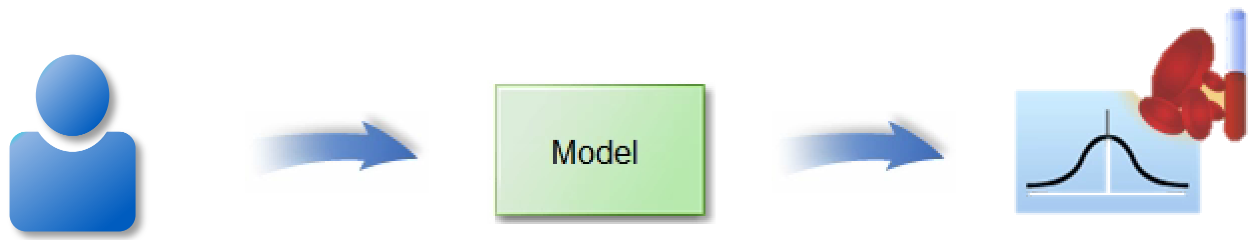
H_0: $\boldsymbol{\mu_1 = \mu_2 = \mu_3}$

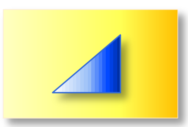
Predictor Variable





One-Way ANOVA: The ANOVA model



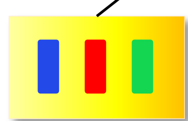


T-Cell Count = Base Level + Medication + Unaccounted for Variation

$$Y_{ik} = \mu + \tau_i + \varepsilon_{ik}$$

Error term

effect



$\tau_1 = \mu_1 - \mu$

$\tau_2 = \mu_2 - \mu$

$\tau_3 = \mu_3 - \mu$

i: indexes treatments
(three types of medication)
K: observation number

μ: overall population mean

One-way ANOVA

One-Way ANOVA: Sums of Squares

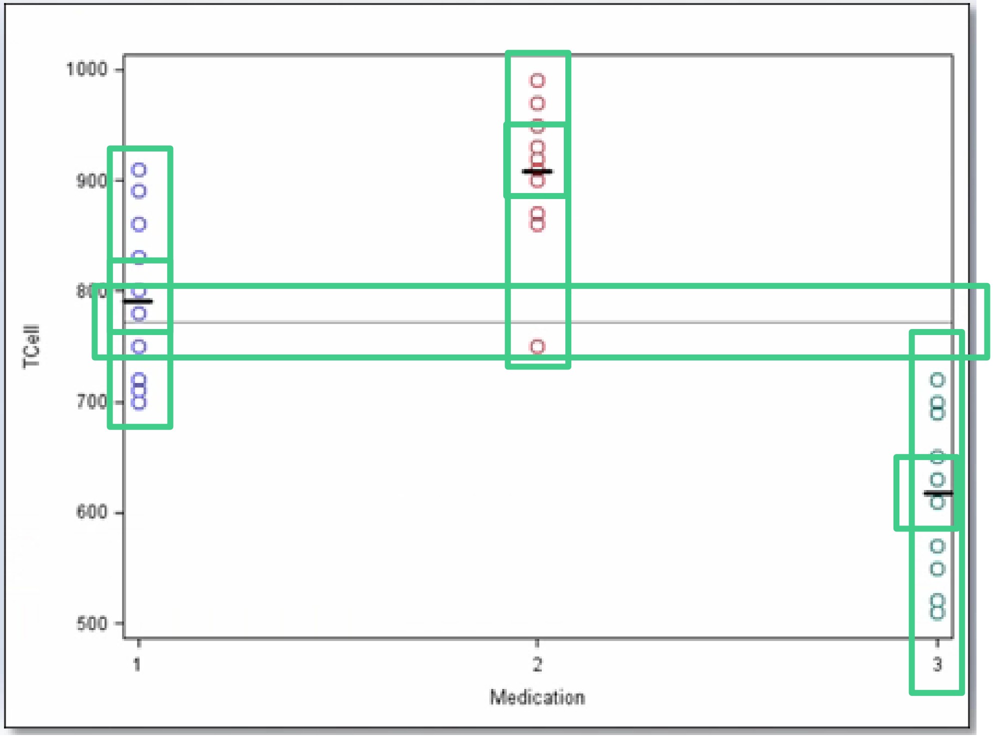
H_0: $\boldsymbol{\mu_1} = \boldsymbol{\mu_2} = \boldsymbol{\mu_3}$

Variability between groups **>** Variability within groups

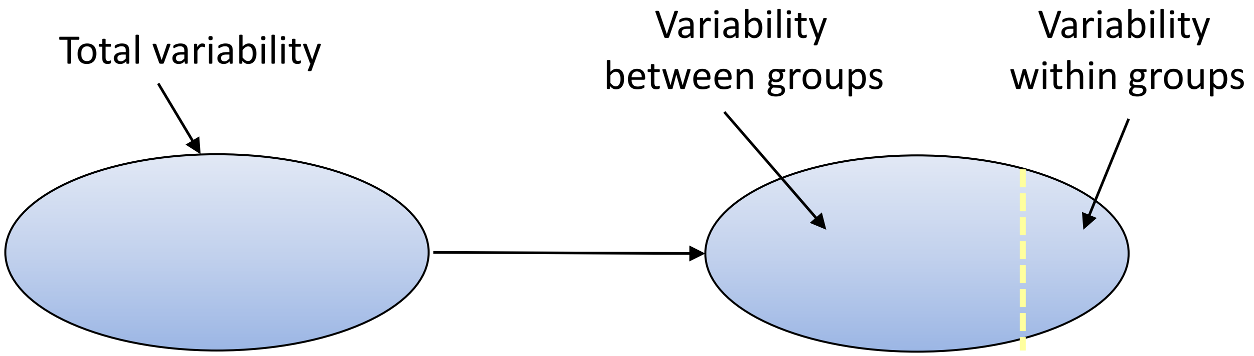
By ratio

By ratio

One-Way ANOVA: Sums of Squares



One-Way ANOVA: Sums of Squares



SS_T = SS_M + SS_E

$SS_T = \sum\sum(Y_{ij} - \bar{\bar{Y}})^2$ Total Sum of Squares

$SS_M = \sum n_i(\bar{Y}_i - \bar{\bar{Y}})^2$ Model Sum of Squares

$SS_E = \sum\sum(Y_{ij} - \bar{Y}_i)^2$ Error Sum of Squares

One-Way ANOVA: F statistic

F statistic and Critical Value at α=0.05

$$F(\text{Model df}, \text{Error df}) = MS_M / MS_E = \frac{SS_M/_{df_M}}{SS_E/_{df_E}}$$

MS_M : Model Mean Square
MS_E : Error Mean Square

In general, *degree of freedom* (DF) can be thought of as the number of independent pieces of information.

1. Model DF is the number of treatment minus 1.
2. Corrected total DF is the sample size minus 1.
3. Error DF is the sample size minus the number of treatments (or the difference between the corrected total DF and the Model DF)

One-Way ANOVA: Coefficient of Determination

Coefficient of Determination:

$$R^2 = \frac{SS_M}{SS_T}$$

"proportion of variance accounted for by the model "

One-Way ANOVA: Assumptions for ANOVA

Assumptions for ANOVA

1. Independent observations
2. Error terms are normally distributed
3. Error terms have equal variances

Assessing ANOVA Assumptions

1. Good data collection methods help ensure the independence assumption.
2. Diagnostic plots can be used to verify the assumption that the error is approximately normally distributed
3. The Linear Models task can produce a test of equal variance. H0 for this hypothesis test is that the variances are equal for all populations.

One-Way ANOVA: Predicted and Residual values

T-Cell Count	=	Base Level	+	Medication	+	Unaccounted for Variation
Y_{ik}	$=$	μ	$+$	τ_i	$+$	ε_{ik}

Estimates of the error term

Observation - Group mean = Residuals

One-Way ANOVA: Predicted and Residual values

Obs	Medication	TCellCount	Predicted	Residual
1	1	700	795	-95
2	2	750	905	-155
3	3	550	615	-65
4	1	800	795	5
5	2	930	905	25

-

+

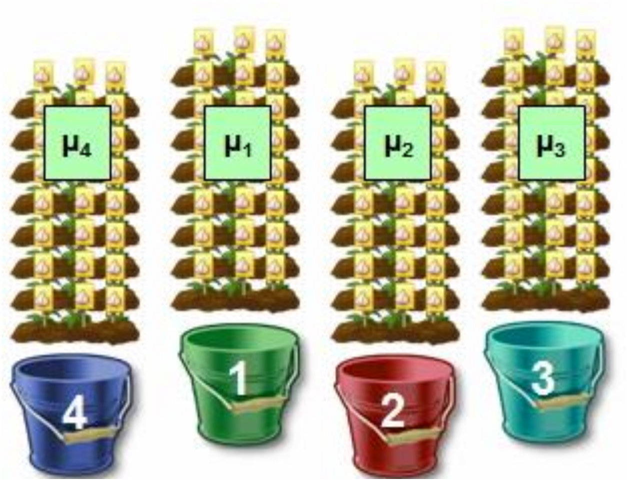
The **predicted value** in ANOVA is the ***group mean***.
A ***residual*** is the difference between the observed value of the response and the predicted value of the response variable.

What is the predicted value for medication 2 in observations 2 and 5?

One-Way ANOVA:

Scenario: Comparing Group Means with One-Way ANOVA

Garlic



Response Variable

Bulb Weight

Predictor Variable

Fertilizer

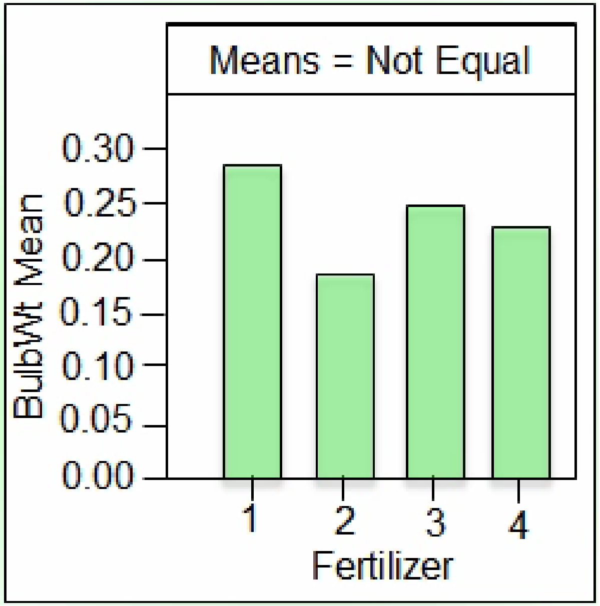
One-Way ANOVA:

Scenario: Comparing Group Means with One-Way ANOVA

ANOVA

H$_0$: $\mu_1 = \mu_2 = \mu_3 = \mu_4$

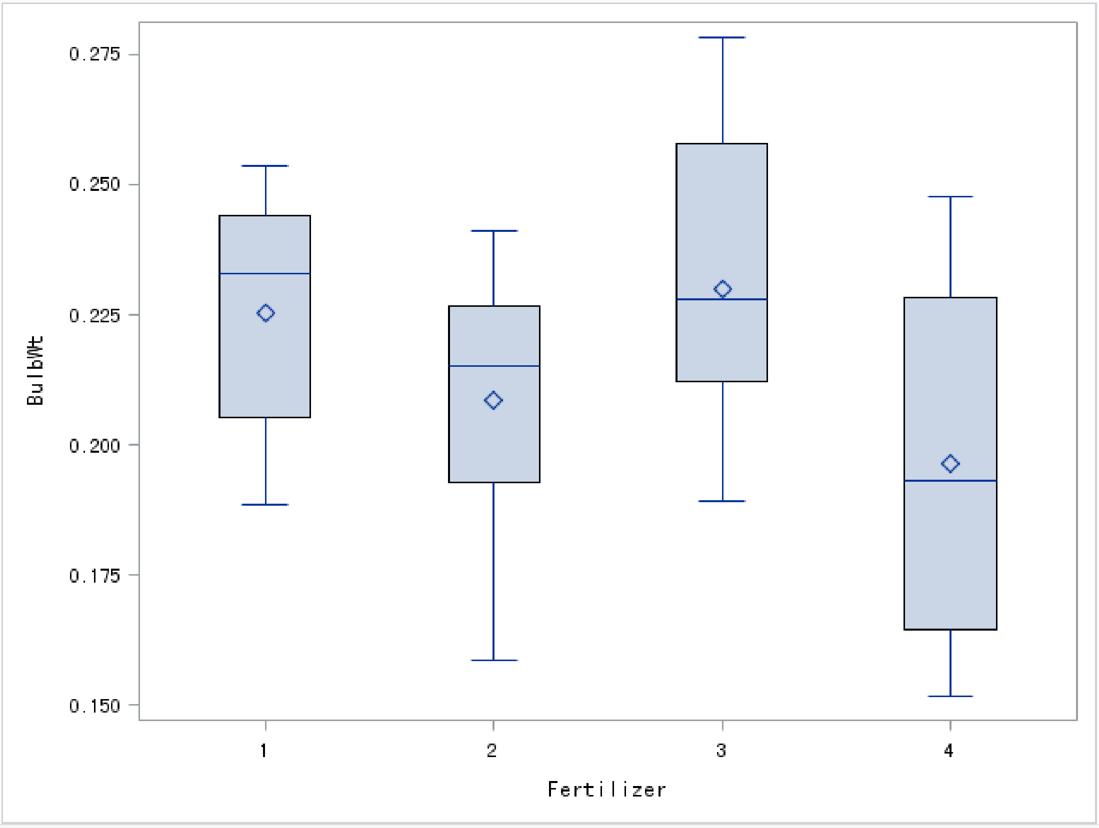
H$_a$: at least one is different



One-Way ANOVA:

Scenario: Comparing Group Means with One-Way ANOVA

Description check of MGGARLIC > Summary Statistic



One-Way ANOVA:

Question 1.

You have 20 observations in you ANOVA and you calculate the residuals. What will they sum to?

a) -20
b) 20
c) 400
d) 0
e) Need more information

Answer: d

One-Way ANOVA:

Question 2.

Which of the following phrases describes the model sums of squares, or SSM?

a) The variability between the groups
b) The variability within the groups
c) The variability explained by the error terms

Answer: a

One-Way ANOVA:

Question 3.

Match the null hypothesis to the correct SAS output

b $H_0: \sigma_1^2 = \sigma_2^2$

a $H_0: \mu_1 = \mu_2$

a)

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	1	0.03033816	0.03033816	51.02	<.001
Error	79	0.04638442	0.00059467		
Corrected Total	79	0.07672257			

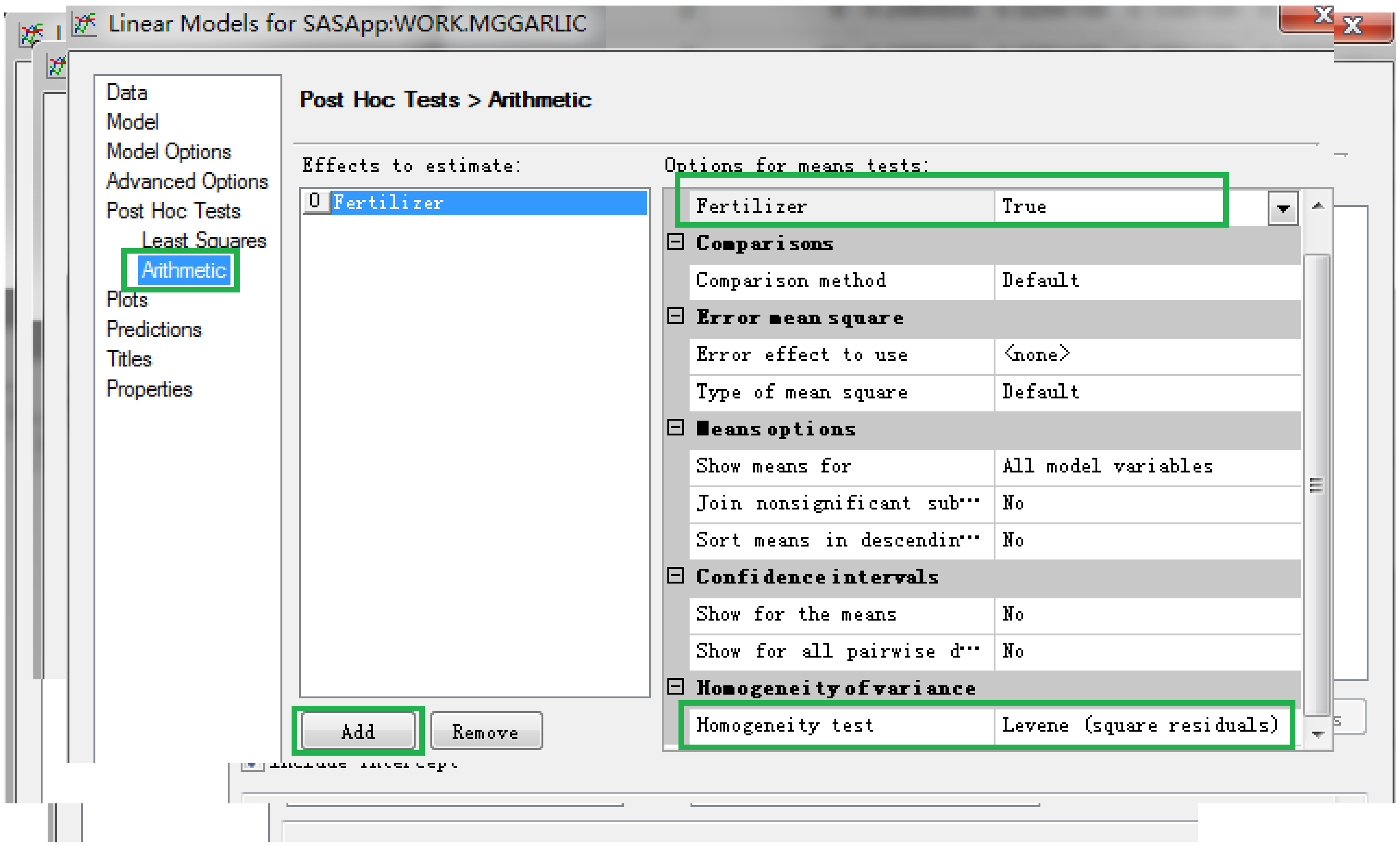
b)

Levene's Test for Homogeneity of Weight Variance ANOVA of Squared Deviations from Group Means					
Source	**DF**	**Sum of Squares**	**Mean Square**	**F Value**	**Pr > F**
Brand	1	9.237E-7	9.237E-7	1.12	0.2942
Error	78	0.000065	8.283E-7		

One-Way ANOVA:

Scenario: Comparing Group Means with One-Way ANOVA

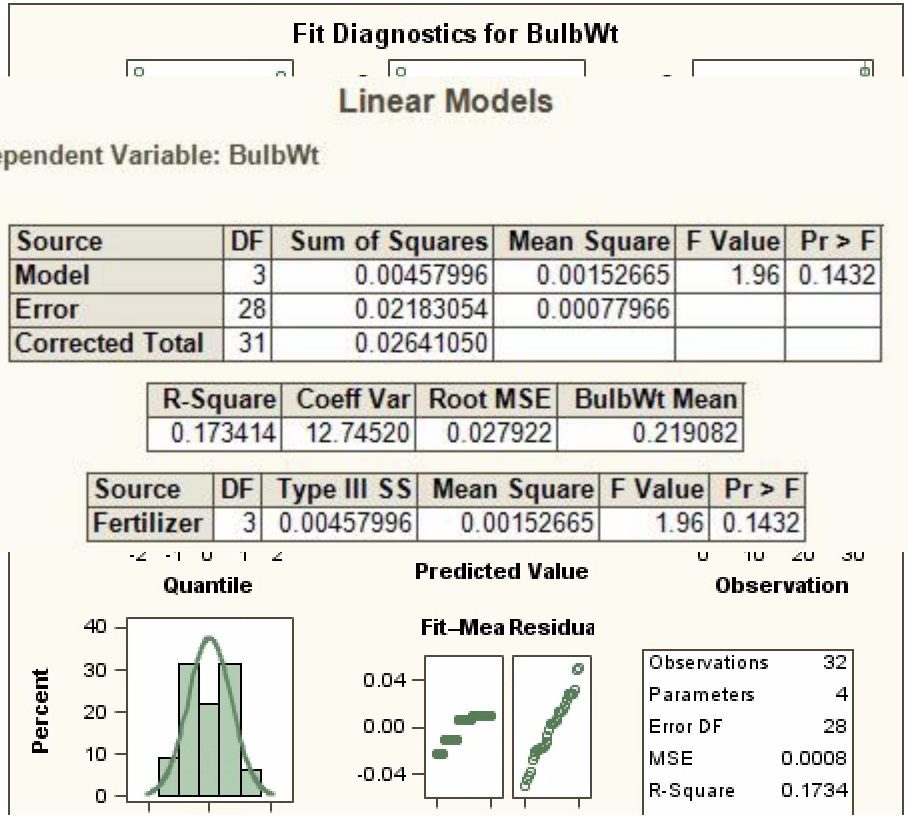
Task> ANOVA>Linear Models, with MGGARLIC data



One-Way ANOVA:

Scenario: Comparing Group Means with One-Way ANOVA

Task> ANOVA>Linear Models



Linear Models

Dependent Variable: BulbWt

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	0.00457996	0.00152665	1.96	0.1432
Error	28	0.02183054	0.00077966		
Corrected Total	31	0.02641050			

R-Square	Coeff Var	Root MSE	BulbWt Mean
0.173414	12.74520	0.027922	0.219082

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Fertilizer	3	0.00457996	0.00152665	1.96	0.1432

One-Way ANOVA: Summary

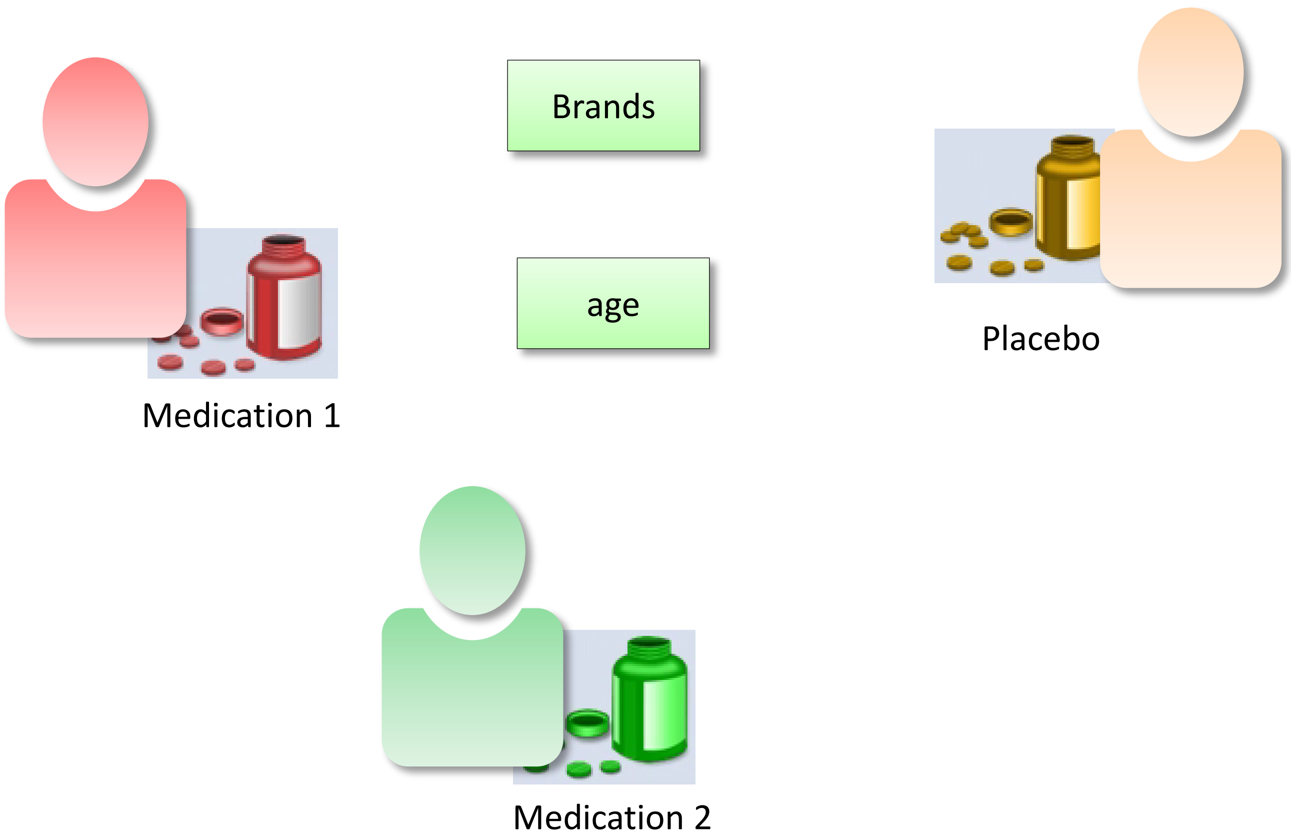
Null Hypothesis:	All means are equal
Alternative Hypothesis:	at least one mean is different

1. Produce descriptive statistics.
2. Verify assumptions.
    - Independence
    - Normality
    - Equal variance
3. Examine the *p*-value in the ANOVA table. If the *p*-value is less than alpha, reject the null hypothesis.

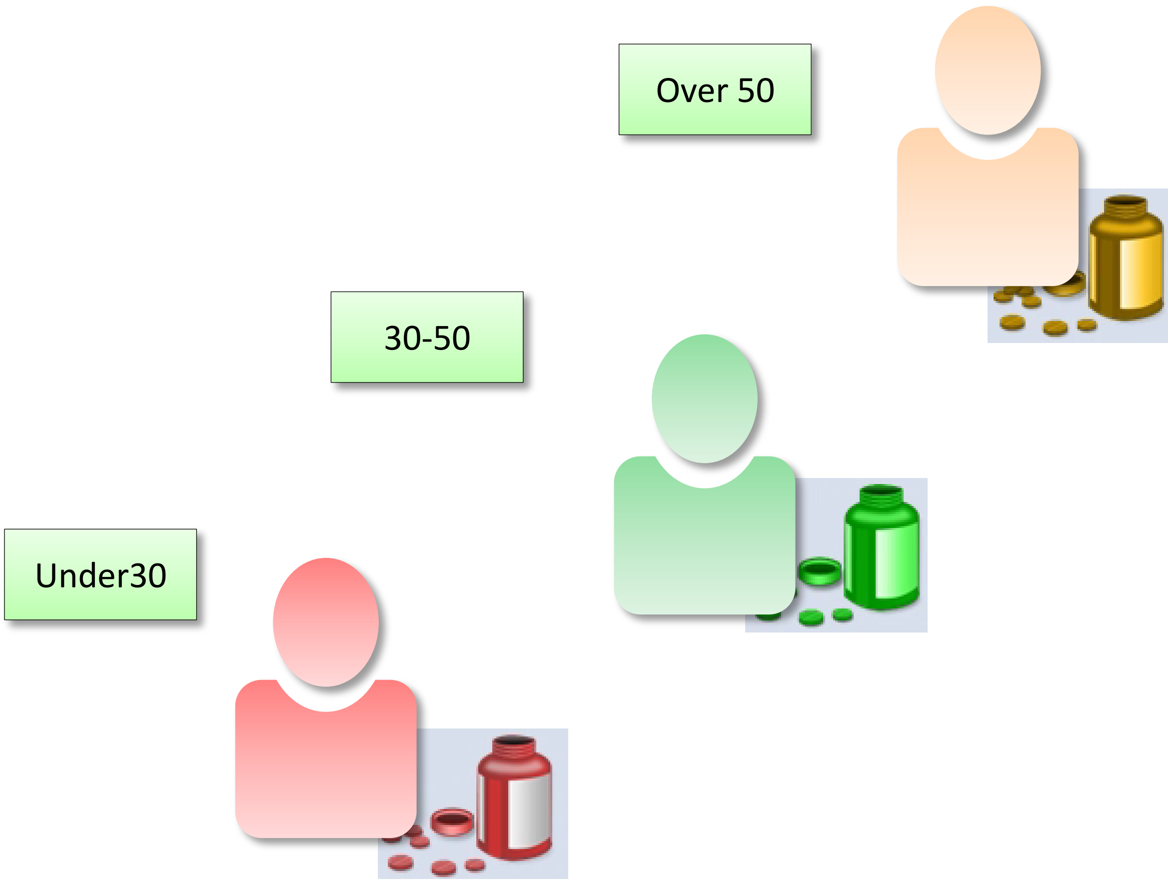
Agenda

- 0. Lesson overview
- 1. Two-Sample *t*-Tests
- 2. One-Way ANOVA
- 3. **ANOVA with Data from a Randomized Block Design**
- 4. ANOVA Post Hoc Tests
- 5. Two-Way ANOVA with Interactions
- 6. Summary

ANOVA with Data from a Randomized Block Design: Introduction



ANOVA with Data from a Randomized Block Design: Introduction



ANOVA with Data from a Randomized Block Design: Objective

- Recognize the difference between a completely randomized design and a randomized block design.
- Differentiate between observed data and designed experiments.
- Use the Linear Models task to analyze data from a randomized block design

ANOVA with Data from a Randomized Block Design: Observational Studies

Groups can be naturally occurring.

Gender and ethnicity

Random assignment might be unethical or untenable

Smoking or credit risk groups

In **Observational or Retrospective studies**, the data values are observed as they occur, not affected by an experimental design.

ANOVA with Data from a Randomized Block Design: Controlled Experiments

1. Random assignment might be desirable to eliminate selection bias.
2. You often wan to look at the outcome measure prospectively.
3. You can manipulate the factors of interest and can more reasonably claim causation.
4. You can design your experiment to control for other factors contributing to the outcome measure.

ANOVA with Data from a Randomized Block Design

Question 3.

Can you determine a cause-and–effect relationship in an observational study?

a) Yes
b) No

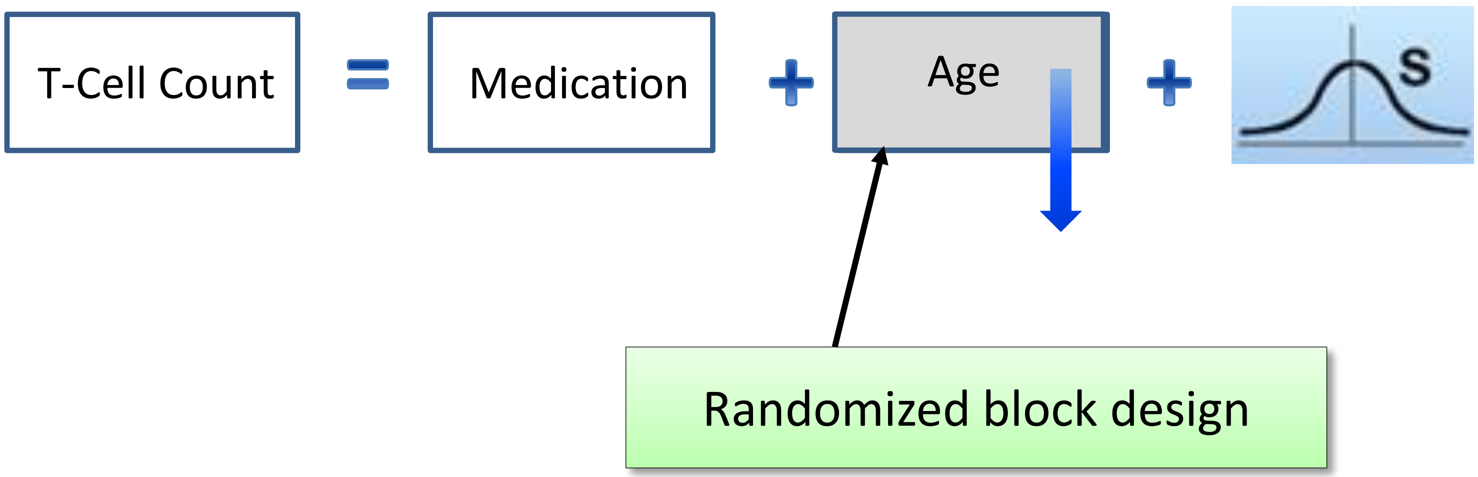
Answer: b

observational study ⟷ controlled study

In an observational study, you often examine what already occurred, and therefore have little control over factors contributing to the outcome.
In a controlled experiment, you can manipulate the factors of interest and can more reasonably claim causation.

ANOVA with Data from a Randomized Block Design: Nuisance Factors

Nuisance Factors are factors can affect the outcome but are not of interest in the experiment.



ANOVA with Data from a Randomized Block Design

Question 4.

Which part of the ANOVA tables contains the variation due to nuisance factors?

a) Sum of Squares Model
b) Sum of Squares Error
c) Degrees of Freedom

Answer: b

ANOVA with Data from a Randomized Block Design: Including a Blocking Variable in the Model

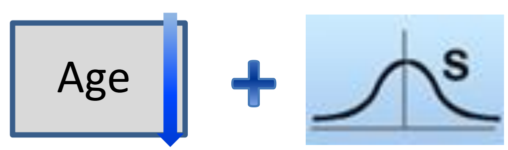
T-Cell Count	=	Base Level	+	Age	+	Medication	+	Unaccounted for Variation
Y_{ijk}	$=$	μ	$+$	α_i	$+$	τ_j	$+$	ε_{ijk}

T-Cell Count	=	Base Level	+	Medication	+	Unaccounted for Variation
Y_{ik}	$=$	μ	$+$	τ_i	$+$	ε_{ik}

Age

ANOVA with Data from a Randomized Block Design: Including a Blocking Variable in the Model

T-Cell Count	=	Base Level	+	Age	+	Medication	+	Unaccounted for Variation
Y_{ijk}	$=$	μ	$+$	α_i	$+$	τ_j	$+$	ε_{ijk}



ANOVA with Data from a Randomized Block Design: More ANOVA Assumptions

ANOVA

The treatment are randomly assigned to each block

Independent observations

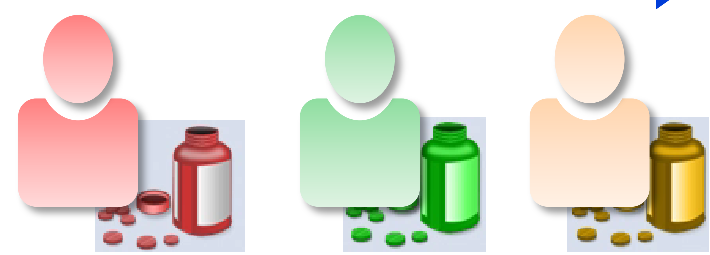
Normally distributed data

Equal variances

Under30

30-50

Over 50

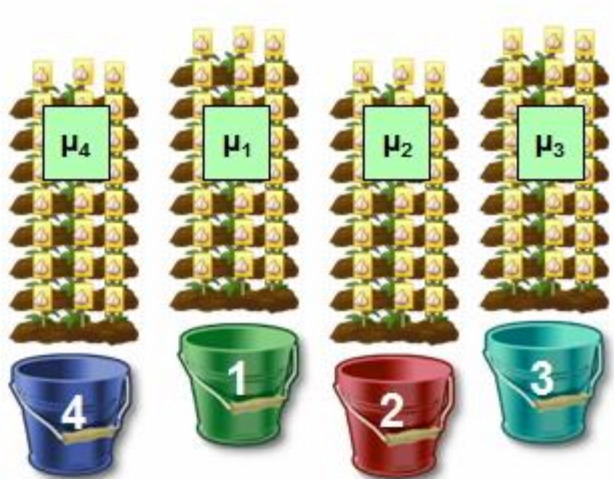


No interaction

ANOVA with Data from a Randomized Block Design

Scenario: Creating a Randomized Block Design

Garlic



H$_0$: $\mu_1 = \mu_2 = \mu_3 = \mu_4$

H$_a$: at least one is different

What's the nuisance factors in this case

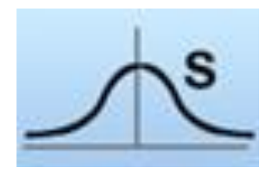
Sun

PH level of soil

Rain

ANOVA with Data from a Randomized Block Design

Scenario: Creating a Randomized Block Design

Bulb Weight = fertilizer + Sun / PH of the soil / Rain + 

ANOVA with Data from a Randomized Block Design

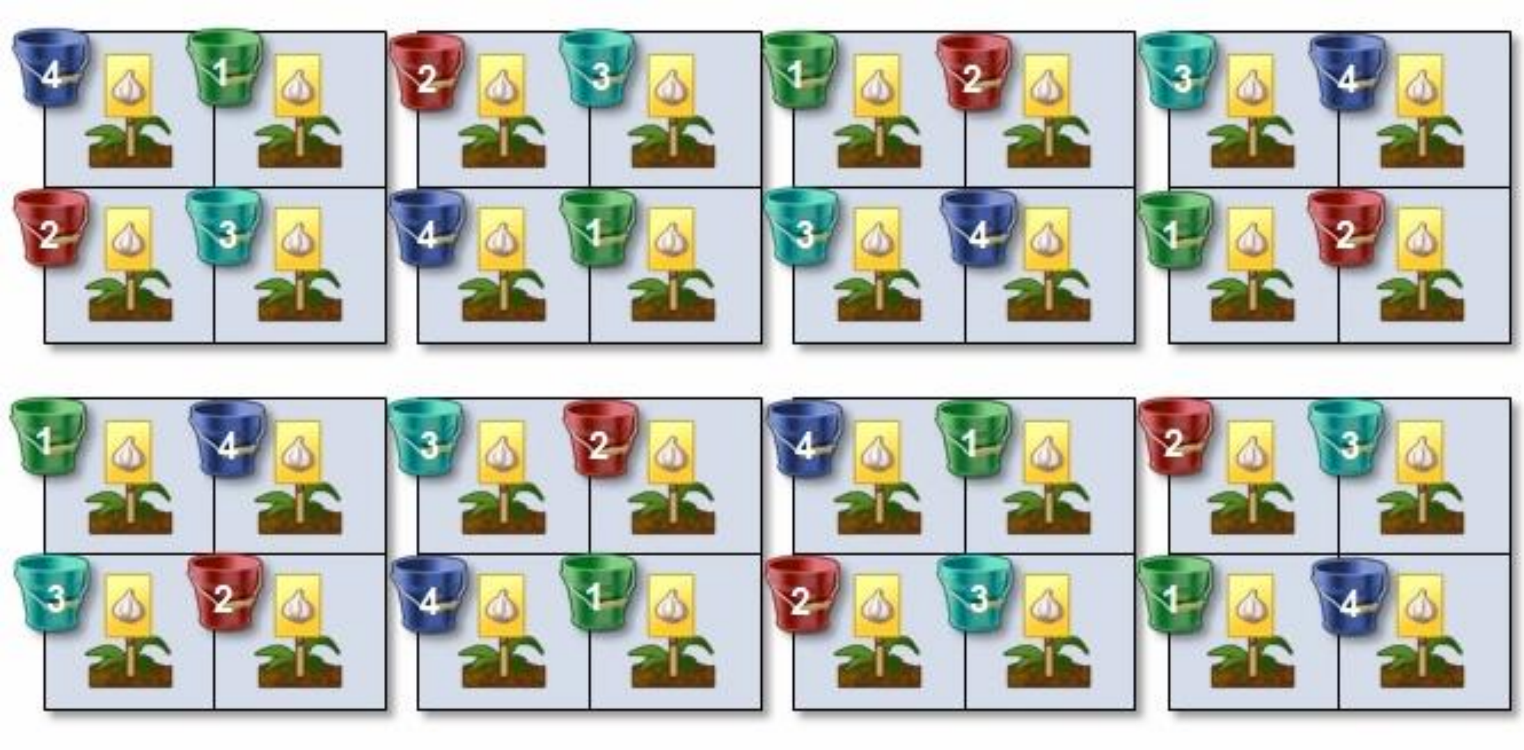
Scenario: Creating a Randomized Block Design

Randomized block design

T-Cell Count	=	Base Level	+	Sector	+	Fertilizer Type	+	Unaccounted for Variation
Y_{ijk}	$=$	μ	$+$	$\boxed{\alpha_i}$	$+$	τ_j	$+$	ε_{ijk}

ANOVA with Data from a Randomized Block Design

Scenario: Creating a Randomized Block Design



ANOVA with Data from a Randomized Block Design

Question 5.

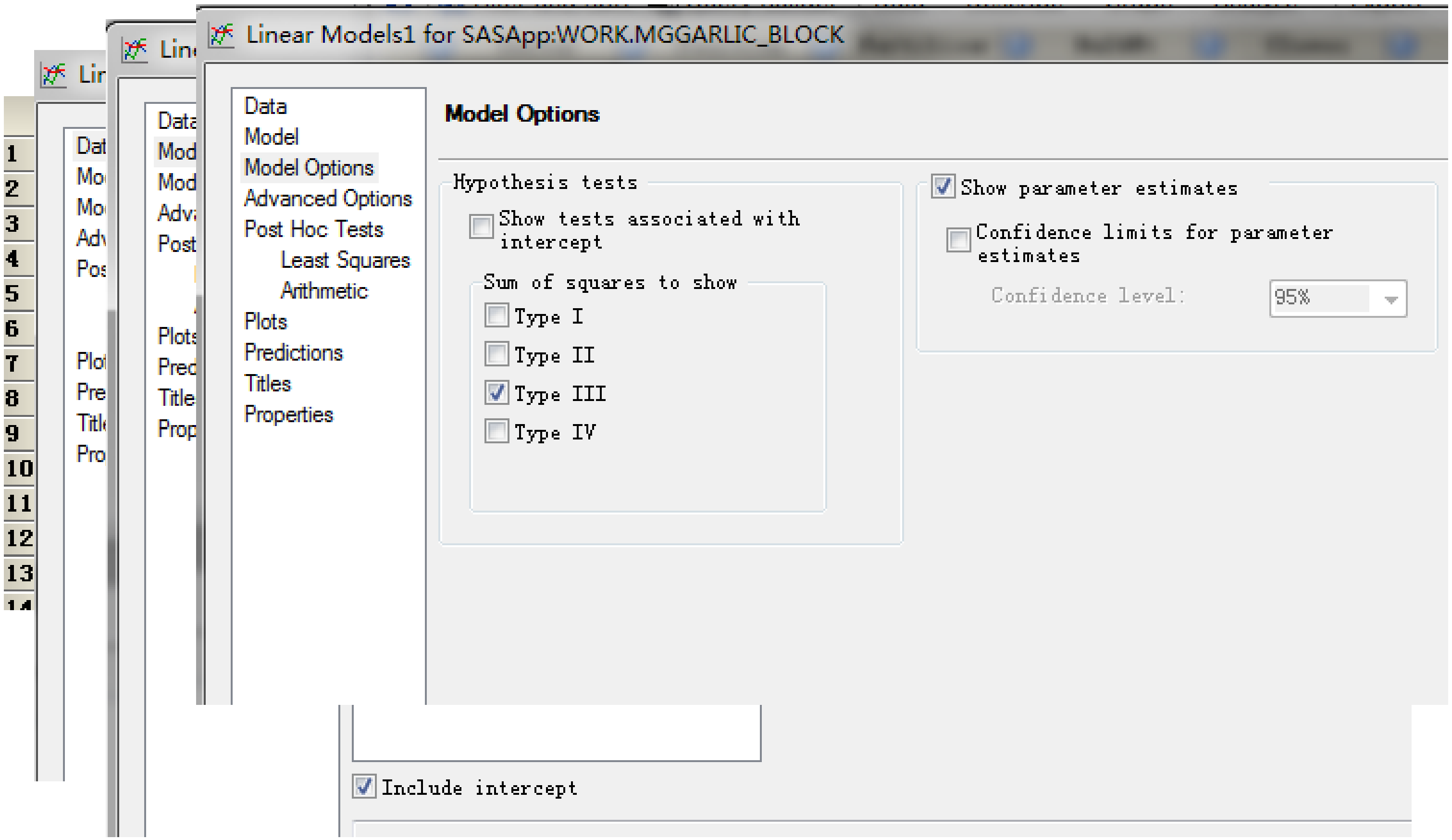
In a block design, Which part of the ANOVA tables contains the variation due to nuisance factors?

a) Sum of Squares Model
b) Sum of Squares Error
c) Degrees of Freedom

Answer: a

ANOVA with Data from a Randomized Block Design: Performing ANOVA with Blocking

Task> ANOVA>Linear Models, with MGGARLIC_BLOCK data



ANOVA with Data from a Randomized Block Design: Performing ANOVA with Blocking

Task> ANOVA>Linear Models, with MGGARLIC_BLOCK data

Fit Diagnostics for BulbWt

ANOVA for Randomized Block Design

Dependent Variable: BulbWt

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	10	0.02307263	0.00230726	5.86	0.0003
Error	21	0.00826745	0.00039369		
Corrected Total	31	0.03134008			

R-Square	Coeff Var	Root MSE	BulbWt Mean
0.736202	9.085064	0.019842	0.218398

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Sector	7	0.01798632	0.00256947	6.53	0.0004
Fertilizer	3	0.00508630	0.00169543	4.31	0.0162

Parameter	Estimate		Standard Error	t Value	Pr > |t|
Intercept	0.1915259375	B	0.01163316	16.46	<.0001
		B	0.01403012	2.80	0.0106
		B	0.01403012	-0.91	0.3743
		B	0.01403012	1.77	0.0913

R-Square 0.7362

Bulb Weight = Base Level + Sector + Fertilizer Type + Unaccounted for Variation

$$Y_{ijk} = \mu + \alpha_i + \tau_j + \varepsilon_{ijk}$$

ANOVA with Data from a Randomized Block Design

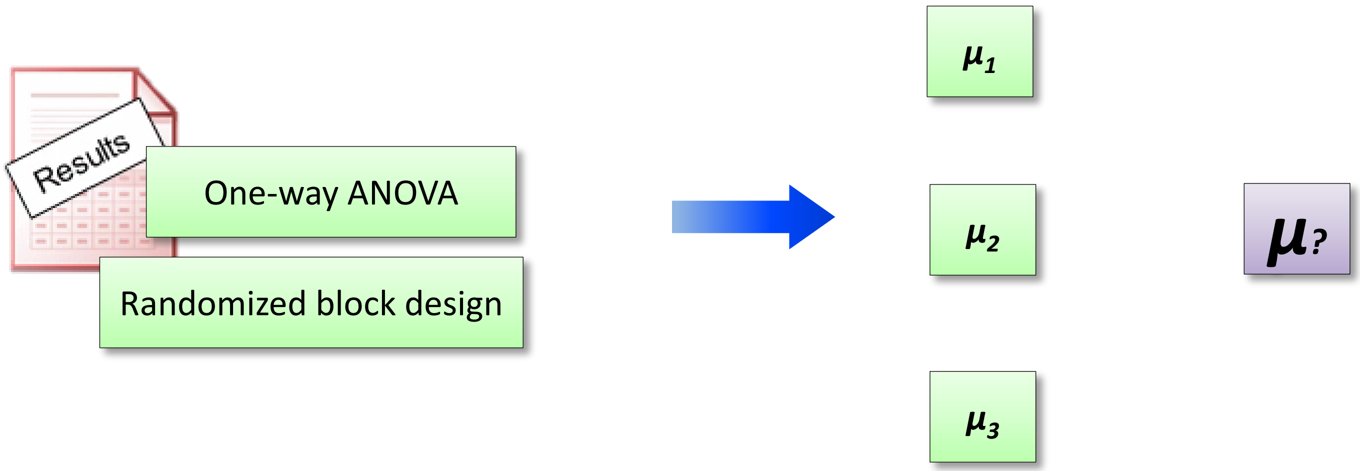
My groups are different. What next?

- The *p*-value for **Fertilizer** indicates you should reject the H_0 that all groups are the same.
- From which pairs of fertilizers, are garlic bulb weights different from one another?
- Should you go back and do several *t*-tests?

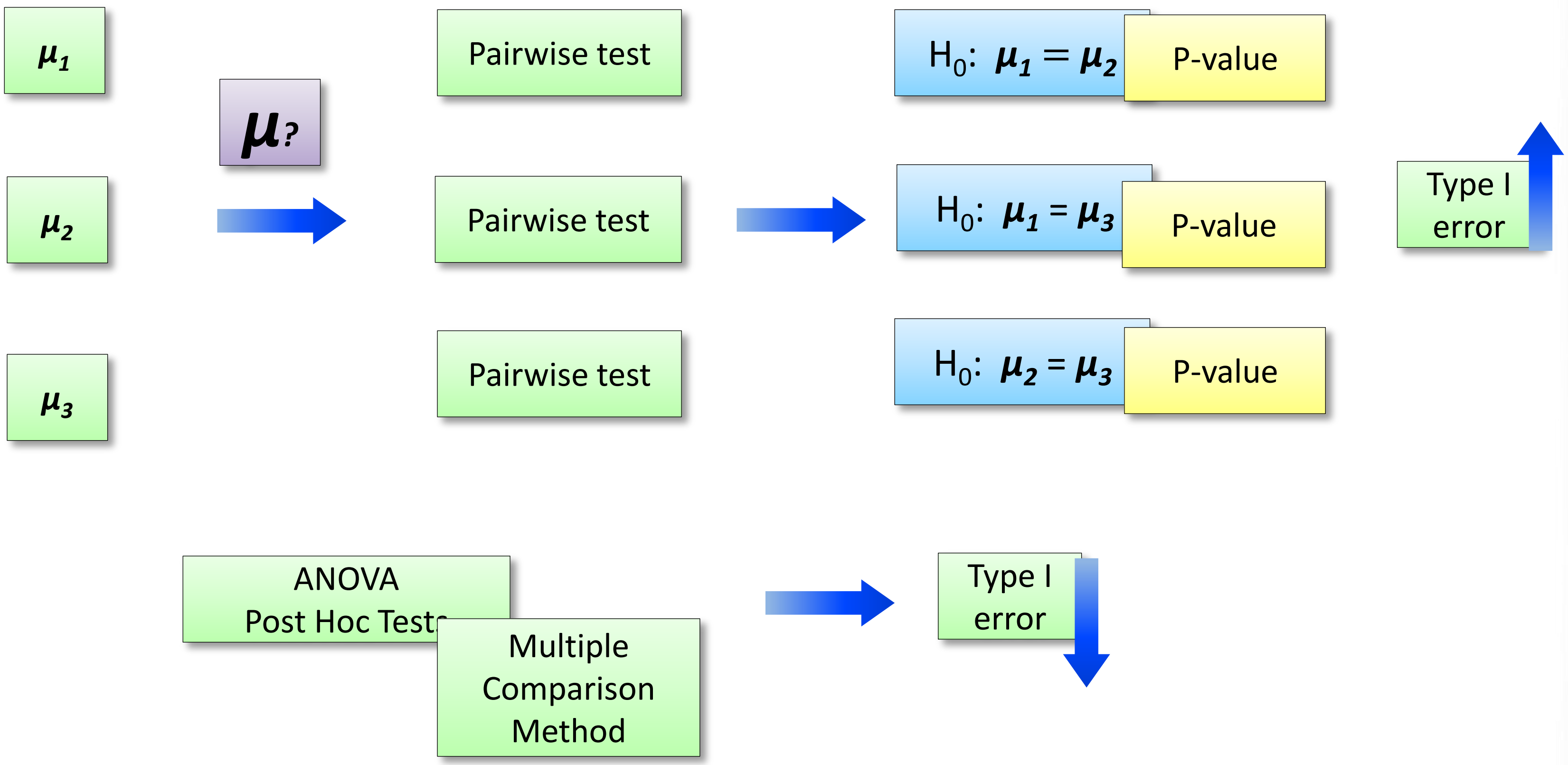
Agenda

- 0. Lesson overview
- 1. Two-Sample *t*-Tests
- 2. One-Way ANOVA
- 3. ANOVA with Data from a Randomized Block Design
- 4. **ANOVA Post Hoc Tests**
- 5. Two-Way ANOVA with Interactions
- 6. Summary

ANOVA Post Hoc Tests: Introduction



ANOVA Post Hoc Tests: Introduction



ANOVA Post Hoc Tests:
Multiple Comparison Methods

Question 7.

With a fair coin, your probability of getting heads on one flip is 0.5. if you flip a coin and got heads, what is the probability of getting heads on the second try?

a) 0.5
b) 0.25
c) 0.00
d) 1.00
e) 0.75

Answer: a

ANOVA Post Hoc Tests: Multiple Comparison Methods

Question 8.

With a fair coin, your probability of getting heads on one flip is 0.5. If you flip a coin twice, what is the probability of getting ***at least*** one head out of two?

a) 0.5
b) 0.25
c) 0.00
d) 1.00
e) 0.75

Answer: e

ANOVA Post Hoc Tests:
Multiple Comparison Methods

α=0.05

Type I Error

H$_0$: $\boldsymbol{\mu_1 = \mu_2}$

H$_0$: $\boldsymbol{\mu_1 = \mu_3}$

H$_0$: $\boldsymbol{\mu_2 = \mu_3}$

ANOVA Post Hoc Tests:
Multiple Comparison Methods

Comparisonwise Error Rate	Number of Comparisons	Experimentwise Error Rate
0.05	1	0.05
0.05	3	0.14
0.05	6	0.26
0.05	10	0.40

Type I Error

Pairwise t-test

$$EER = 1 - (1 - \alpha)^{nc}$$

nc: Number of comparisons

ANOVA Post Hoc Tests:
Tukey's Multiple Comparison Method

Tukey Method → EER ↓

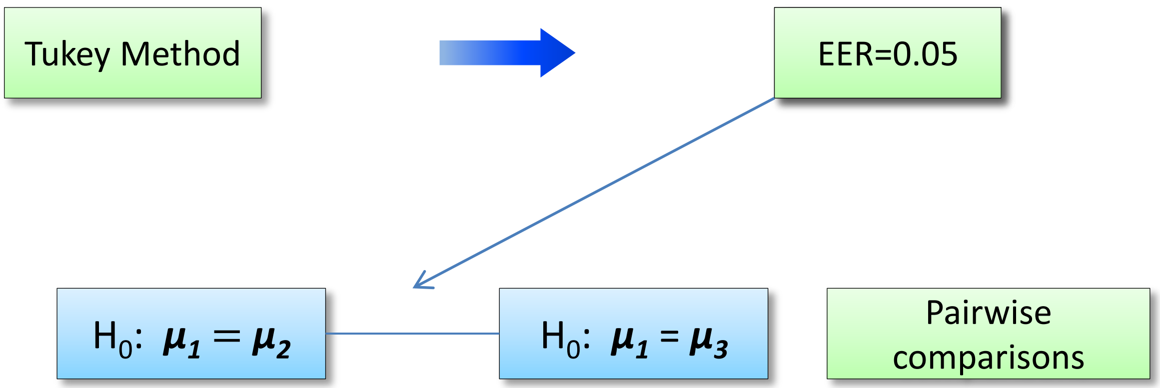
H_0: $\mu_1 = \mu_2$

H_0: $\mu_1 = \mu_3$

Pairwise comparisons

H_0: $\mu_2 = \mu_3$

ANOVA Post Hoc Tests: Tukey's Multiple Comparison Method



ANOVA Post Hoc Tests:
Tukey's Multiple Comparison Method

This method is appropriate when you consider pairwise comparisons only.

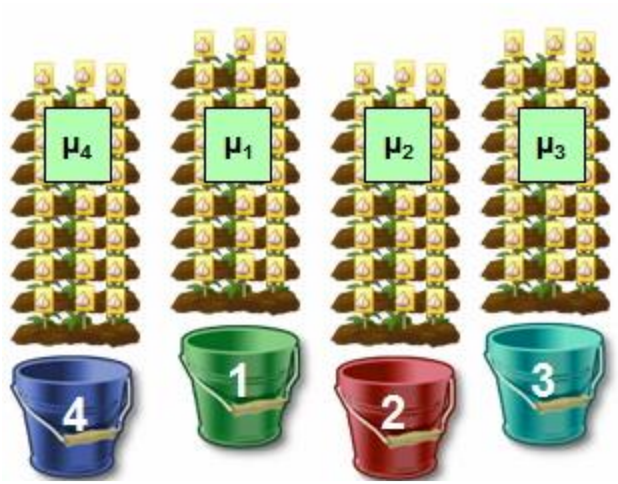
The Experimentwise Error Rate is:

– Equal to alpha when ***all*** pairwise comparisons are considered

– Less than alpha when ***fewer*** than all pairwise comparisons are considered

ANOVA Post Hoc Tests:
scenario: determine which mean is different

Garlic

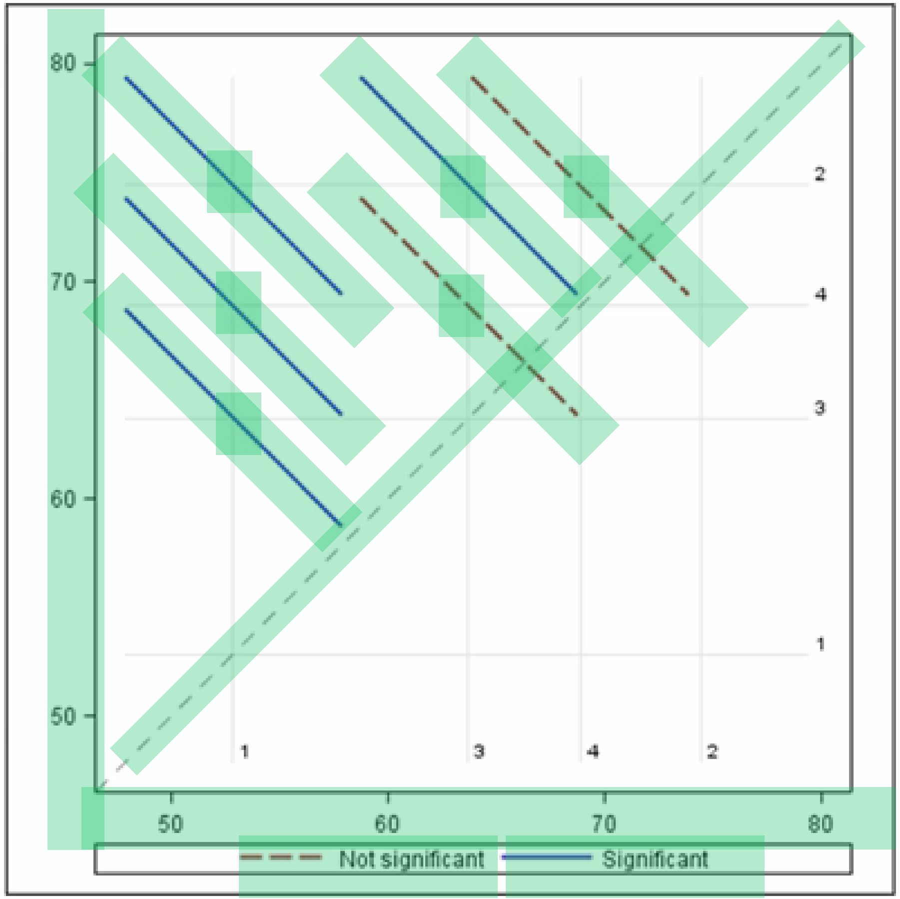


Fertilizers: three organics, one control

H_a: at least one is different

ANOVA Post Hoc Tests: Diffograms and the Tukey Method

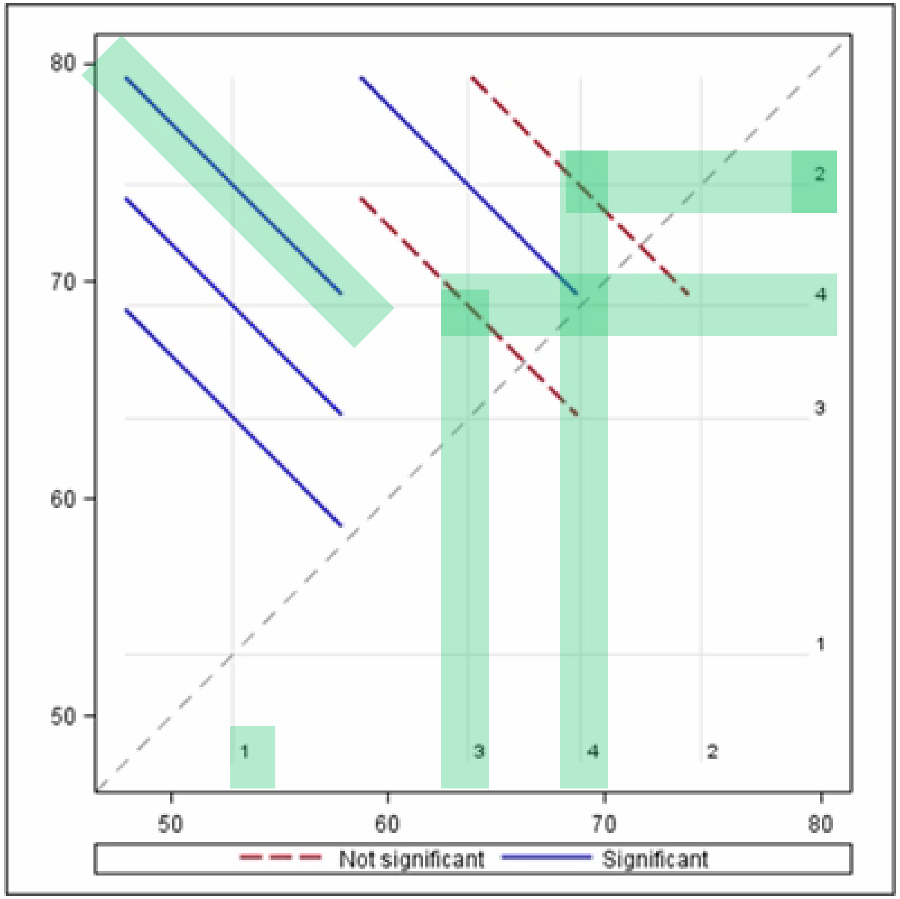
least square mean
by
least square mean



Difference between the means

Equality of the means

ANOVA Post Hoc Tests: Diffograms and the Tukey Method



Is there the diff between the treatments 1 and 2?

Can you identify the pairwise comparisons that do not have significant diff means?

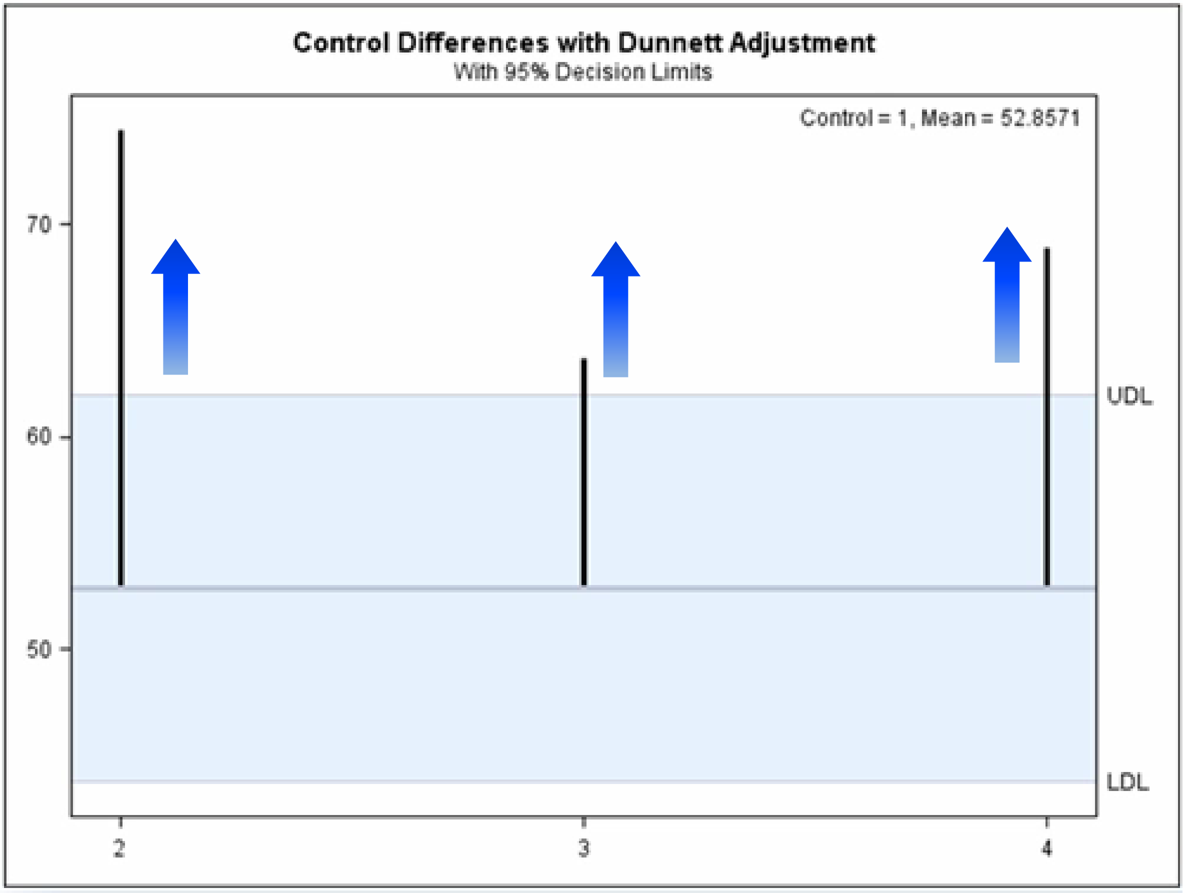
ANOVA Post Hoc Tests:
Dunnett's Multiple Comparison Method

Special Case of Comparing to a Control

Comparing to a control is appropriate when there is a natural reference group, such as a placebo group in a drug trial.

- Experimentwise Error Rate is no greater than the stated alpha
- Comparing to a control takes into account the correlations among tests
- One-sided hypothesis test against a control group can be performed
- Control comparison computes and tests k-1 GroupWise differences, where k is the number of levels of the classification variable.
- An example is the Dunnett method

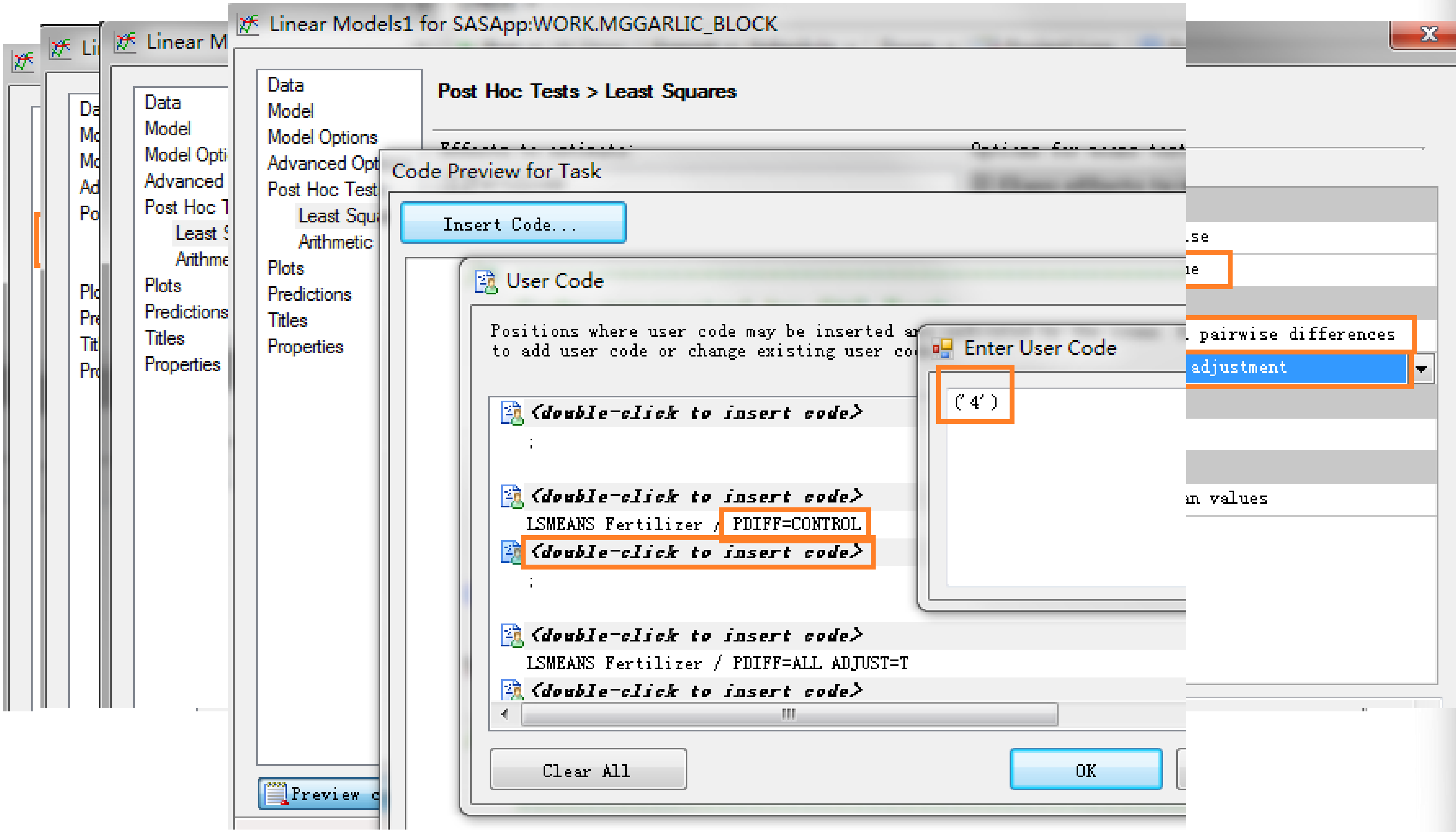
ANOVA Post Hoc Tests: Control Plots and the Dunnett Method



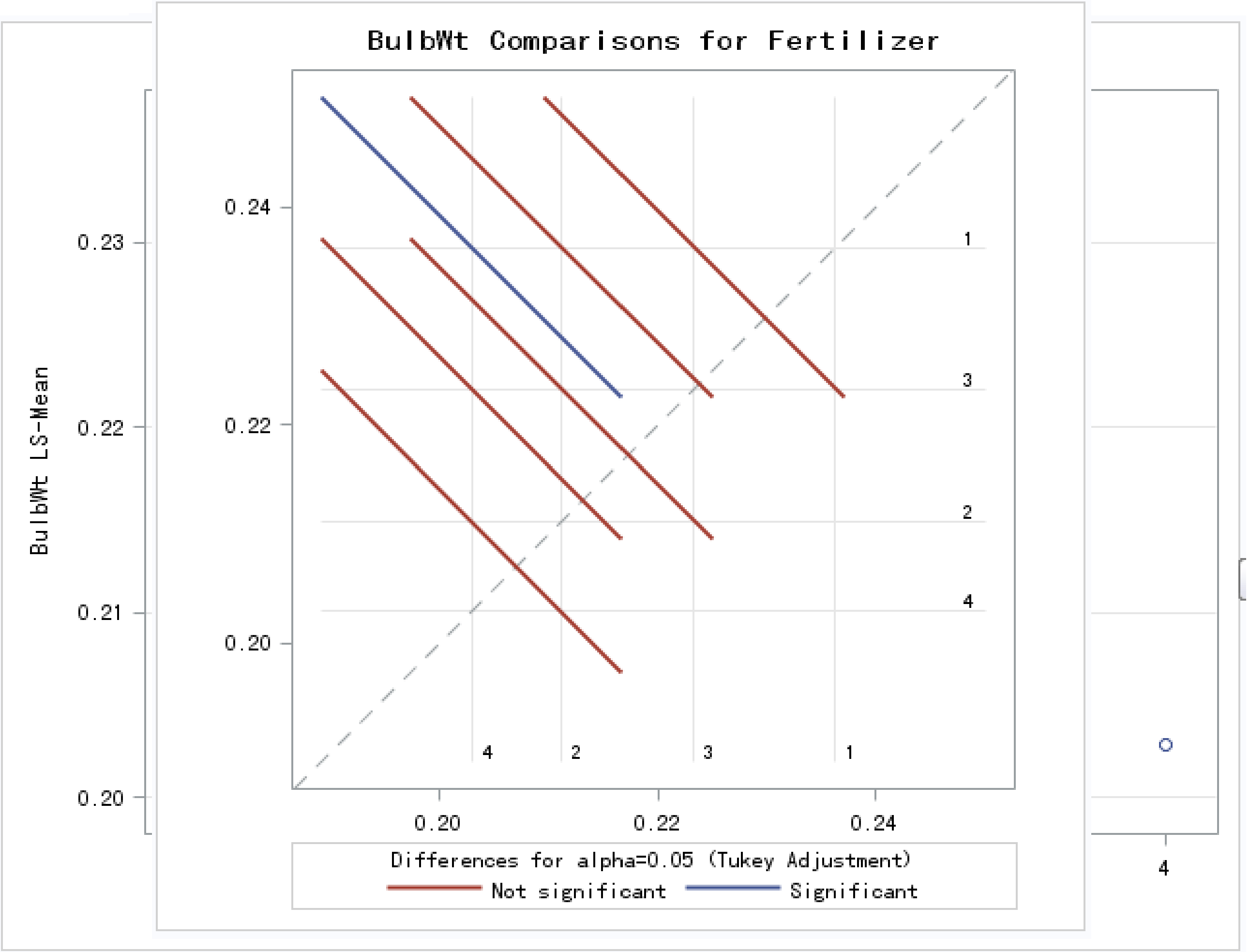
Upper decision limit

Lower decision limit

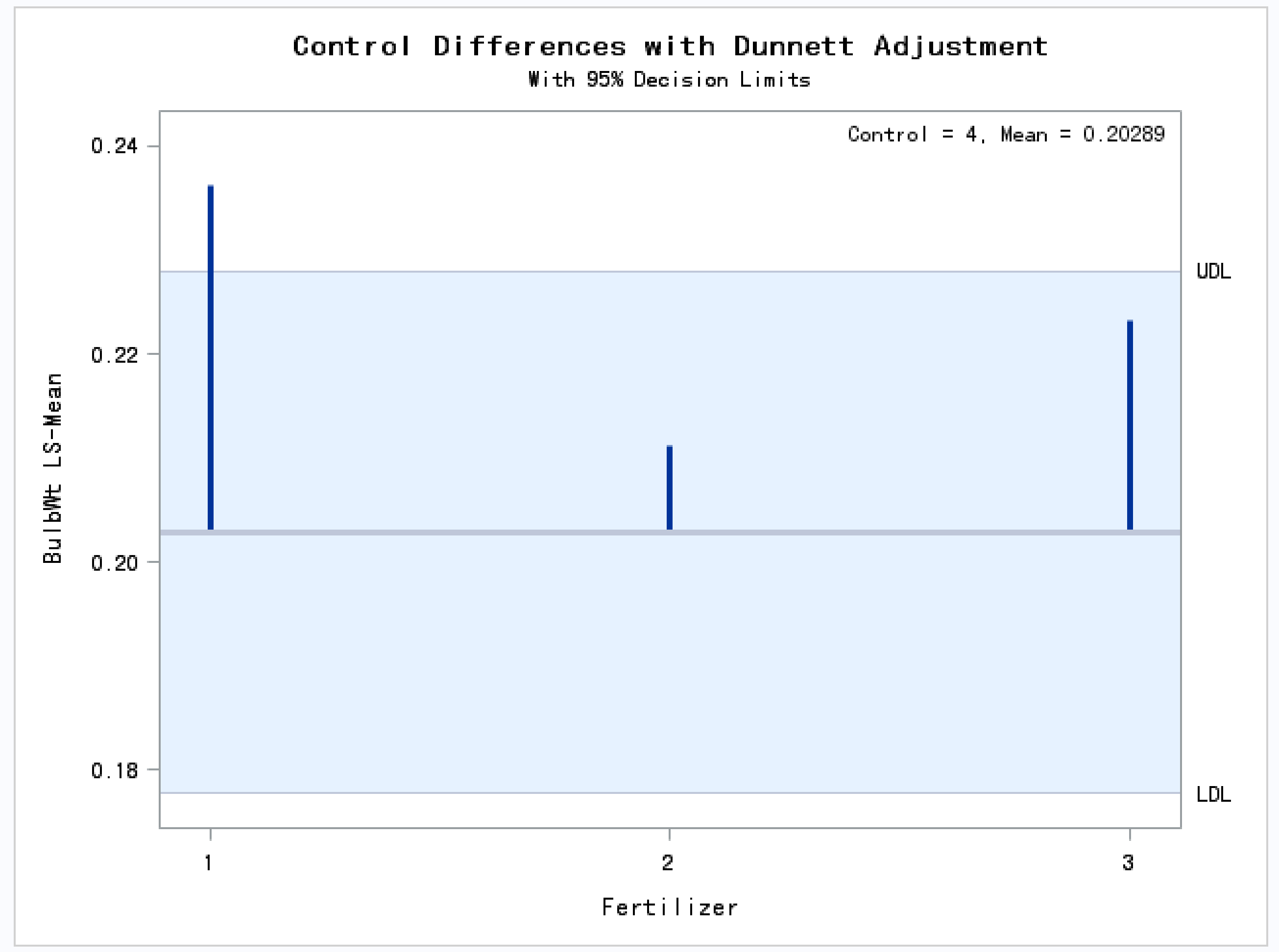
ANOVA Post Hoc Tests: Performing a Post Hoc Tests



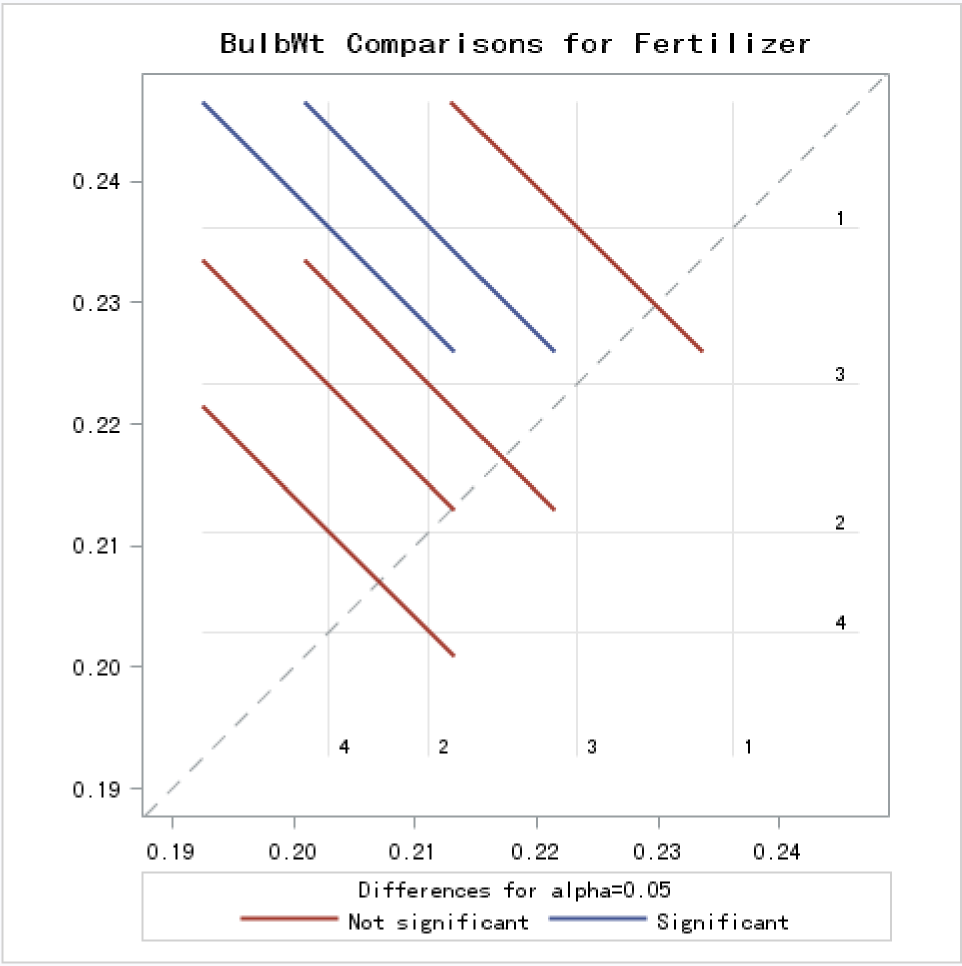
ANOVA Post Hoc Tests:
Performing a Post Hoc Tests: Turkey



ANOVA Post Hoc Tests: Performing a Post Hoc Tests: Dunnett



ANOVA Post Hoc Tests: Performing a Post Hoc Tests: *t*-test



Agenda

- 0. Lesson overview
- 1. Two-Sample *t*-Tests
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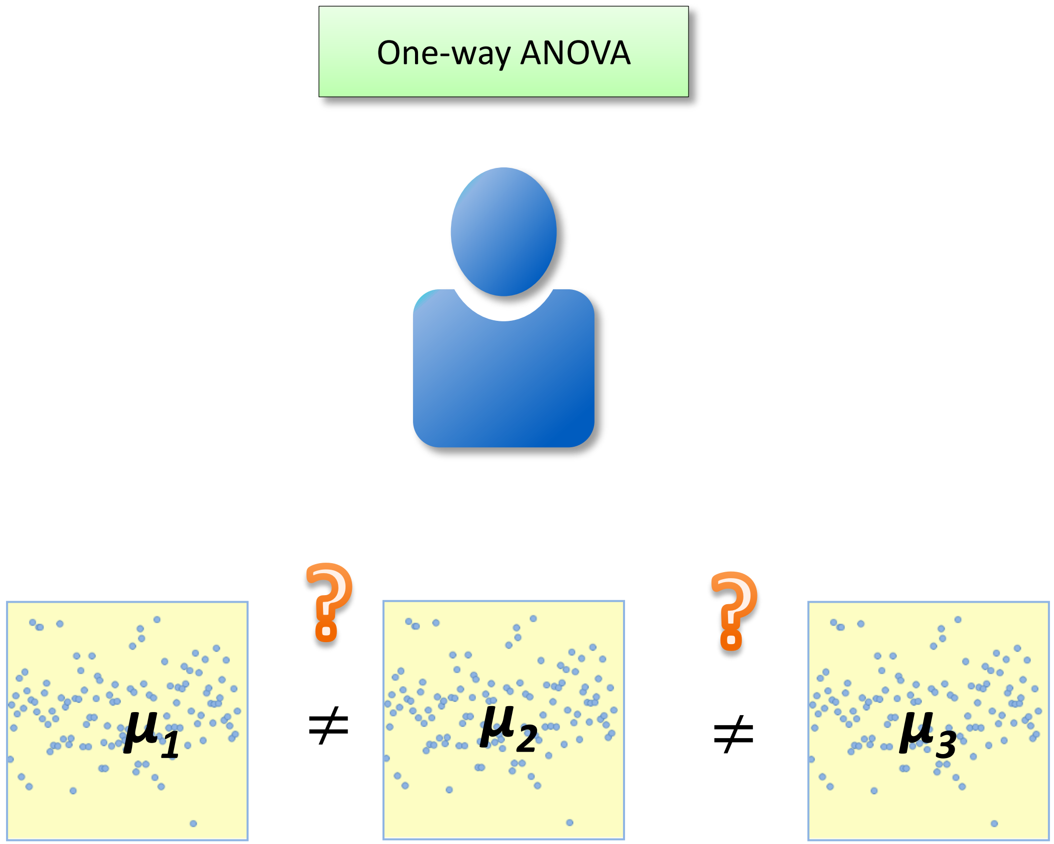
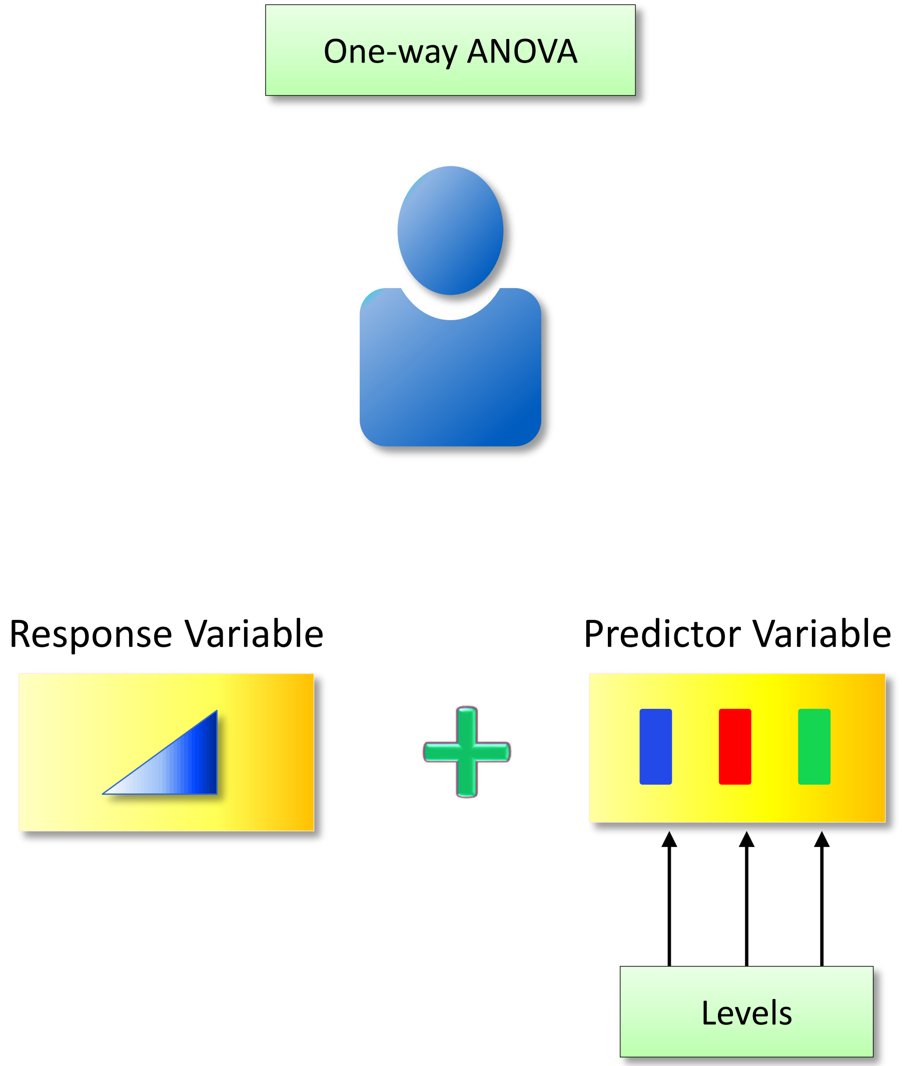
- 3. ANOVA with Data from a Randomized Block Design
- 4. ANOVA Post Hoc Tests
- 5. **Two-Way ANOVA with Interactions**
- 6. Summary

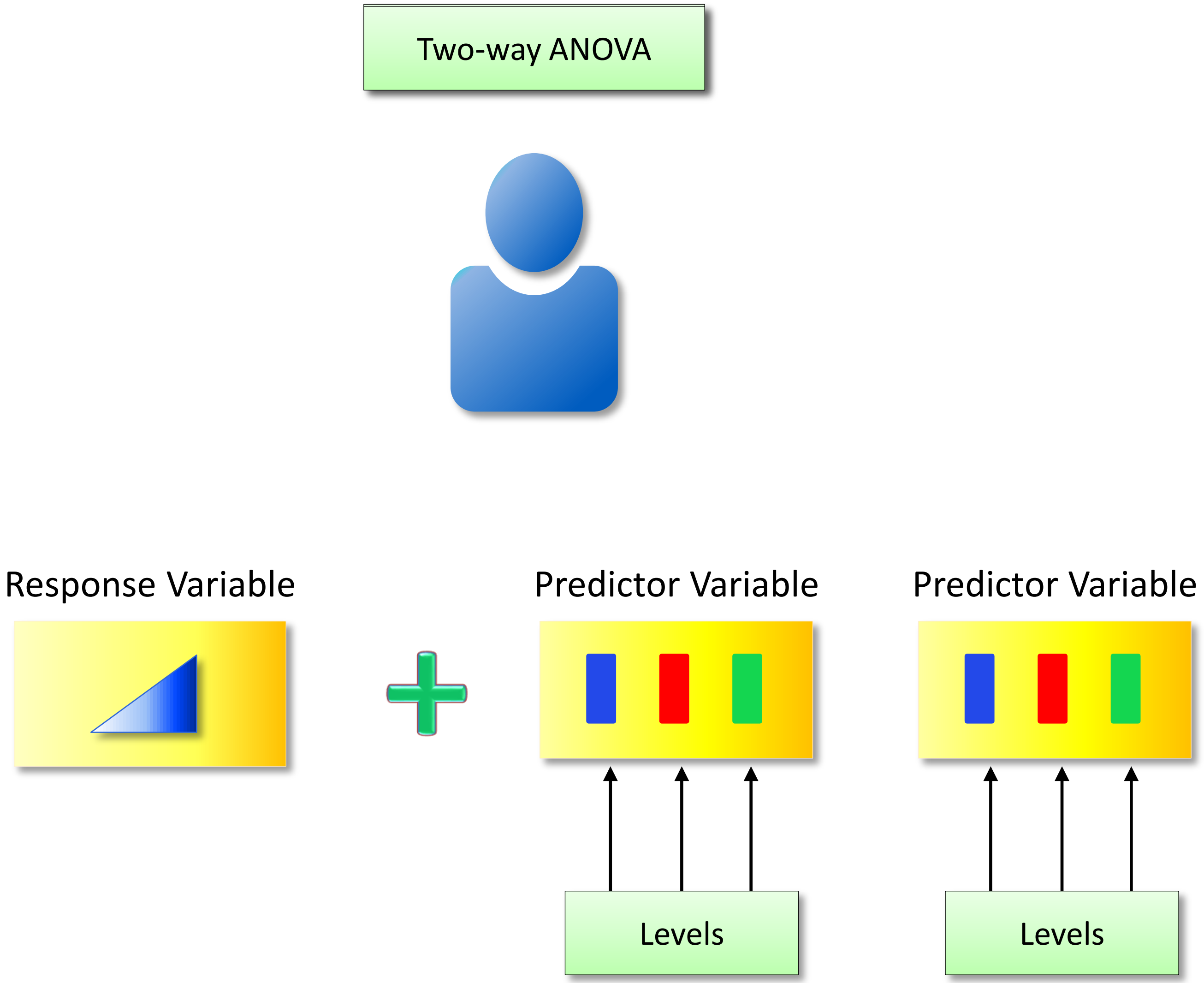
Two-Way ANOVA with Interactions: Introduction



Two-Way ANOVA with Interactions: Introduction



Two-Way ANOVA with Interactions: Introduction



Two-Way ANOVA with Interactions: Introduction

Two-way ANOVA

High alloy

Low alloy

Heat 1

Heat 2

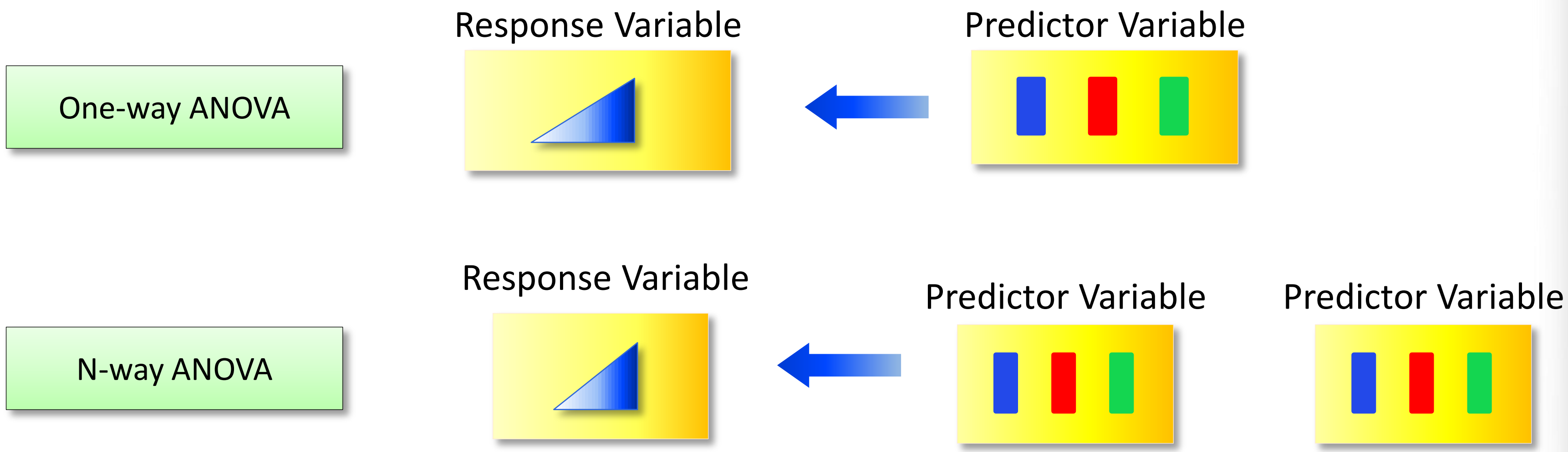
Heat 3

Heat 4

Two-Way ANOVA with Interactions: Objective

- Fit a two-way ANOVA
- Detect interactions between factors
- Analyze the treatments when there is a significant interaction

Two-Way ANOVA with Interactions: n-Way ANOVA



More than one Predictor Variable

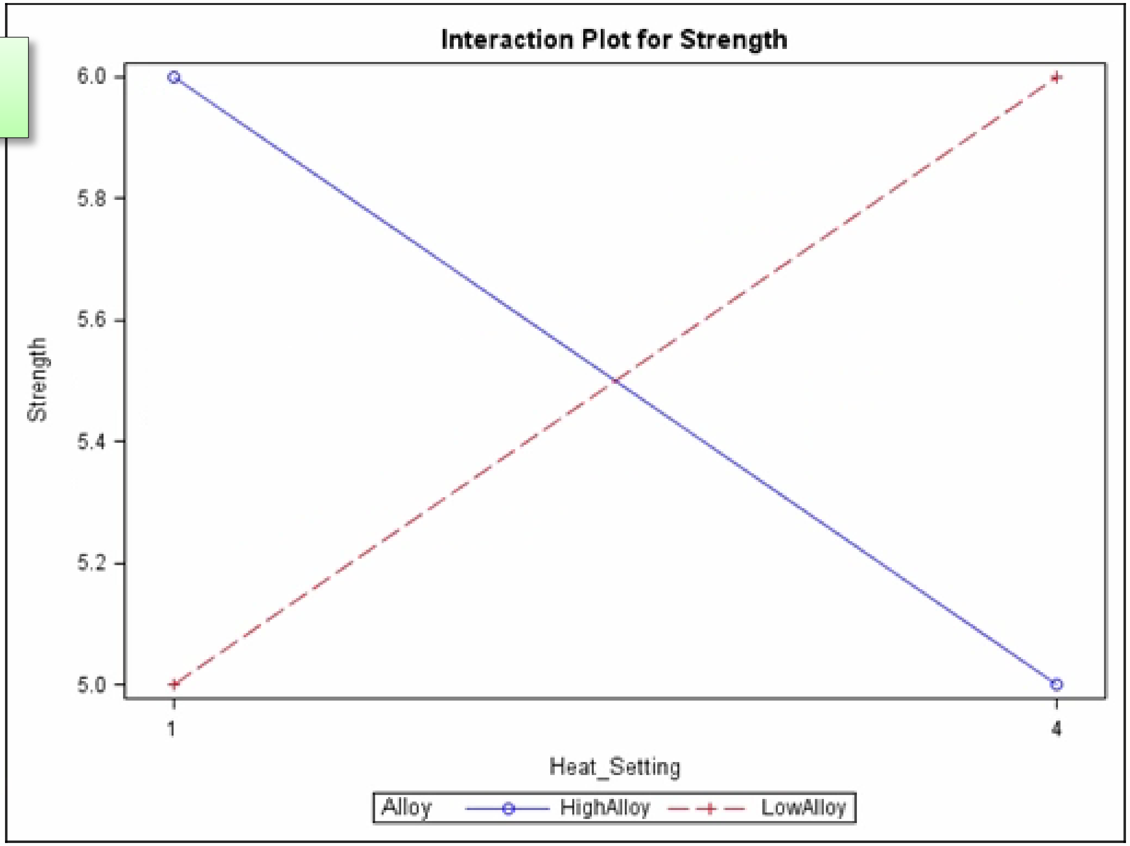
Two-Way ANOVA with Interactions: n-Way ANOVA

Randomized block design $\approx$ N-way ANOVA

$$Y_{ijk} = \mu + \underbrace{\boxed{\alpha_i}}_{\text{blocking factor}} + \underbrace{\boxed{\tau_j}}_{\text{interested factor}} + \varepsilon_{ijk}$$

Two-Way ANOVA with Interactions: interactions

Interaction



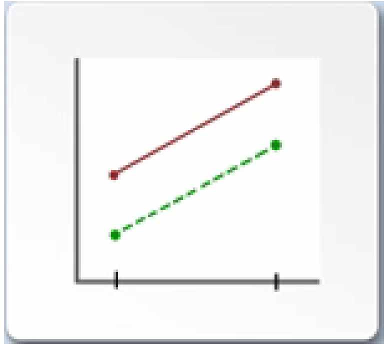
Two-Way ANOVA with Interactions: interactions

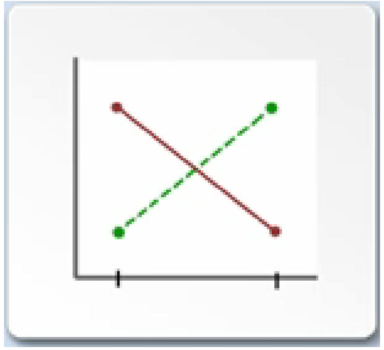
Two-way ANOVA

Interactions

?

Alloys





Heat setting

?

Two-Way ANOVA with Interactions:
The Two-Way ANOVA Model

Two-way ANOVA

When non-significant

Breaking Strength	=	Base Level	+	Alloy	+	Heat Setting	+	Alloy and Heat Setting	+	Unaccounted for Variation

$$Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk}$$

μ: overall population mean,
Regardless of alloy and heating

effect

$\alpha_i = \mu_i - \mu$

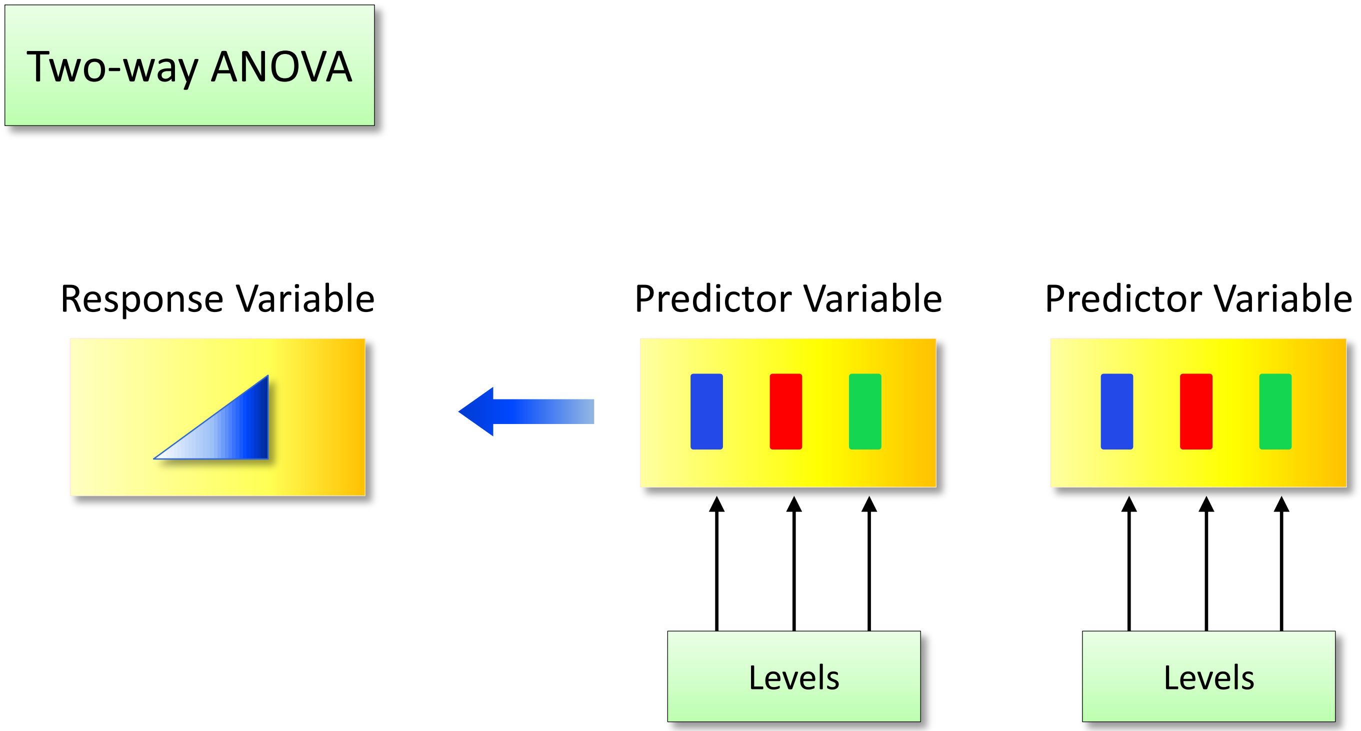
effect

$\beta_j = \mu_j - \mu$

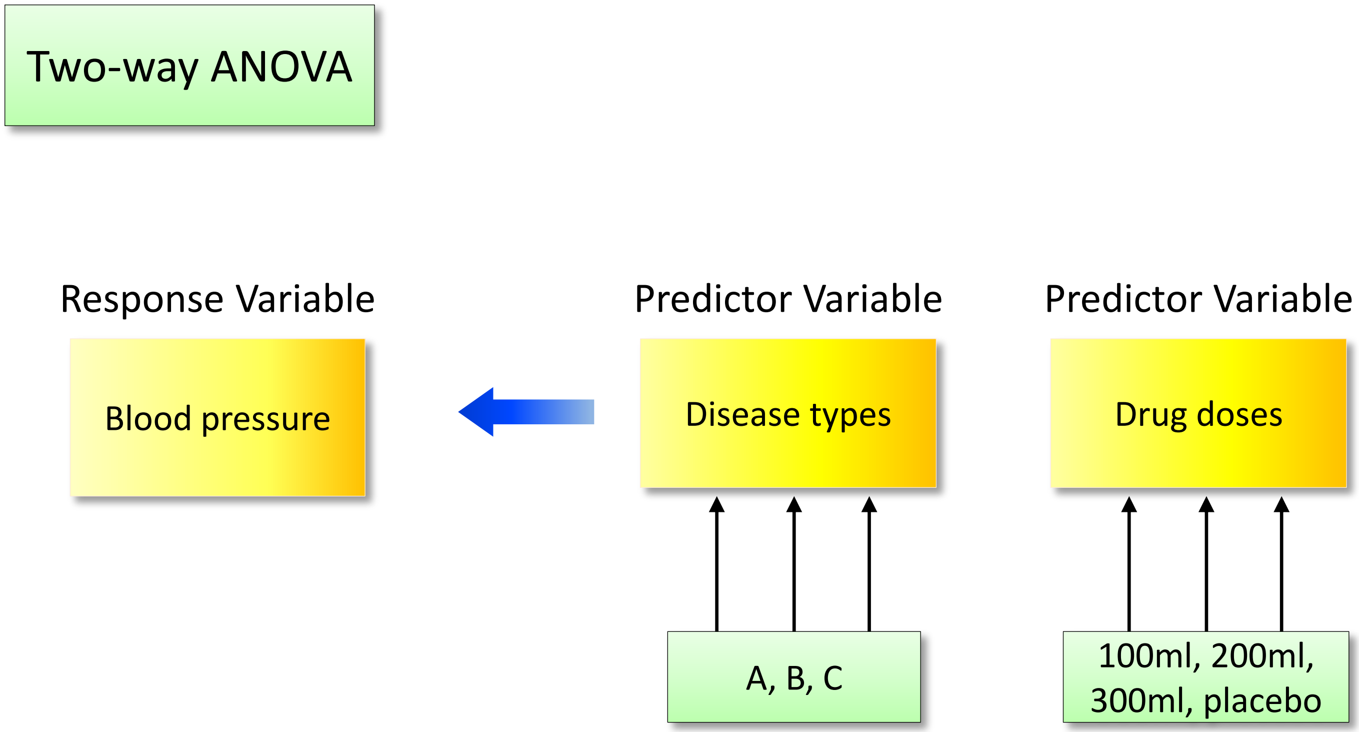
Effect of interaction

Error term

Two-Way ANOVA with Interactions: Scenario: Using a Two-Way ANOVA



Two-Way ANOVA with Interactions: Scenario: Using a Two-Way ANOVA



Two-Way ANOVA with Interactions: Identify the data

Drug

	PatientID	DrugDose	Disease	BloodP
1	69	2	B	13
2	162	4	A	-47
3	181	1	B	12
4	209	4	A	-4
5	308	2	A	4
6	331	4	C	37
7	340	4	C	-19
8	350	1	B	-9
9	360	2	B	-17
10	363	4	A	-41

Two-Way ANOVA with Interactions: Applying the model

Two-way ANOVA assumptions:

Independent observations

Normally distributed data

Equal variances

Two-Way ANOVA with Interactions: The Two-Way ANOVA Model

Two-way ANOVA

BloodP = Base Level + Disease + DrugDose + DrugDose and Disease + Unaccounted for Variation

$$Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk}$$

Observed **BllodP** for each patient

Overall mean of **BllodP**

effect

$\alpha_i = \mu_i - \mu$

effect

$\beta_j = \mu_j - \mu$

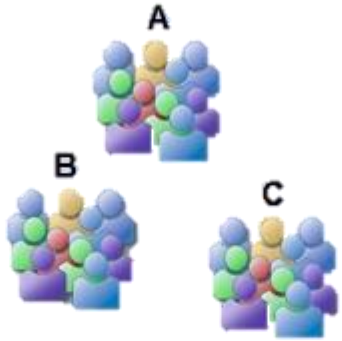
Effect of interaction

Error term

Two-Way ANOVA with Interactions: The Two-Way ANOVA Model

Two-way ANOVA

H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5 = \mu_6 = \mu_7 = \mu_8 = \mu_9 = \mu_{10} = \mu_{11} = \mu_{12}$

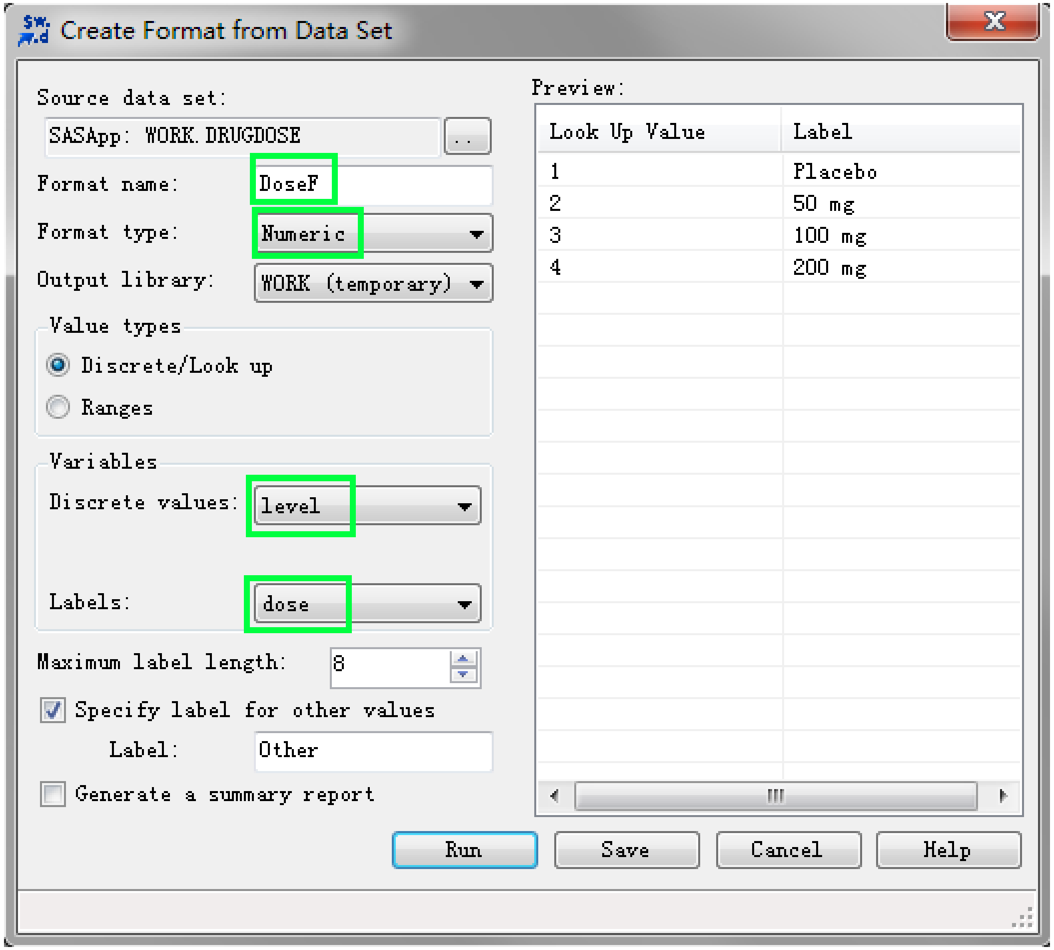


Two-Way ANOVA with Interactions: Examining Your Data

```
/* Create format, Method I,
via EG UI */
data drugdose;
input dose $ 8. level;
cards;
Placebo 1
50 mg   2
100 mg  3
200 mg  4
;
run;



/*Method II, by code*/
proc format library=work;
    value dosefmt
        1='Placebo'
        2='50 mg'
        3='100 mg'
        4='200 mg';
run;
```

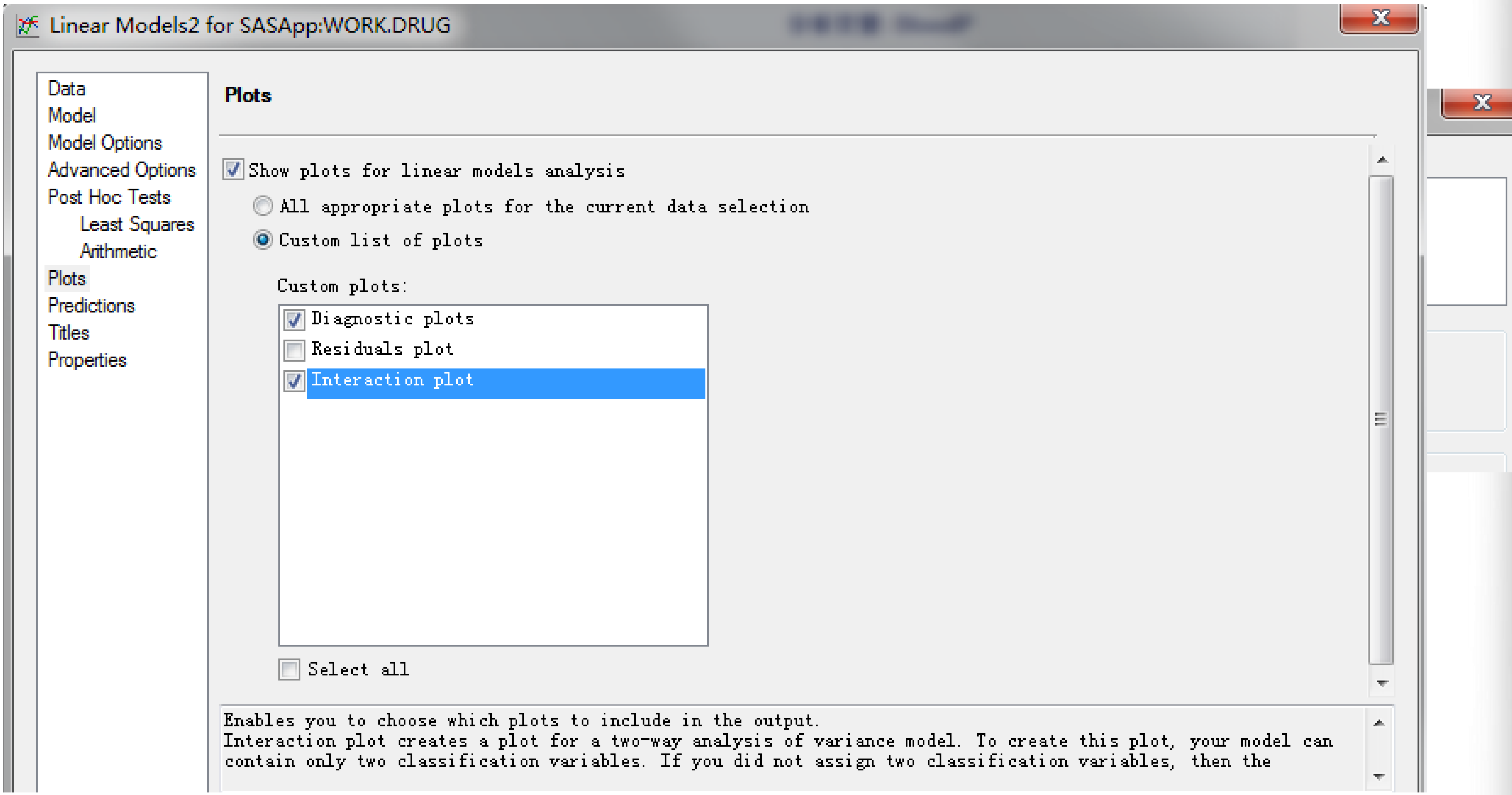


Two-Way ANOVA with Interactions: Examining Your Data

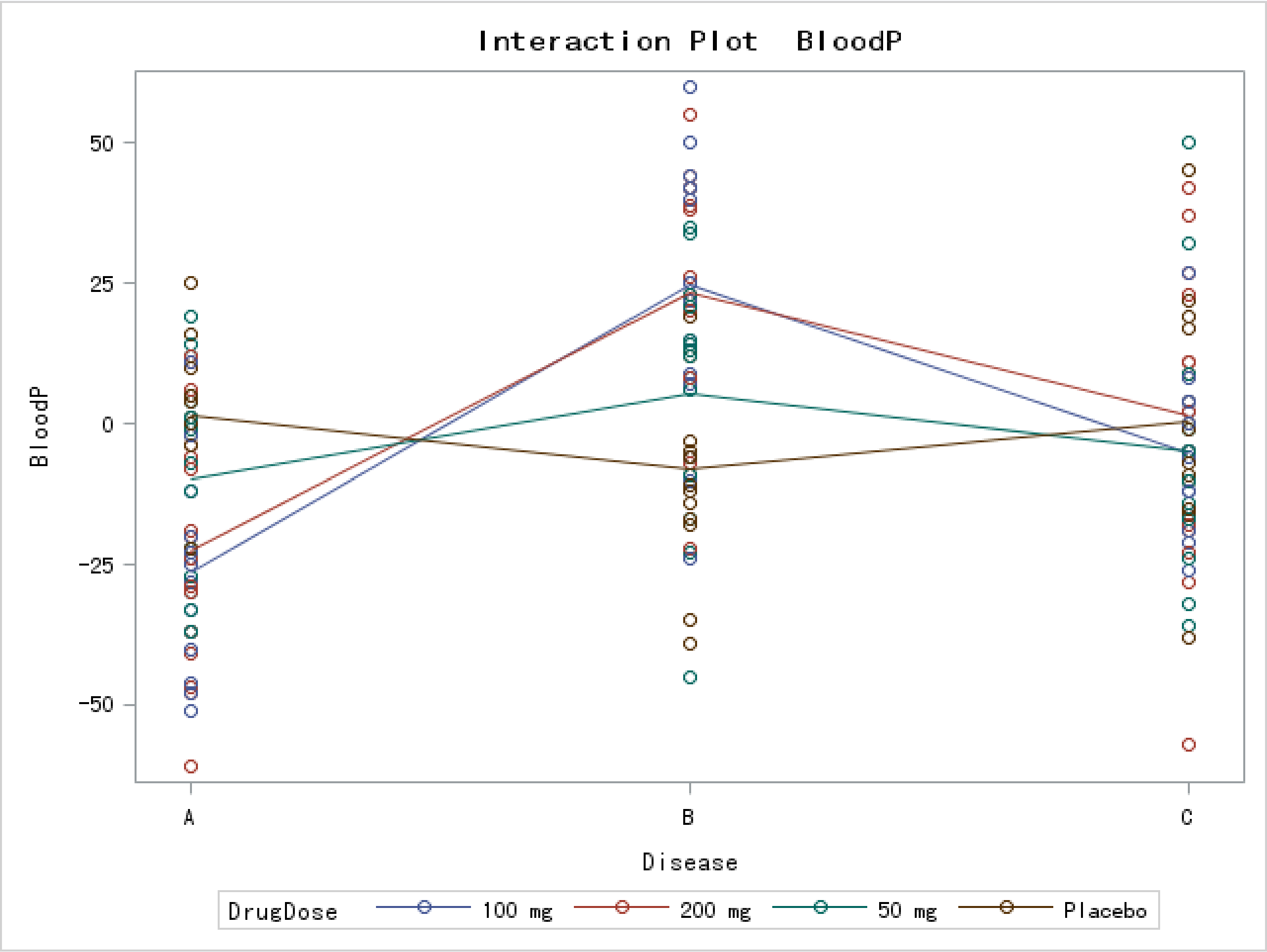
Analysis Variable : Bloo							
Disease	DrugDose	N Obs	Mean	Std De	[illegible]	[illegible]	N
A	Placebo	12	1.3333333	13.533348	[illegible]	[illegible]	12
	100mg	16	-9.6875000	18.888157	[illegible]	[illegible]	16
	200mg	13	-26.2307692	18.1390640	-51.0000000	11.0000000	13
	500mg	18	-22.5555556	21.0970369	-61.0000000	12.0000000	18
B	Placebo	15	-8.1333333	16.9109714	-39.0000000	22.0000000	15
	100mg	15	5.4000000	21.8886794	-45.0000000	35.0000000	15
	200mg	14	24.7857143	23.7427838	-24.0000000	60.0000000	14
	500mg	13	23.2307692	23.5872630	-22.0000000	55.0000000	13
C	Placebo	14	0.4285714	20.2929100	-38.0000000	45.0000000	14
	100mg	13	-4.8461538	24.0341637	-36.0000000	50.0000000	13
	200mg	14	-5.1428571	13.9827209	-26.0000000	27.0000000	14
	500mg	13	1.3076923	28.7847894	-57.0000000	42.0000000	13

In which disease type does the drug dose appear to be most effective?

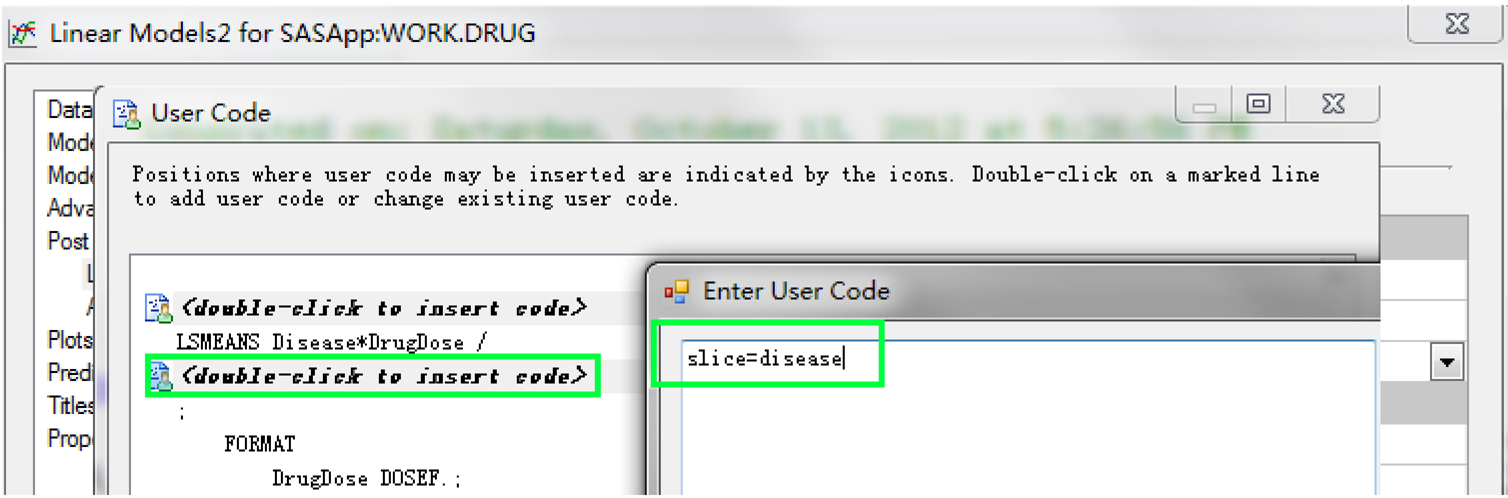
Two-Way ANOVA with Interactions: Performing Two-Way ANOVA with Interactions



Two-Way ANOVA with Interactions: Performing Two-Way ANOVA with Interactions



Two-Way ANOVA with Interactions: Performing a Post Hoc Pairwise Comparison



SLICE= specifies effects within which to test for differences between interaction LS-mean effects. This can produce what are known as *tests of simple effects* (Winer 1971). For example, suppose that A*B is significant and you want to test for the effect of A within each level of B. The appropriate LSMEANS statement is as follows:

```
lsmeans A*B / slice=B;
```

Two-Way ANOVA with Interactions: Performing a Post Hoc Pairwise Comparison

Linear Models

Least Squares Means

Disease	DrugDose	BloodP LSMEAN
A	100mg	-9.6875000
A	200mg	-26.2307692
A	500mg	-22.5555556
A	Placebo	1.3333333
B	100mg	5.4000000
B	200mg	24.7857143
B	500mg	23.2307692
B	Placebo	-8.1333333
C	100mg	-4.8461538
C	200mg	-5.1428571
C	500mg	1.3076923
C	Placebo	0.4285714

Dis	BloodP	
Diseas	ue	Pr > F
A	87	0.0029
B	14	<.0001
C	36	0.7815

Agenda

- 0. Lesson overview
- 1. Two-Sample *t*-Tests
- 2. One-Way ANOVA
- 3. ANOVA with Data from a Randomized Block Design
- 4. ANOVA Post Hoc Tests
- 5. Two-Way ANOVA with Interactions
- 6. **Summary**

Question 9.

If you want to compare the average monthly spending for males versus females which statistical method should you choose?

a) One-Sample t-Tests
b) One-Way ANOVA
c) Two-Way ANOVA

Answer: b

Home Work: Exercise 1

1.1 Using the t Test for Comparing Groups

Elli Sageman, a Master of Education candidate in German Education at the University of North Carolina at Chapel Hill in 2000, collected data for a study: she looked at the effectiveness of a new type of foreign language teaching technique on grammar skills. She selected 30 students to receive tutoring; 15 received the new type of training during the tutorials and 15 received standard tutoring. Two students moved away from the district before completing the study. Scores on a standardized German grammar test were recorded immediately before the 12–week tutorials and then again 12 weeks later at the end of the trial. Sageman wanted to see the effect of the new technique on grammar skills. The data are in the GERMAN data set.

Change	change in grammar test scores
Group	the assigned treatment, coded **Treatment** and **Control**

Assess whether the **Treatment** group changed the same amount as the **Control** group. Use a two-sided *t*-test.

a. Analyze the data using the t Test task. Assess whether the **Treatment** group improved more than the **Control** group.
b. Do the two groups appear to be approximately normally distributed?
c. Do the two groups have approximately equal variance?
d. Does the new teaching technique seem to result in significantly different change scores compared with the standard technique?

Home Work: Exercise 2

2.1 Analyzing Data in a Completely Randomized Design

Consider an experiment to study four types of advertising: local newspaper ads, local radio ads, in-store salespeople, and in-store displays. The country is divided into 144 locations, and 36 locations are randomly assigned to each type of advertising. The level of sales is measured for each region in thousands of dollars. You want to see whether the average sales are significantly different for various types of advertising. The **Ads** data set contains data for these variables:

Ad type of advertising

Sales level of sales in thousands of dollars

a. Examine the data. Use the Summary Statistics task. What information can you obtain from looking at the data?
b. Test the hypothesis that the means are equal. Be sure to check that the assumptions of the analysis method that you choose are met. What conclusions can you reach at this point in your analysis?

Home Work: Exercise 3

3.1 Analyzing Data in a Randomized Block Design

When you design the advertising experiment in the first question, you are concerned that there is variability caused by the area of the country. You are not particularly interested in what differences are caused by **Area**, but you are interested in isolating the variability due to this factor. The **ads1** data set contains data for the following variables:

Ad type of advertising
Area area of the country
Sales level of sales in thousands of dollars

a. Test the hypothesis that the means are equal. Include all of the variables in your model.
b. What can you conclude from your analysis?
c. Was adding the blocking variable **Area** into the design and analysis detrimental to the test of **Ad**?

Home Work: Exercise 4

4.1 post Hoc Pairwise Comparisons

Consider again the analysis of Ads1 data set. There was a statistically significant difference among means for sales for the different types of advertising. Perform a post hoc test to look at the individual differences among means for the advertising campaigns.

a. Conduct pairwise comparisons with an experiments error rate of a=0.05. (use the Tukey adjustment) which types of advertising are significantly different?
b. Use display (case sensitive) as the control group and do a Dunnett comparison of all other advertising methods to see whether those methods resulted in significantly different amounts of sales compared with display advertising in stores?

Home Work: Exercise 5

5.1 Performing Two-Way ANOVA

Consider an experiment to test three different brands of concrete and see whether an additive makes the cement in the concrete stronger. Thirty test plots are poured and the following features are recorded in the Concrete data set:

Strength	the measured strength of a concrete test plot
Additive	whether an additive was used in the test plot
Brand	the brand of concrete being tested

a. Use the Summary Statistics task to examine the data, with Strength as the analysis variable and **Additive** and **Brand** as the classification variables. What information can you obtain from Looking at the data?
b. Test the hypothesis that the means are equal, making sure to include an interaction term if the results from the Summary Statistics output indicate that would be advisable. What conclusions can you reach at this point in your analysis?
c. Do the appropriate multiple comparisons test for statistically significant effects?

Thank you!