ADS Homework 2. part 1

1: Classification of Hand-Written Digits

Sklearn provides a dataset of handwritten digits as one of the examples. This data includes impages of 1787 digits. For each digit, we have 64 variables x1,x2,...x64, each one representing a pixel (grayscale from 0 to 16) on the 8x8 image. The label (y) for each image is the actual digit it represents from 0 to 9.

In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
import matplotlib.pyplot as plt
%matplotlib inline
import statsmodels.formula.api as smf
import statsmodels.api as sm
from scipy.stats import t

import warnings
warnings.filterwarnings('ignore')
```

In [2]:
```
#Load the data
from sklearn.datasets import load_digits
digits = load_digits()
X = digits.data # the digits data
y = digits.target # the labels for the digits. This label is only used for colo
#Make a dataframe
data=pd.DataFrame(np.concatenate((y.reshape(len(y),1),X),axis=1))
data.columns=["y"]+["x{}".format(i) for i in range(1,65)]
data.head()
```

Out[2]:

	y	x1	x2	x3	x4	x5	x6	x7	x8	x9	...	x55	x56	x57	x58	x59	x60	x61	x6
0	0.0	0.0	0.0	5.0	13.0	9.0	1.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	6.0	13.0	10.0	(
1	1.0	0.0	0.0	0.0	12.0	13.0	5.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	11.0	16.0	1(
2	2.0	0.0	0.0	0.0	4.0	15.0	12.0	0.0	0.0	0.0	...	5.0	0.0	0.0	0.0	0.0	3.0	11.0	16
3	3.0	0.0	0.0	7.0	15.0	13.0	1.0	0.0	0.0	0.0	...	9.0	0.0	0.0	0.0	7.0	13.0	13.0	9
4	4.0	0.0	0.0	0.0	1.0	11.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	2.0	16.0	4

5 rows × 65 columns

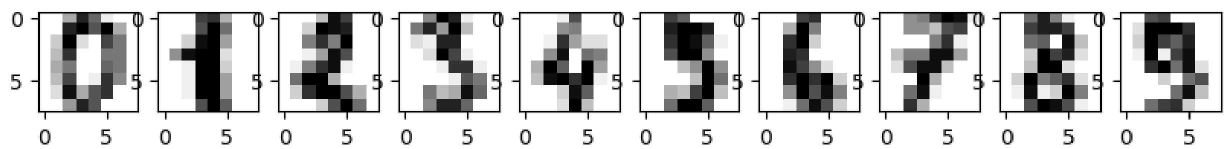
In [3]:
```
#original image data is a 2D array
X.shape
```

Out[3]:
```
(1797, 64)
```

If we take the first 10 observations and reconstruct the graph and we could see:

In [4]:
```
#plot the digits using imshow from matplotlib
plt.figure(figsize=(10,1))
print("Label:{}".format(list(y[:10])))
for i in range(10):
    plt.subplot(1,10,i+1)
    plt.imshow(X[i].reshape(8,8), cmap=plt.cm.gray_r)
plt.show()
```

```
Label:[0, 1, 2, 3, 4, 5, 6, 7, 8, 9]
```



In [5]:
```
# PCA module
from sklearn.decomposition import PCA
from sklearn.model_selection import train_test_split
from mpl_toolkits.mplot3d import Axes3D
from sklearn import datasets
from sklearn import linear_model
from sklearn import preprocessing
from sklearn.metrics import r2_score
```

Task 1: Apply PCA to see if we can describe the data with a smaller number of most significant features

In [6]:
```
#set target variable (y) as index
data
```

Out[6]:

	y	x1	x2	x3	x4	x5	x6	x7	x8	x9	...	x55	x56	x57	x58	x59	x60	x6
0	0.0	0.0	0.0	5.0	13.0	9.0	1.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	6.0	13.0	10.
1	1.0	0.0	0.0	0.0	12.0	13.0	5.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	11.0	16.
2	2.0	0.0	0.0	0.0	4.0	15.0	12.0	0.0	0.0	0.0	...	5.0	0.0	0.0	0.0	0.0	3.0	11.
3	3.0	0.0	0.0	7.0	15.0	13.0	1.0	0.0	0.0	0.0	...	9.0	0.0	0.0	0.0	7.0	13.0	13.
4	4.0	0.0	0.0	0.0	1.0	11.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	2.0	16.
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	.
1792	9.0	0.0	0.0	4.0	10.0	13.0	6.0	0.0	0.0	0.0	...	4.0	0.0	0.0	0.0	2.0	14.0	15.
1793	0.0	0.0	0.0	6.0	16.0	13.0	11.0	1.0	0.0	0.0	...	1.0	0.0	0.0	0.0	6.0	16.0	14.
1794	8.0	0.0	0.0	1.0	11.0	15.0	1.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	2.0	9.0	13.
1795	9.0	0.0	0.0	2.0	10.0	7.0	0.0	0.0	0.0	0.0	...	2.0	0.0	0.0	0.0	5.0	12.0	16.
1796	8.0	0.0	0.0	10.0	14.0	8.0	1.0	0.0	0.0	0.0	...	8.0	0.0	0.0	1.0	8.0	12.0	14.

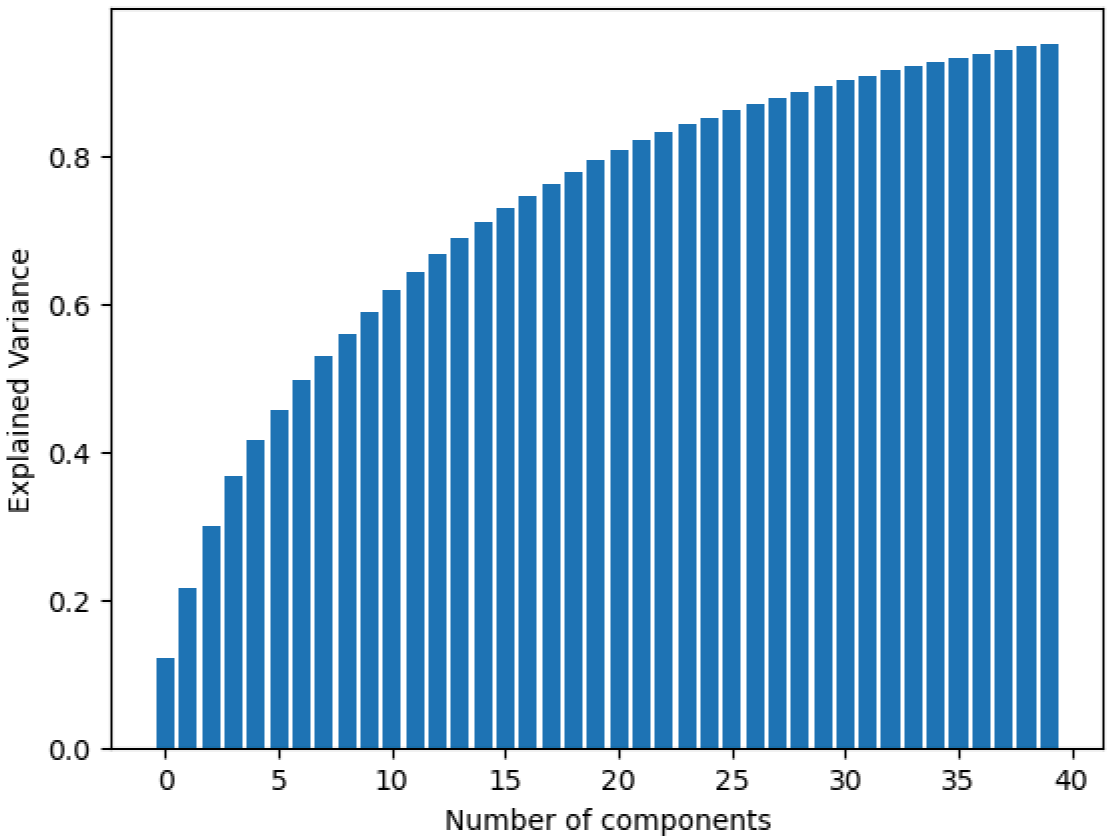
1797 rows × 65 columns

In [7]:
```
data.set_index('y',inplace=True)
```

In [8]:
```
# apply standardization to the pixels data (even though the features are graysc
data2S=(data-data.mean())/data.std()
data2S_clean = data2S.drop(columns='x1')
data2S_clean=data2S_clean.dropna(axis=1)
```

In [9]:
```
#perform PC decomposition over data
#Plot explained variance per number of leading PC's
pca = PCA(data2S_clean.shape[1])
```

In [10]:
```
pca = PCA(data2S_clean.shape[1])
dataP=pca.fit_transform(data2S_clean)
eigenvalues = pca.explained_variance_ratio_
#plot explained variance over the number of compinents
n=40
plt.bar(np.arange(n), eigenvalues[:n].cumsum())
plt.xlabel("Number of components")
plt.ylabel("Explained Variance")
plt.show()
```



describe in a few words what you can infer from the above plot

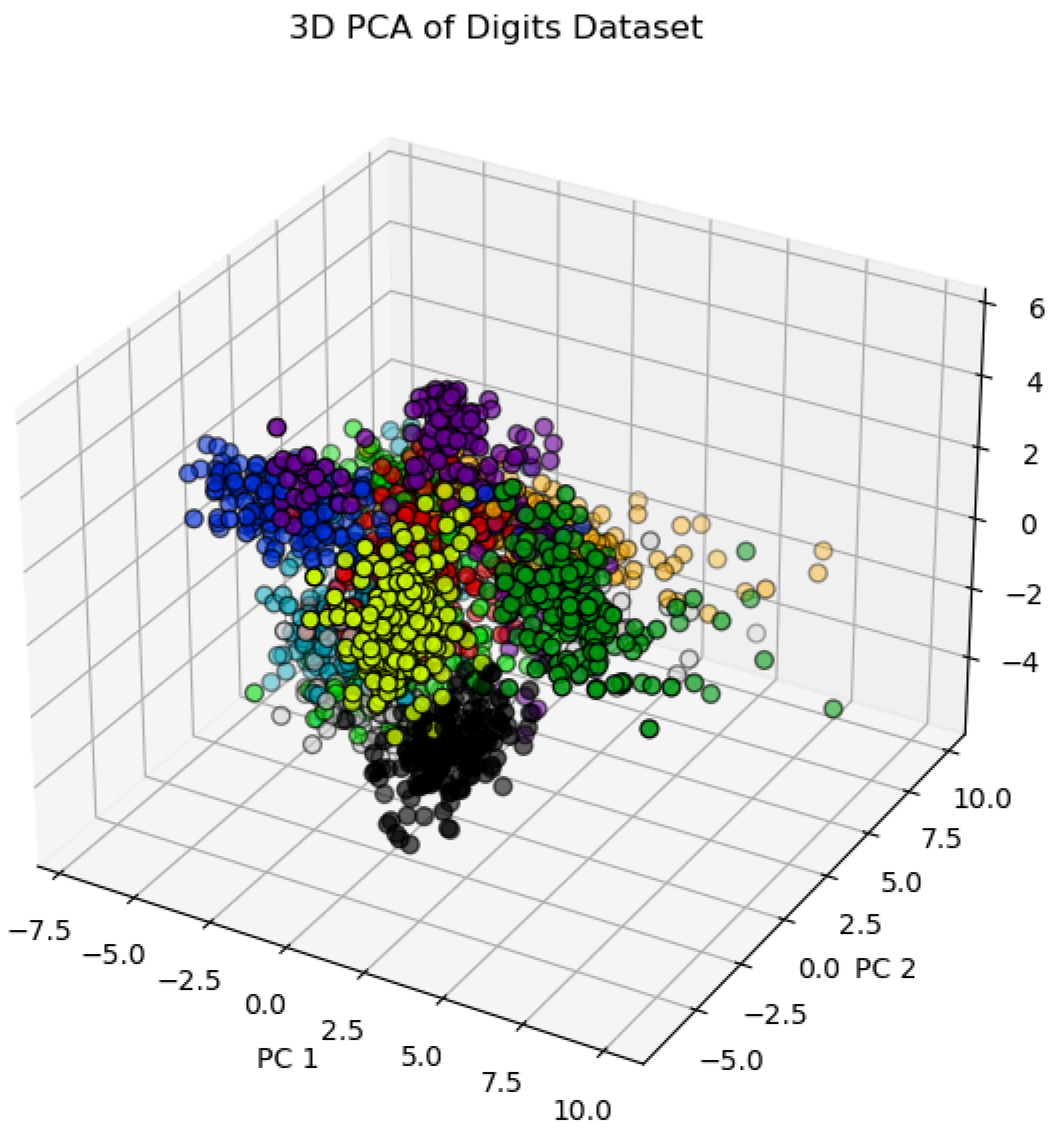
a small number of principal components capture the majority of the data's variance.

Task 2: Now use three leading PCs to visualize the digits (a 3d scatterplot in PC space with different colors representing different

digits)

In [12]:
```
#reapply PCA with three components so we can visualize in 3 dimensions
pca_3d = PCA(n_components=3)
data_3d = pca_3d.fit_transform(data2S_clean)
```

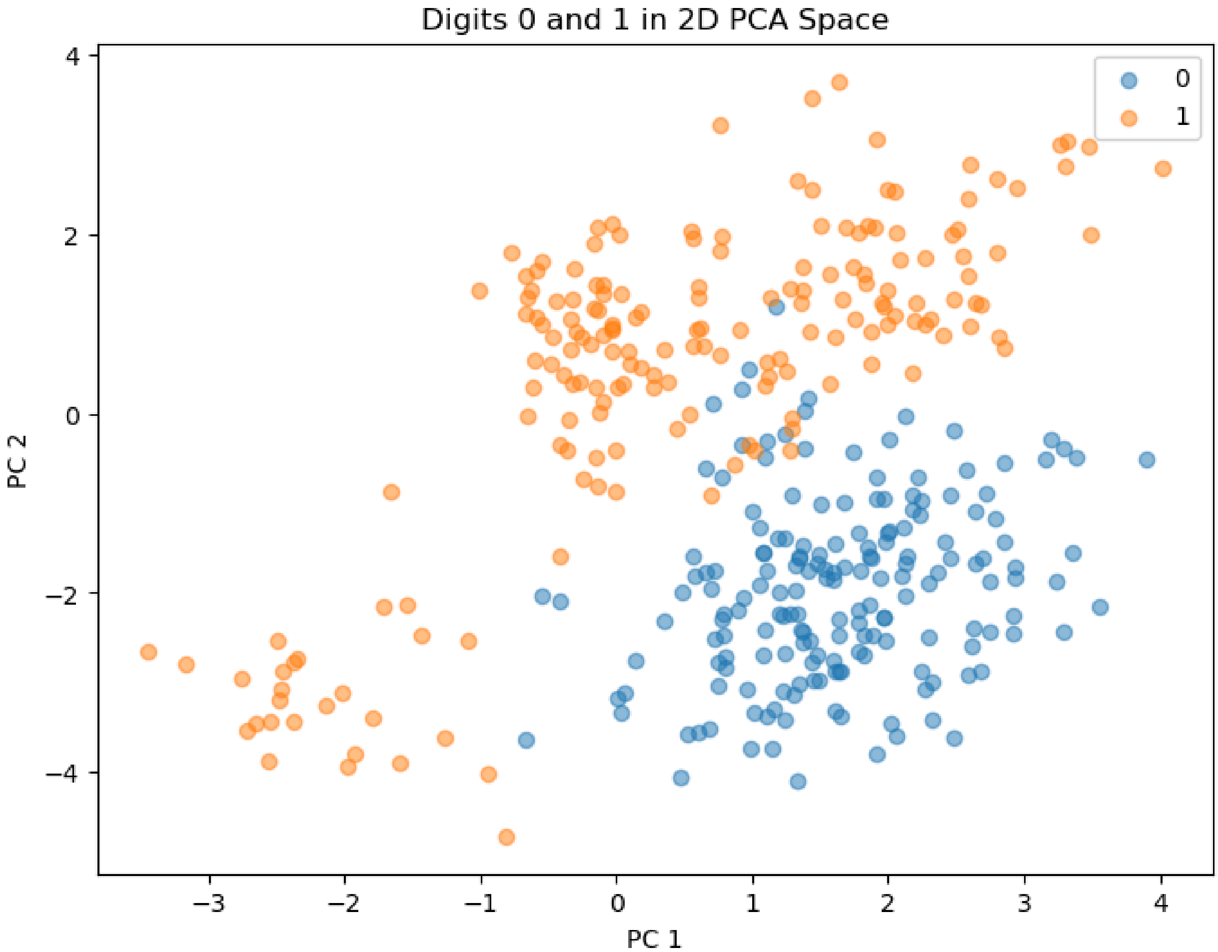
In [13]:
```
# use matplotlib 3D scatter plot: https://matplotlib.org/mpl_toolkits/mplot3d/
fig = plt.figure(figsize=(10, 7))
ax = fig.add_subplot(111, projection='3d')

# Using the original 'y' as color
ax.scatter(data_3d[:, 0], data_3d[:, 1], data_3d[:, 2], c=digits.target, cmap=
ax.set_title("3D PCA of Digits Dataset")
ax.set_xlabel("PC 1")
ax.set_ylabel("PC 2")
ax.set_zlabel("PC 3")

plt.show()
```

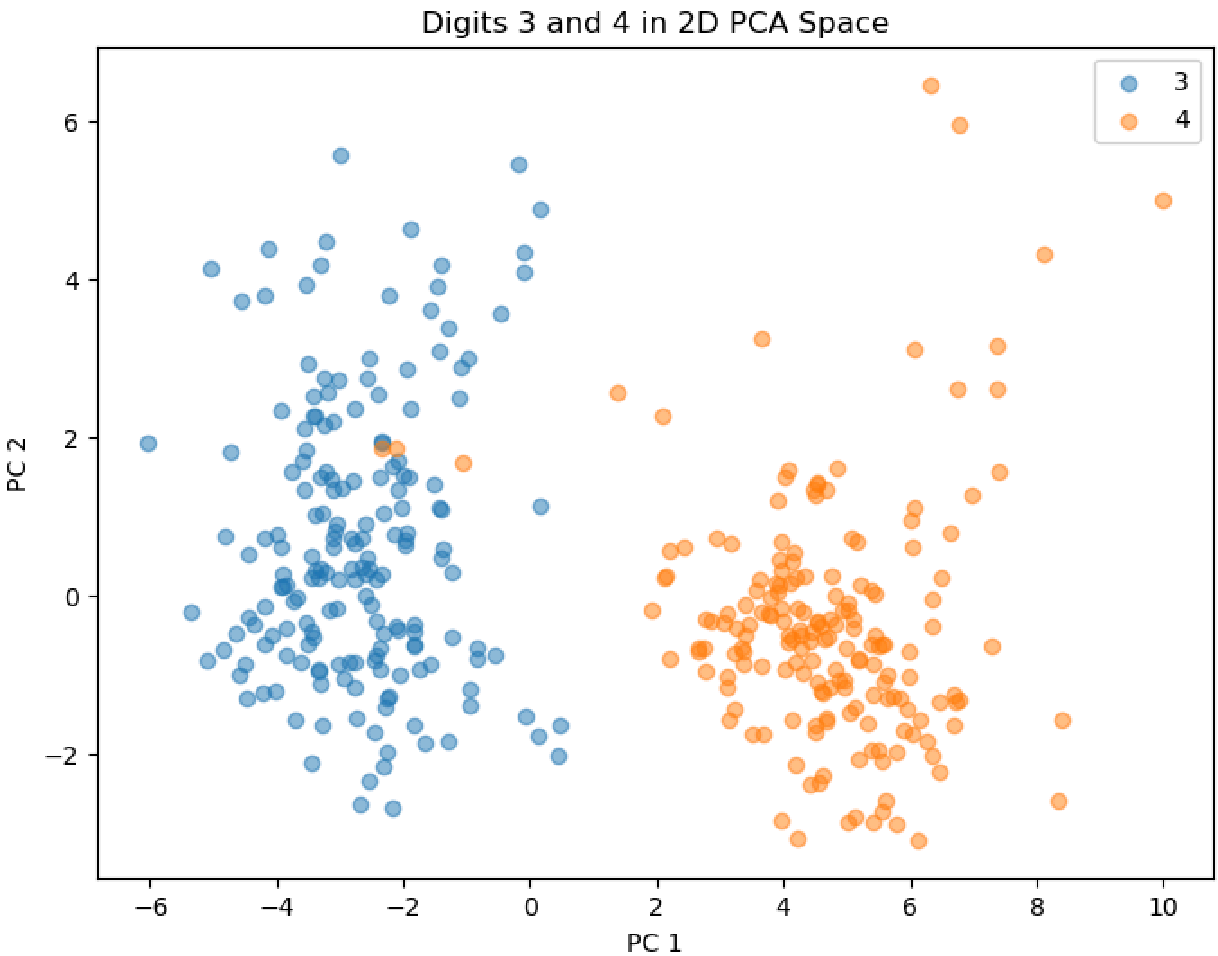


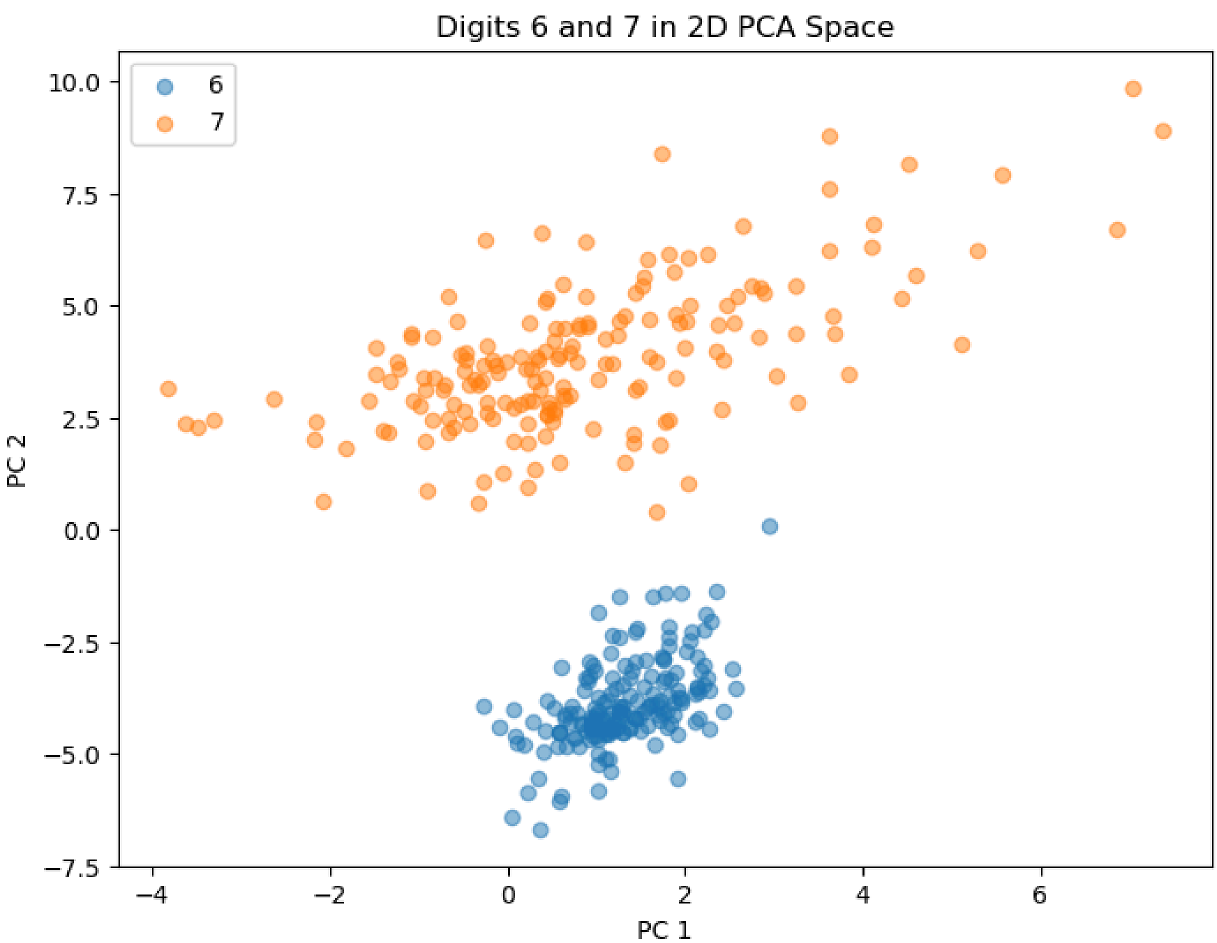
Task 3: Find examples of the pairs of digits which a) can and b) can not be clearly distinguished on a 2D plot of first two PCs

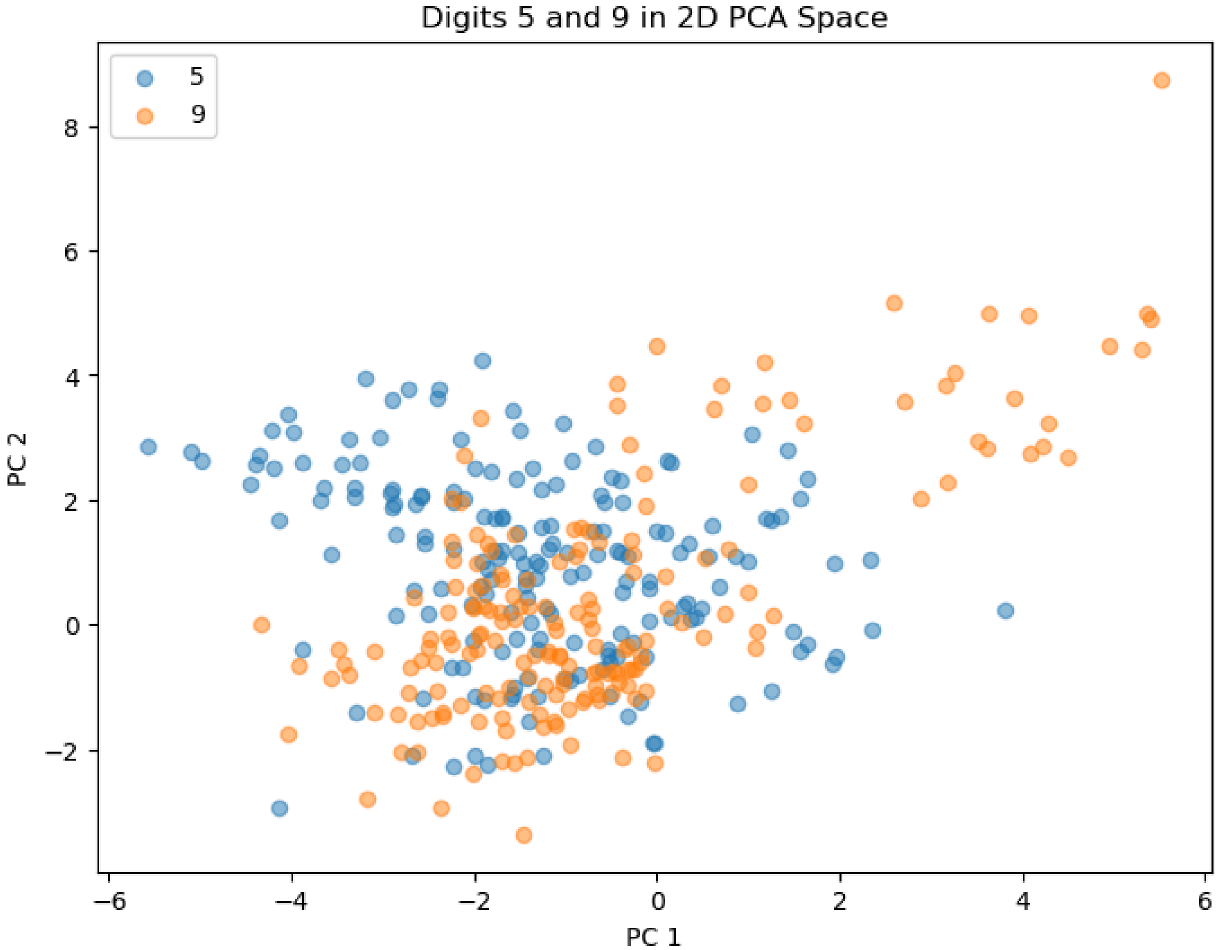
(provide 3 examples each on separate plots)

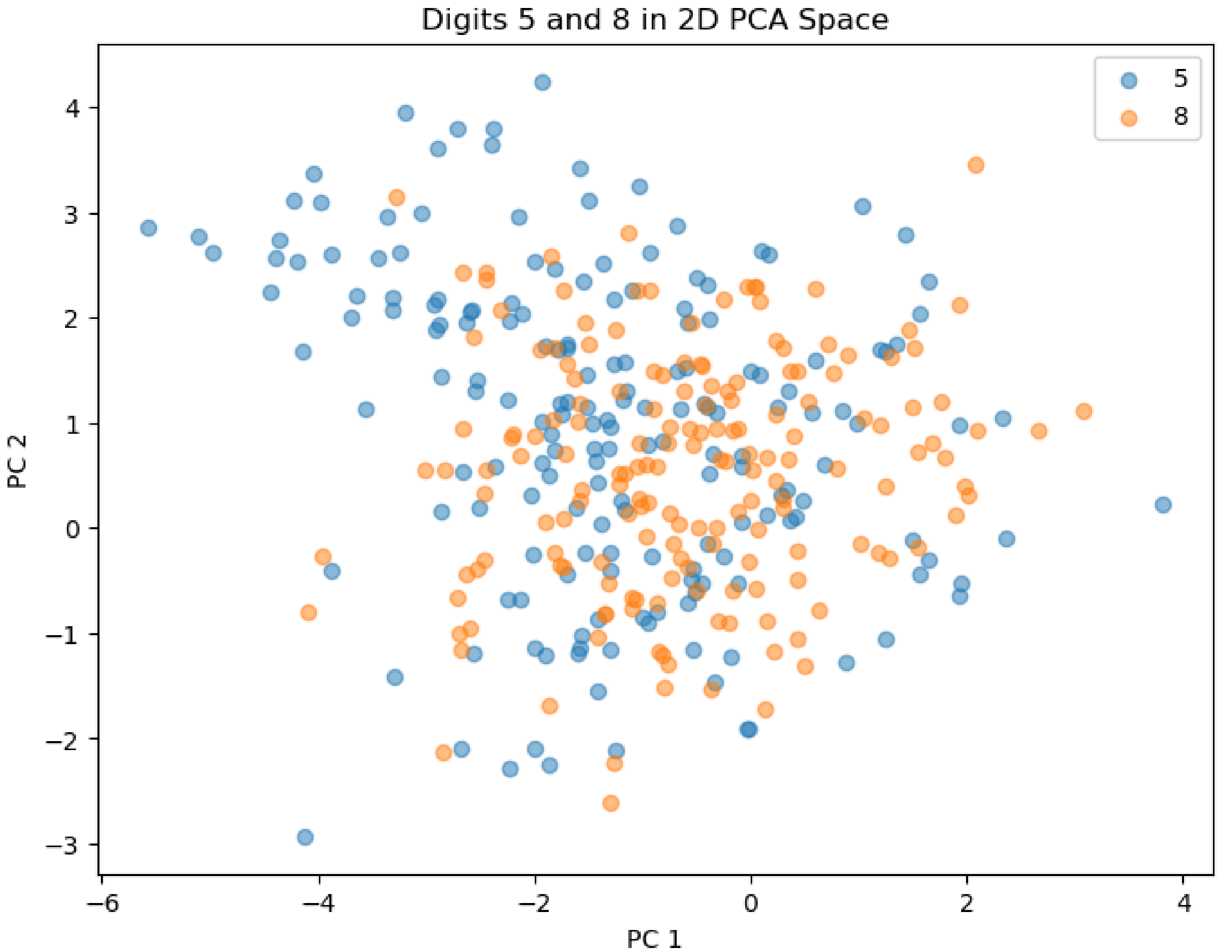
In [14]:
```
# Applying PCA with two components
pca_2d = PCA(n_components=2)
data_2d = pca_2d.fit_transform(data2S_clean)

# Function to plot 2D scatter for specific digit pairs
def plot_digit_pairs(digit1, digit2):
    idx1 = digits.target == digit1
    idx2 = digits.target == digit2

    plt.figure(figsize=(8, 6))
    plt.scatter(data_2d[idx1, 0], data_2d[idx1, 1], label=str(digit1), alpha=0
    plt.scatter(data_2d[idx2, 0], data_2d[idx2, 1], label=str(digit2), alpha=0
    plt.legend()
    plt.xlabel('PC 1')
    plt.ylabel('PC 2')
    plt.title(f'Digits {digit1} and {digit2} in 2D PCA Space')
    plt.show()

# Can be distinguished
plot_digit_pairs(0, 1)
plot_digit_pairs(3, 4)
plot_digit_pairs(6, 7)

# Cannot be clearly distinguished
plot_digit_pairs(5, 9)
plot_digit_pairs(5, 8)
plot_digit_pairs(3, 8)
```

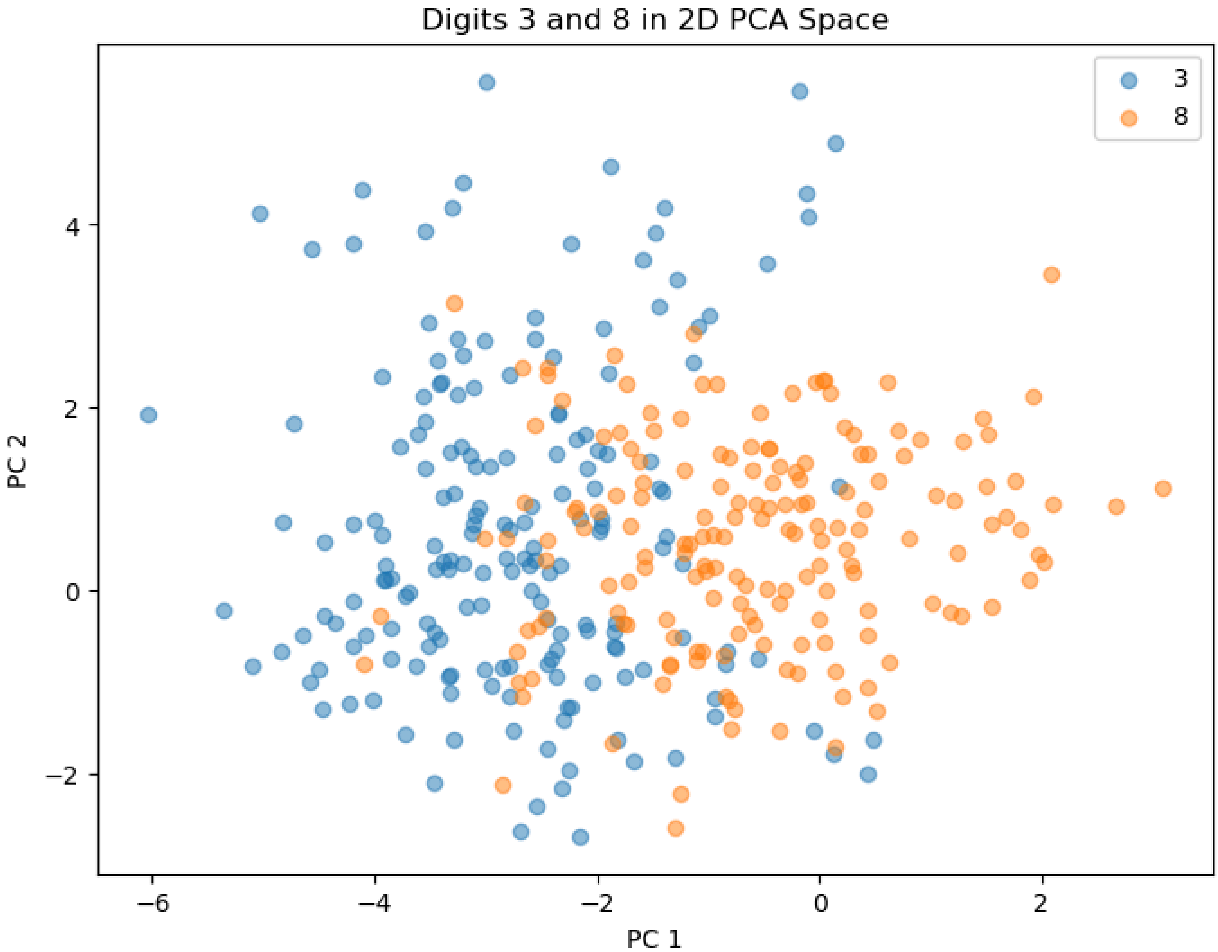

Digits 0 and 1 in 2D PCA Space
0
1
PC 1
PC 2
−3
−2
−1
0
1
2
3
4
−4
−2
0
2
4


Digits 3 and 4 in 2D PCA Space
3
4
PC 2
PC 1
6
4
2
0
−2
−6
−4
−2
0
2
4
6
8
10


Digits 6 and 7 in 2D PCA Space
6
7
PC 2
PC 1
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−4
−2
0
2
4
6


Digits 5 and 9 in 2D PCA Space
5
9
PC 2
PC 1
8
6
4
2
0
−2
−6
−4
−2
0
2
4
6


Digits 5 and 8 in 2D PCA Space
5
8
PC 2
PC 1
4
3
2
1
0
−1
−2
−3
−6
−4
−2
0
2
4




```
In [ ]:
```