KASASIRA CHARLES DERRICK

Reg No. 21/U/05662/EVE Student No. 2100705662.

SECTION A.

1. The limit of f(x) is the value of f(x) as x approaches **a** that is to say if the limit is L, we represent it as

$$\lim_{x \to a} f(x) = L$$

2. $\lim_{x \to 1} \frac{2x+4}{x+1} = 3$

means $\forall x\ \delta > 0, \exists\ \varepsilon > 0$

s.t $|x-a| < \delta \Rightarrow |f(x) - L| < \varepsilon$

$$|x-1| < \delta \Rightarrow \left| \frac{2x+4}{x+1} - 3 \right| < \varepsilon$$

$$\frac{2x+4-3(x+1)}{x+1} < \varepsilon$$

$$\frac{-x+1}{x+1} < \varepsilon$$

$$\frac{-1(x-1)}{x+1} \times \frac{-1}{-1} < \varepsilon$$

$$\left| \frac{x-1}{-1(x+1)} \right| < \varepsilon$$

$$\left| \frac{x-1}{x+1} \right| < \varepsilon$$

Since $|x-1|$ is a small number $|x+1| \approx 2 \Rightarrow x+1 = 2+\lambda$

$$\frac{|x-1|}{(2+\lambda)} < \varepsilon$$

$$|x-1| < (2+\lambda)\varepsilon$$

$$\Rightarrow \delta = (2+\lambda)\varepsilon$$

For $f(x)$ to be continous at a point $x = a$

$\lim_{x \to}$ $f(a)$ must be defined.

$\lim_{x \to a} f(x)$ must exist.

$$\lim_{x \to a} f(x) = f(a)$$

Show that $f(x) = c^x$ where $c \neq 0$,

point $x \in (-\infty, +\infty)$

$f(\infty) = c^\infty = \infty$

$$\lim_{x \to \infty} \cancel{f(x)}\ f(x) = \lim_{x \to \infty} c^x = c^\infty = \infty$$

Since $f(\infty) = \lim_{x \to \infty} c^x$ at a point $x \in (-\infty, +\infty)$

$\therefore f(x) = c^x$ is continous

SECTION A. 4 a)

4 L'hopital's law states that

$$\therefore \lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

a) $\lim_{x \to 0} \frac{x \sin 3x}{\tan x}$

from L'hopital's law $\lim_{x \to 0} \frac{\cancel{x \sin 3x}\, f(x)}{\cancel{\tan x}\, g(x)} = \frac{f'(x)}{g'(x)}$

If $f(x) = x \sin 3x$.

$\cancel{f'(x) =}$

Using Product rule to find the derivative of $x \sin 3x$

$(fg)' = f'g + fg'$

$$f'(x) = \left(\frac{dx}{dx}\right) \sin 3x + x \left(\frac{d \sin 3x}{dx}\right)$$

$$= \sin 3x + 3x \cos 3x$$

$$\frac{d(\tan x)}{dx} = \sec^2 x$$

$$\frac{f'(x)}{g'(x)} = \frac{\sin(3x) + 3x \cos(3x)}{\sec^2 x}$$

as x approaches 0

$$= \frac{\sin(3 \times 0) + 3 \times 0 \times \cos(3 \times 0)}{\sec^2(0)}$$

$$= 0$$

$$\therefore \lim_{x \to 0} \frac{x \sin 3x}{\tan x} = 0$$

SECTION A 4 b)

$$\lim_{x \to 0} \frac{\sin^{-1} 2x}{\sin^{-1} 4x}$$

Apply L'Hopital's rule $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{f'(x)}{g'(x)}$

$$\frac{d}{dx}(\sin^{-1} 2x) = \frac{2}{\sqrt{1-4x^2}} \quad \frac{d}{dx}(\sin^{-1} 4x) = \frac{4}{\sqrt{1-16x^2}}$$

$$= \frac{2}{\sqrt{1-4x^2}}$$

$$= \lim_{x \to 0} \left(\frac{\frac{2}{\sqrt{1-4x^2}}}{\frac{4}{\sqrt{1-16x^2}}} \right)$$

$$= \left(\frac{\frac{2}{\sqrt{1-4(0)^2}}}{\frac{4}{1-16(0)^2}} \right)$$

$$= \frac{1}{2}$$

$$\therefore \lim_{x \to 0} \frac{\sin^{-1} 2x}{\sin^{-1} 4x} = \frac{1}{2}$$

Section A 5

Squeeze theorem states that if three functions are in such away that $g(x) \leq f(x) \leq h(x)$ for acertain neighborhood of a and $\lim_{x \to a} g(x) = \lim_{x \to a} = L$, the $\lim_{x \to a} f(x) = L$

$$\lim_{x \to \infty} \frac{3x((\cos^3 x + \cos 3x)}{x^2 + 2^x}$$

Since for cosine: $-1 \leq \cos\theta \leq 1$

$$\therefore \quad -2 \leq \cos\theta + \cos\alpha \leq 2$$

$$\therefore \frac{3x \cdot -2}{x^2+2^x} \leq \frac{3x(\cos^3 x + \cos 3x)}{x^2+2^x} \leq \frac{3x \cdot 2}{x^2+2^x}$$

and $\lim_{x \to \infty} \frac{3x \cdot -2}{x^2+2^x} = \lim_{x \to \infty} \frac{3x \cdot 2}{x^2+2^x} = 0$

applying squeeze law

$$\lim_{x \to \infty} \frac{3x(\cos^3 x + \cos 3x)}{x^2+2^x} = 0$$

Section B 1.

$$\lim_{x \to 2} \frac{\sqrt{x}}{\sqrt{x-2}}$$

Right-side limit.

$$\lim_{x \to 2^+} \left(\frac{\sqrt{x}}{\sqrt{x-2}} \right) =$$

2.1	4.5826
2.01	14.1774
2.001	44.7325

$$\therefore \lim_{x \to 2^+} \left(\frac{\sqrt{x}}{\sqrt{x-2}} \right) = \infty$$

left-side limit.

$$\lim_{x \to 2^-} \left(\frac{\sqrt{x}}{\sqrt{x-2}} \right)$$

1.9 Does not exist (DNE).

Since $\lim_{x \to 2^+} \left(\frac{\sqrt{x}}{\sqrt{x-2}} \right) \neq \lim_{x \to 2^-} \left(\frac{\sqrt{x}}{\sqrt{x-2}} \right)$ the limit does not Exist

$\lim_{x \to 2} \left(\frac{\sqrt{x}}{\sqrt{x-2}} \right)$ Does not exit.

Section B 2.

$$\lim_{x \to \infty} \frac{x^3}{2x^3 + 1}$$

Divide by highest denominator power.

$$\left(\frac{x^3}{x^3}\right) \Big/ \left(\frac{2x^3}{x^3} + \frac{1}{x^3}\right) = \frac{1}{2 + 1/x^3}$$

$$\lim_{x \to \infty} \left(\frac{1}{2 + 1/x^3}\right)$$

from $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$

$$= \frac{\lim_{x \to \infty} 1}{\lim_{x \to \infty} \left(2 + \frac{1}{x^3}\right)}$$

$$= \frac{1}{2 + 0}$$

$$= \frac{1}{2}$$

$$\therefore \lim_{x \to \infty} \frac{x^3}{2x^3 + 1} = \frac{1}{2}$$

Section B 3

$$\lim_{x \to 0} \frac{x \cos x}{e^x}$$

from $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$

$$= \frac{\lim_{x \to 0} x \cos x}{\lim_{x \to 0} e^x}$$

$$\lim_{x \to 0} e^x = e^0 = 1$$

$$\lim_{x \to 0} x \cos x = 0 . \cos(0) = 0$$

$$= \frac{0}{1}$$

$$= 0$$

$$\therefore \lim_{x \to 0} \frac{x \cos x}{e^x} = 0$$

Section B 4.

$$\lim_{x \to 0} \cot x$$

Right side limit.

$$\lim_{x \to 0^+} \cot(x) = \infty$$

Left side limit

$$\lim_{x \to 0^-} \cot(x) = -\infty$$

Since $\lim_{x \to 0^+} \neq \lim_{x \to 0^-}$

$\therefore \lim_{x \to 0} \cot x$ = Does not Exist

Section B 5

$$\lim_{x \to 1} \frac{x^2-1}{x-1}$$

$$= \lim_{x \to 1} \frac{(x^2-1^2)}{(x-1)}$$

$$= \lim_{x \to 1} \frac{(x+1)(x-1)}{(x-1)}$$

$$= \lim_{x \to 1} \frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}}$$

$$= \lim_{x \to 1} x+1$$

$$= 1+1$$

$$= 2$$

$$\therefore \lim_{x \to 1} \frac{x^2-1}{x-1} = 2$$