CSE 326/426 (Fall 2022) Homework 8

Due on 11:55pm, Fri, Nov 4, 2022

Grading: All questions have the same points (10 each). Only questions with $*$ will be graded (undergraduates: do not need to answer the graduate-only question).

Submitting: Only electronic submissions in PDF format on Coursesite are accepted. Name your file as

<Your LIN>_HW8.pdf

A few ways to create a single PDF file. i) Use Microsoft Word and insert your writing or any figure taken, scanned, or plotted. Then choose "File→Print" in the main menu and you will find an option for outputting the file to a PDF file. ii) Use Latex to write your solution and include any figures. iii) Use the online Google Doc as an alternative of Word. It has sufficient features to combine multiple images and texts. Exporting to PDF files is similar as with Word.

Please DO NOT compress the PDF file as that will slow down the processing of your submission and the grading.

Questions:

1. $*$ In the PCA maximum-variance formulation, the objective function to maximize is
$$J(\mathbf{u}_1) = \mathbf{u}_1^\top S \mathbf{u}_1 \tag{1}$$
where $\mathbf{u}_1$ is the eigenvector of the covariance matrix S with the maximual eigenvalue λ_1. Prove that the objective function J is
$$J(\mathbf{u}_1) = \lambda_1. \tag{2}$$
(*Hints: use the eigen-decomposition of S and use the orthonormality of the eigenvectors.*)

2. Given the training data $\mathbf{x}^{(1)} = [1, 1]^\top$, $y^{(1)} = 1$, $\mathbf{x}^{(2)} = [1, 0]^\top$, $y^{(2)} = 1$, $\mathbf{x}^{(3)} = [-1, 0]^\top$, $y^{(3)} = 0$, and $\mathbf{x}^{(4)} = [-1, -1]^\top$, $y^{(4)} = 0$, find the within-class and between-class scatter matrices S_W and S_B for linear discriminant analysis. Don't just give the final matrices, but show step-by-step derivations going from $\mathbf{x}$ to S_0, S_1, S_W, $\mathbf{m}_0$, $\mathbf{m}_1$, and S_B.

3. $*$ Prove that the matrix $S = \mathbf{x}\mathbf{x}^\top$ is of rank 1, where $\mathbf{x}$ is a n-dimensional column vector. (*Hint: prove that any column of S is linearly dependent of the first column.*).

4. Given data points $\mathbf{x}^{(1)} = [1, 1, 1]^\top$, $\mathbf{x}^{(2)} = [1, -1, 0]^\top$, and $\mathbf{x}^{(3)} = [0, 2, 2]^\top$, compute the 3×3 covariance matrix $S = \frac{1}{3}\sum_{i=1}^{3} \mathbf{x}^{(i)} {\mathbf{x}^{(i)}}^\top$.

5. $*$ (Graduate only) List one commonality and one difference between LDA (linear discriminant analysis) and SVM.