APPLIED DATA SCIENCE

Dimensionality reduction. Principal Component Analysis

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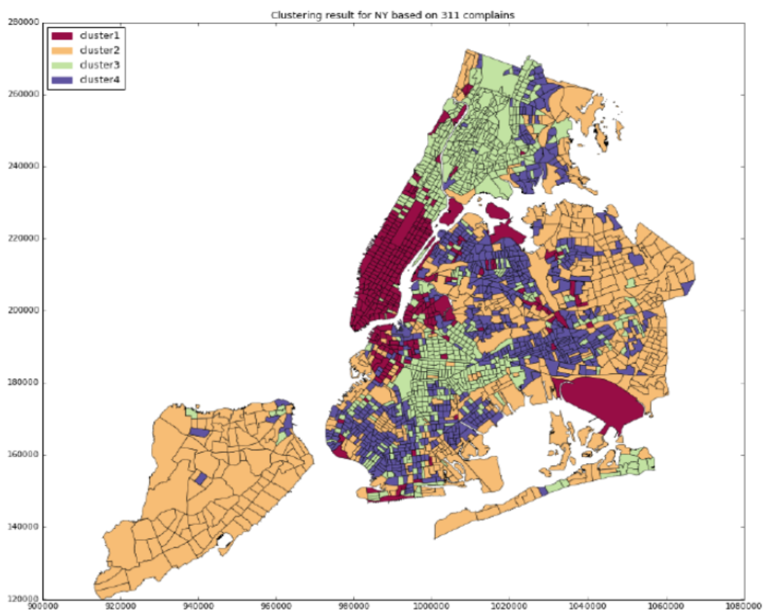
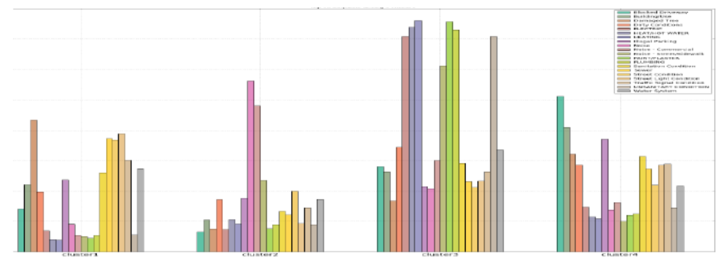
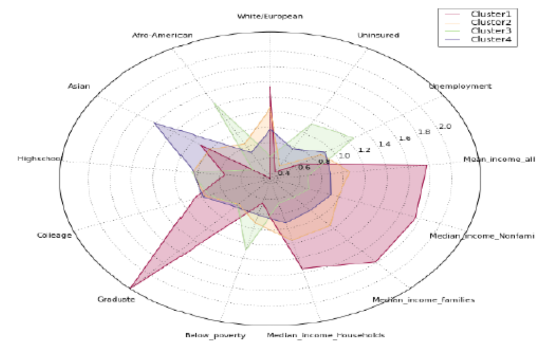
Video lectures

Principal Component Analysis

Characterize urban neighborhoods with their 311 activity
Model income, unemployment or average real estate prices

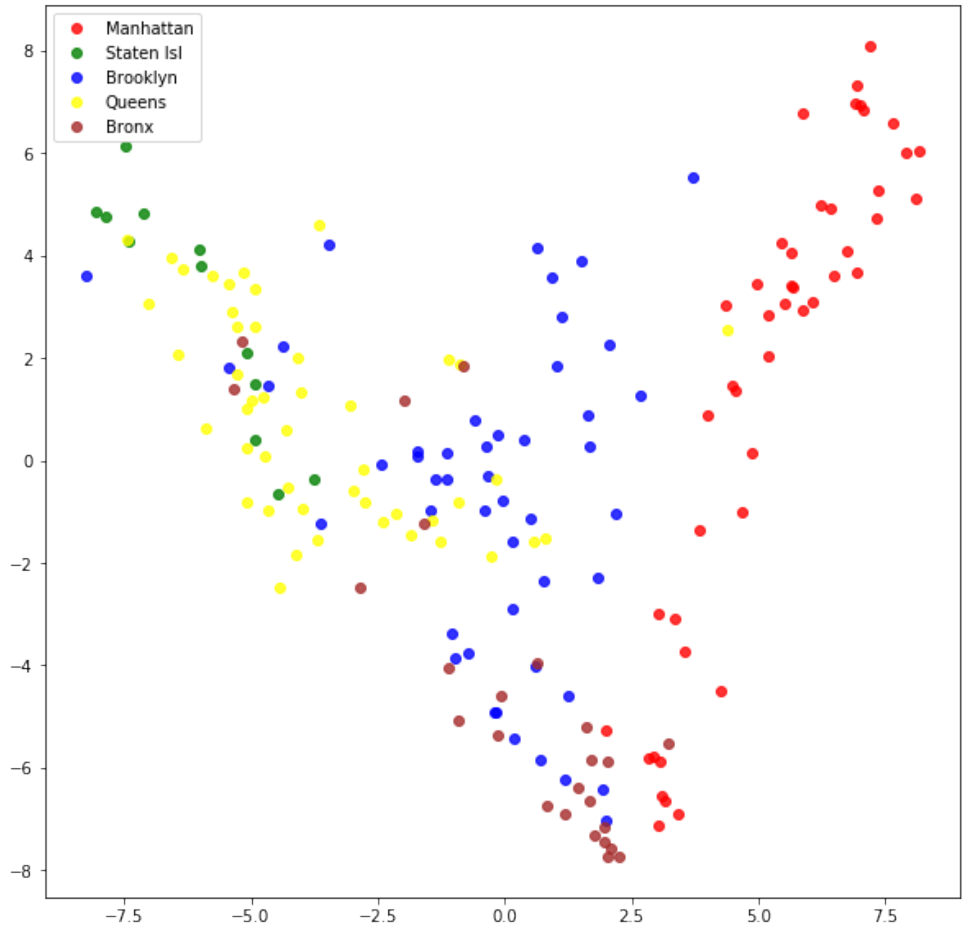






Wang L, Qian C, Kats P, Kontokosta C, Sobolevsky, S. (*corresp.) (2017) Structure of 311 service requests as a signature of urban location. PloS ONE. 12(10), e0186314.

Principal Component Analysis



Issues with multi-dimensional data

• How to analyze/visualize it?

$$x = (x_1, x_2, x_3, ..., x_n)$$

In case of regression:

$$y = f(x)$$

- complexity
- irrelevant information
- overfitting
- multi-collinearity

Feature selection vs dimensionality reduction

$$y = f(x) \qquad\qquad x = (x_1, x_2, x_3, ..., x_n)$$

feature selection reduces dimensionality of x by removing less relevant components

$$(x_1, x_2, x_3, x_4, x_5) \rightarrow (x_1, x_3, x_5)$$

dimensionality reduction looks for more general mapping

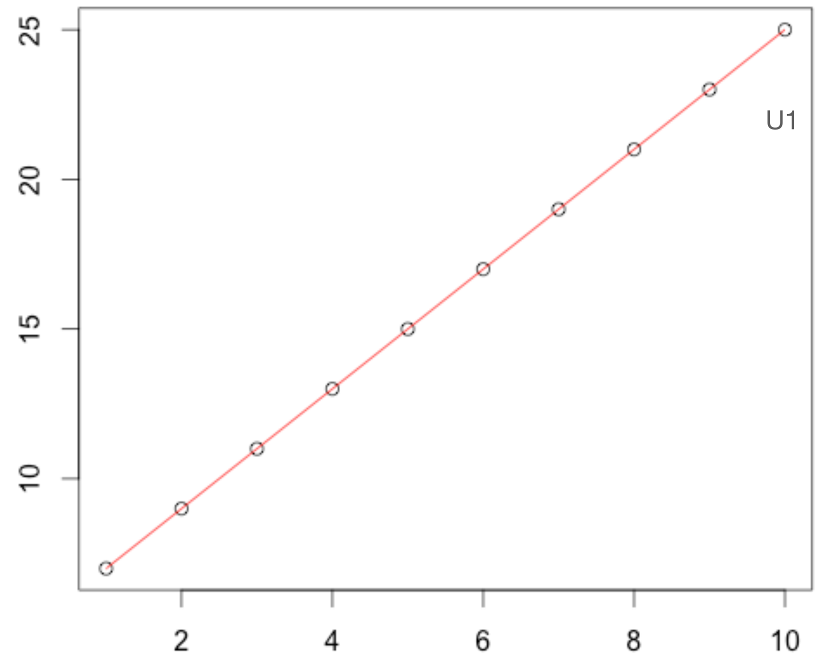
$$(x_1, x_2, x_3, ..., x_n) \rightarrow (x'_1, x'_2, x'_3, ..., x'_m),\ \ m < n$$

$$y = f(x')$$

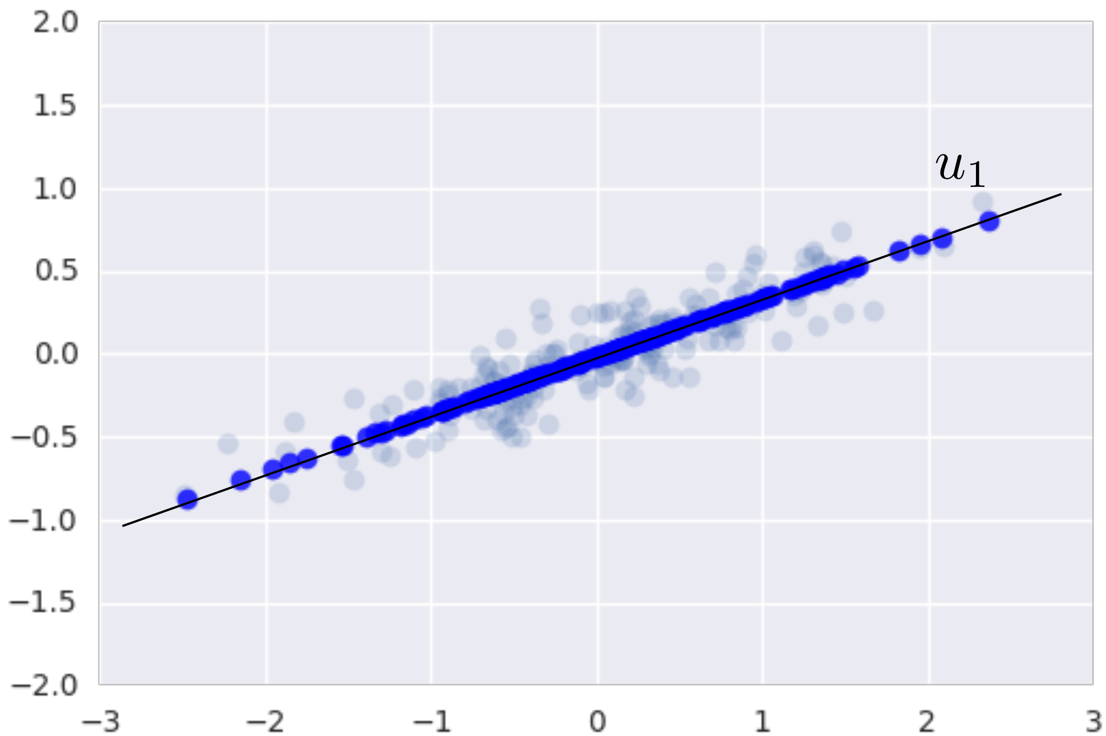
$$(x_1, x_2, x_3, x_4, x_5) \rightarrow x' = (x_1 + x_2 + x_3 + x_4 + x_5, x_1 x_2 x_3 x_4 x_5)$$

Pareto rule: 20% information often provides 80% of value

Same information with smaller number of parameters



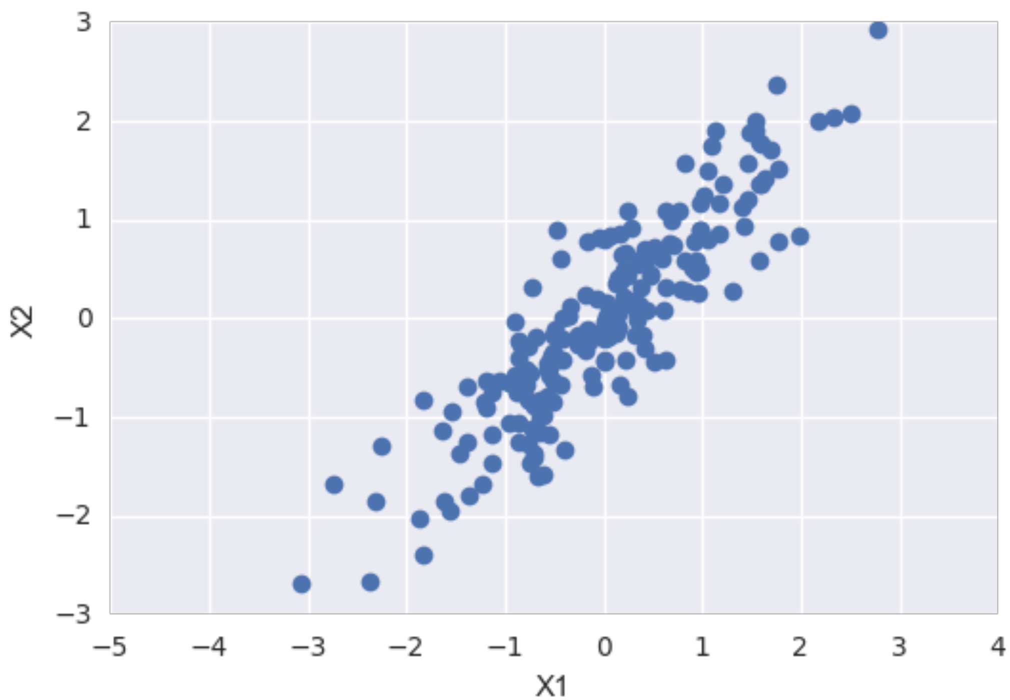
Almost the same information



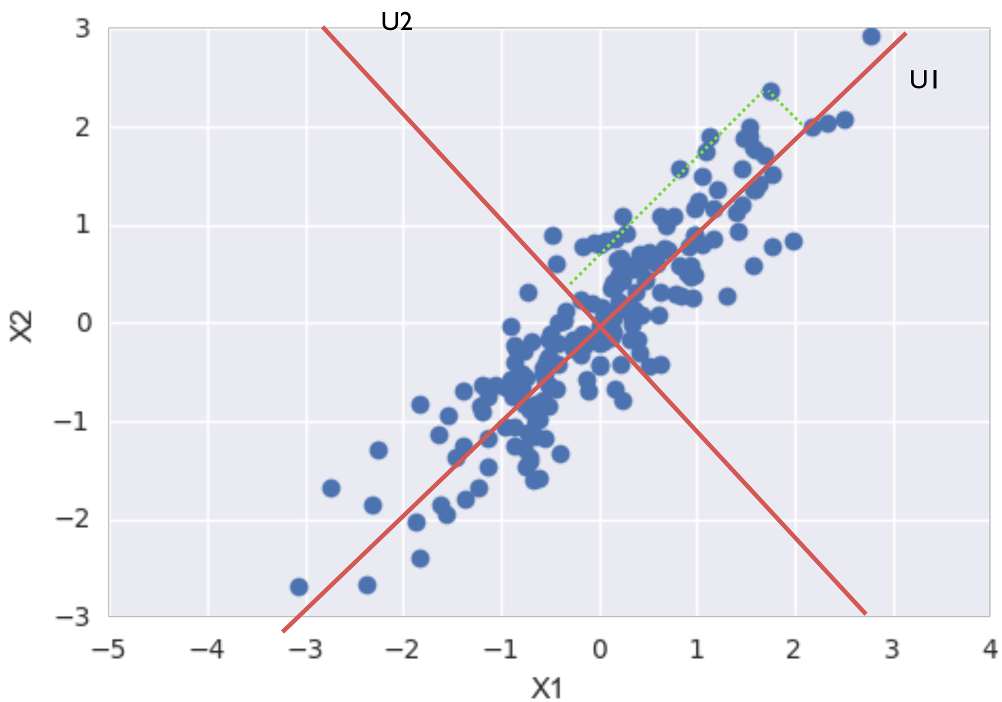
correlated features to uncorrelated

$$(x_1, x_2, x_3, ..., x_n) \rightarrow (u_1, u_2, u_3, ..., u_n)$$

Original data

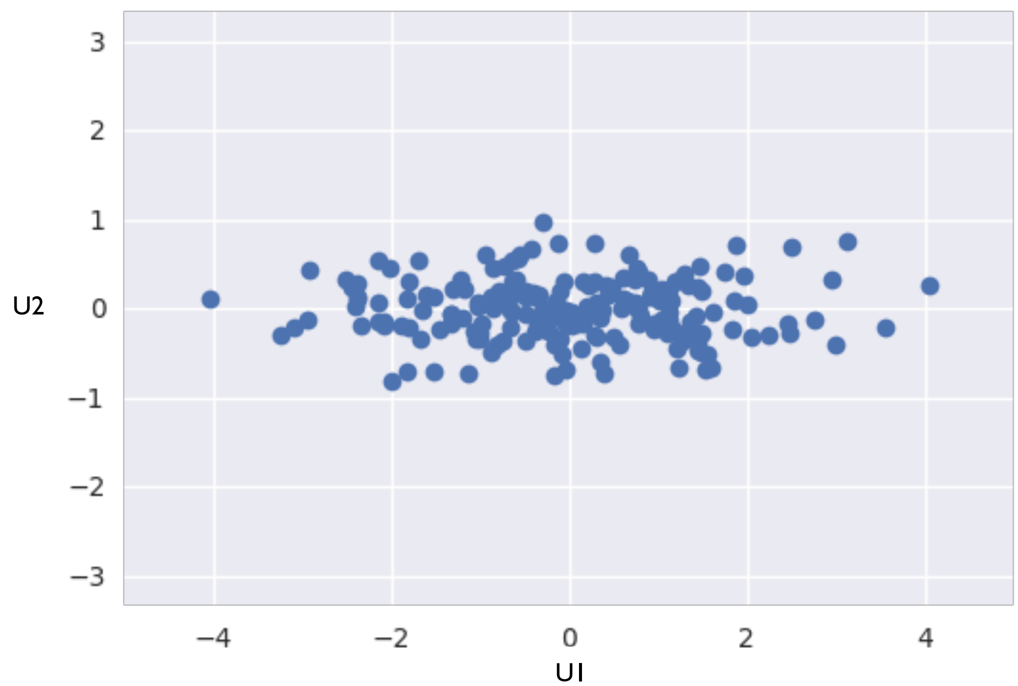


Original data - new system of coordinates

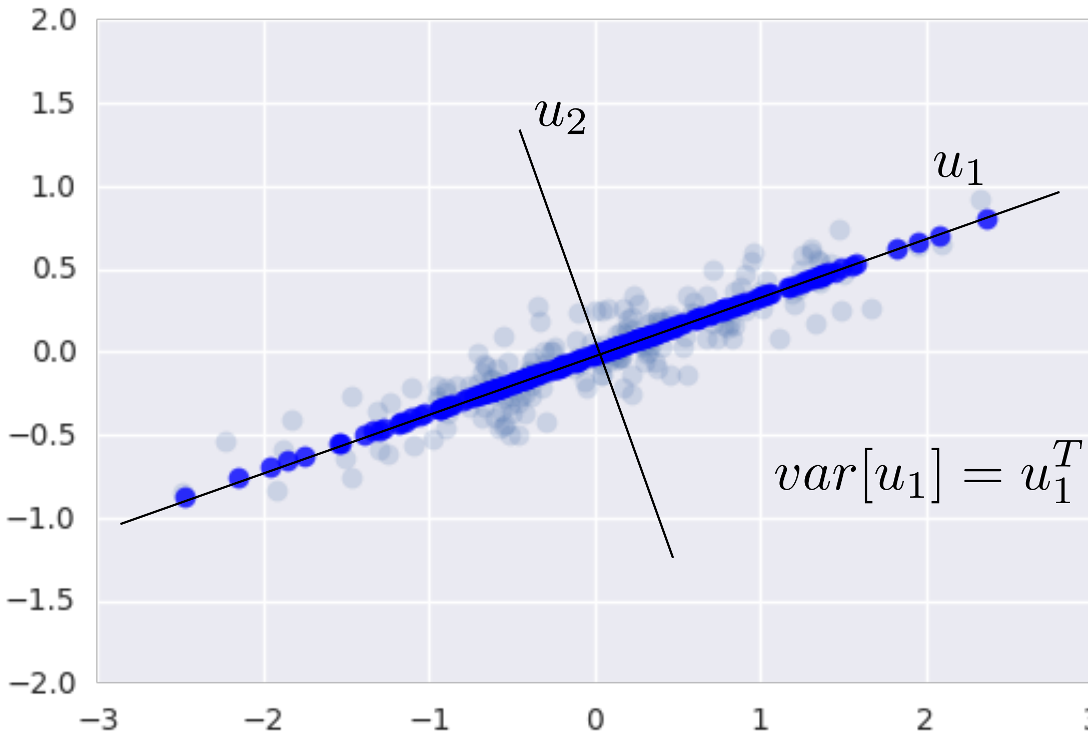


Uncorrelated data (rotation)

$$U = XV$$



Almost the same information



$$var[u_1] = u_1^T u_1 \rightarrow max$$

Principal components - maths

Given the standardized data $X = \{x_i^j, i = 1..n, j = 1..N\}$

Find uncorrelated latent features

$$u_j = x_1 v_j^1 + x_2 v_j^2 + ... + x_n v_j^n$$

$$Var[u_1] \geq Var[u_2] \geq \ldots \geq Var[u_n]$$

Learn matrix V implementing the rotation transform

$$u_i = X v_i \qquad U = XV$$

V - n x n
U - N x n

V - matrix of eigenvectors of $X^T X$

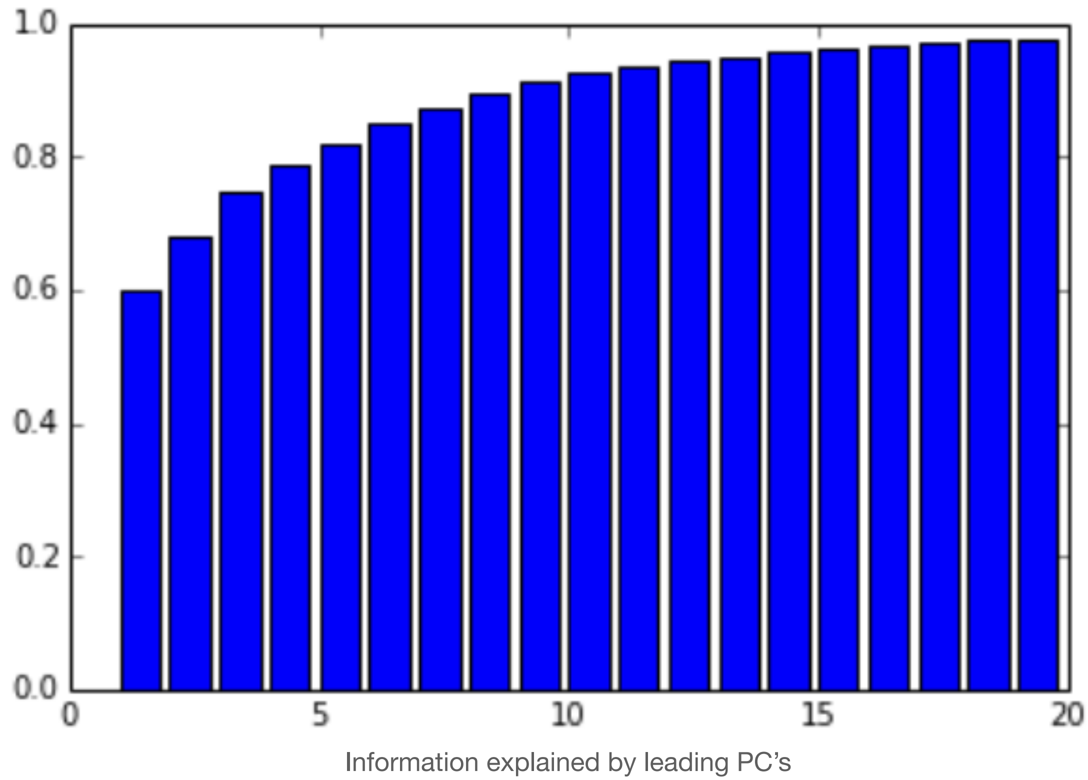
Principal components - select by variation

Information explained by a PC u_i

$$Var[u_i] = \lambda_i$$

% of info explained by leading k components

$$\frac{\sum_{i=1}^{k} \lambda_i}{\sum_{i=1}^{n} \lambda_i}$$



Leading PCs - uncorrelated low-dimensional feature space:

- Data exploration (e.g. visualization)
- Modeling

Limitations:

- Relevance to a particular target variable
- Interpretability
- Linear