AN ILLUSTRATION OF SOME BASIC PROBABILITY CONCEPTS: DETERMINING PROBABILITIES OF WINNING IN SINGLE ELIMINATION TOURNAMENTS

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ABSTRACT

Single elimination tournaments are used as a context for illustrating some basic probability concepts. An algorithm which automatically constructs a tournament, given team seeding, is presented. Probabilities of winning are calculated as a function of team seed and strength, both of which are selected by the user. This technique allows for easy experimentation to see how the interaction of seeding and strength affects probabilities of winning. Users may also select a particular team to win up to a predetermined round in order to see the resulting changes in probabilities. Some examples with surprising results are used to illustrate how probabilities can change.

1. INTRODUCTION

Teachers at the high school or beginning college level can use the widespread interest in sports to illustrate practical uses of probability. This work is intended to provide such illustration. The NCAA basketball tournament has been used to motivate and illustrate statistics education [1, 2, 3]. Data from past tournaments were used to reinforce basic principles of probability, normal distributions, and linear regression. A different approach is described in this paper. The user must input tournament seeding and relative team strengths. From these inputs probabilities of winning are calculated. The procedure described could be used to compare the effects of tournament seed vs. team strength on the probabilities of winning each round of a single elimination tournament.

The design of tournaments in sports is of interest not only to athletic administrators and coaches, but to experimental statisticians and others. Bradley and Terry [4] provided a method of analysis of paired comparisons which permits an estimation of treatment preferences. Ranks are used in incomplete block designs of size 2. They state that their methods are particularly useful in problems with subjective ranking. An illustration of the use of their methods was given for a taste-test experiment of pork roasts compared by ranking pairs. Kendall [5] noted that round-robin tournaments are analogous to balanced pairwise experiments. Single elimination tournaments are similar to choosing one experimental condition from many when only one pair at a time can be evaluated, as in sensory evaluations [6]. In communications network routing, single elimination tournaments are analogous to the knockout switching scheme for deciding among packets contending for a circuit. This paper will use the context of sports single elimination (knockout) tournaments as an illustration to calculate probabilities of interest in paired comparison competition.

Single elimination tournaments are common in several sports. For example, in college basketball post season conference, NIT, and NCAA tournaments, in football tournaments, and in tennis tournaments, teams or individuals are ranked by some method before the completion begins.

2. COACHING DECISIONS

Coaches of athletic teams face decisions related to strategy, both for particular games and for the season. For example, a basketball team may have a young player who needs game experience to improve and eventually strengthen the team; but, while this player is gaining experience, the team might lose games that it would have won using other players. The team might also be strengthened late in the season by a new or previously injured returning player. A coach might also employ new plays or strategies in games in order to gain proficiency that will be of value at the end of the season, but because of a learning period might cost games during the regular season. At a season ending tournament, the team will have a greater chance of winning with the now more experienced or new player or new plays, but because of the extra regular season losses, may start the tournament with a lower seed. Weighted against entering a tournament with a stronger team is the assumed decreased probability of winning from a lower tournament seeding. This paper describes a method that can help coaches make decisions by giving them probabilities of winning tournaments under various tournament seeding and team strengths. Using this knowledge a coaching staff can make better decisions affecting the balance between higher tournament seeding and greater strength at season end.

3. RANK ORDER VS. SLOT ORDER

In a standard single elimination tournament structure the teams or players are rank ordered. The rank position is also called the seed. The ranking may be determined in several ways, but the two most common are by won-lost record or by some group's opinion as to relative strength of each team. After selecting the seeding, all the possible game pairings are determined. Most commonly, a "folding" procedure determines these match-ups. Assuming the number of teams is a power of 2, the 1^{st} ranked team plays the last ranked team, the 2^{nd} ranked team plays the next to last, etc. Thus, the seeded list of teams is folded in the middle to assign the match pairs. These initial pairings are then rearranged such that if the higher ranked team always wins, then the top four teams will be in the semifinal and the top two teams will meet in the tournament final. This rearrangement will be called slot order. In an 8 team tournament, slot 1 will be assigned to the team ranked 1^{st}, slot 2 the team ranked 8^{th}, slot 3 the 4^{th}, slot 4 the 5^{th}, slot 5 the 2^{nd}, slot 6 the 7^{th}, slot 7 the 3^{rd}, and slot 8 the 6^{th}. The assignment of ranked teams to slots for any size tournament field can be accomplished by a folding algorithm as described below.

If teams are not ranked, but are seeded randomly, then the final will most likely match the 1^{st} ranked team against a team ranked about 3^{rd} [7]. While this observation indicates an inherent problem in a knockout tournament as compared to a round robin tournament, where every team plays every other team, it provides a possible motivation for a team to "shake up" the rankings by increasing their strength entering a tournament relative to higher seeded teams and not necessarily to strive for a higher ranking prior to the tournament.

4. FAIRNESS IN TOURNAMENT STRUCTURE

The first objective of a tournament is normally to structure the tournament so that the highest ranked team has the highest probability of winning the tournament championship and other highly ranked teams have the best chance of advancing. Of several alternative tournament designs, Appleton [8] found that for closely matched teams, the round robin tournament played twice is the most effective at allowing the best team to win. However, single elimination tournaments produce a champion with fewer games and provide the excitement that a lower ranked team has an increased change of winning. They also provide a schedule such that the two top ranked teams will be able to meet in a championship final.

Horen and Riezman [6] judged various single elimination tournament structures under four criteria. These were: (1) Does the structure (or draw) maximize the probability that the best team will win? (2) Is it fair, i.e. does a better team have a higher probability of winning? (3) Is the probability maximized that the two best teams will meet in the final? and (4) Is the expected value of the winning team maximized? They found that the standard structure does meet these criteria for a four team tournament. However, looking at the first two criteria for an eight team tournament, the normal tournament structure is not the best. Eight other draws maximize the probability that the best team wins, i.e. meet that criteria better than the standard tournament structure. The slot order produced by the folding algorithm is probably the most popular structure in practice because it has the appearance of fairness to all teams.

After the first round, a new list is made of the winners from the previous round. This new list is again folded in the middle. Hwang [9] shows that before each round if the remaining teams are reordered by seeding, then the new seeded draw would be fair. This reordering is not commonly done and is not assumed here. The process continues until there is one game for the tournament championship. After each round the teams have, in effect, reseeded themselves, but only to predetermined positions.

McGarry and Schutz [10] found for eight team tournaments, that the knockout "tournament's efficacy is notably enhanced, however, in some cases beyond that of the RR [round robin] tournament if double elimination procedures are used and the seeding is reasonably accurate." [10, p. 65] They conclude that there is no single, best tournament structure. Methods for labeling and counting double-elimination tournaments have been proposed by Edwards [11].

5. SETTING THE TOURNAMENT FIELD AND THE FOLDING ALGORITHM

In the program used to calculate winning probabilities team ranks and strengths are entered by the user. These strengths may be subjective, or they may be derived based on team records as in Kaplan and Garstka [12]. The user gives a relative numerical strength estimate for each team. This format allows the user to easily vary team ranks and relative strengths in order to study how probabilities of winning may vary under a range of tournament setups.

Several assumptions are made. The games are independent, and the probability of one team beating another remains constant throughout the tournament. There is one tournament structure as determined by the folding algorithm. The tournament is a single elimination tournament.

The program also allows the user to select a team to win up to a given round. This permits the user to ask questions such as, "Suppose my team wins the first 2 rounds, how does that change the probability of winning the tournament?"

Single elimination tournaments are easiest to schedule if the number of teams in the tournament is a power of two. If the number of teams is not a power of two, one or more of the top seeded teams will get a "bye" in the first round. The number of teams with byes is such that the second round will have no byes and the number of teams left after the first round will be a power of two. The starting number of slots in the tournament will be the smallest number which is a power of two equal to or greater than the number of teams in the tournament. For example, if there are 6 teams, the number of slots is $2^3 = 8$. The number of byes is $8 - 6 = 2$; therefore, the first 2 seeded teams will receive byes in the first round. The last 4 seeded teams will play in the first round, leaving 2 for the next round. Together with the first two teams, this will mean a full round of 4 teams will play in round 2. The number of rounds in the tournament is equal to the power to which 2 is raised to get the number of slots in the tournament. If N is the number of teams in the tournament, then R, the number of rounds, may be found by

$$R = \left\lceil \frac{\log_2 N}{\log_2 2} \right\rceil = \lceil \log_2 N \rceil, \quad \text{since } \log_2 2 = 1$$

where $\lceil x \rceil$ is the smallest integer greater than or equal to x. The number of slots, S, in the tournament is $S = 2^R$.

Figure 1 gives the form in which tournaments pairings are typically given for an eight team field. This form presents a clear picture of the games each team must win and the other possible teams that any team may play at each round of the tournament. The folding algorithm determines the initial placement, or slot, of each team in the figure.

Once the teams have been placed in rank order, they must be rearranged in the proper order for tournament pairings, as in Figure 1. A list of N teams in rank order, where $N = 2^R$, is "folded" in the middle such that team 1 is paired with N, 2 with $N-1$, ..., $\frac{N}{2}$ with $\frac{N}{2}+1$. This folding will produce the correct pairings for the first round of a single elimination tournament; however, the teams are not in the proper slots for the entire tournament and will not create a structure such that the 1st ranked team will play the 2nd ranked team in the final game, assuming they both win all their previous games. Rearranging the rank order into the proper slot order for an entire single elimination tournament can be accomplished by folding the rank ordered teams R times. For example, in an 8 team field, there must be 3 folds, as shown in Figure 2. The order index number of each team after the R^{th} fold is the correct slot number for the normal tournament structure.

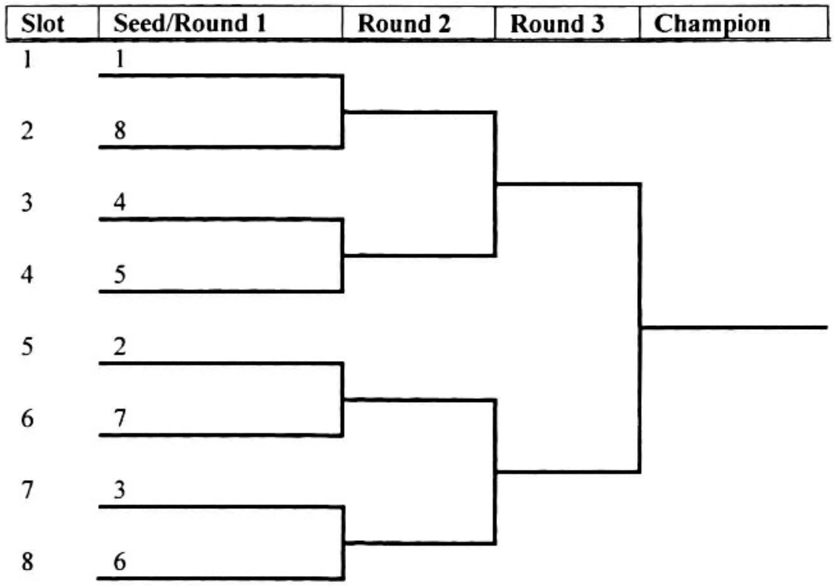

Figure 1. Common structure for an eight team single elimination tournament.

Teams order by Rank	First fold	Second fold	Third fold
1	1, 8	1, 8, 4, 5	1, 8, 4, 5, 2, 7, 3, 6
2	2, 7	2, 7, 3, 6	
3	3, 6		
4	4, 5		
5			
6			
7			
8			

Figure 2. Changing from rank order to tournament slot order for an 8 team tournament by the folding procedure

An algorithm which will assign teams to their proper slots, starting with a list of teams indexed in rank order, is shown in Figure 3 (next page). In the algorithm, a Rank (R) to Slot (S) matrix, $RS(i, j)$ is a two dimensional matrix in which the folding will occur. The size of the matrix is $S \times S$. Initially, column 1 will contain the N teams in rank order and $S - N$ byes. After completion of the algorithm, row 1 of RS will be the list of teams in slot order for the tournament.

6. PROBABILITIES OF WINNING

All the probabilities calculated by the program are based on the seed and slot of each team and a value for relative strength of each team, as determined by the program user. This strength must be a positive number, for example, a value between 1 and 1000, where 1 is the weakest possible

```
Get the number of folds, R (will be the number of iterations of the algorithm)
Set the (number of slots) to 2^R
Initialize the (number of columns) to 0
FOR I = 1 TO R
    Set (number of slots) = (number of slots)/2
    Set n = 0
    FOR j = 2* (number of slots) TO (number of slots) + 1 IN STEPS OF -1
        Set n = n + 1
        Set c = (number of columns) + 1
        FOR k = 1 TO (number of columns)
            Set RS(n, c) = RS(j, k)
            Set c = c + 1
        ENDFOR
    ENDFOR
    Set (number of columns) = (number of columns) * 2
ENDFOR
```

Figure 3. The folding algorithm to convert rank order to slot order (Pseudo code).

strength and 1000 is the strongest. A value of 0 is automatically assigned to any bye slot. Normally, the highest seeded team will have the highest strength value associated with it and the lowest seeded team the lowest strength. This ordering is not a requirement by the program, however. The user may give any relative strength desired to any team in the tournament. If the user feels that team 6 is stronger than teams 4 and 5, it should be given a higher value then those two teams.

The probability of team i beating team j is,

$$P_{ij} = \frac{\text{strength}(i)}{\text{strength}(i)+\text{strength}(j)}.$$

Note that $P_{ij} = 1 - P_{ji}$. This is the most commonly used method of calculating P_{ij} [4], although other models have been used [1]. Since it is assumed that a team's strength will not change over the course of the tournament, these probabilities do not change, so a table of all possible matches may be easily constructed prior to calculating probabilities of a team winning rounds in the tournament.

If the strength(A) = 400 and strength(B) = 200, then team A is viewed as twice as strong as team B. Team A's probability of beating B is

$$P_{AB} = 400/(400+200) = 0.6667 \text{ and } P_{BA} = 1 - P_{AB} = 0.3333.$$

That is, team A is twice as likely to beat team B as team B is to beat team A.

The program gives probabilities of each team winning each round in the tournament. This would include the probability of winning the championship for each team. The probability of team i winning through round r, W_{ir}, is the probability that i won through round $r-1$ times the probability that i beats its opponent in round r. In round one there is one possible opponent, but the number of possible opponents increases in later rounds, up to half the total number of slots in the championship round. Therefore, the opponent of team i is conditional on winning the previous rounds. In general, for a single elimination tournament

$$W_{ir} = W_{i,r-1}\left[\sum_{k=v}^{u} P_{ik}W_{k,r-1}\right], \text{ where } W_{i,0} = 1 \text{ and } r > 0$$

v and u are the lower and upper limits of the slot numbers of the possible opponents. In round r, the lowest slot number of all the potential opponents of the team in slot i is [11]

$$S(i,r) = 1 + 2^{r-1} + 2^{r+1}\left\lfloor \frac{i-1}{2^r} \right\rfloor - 2^{r-1}\left\lfloor \frac{i-1}{2^{r-1}} \right\rfloor$$

where $\lfloor x \rfloor$ is the largest integer value less than or equal to x. The other potential opponents for i in the same round are the next $2^{r-1}-1$ slots. Therefore, $v = S(i,r)$ and $u = v + 2^{r-1} - 1$.

The program also allows the user to assume a selected team makes it to a selected round of the tournament. A probability of 1 is assigned to the team for each of the rounds before the selected round and all the potential opponents' probabilities are changed to 0.

7. AN EXAMPLE

When execution of the program begins, the following form appears.

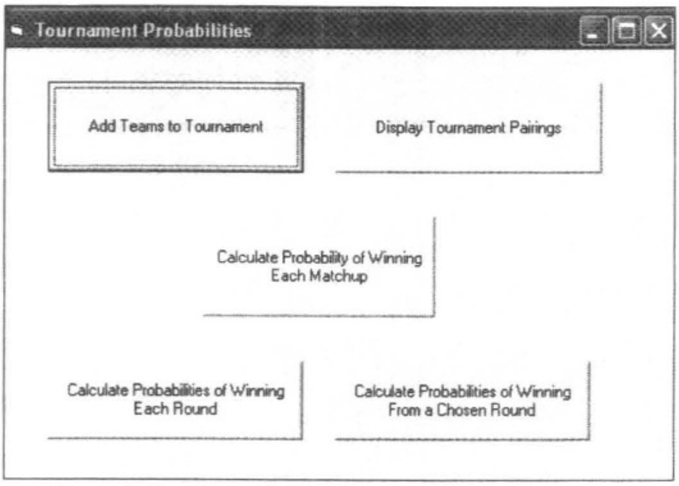

Figure 4. First Form Probabilities of Winning Program.

Selecting the first command, "Add Teams to Tournament", allows the user to input the number of teams in the tournament and add the teams one at a time, giving the team rank, name, and strength. A summary of this information can then be displayed and/or printed, as shown is Figure 5. The number of teams does not have to be a power of 2. The program will fill in "byes", with a strength of 0, to complete the structure.

Rank	Team Name	Strength
1	North Carolina State	100
2	Virginia	90
3	Duke	80
4	Maryland	70
5	Wake Forest	60
6	Georgia Tech	50
7	Clemson	40
8	North Carolina	30

Figure 5. Rank, teams and strengths of an example tournament.

The second button, "Display Tournament Pairings," will run the folding algorithm and give the entire tournament in slot order, as in Figure 6.

Slot	Team Name	Rank
1	North Carolina State	1
2	North Carolina	8
3	Maryland	4
4	Wake Forest	5
5	Virginia	2
6	Clemson	7
7	Duke	3
8	Georgia Tech	6

Figure 6. The example tournament structure.

Figure 7 is the probability matrix, P_{ij}, of all possible matches and results from selecting the third button, "Calculate Probability of Winning Each Matchup."

Team Name	Rank	1	2	3	4	5	6	7	8
North Carolina State	1		0.5263	0.5556	0.5882	0.6250	0.6667	0.7143	0.7692
Virginia	2	0.4737		0.5294	0.5625	0.6000	0.6429	0.6923	0.7500
Duke	3	0.4444	0.4706		0.5333	0.5714	0.6154	0.6667	0.7273
Maryland	4	0.4118	0.4375	0.4667		0.5385	0.5833	0.6364	0.7000
Wake Forest	5	0.3750	0.4000	0.4286	0.4615		0.5455	0.6000	0.6667
Georgia Tech	6	0.3333	0.3571	0.3846	0.4167	0.4545		0.5556	0.6250
Clemson	7	0.2857	0.3077	0.3333	0.3636	0.4000	0.4444		0.5714
North Carolina	8	0.2308	0.2500	0.2727	0.3000	0.3333	0.3750	0.4286	

Figure 7. Example of individual match-up probabilities of winning.

The first three command buttons must be selected in order to get the tournament structure and P_{ij} matrix necessary for the last two buttons.

"Calculate Probabilities of Winning Each Round" is selected in order to get the probabilities of every team winning each round, up to and including the tournament championship. See Figure 8.

Slot	Team name	Rank	1	2	3
1	North Carolina State	1	0.7692	0.4655	0.2701
2	North Carolina	8	0.2308	0.0728	0.0217
3	Maryland	4	0.5385	0.2575	0.1271
4	Wake Forest	5	0.4615	0.2041	0.0931
5	Virginia	2	0.6923	0.3967	0.2152
6	Clemson	7	0.3077	0.1157	0.0405
7	Duke	3	0.6154	0.3267	0.1679
8	Geogia Tech	6	0.3846	0.1608	0.0645

Figure 8. Example probabilities of each team winning each round.

Since the rank order of teams is the same as the strength order in this example, we would expect the probabilities of winning the tournament also to be in the same order. The results shown in Figure 8 are consistent with this expectation.

The last button, "Calculate Probabilities of Winning From a Chosen Round", is used to guarantee that a selected team wins up to a user chosen round. For example, in Figure 9 we can see how winning probabilities change if we select Duke to make it to the championship round. Notice that Duke's probability of winning the first and second round has changed to 1 and the appropriate opponent's probability in each of the first two rounds has changed to 0. Duke's probability of winning the tournament goes from 0.1679 to 0.5138.

Slot	Team name	Rank	1	2	3
1	North Carolina State	1	0.7692	0.4655	0.2586
2	North Carolina	8	0.2308	0.0728	0.0198
3	Maryland	4	0.5385	0.2575	0.1202
4	Wake Forest	5	0.4615	0.2041	0.0875
5	Virginia	2	0.6923	0.0000	0.0000
6	Clemson	7	0.3077	0.0000	0.0000
7	Duke	3	1.0000	1.0000	0.5138
8	Geogia Tech	6	0.0000	0.0000	0.0000

Figure 9. Example where the 3 seed selected to make the 3rd round.

8. EXPLORING THE EXAMPLE

The power of creating a program such as the one used for this work is in facilating the exploration of alternative scenarios. Coaches or other experimenters can understand what happens when a team's strength is changed from what might be expected by its seed in the tournament. For example, what would be the minium strength each team needs, given their current seed, in order to have a higher probability of winning the tournament than any other team?

Start with the example above, where the top ranked team has a relative strength of 100 and the strength of each succeeding rank is reduced by 10. Table 1 gives the minimum strengths needed for each team to increase its probability of winning the tournament to a value greater than any other team. Strengths are given in integer values and PWT is the Probability of Winning the Tournament.

There are some interesting observations that can be made. In general, as the rank of the team whose strength is being adjusted goes down, the higher the strength needed to get the highest PWT. One exception is the lowest ranked team. It only needs a strength of 101 to make $P_{8,1} > 0.5$, which is the 8th ranked team's 1st round match. To win the tournament the 8th ranked team needs only a relative strength of 112 to have the highest PWT. This is a value less than any other team needs except the 2nd ranked team.

Notice also that the needed increase in strength is not constant as the seeds get lower. The 3rd and 4th seeded teams, Duke and Maryland, need the same strength to gain the highest PWT. The 5th team, Wake Forest, needs only 3 more than the 4th team to have the highest PWT, but the 6th team needs 10 more than the 5th. These seeming inconsistencies are a result of the structure of the tournament. Even though there is a constant difference between succeeding ranks, there is no such constant difference in necessary strength increases.

Not seen in Table 1 is an interesting fact observed in the winning probability tables. While the strength has been adjusted to give each team in succession the highest probability of winning the tournament, they still have a lower probability of winning each of the first two rounds than the second highest PWT team.

The strength adjustments needed to gain the highest PWT are not only dependent on the slot of each team, but on the relative strengths of other teams. Strength adjustments could vary a great deal with changes in the strength estimates of other teams.

Team to Increase Strength	Seed	Minimum strength to have highest PWT	PWT	Second highest PWT	Team with second highest PWT
Virginia	2	107	0.2611	0.2587	N. C. State
Duke	3	113	0.2535	0.2507	N. C. State
Maryland	4	113	0.2349	0.2336	N. C. State
Wake Forest	5	116	0.2288	0.2277	N. C. State
Georgia Tech	6	126	0.2426	0.2401	N. C. State
Clemson	7	128	0.2408	0.2407	N. C. State
North Carolina	8	112	0.2026	0.2001	Virginia

Table 1. Minimum strength needed for each team to get highest probability of winning the tournament. PWT is Probability of Winning the Tournament.

As an example, consider the 4^{th} seeded team. With the existing set of team strengths, Maryland needs to increase its strength to 113 to get the highest PWT. If the last seeded team, North Carolina, has its strength changed from 30 to 10, then we find that Maryland needs to increase its strength to 130 in order to have the highest PWT. This is a surprisingly large increase. It may seem counterintuitive that if a potential tournament opponent's strength decreases, a team needs to greatly increase its strength in order to get the highest PWT. The explanation is in the structure of the tournament. If the 8^{th} place team is weakened, then the 1^{st} place team has an increased probability of advancing to the next round and presenting a stronger opponent to the 4^{th} seed should that team win its first round match. Even though the result makes sense, it is still surprising.

9. EXTENSIONS

There are many areas for future study. The program used could have enhancements related to input and output and further analytic capabilities. Allowing changes to team strengths during the tournament would be helpful. Team strengths do change in practice with injuries, fatigue, or emotional factors, such as momentum or rivalry inspired motivation. It would be helpful to capture these changes. Analytic work, while complex, could calculate break-even points for changing winning probabilities versus team strengths. Computer simulations might also be constructed to study the effect of factors such as team strengths and tournament rankings. Each team's strength might be drawn from a random distribution with distribution parameters as inputs, rather than relative strength. Repeated simulations would be run to get confidence intervals on probability outputs. The program could be used to further enhance basic statistics instruction if team strengths were set as random variates from given distributions.

10. CONCLUSION

The folding algorithm and probability and slot calculations presented, when incorporated in a program, give experimenters a tool to study possible tournament outcomes for varying team strengths and rankings. The probability results of various rank and strength settings may be used to make player and strategic decisions during the sport's season. The program may also be used in other paired experimental contexts to calculate probabilities directly or to confirm analytic results. Instructors of beginning statistics may also find it useful to motivate and illustrate course topics.

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Author: Bengtson, Neal M.

Title: An Illustration of Some Basic Probability Concepts: Determining Probabilities of Winning in Single Elimination Tournaments

Source: Math Comput Educ 44 no2 Spr 2010 p. 92-104
ISSN: 0730-8639

Publisher: M A T Y C Journal, Inc.
Box 158, Old Bethpage, NY 11804