Math339 **Fall 2022**

HW03

Trun-in needs to be submitted through the coursesite link in PDF format, which could be done by

- *Scan the manual write-up into PDF file.*
- *Typeset your turn-in in editors and convert into PDF.*
- *Prepare your turn-in through software like OneNote, Notable, ... and then save/export as PDF.*

1. Exercise 3.7 in the text.
2. Given a process $y_T = \beta_0 + \beta_1 t + \varepsilon_t,\ \varepsilon_t \overset{\text{uncorr.}}{\sim} (0, \sigma^2)$. Show that the second-order exponentially smoothed estimate, $\hat{y}_T = 2\tilde{y}_T^{(1)} - \tilde{y}_T^{(2)}$ (as in the equation (4.23) of the text), is an unbiased estimate of $\mathrm{E}(y_T)$.
3. (Example 4.2) Referring to the dataset, **CPI.xlsx**, posted in HW03 link on coursesite,
   (a) Visualize the series by generating the time-series plot for "CPI"series that is to be indexed by "Month"series. Comment on whether there exists a linear trend.
   (b) Find and plot the first-order exponentially smoothed estimates (using $\tilde{y}_0^{(1)} = y_1$, $\lambda = 0.3$) overlapped with the original series. Comment on the fitness of the first-order exponential smoother on the CPI series.
   (c) Find and plot the second-order exponentially smoothed estimates (using $\tilde{y}_0^{(1)} = y_1$, $\tilde{y}_0^{(2)} = \tilde{y}_0^{(1)}$, $\lambda = 0.3$) overlapped with the original series. Comment on the fitness of the first-order exponential smoother on the CPI series.
   (d) Implement the second-order exponential smoother to incorporate (4.24) as a new **function**. Fit the function to the data with keeping $\lambda = 0.3$ and plot the new fit with original data. Comment on the comparison between this revised fit to the fit in (c).