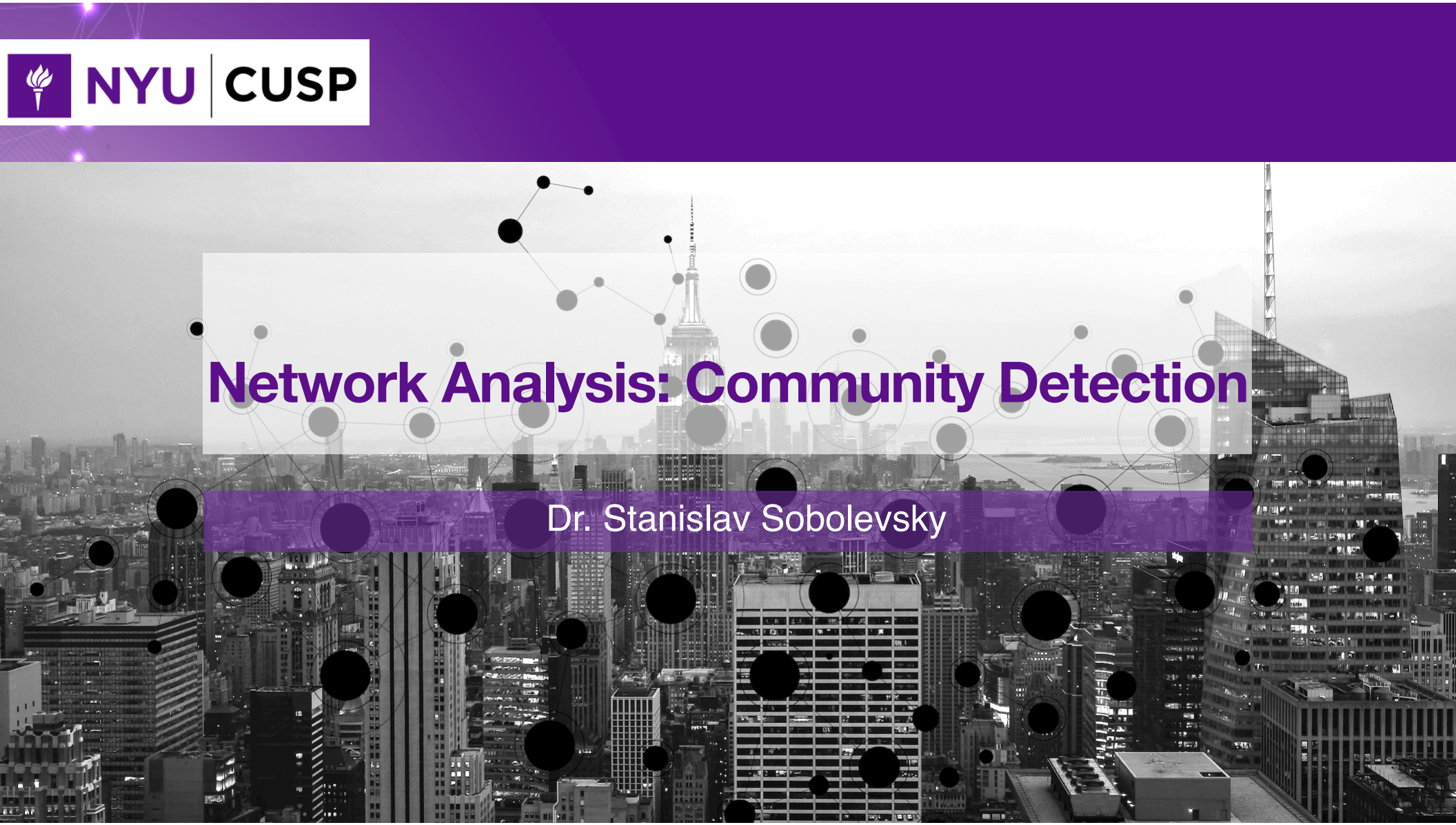
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Network Analysis: Community Detection
Dr. Stanislav Sobolevsky

- ✓Connectivity in Networks
- ✓Shortest path routing
- ✓Network community detection

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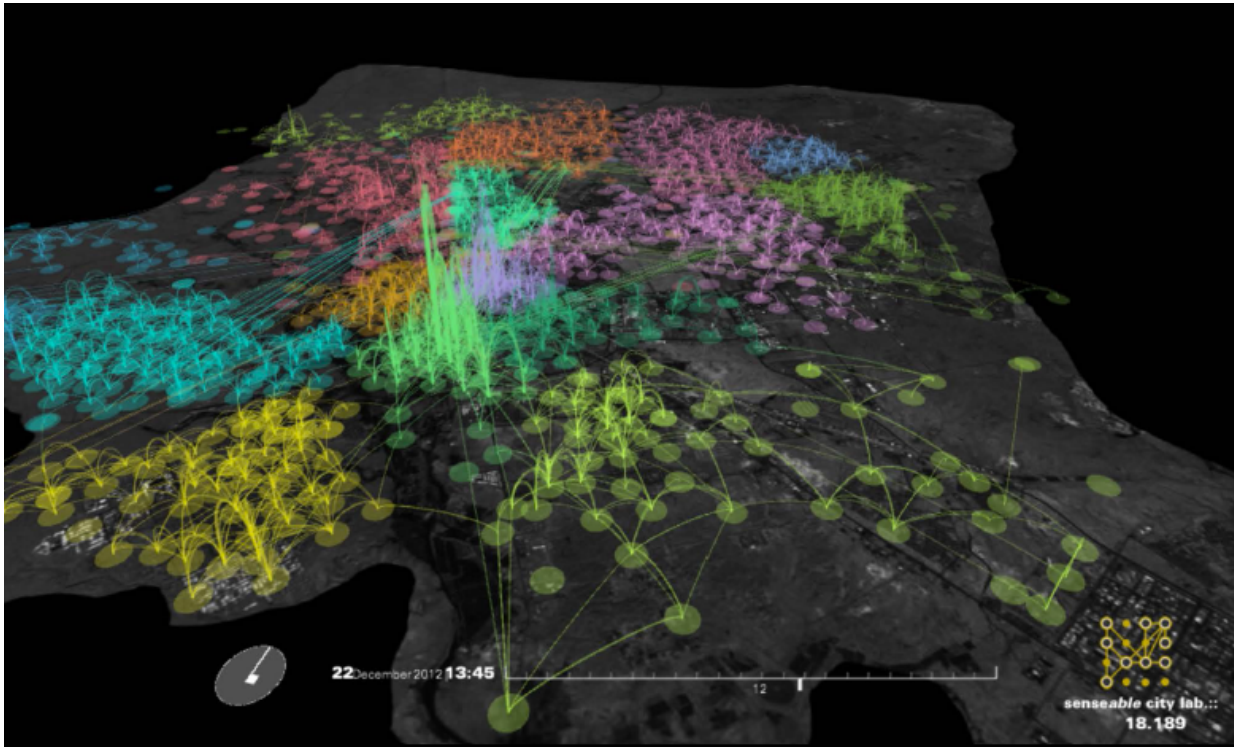
Community detection

Groups of nodes more strongly connected internally vs externally

- Social networks
- Human mobility
- Infrastructural
- Biological
- Economic etc



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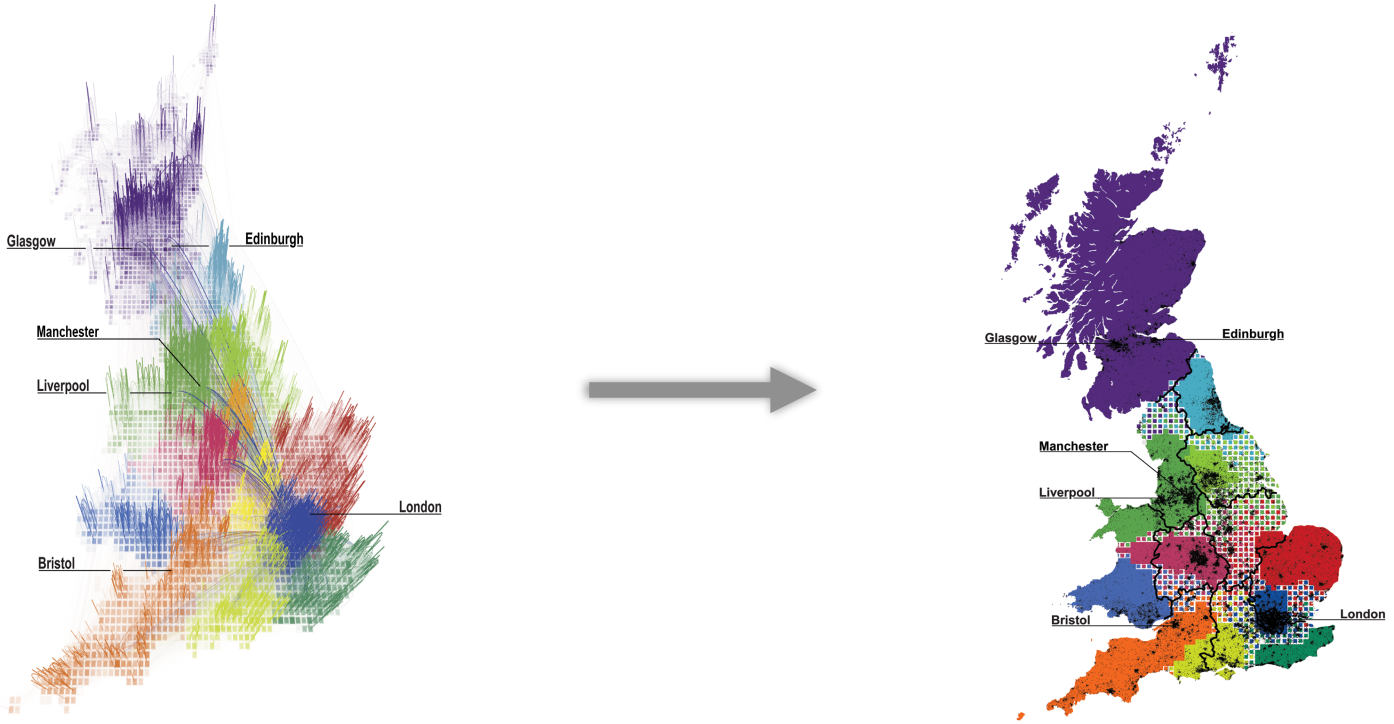
Regional delineation



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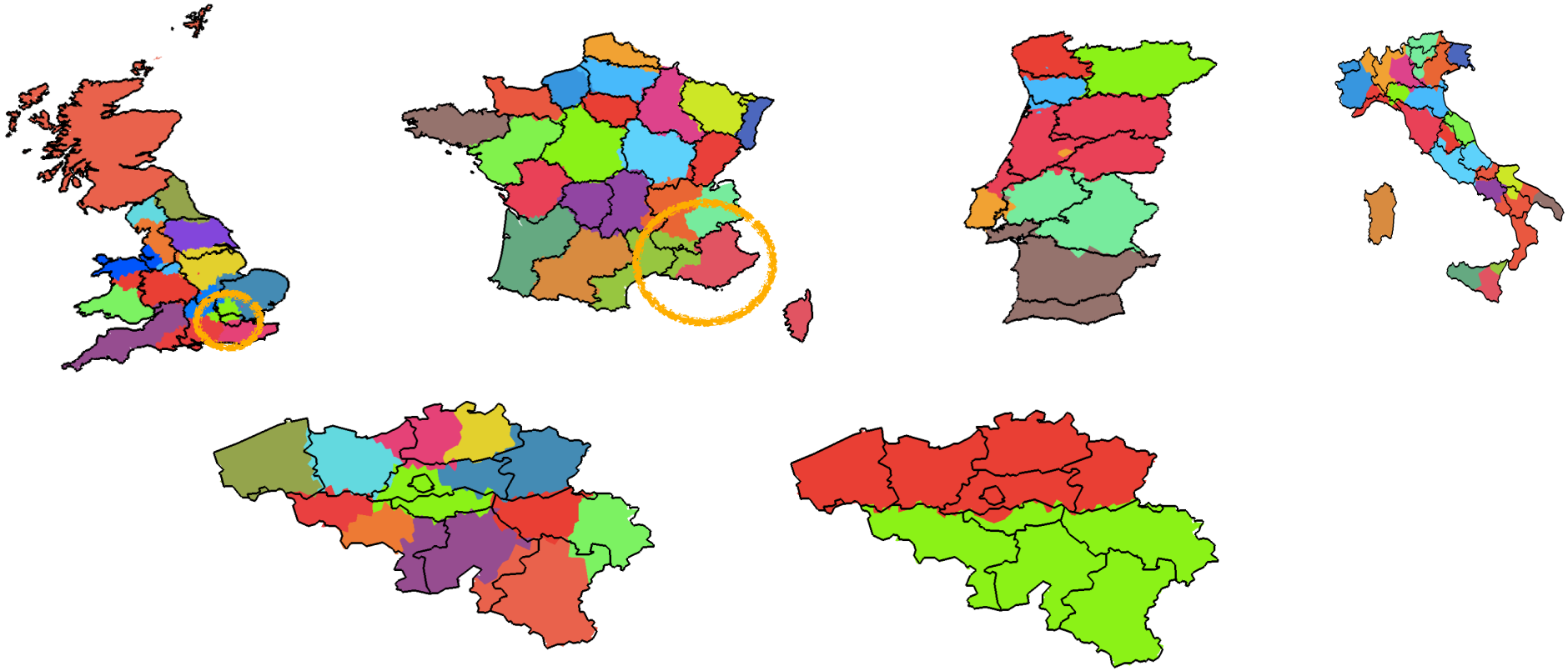
Regional delineation



Sobolevsky S., Szell M., Campari R., Couronne T., Smoreda Z., Ratti C. (2013) Delineating geographical regions with networks of human interactions in an extensive set of countries. PLoS ONE 8 (12), e81707

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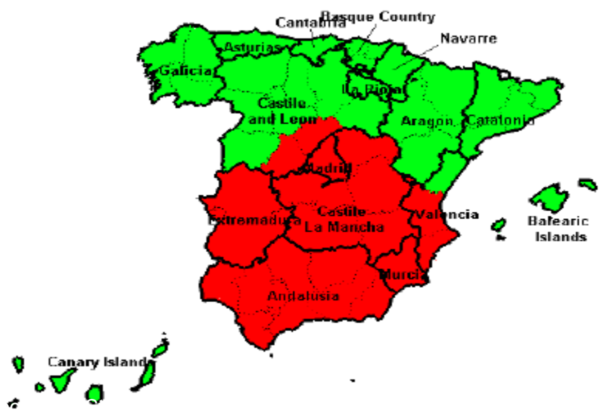
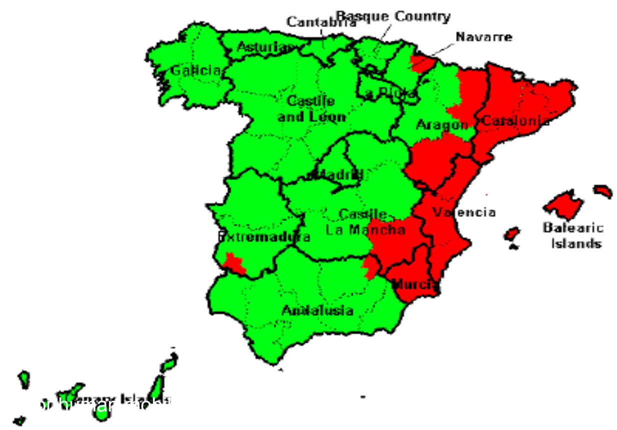
Credit cards





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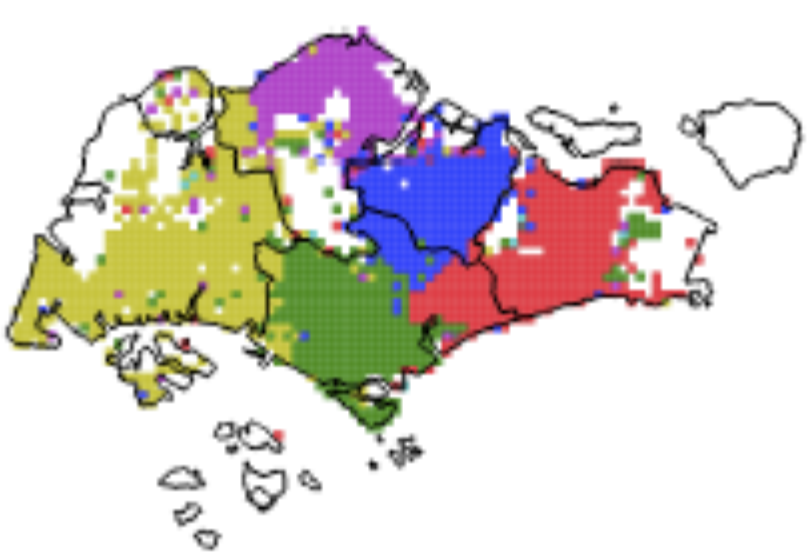
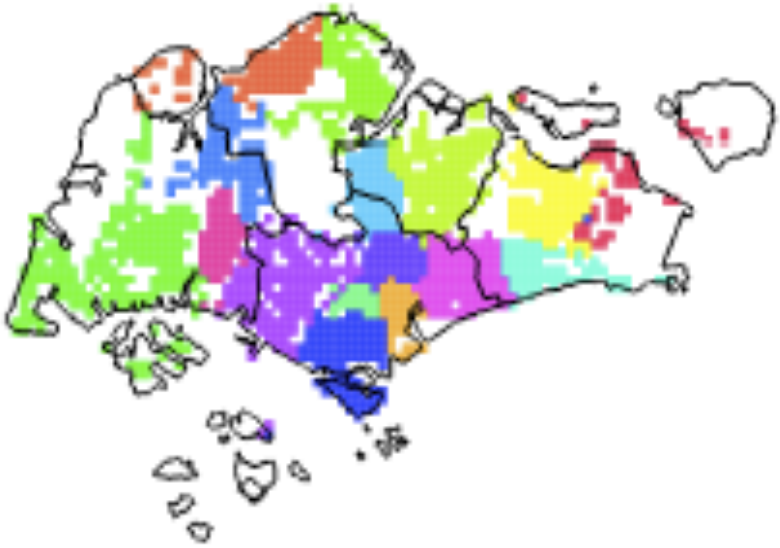
Taxi trips





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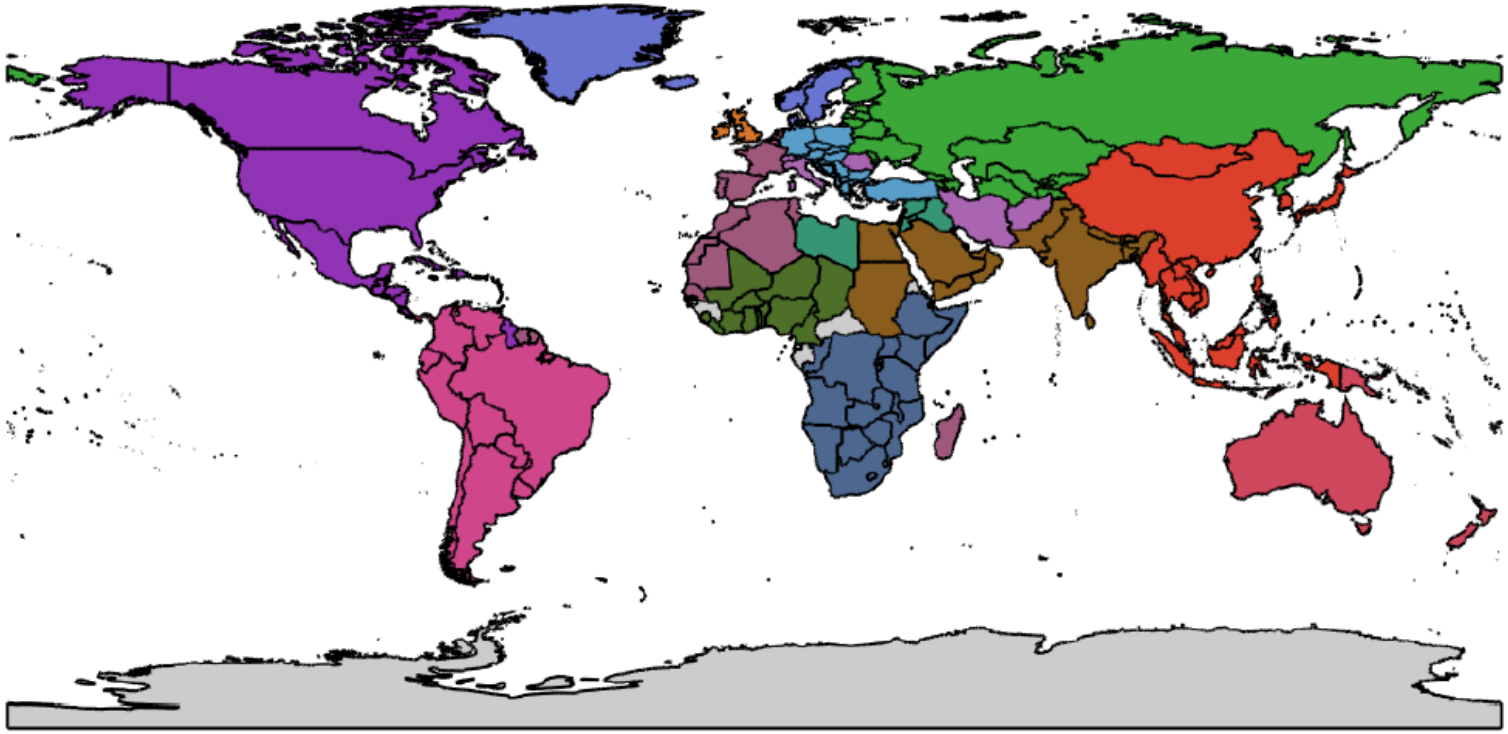
Global: social media



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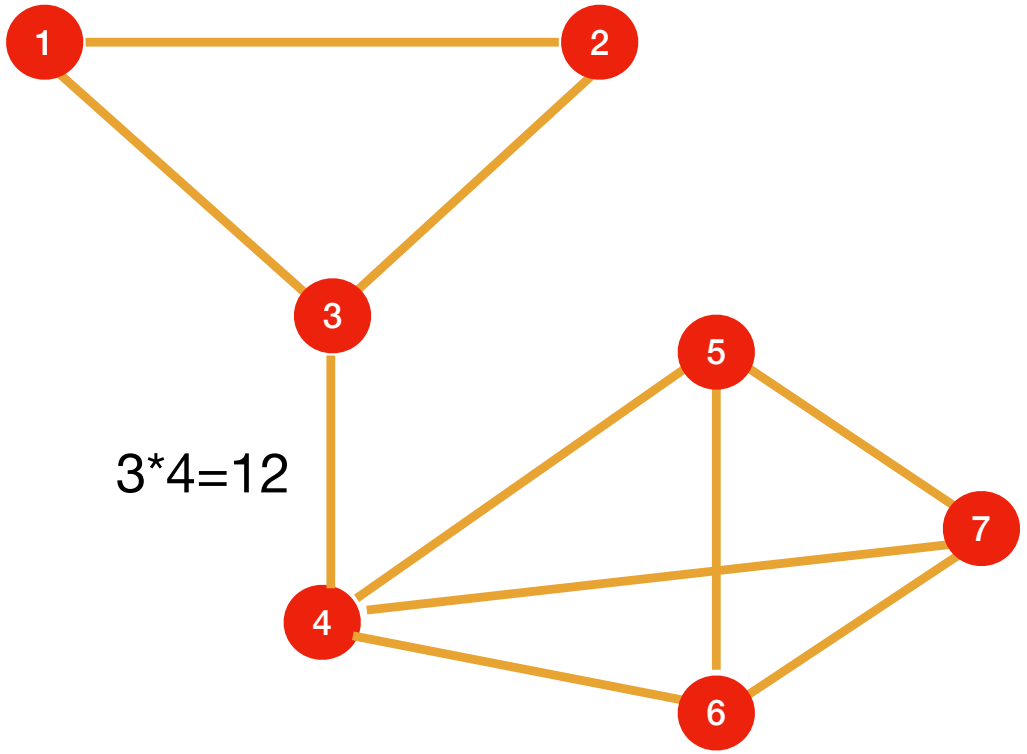
Girvan-Newman algorithm, 2002



3*4=12

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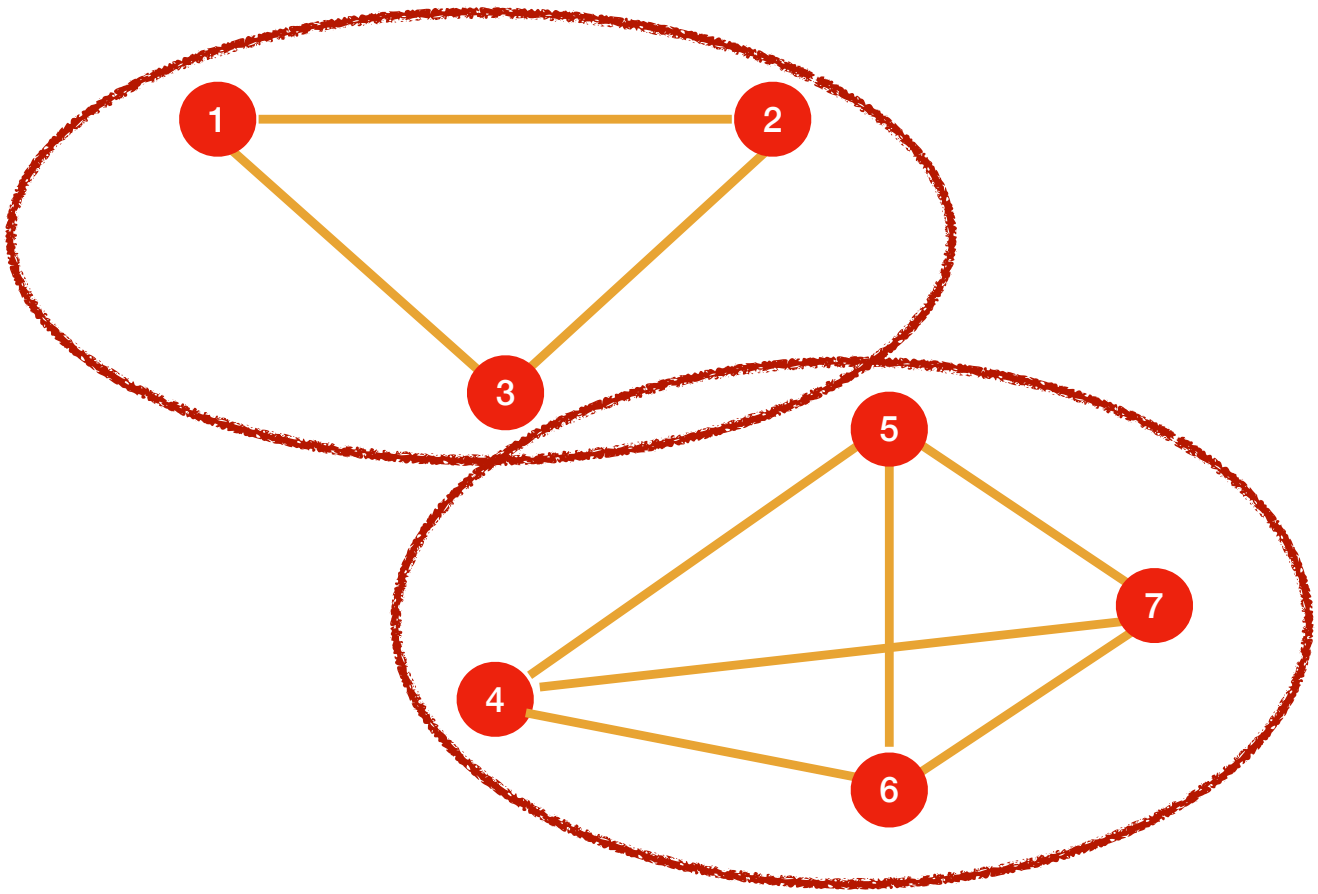
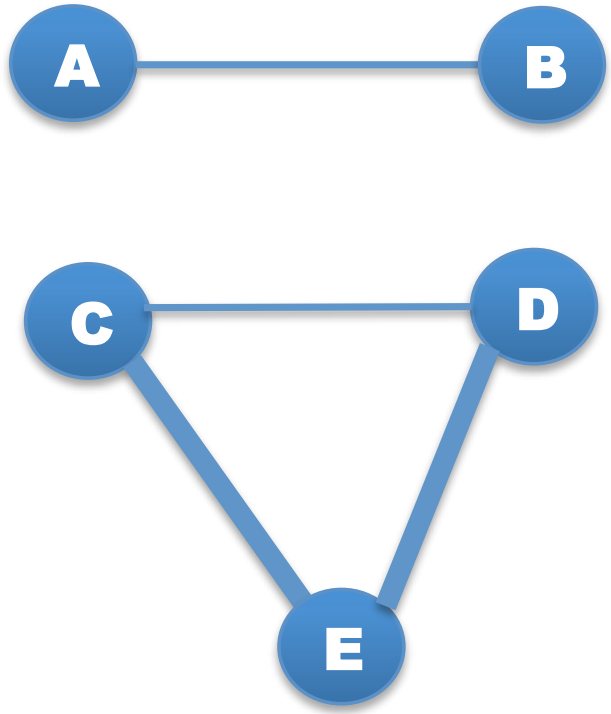
Girvan-Newman algorithm, 2002



Modularity



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Modularity

$$q(x,y) = \frac{e(x,y)}{T} - \frac{k_x^{out} k_y^{in}}{T^2}$$

$$P = (c_x, x \in N)$$

$$Q(P) = \sum_{x,y,\ c_x = c_y} q(x,y)$$

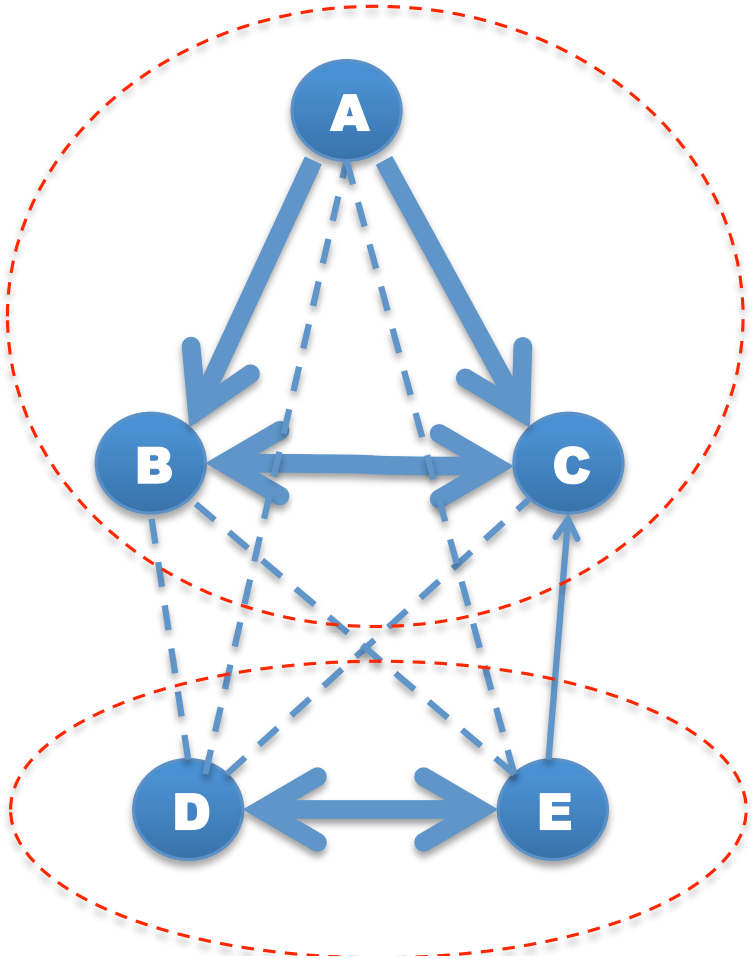
$$Q(P) = \sum_{x,y,\ c_x = c_y} \left[\frac{e(x,y)}{T} - \frac{k_x^{out} k_y^{in}}{T^2} \right]$$

Modularity

$$-1 = -\sum_{x,y} \frac{k_x^{out} k_y^{in}}{T^2} < Q = \sum_{x,y,\ c_x = c_y} \left[\frac{e(x,y)}{T} - \frac{k_x^{out} k_y^{in}}{T^2} \right] < \sum_{x,y} \frac{e(x,y)}{T} = 1$$

$$Q(P_0) = \sum_{x,y} \left[\frac{e(x,y)}{T} - \frac{k_x^{out} k_y^{in}}{T^2} \right] = \sum_{x,y} \frac{e(x,y)}{T} - \sum_{x,y} \frac{k_x^{out} k_y^{in}}{T^2} = 1 - 1 = 0.$$

Modularity



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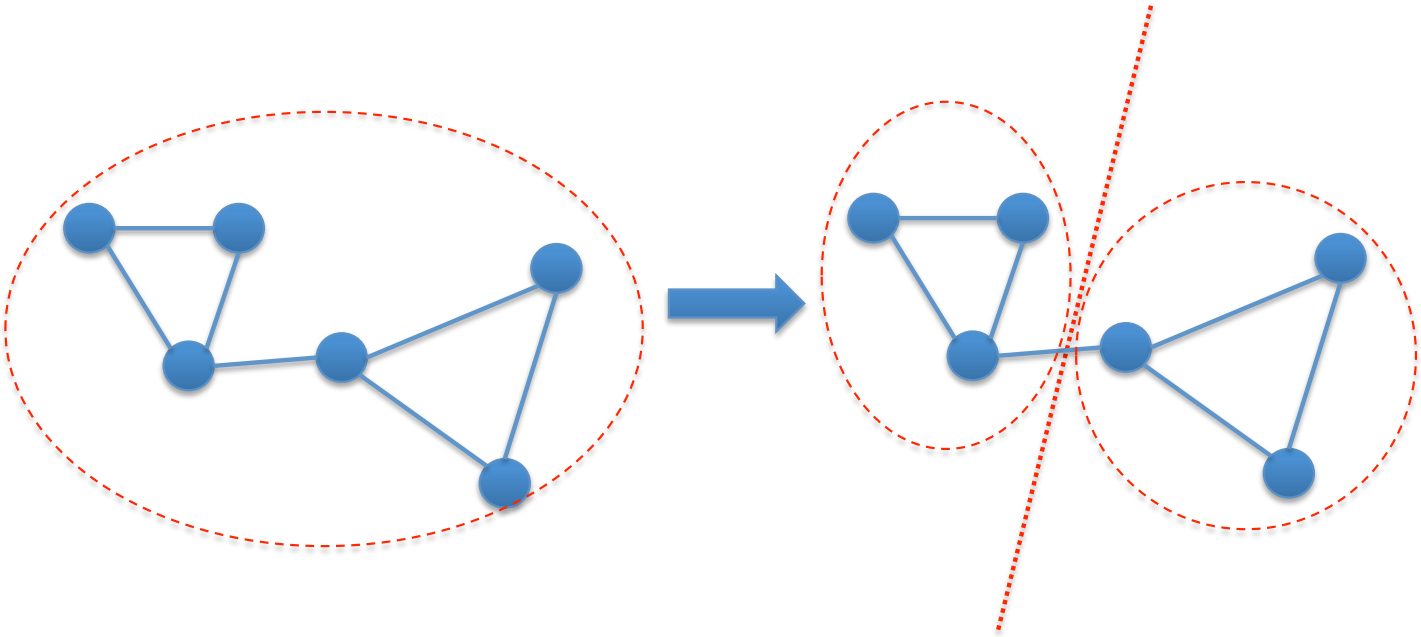
Modularity optimization - splitting



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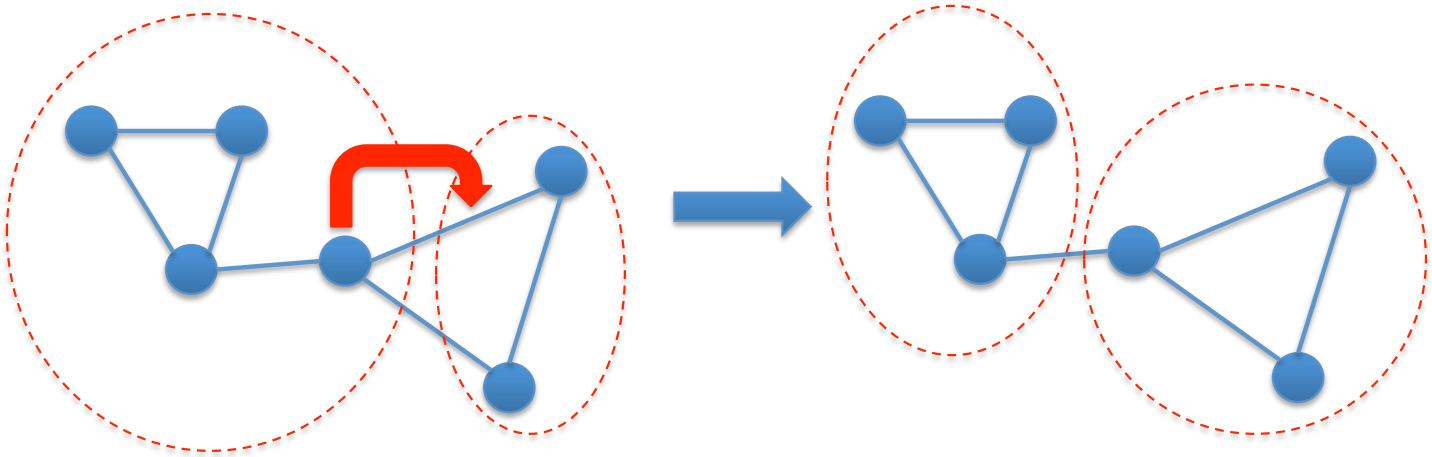
Modularity optimization - shifting



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Modularity optimization approaches

Newman's greedy heuristic	M. E. J. Newman, Phys. Rev. E 69, 066133 (2004), URL http://link.aps.org/doi/10.1103/PhysRevE.69.066133
Clauset-Newman-Moore	A. Clauset, M. Newman, and C. Moore, Phys. Rev. E70 (6), 066111 (2004).
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Louvain method	V. Blondel, J. Guillaume, R. Lambiotte, and E. Lefebvre, J. Stat. Mech 10008 (2008)
Le-Martelot's method	E. Le Martelot and C. Hankin, in Proceedings of the 2011 International Conference on Knowledge Discovery and Information Retrieval (KDIR 2011) (SciTePress, Paris, 2011), pp. 216–225.
Extremal optimization	J. Duch and A. Arenas, Phys. Rev. E 72, 027104 (2005),
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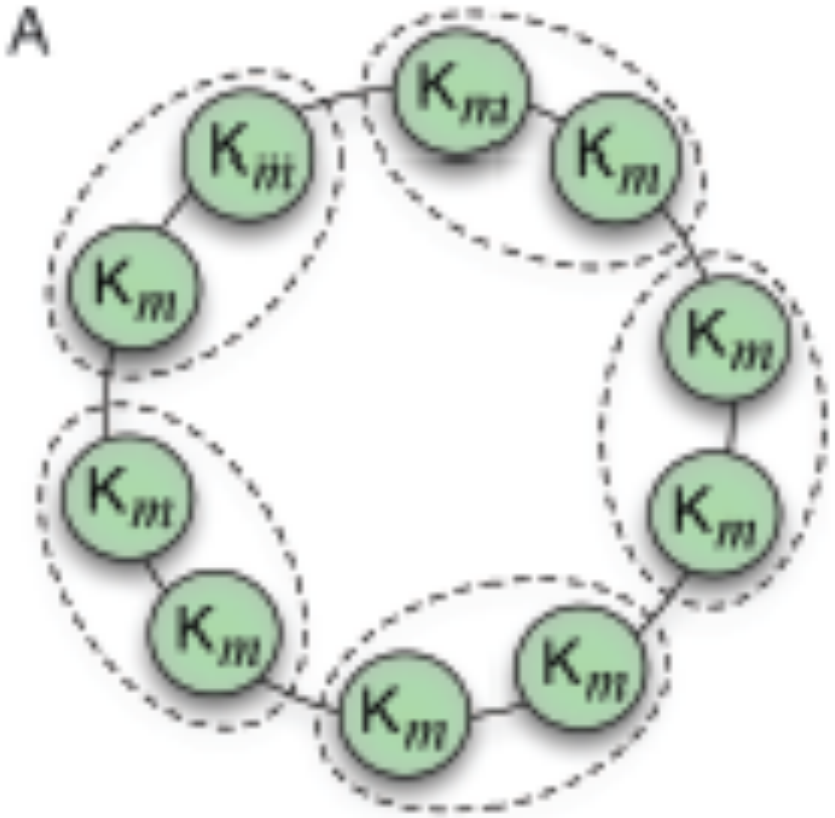
Modularity resolution limit



Fortunato, S., & Barthelemy, M. (2007). Resolution limit in community detection. *Proceedings of the National Academy of Sciences*, *104*(1), 36-41.

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Alternatives to modularity

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Surprise	R. Aldecoa and I. Mar`ın, PLoS ONE 6, e24195 (2011),
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