# FACULTY OF COMPUTING AND INFORMATION TECHNOLOGY

## CSC 1104: COMPUTER ORGANISATION AND ARCHITECTURE

### TEST 3 DAY CLASS 2009 /2010

**1. Define the following:**

**(i) A logical variable**

*A variable that can take on two states e.g. (0, 1, True, false; on/off)*

**(ii) A logical gate**

*A combinatorial circuit with only one output*

**(iii) A Truth Table**

*A table listing all the outputs for the various inputs*

2. (a) Assume that A = 10111 B = 01011 and C = 11100. Find

(i) A **AND** B **AND** C

10111
01011
11100
00000

(ii) A **Excl OR** B **Excl OR** C

10111
01011
11100
11111

(iii) A **OR** B **OR** C

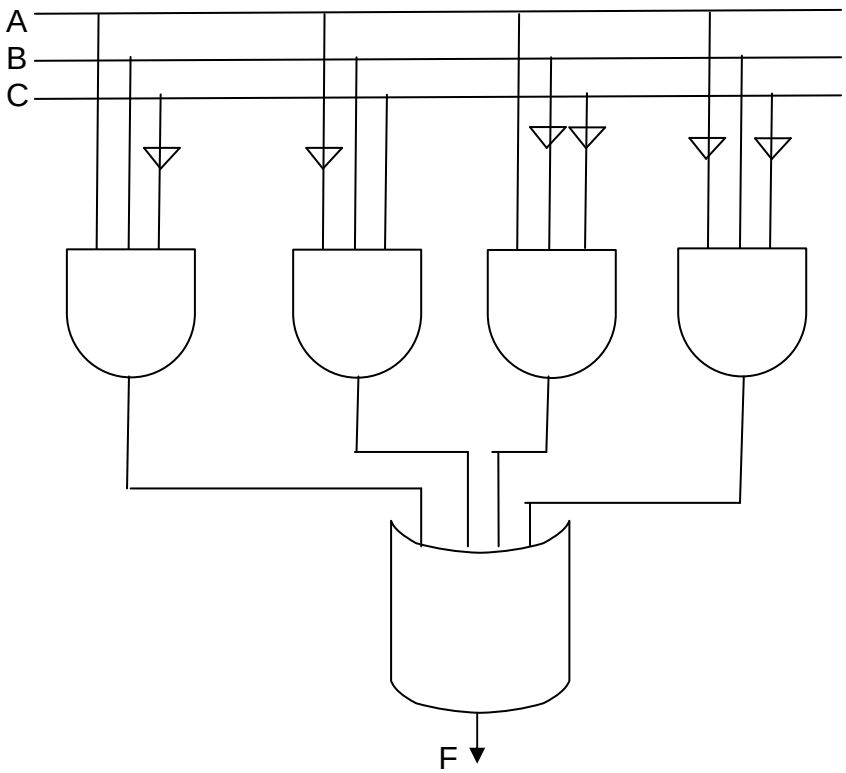
10111
**01011**
11100
11111

3. (a) Given the following Boolean function F = $AB\overline{C} + \overline{A}BC + A\overline{B}\overline{C} + \overline{A}B\overline{C}$

(i) List the truth table of the function.

| **A** | **B** | **C** | **F** |
|---|---|---|---|
| 0 | 0 | 0 | **0** |
| 0 | 0 | 1 | **0** |
| 0 | 1 | 0 | **1** |
| 0 | 1 | 1 | **1** |
| 1 | 0 | 0 | **1** |
| 1 | 0 | 1 | **0** |
| 1 | 1 | 0 | **1** |
| 1 | 1 | 1 | **0** |

(ii) Draw a logic diagram using the original Boolean expression



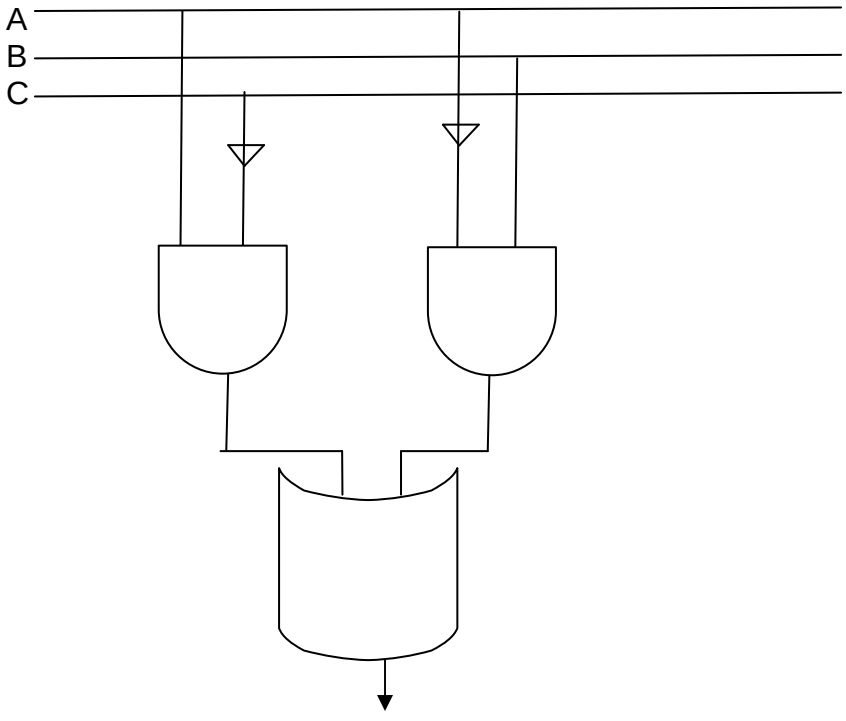

(iii) Simplify the algebraic expression using Boolean algebra or otherwise

$A\bar{B}\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C}$

$= A\bar{B}\bar{C} + A\bar{B}\bar{C} + \bar{A}BC + \bar{A}\bar{B}\bar{C}$

$= A\bar{C}(B + \bar{B}) + \bar{A}B(C + \bar{C})$ **$= A\bar{C} + \bar{A}B$**

(iv) Draw a logic diagram corresponding to the simplified expression



4. (a). Prove that $\overline{ABC}$ is the same as $\bar{A} + \bar{B} + \bar{C}$

| A | B | C | $\bar{A}$ | $\bar{B}$ | $\bar{C}$ | $\bar{A} + \bar{B} + \bar{C}$ | A BC | $\overline{ABC}$ |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 | 1 | **1** | 0 | **1** |
| 0 | 0 | 1 | 1 | 1 | 0 | **1** | 0 | **1** |
| 0 | 1 | 0 | 1 | 0 | 1 | **1** | 0 | **1** |
| 0 | 1 | 1 | 1 | 0 | 0 | **1** | 0 | **1** |
| 1 | 0 | 0 | 0 | 1 | 1 | **1** | 0 | **1** |
| 1 | 0 | 1 | 0 | 1 | 0 | **1** | 0 | **1** |
| 1 | 1 | 0 | 0 | 0 | 1 | **1** | 0 | **1** |
| 1 | 1 | 1 | 0 | 0 | 0 | **0** | 1 | **0** |

(b)

| | | AB | | | |
|---|---|---|---|---|---|
| | | **00** | **01** | **11** | **10** |
| **C** | **0** | 1 | 0 | 0 | 1 |
| | **1** | 0 | 1 | 1 | 0 |

**X**

**(i) Derive the simplified SUM of PRODUCTS for X from the above map**

$\bar{A}\bar{B}\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + ABC = \mathbf{\bar{B}\bar{C} + BC}$

**(ii) Derive the PRODUCT OF SUMS expression for X**

$(A + B + \bar{C})\ (A + \bar{B} + C)\ (\bar{A} + \bar{B} + C)\ (\bar{A} + B + \bar{C})$

**5. (a) List steps that have to be followed when constructing a logical diagram.**

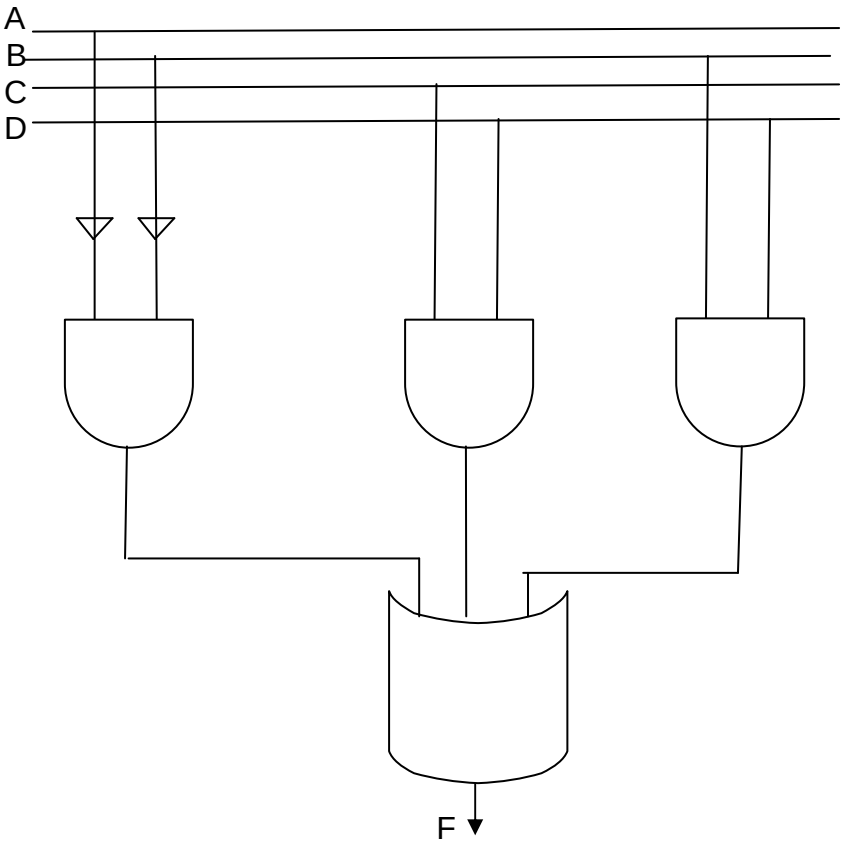
- *Determine all the input/output relationships and put them in a truth table*
- *Use the truth table to find Boolean expressions for each output.*
- *Simplify the Boolean expression*
- *Use the simplified expressions resulting to draw a logical diagram.*

(b) **Design a four input one output circuit that will output a 1 if the decimal number corresponding to the binary input combination is a prime number. Assume also that the combination where all the inputs are similar is not allowed to occur.**

| A | B | C | D | F |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | X |
| 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | X |

| CD \ AB | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | X | 0 | 0 | 0 |
| 01 | 1 | 1 | 1 | 0 |
| 11 | 1 | 1 | X | 1 |
| 10 | 1 | 0 | 0 | 0 |

$$\overline{A}\,\overline{B} + CD + BD$$



# FACULTY OF COMPUTING AND INFORMATION TECHNOLOGY

## CSC 1104: COMPUTER ORGANISATION AND ARCHITECTURE

### TEST 3 EVENING CLASS 2009 /2010

1. **Explain the following briefly.**

**(i) A literal**

*A variable or its complement in an expression*

**(ii) A miniterm**

*A term in the SUM OF PRODUCTS that includes a literal for every input*

(iii) **A logical network**

*A circuit whose inputs and outputs are described by logical variables*

2. (a) **Show that the NOR, AND and OR gates with inverters at their Inputs are equivalent to the AND, NOR and NAND gates respectively.**

| **A** | **B** | $\overline{A}$ | $\overline{B}$ | $\overline{A} + \overline{B}$ | $\overline{\overline{A} + \overline{B}}$ | **AB** |
|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 | 1 |

| **A** | **B** | $\overline{A}$ | $\overline{B}$ | $\overline{A}\,\overline{B}$ | $\overline{A + B}$ |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 |

| **A** | **B** | $\overline{A}$ | $\overline{B}$ | $\overline{A} + \overline{B}$ | $\overline{AB}$ |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 | 0 |

(b) Prove that $\overline{AB}$ is different from $\overline{A}\,\overline{B}$

| **A** | **B** | $\overline{A}$ | $\overline{B}$ | **AB** | $\overline{AB}$ | $\overline{A}\,\overline{B}$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 0 | 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 | 1 | 0 | 0 |

3. (a) **Simplify the expressions below using Boolean Algebra**

**(i)** $\bar{A}BC + AC$

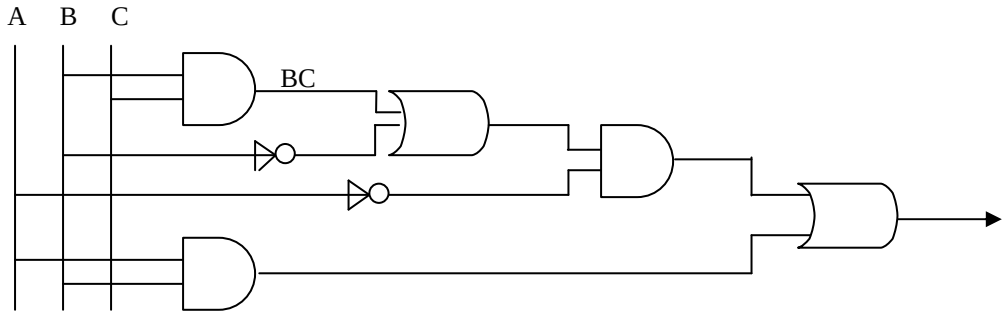
$C(\bar{A}B + A) = C(A + B)$ = **AC + BC**

**(ii)** $\bar{A}B + AB\bar{C} + ABC$

$\bar{A}B + AB(\bar{C} + C) = \bar{A}B + AB = B(\bar{A} + A) =$ **B**

(b) **Draw a logic diagram of a circuit whose output is**

$$X = AB + \bar{A}(BC + \bar{B})$$




(c) **Draw a Karnaugh Map corresponding to the following Boolean Sum of Products expression and use the drawn map to simplify the expression.**

$$\bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}BC\bar{D} + A\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}D$$

| CD \ AB | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | | | | |
| 01 | 1 | 1 | | |
| 11 | 1 | | | 1 |
| 10 | 1 | | | 1 |

$= \bar{B}\bar{C} + \bar{A}\bar{C}D$

4. **Given an expression X = F(A,B,C,D) = Σ(0, 2, 6, 7, 10, 14, 15)**

**(i)** **Derive a corresponding Boolean expression for X.**

**(ii)** **Draw a logical diagram for the expression in part (i) above.**

**(iii)** **Simplify the Boolean expression using Boolean Algebra or otherwise.**

**(iv)** **Draw a logical diagram corresponding to the simplified expression.**

(v) **Assume that there are some *Don't Care Cases* described by the function X = *d(A,B,C,D)* = Σ(3, 8, 11), simplify the above expression again using the Karnaugh Map Method**.

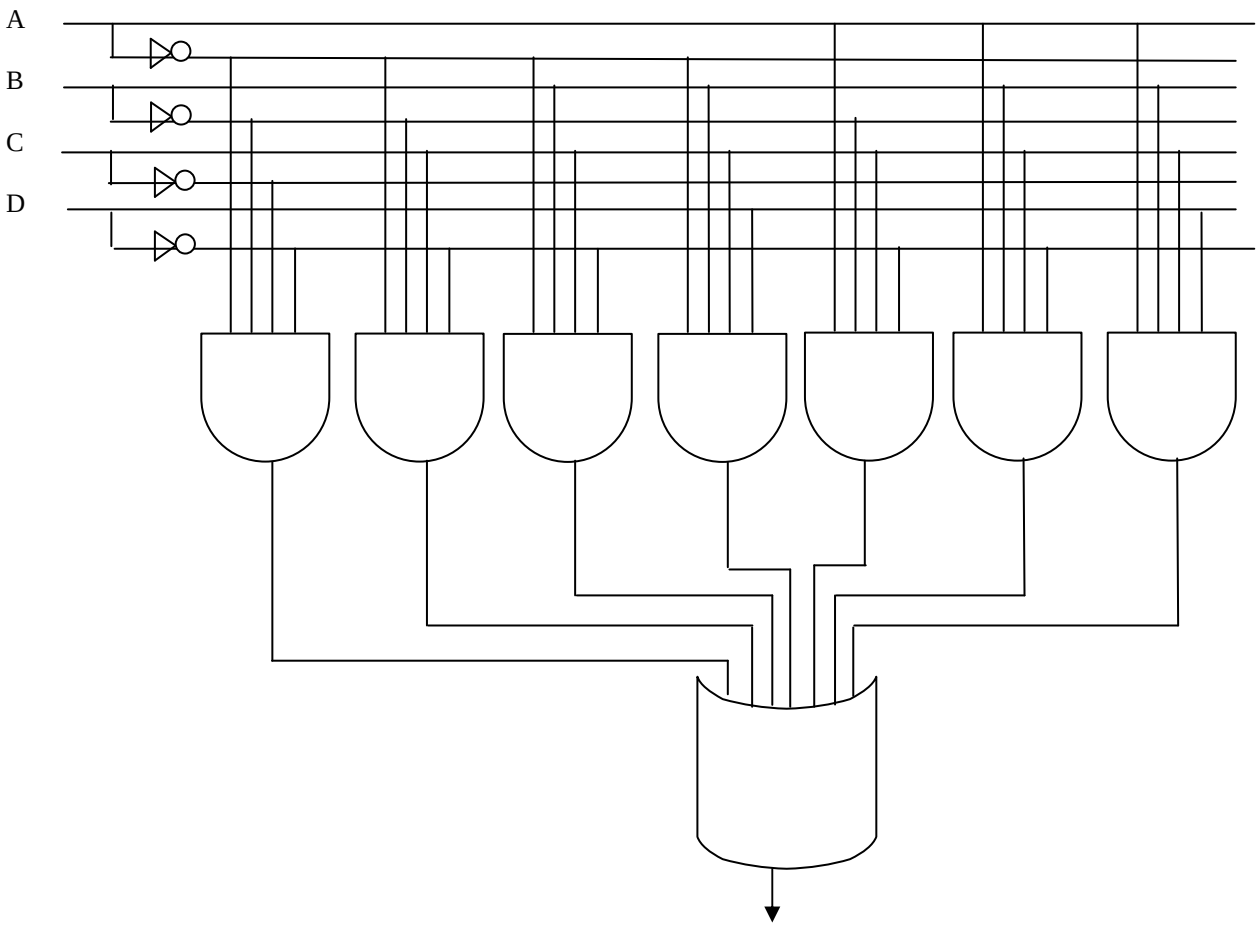
**(i) Derive a corresponding Boolean expression for X.**

**X = F(A,B,C,D) = Σ(0, 2, 6, 7, 10, 14, 15)**

| CD \ AB | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | | | |
| 01 | | | | |
| 11 | | 1 | 1 | |
| 10 | 1 | 1 | 1 | 1 |

$$\bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}B\bar{C}\bar{D} + \bar{A}BC\bar{D} + A\bar{B}C\bar{D} + A\bar{B}CD + ABCD$$

**(ii) Draw a logical diagram for the expression in part (i) above.**

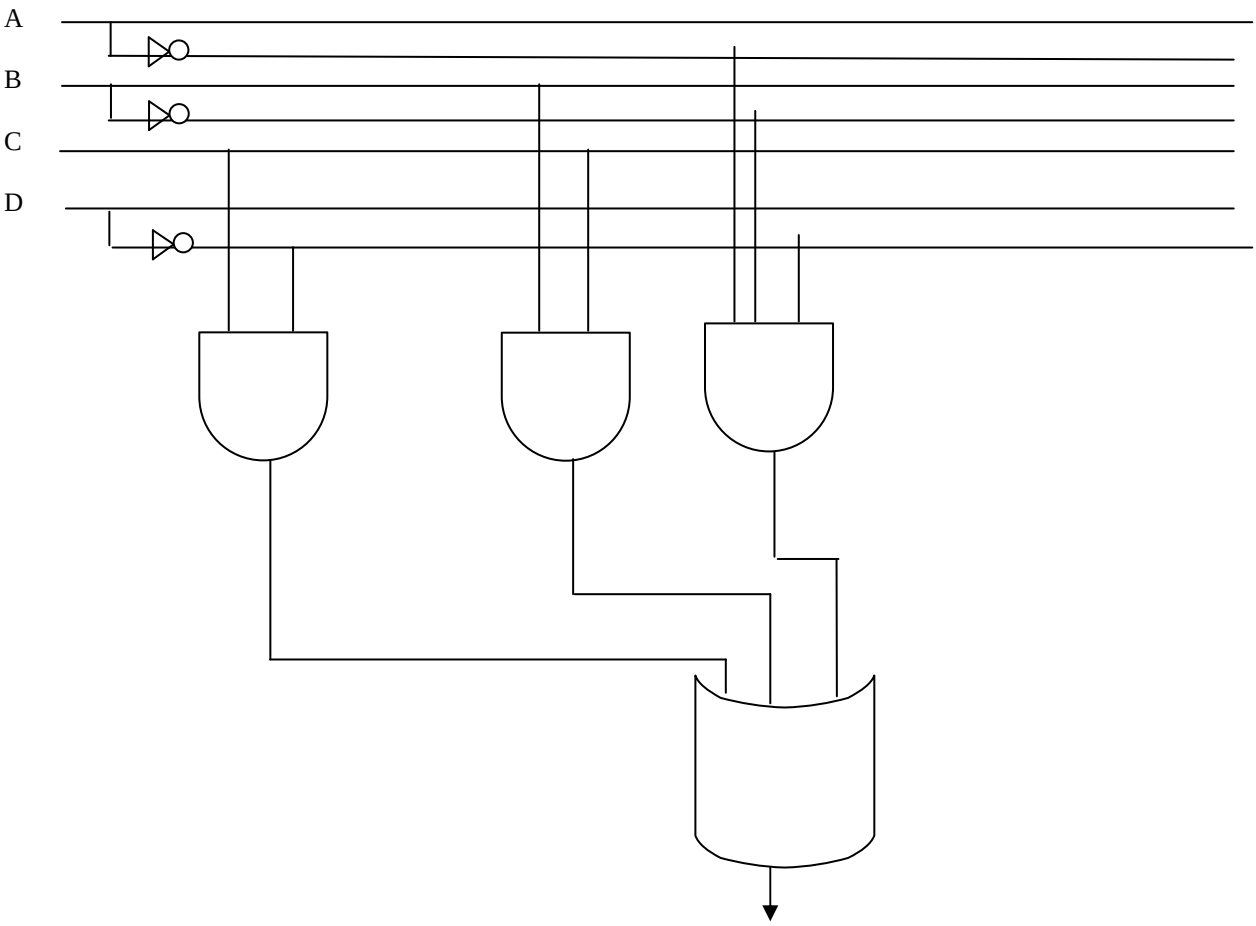



**(iii) Simplify the Boolean expression using Boolean Algebra or otherwise.**

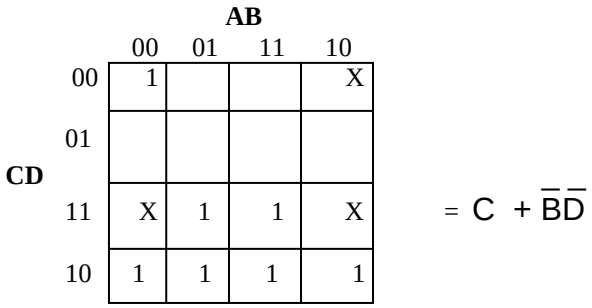
| CD \ AB | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | | | |
| 01 | | | | |
| 11 | | 1 | 1 | |
| 10 | 1 | 1 | 1 | 1 |

$$C\bar{D} + BC + \bar{A}\bar{B}\bar{D}$$

**(iv) Draw a logical diagram for the simplified expression**




(v) **Assume that there are some *Don't Care Cases* described by the function X = *d(A,B,C,D)* = Σ(3, 8, 11), simplify the above expression again using the Karnaugh Map Method**.




$= C + \overline{B}\,\overline{D}$