Recap

“Blend = Cut and Paste images”

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- What to paste? A **source** image
- Where to paste? A **destination** image
- How to cut? A **mask** to cover redundant area

Recap

"Blend = Cut and Paste images"

- What to paste? A **<u>source</u>** image
- Where to paste? A **<u>destination</u>** image
- How to cut? A **<u>mask</u>** to cover redundant area

But the problem is **How to paste?**

How to paste ?

Destination

Gioconda, Monna Lisa

By Leonardo da Vinci

Mask

Facial Feature Area

Source

Lu

How to paste ?

Destination	Mask	Source
		
Gioconda, Monna Lisa *By Leonardo da Vinci*	Facial Feature Area	Lu

How to paste ?

Destination

Gioconda, Monna Lisa

By Leonardo da Vinci

Mask

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Source

Lu

How to paste ?

Destination

Gioconda, Monna Lisa

By Leonardo da Vinci

Mask

Facial Feature Area

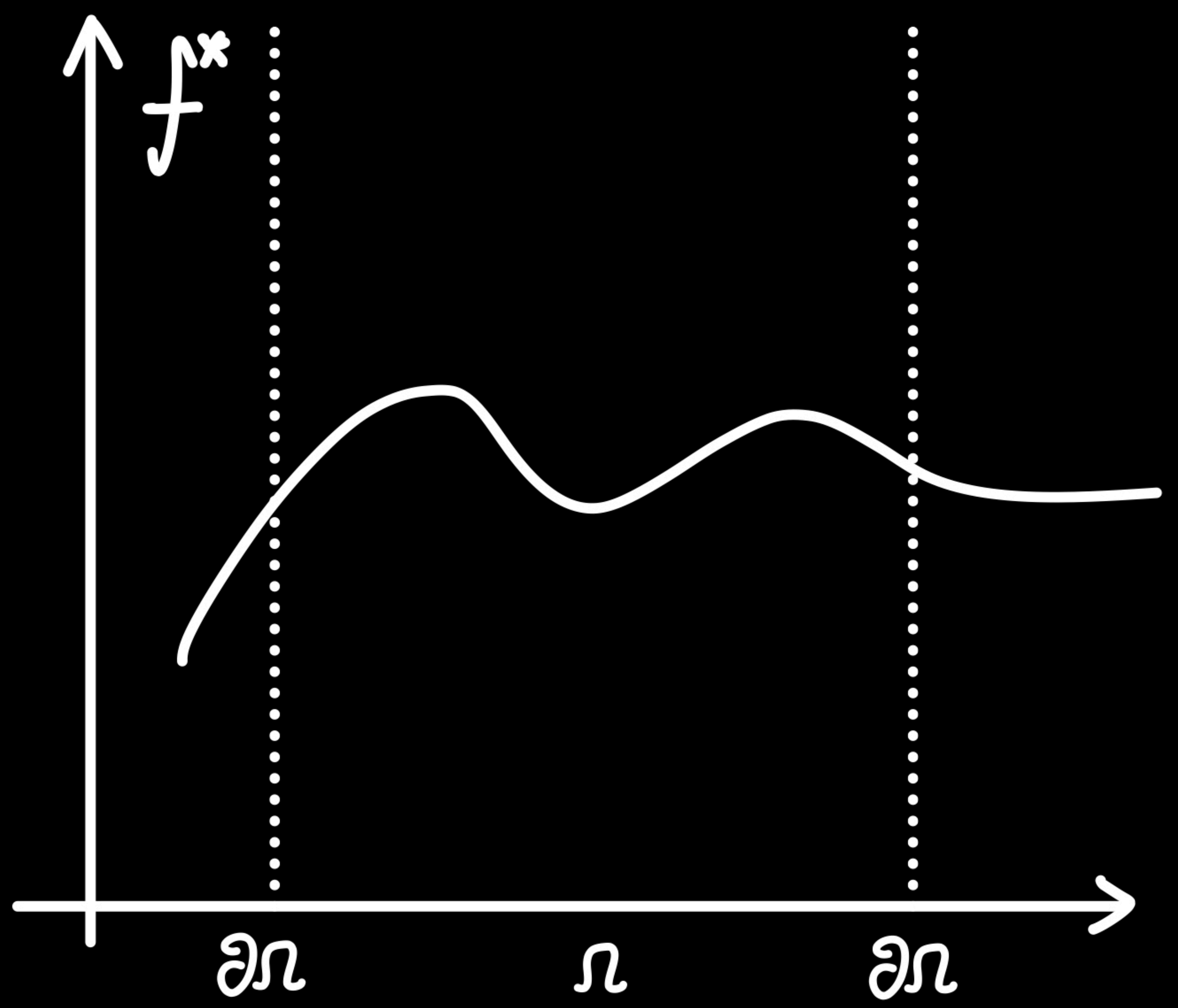
Source

Lu

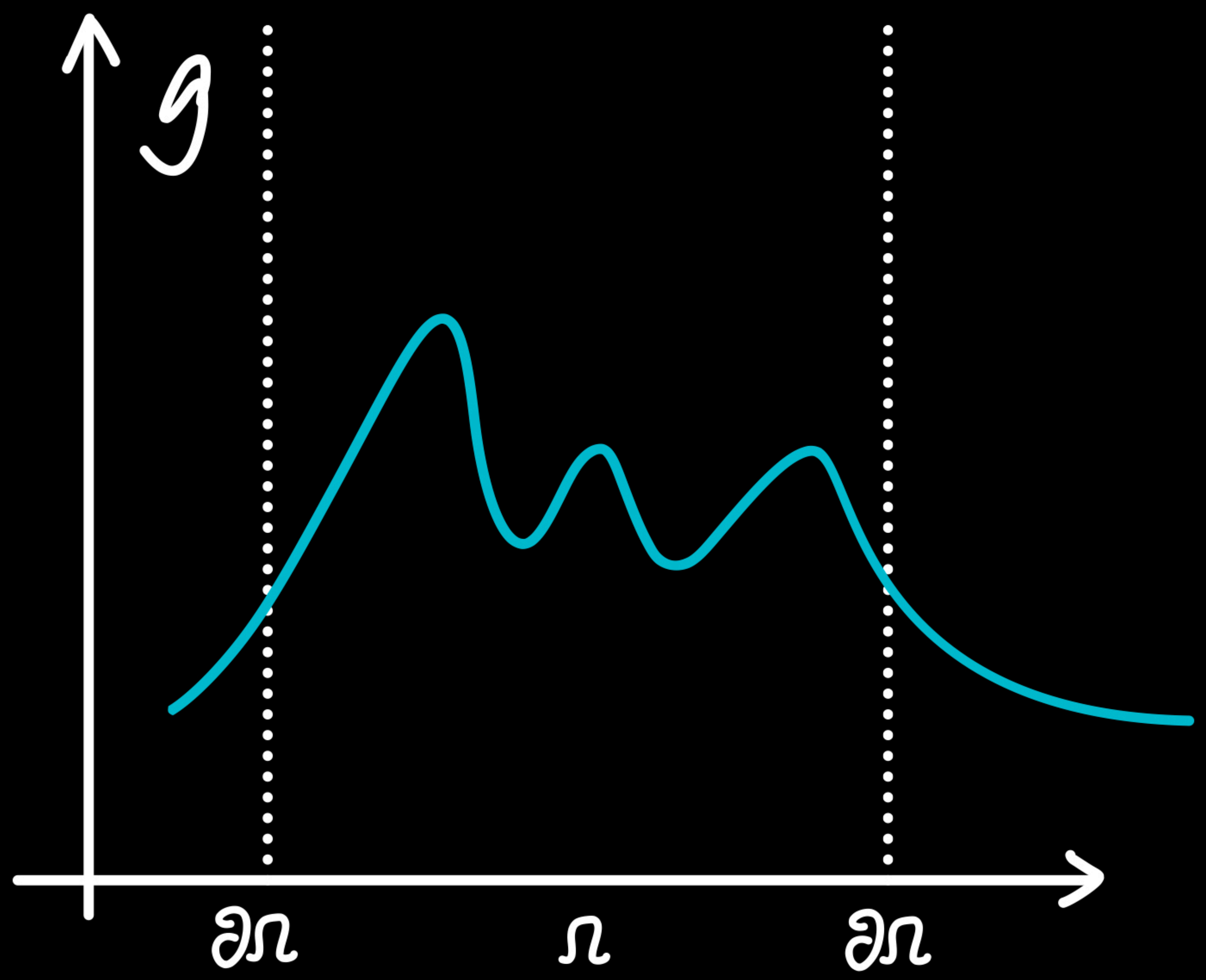
How to paste ?

Direct Copy and Paste

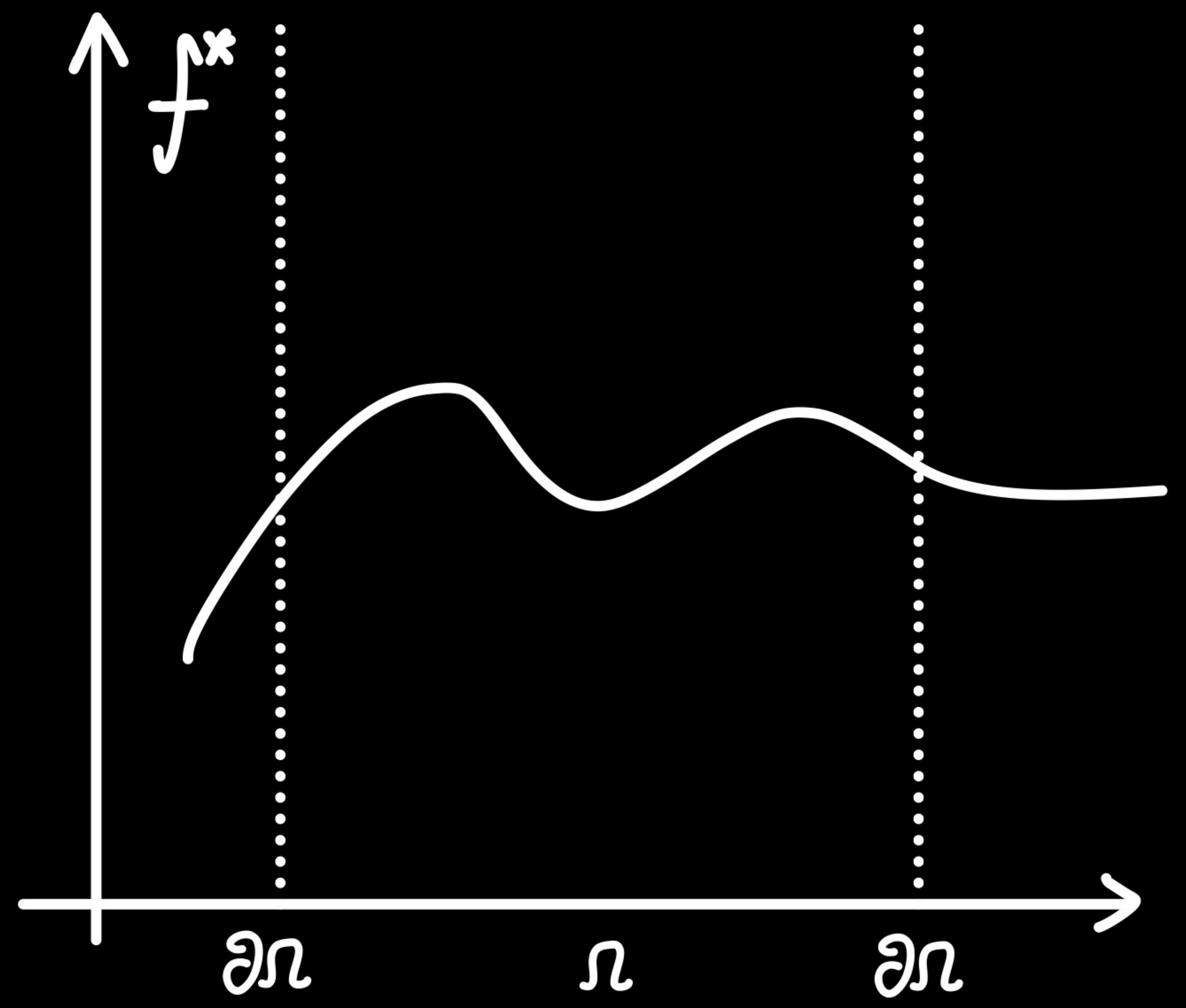
Destination



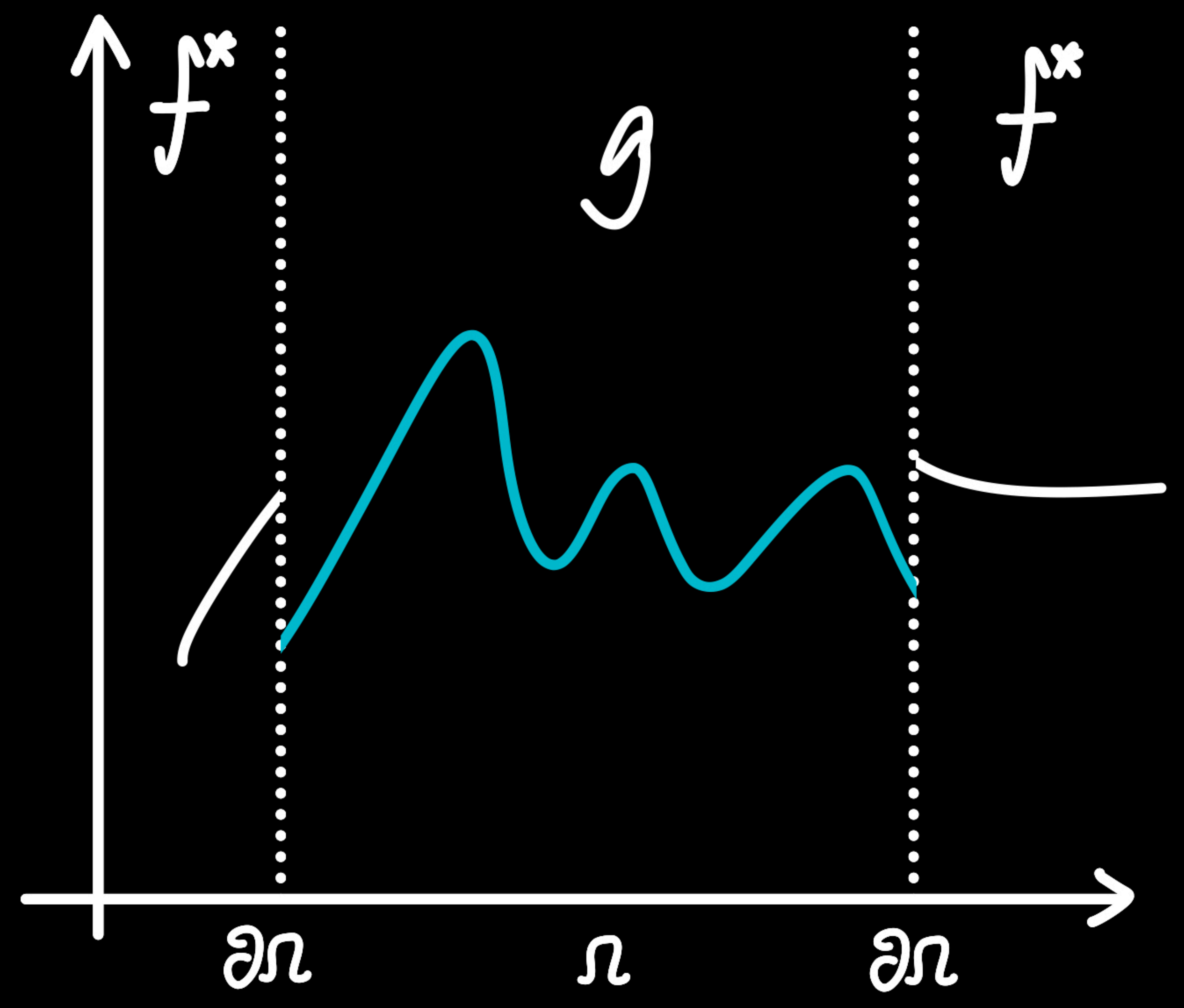
Source



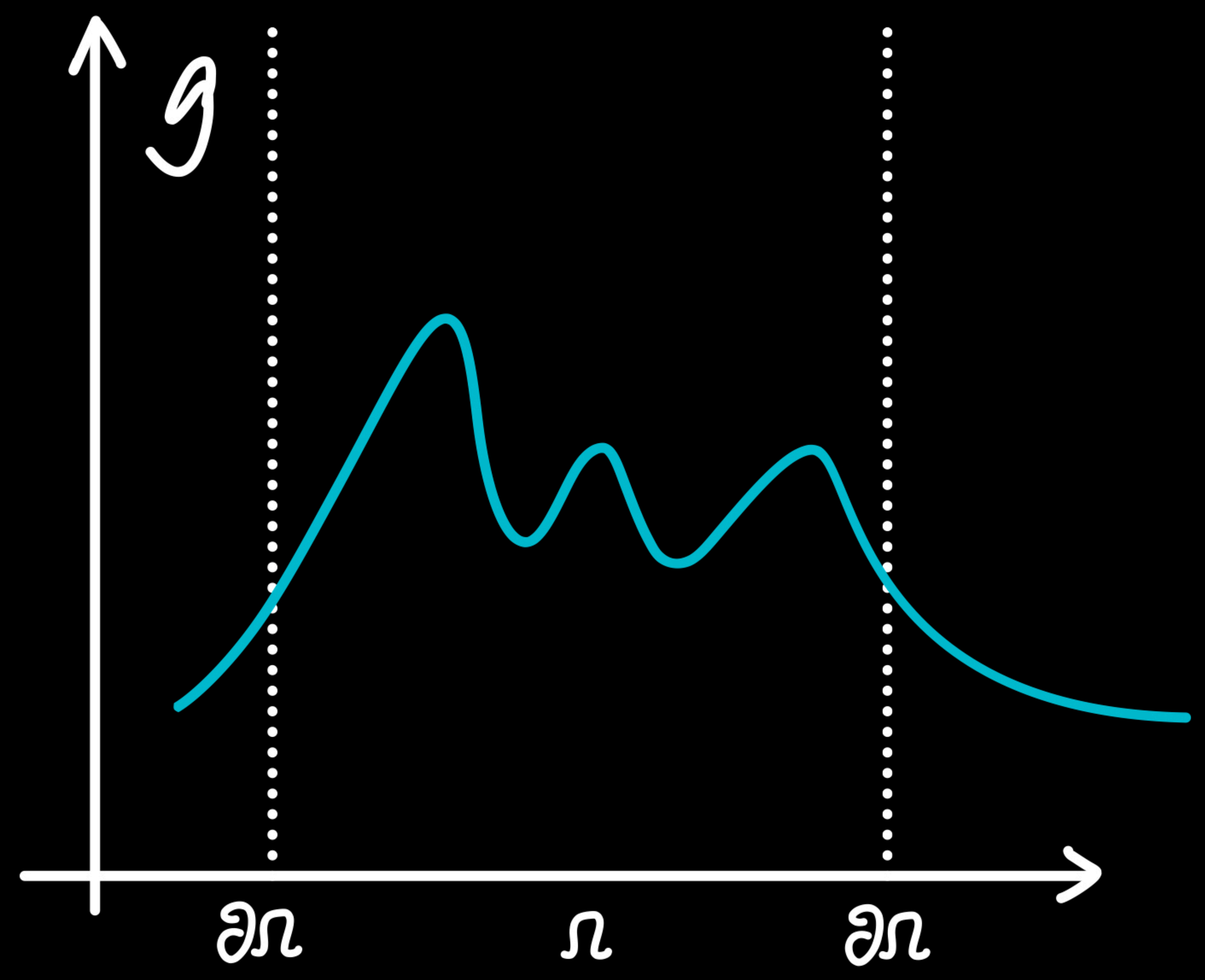
Destination



Direct Copy and Paste



Source



How to paste ?

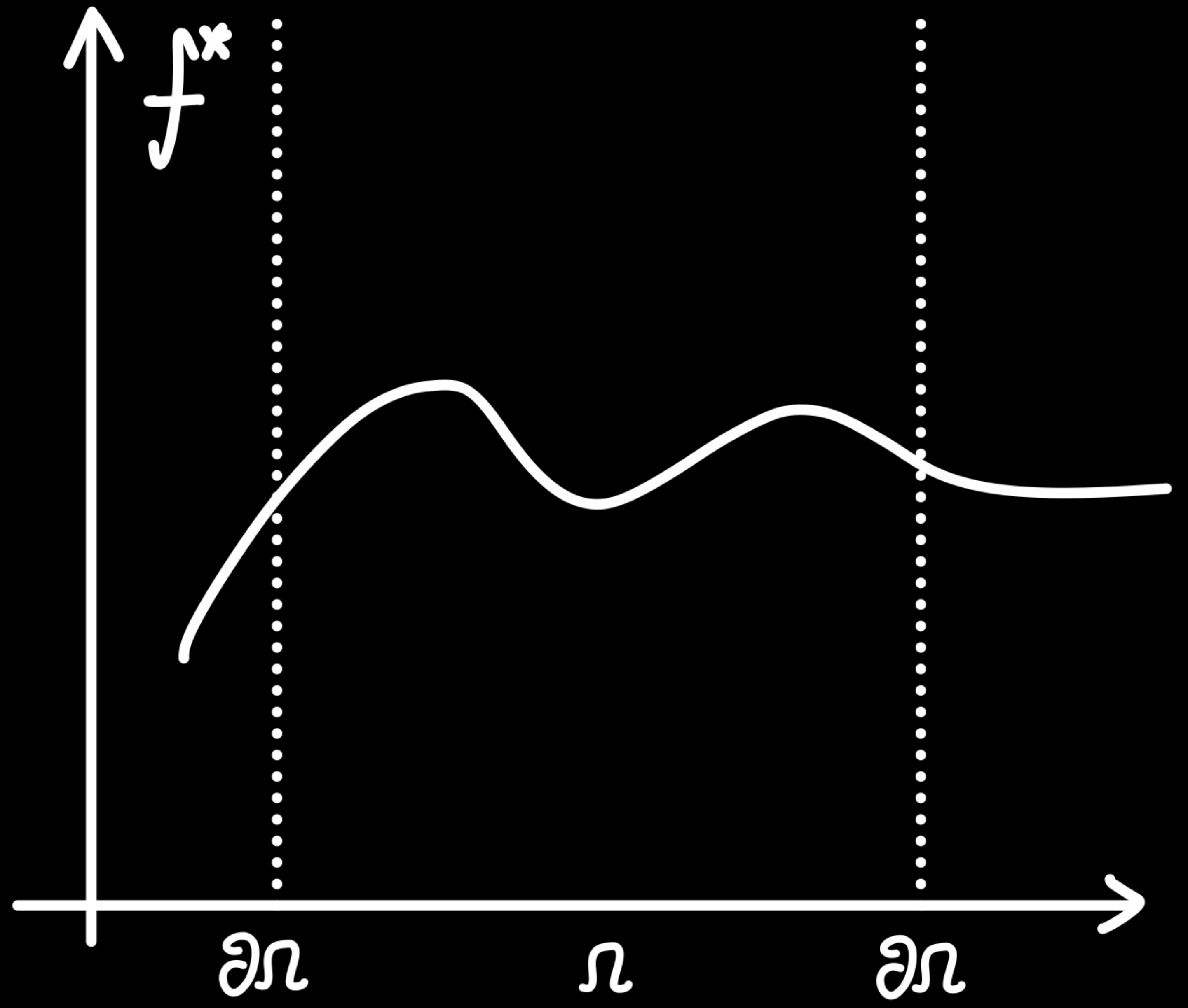
Alpha Blending

How to paste ?

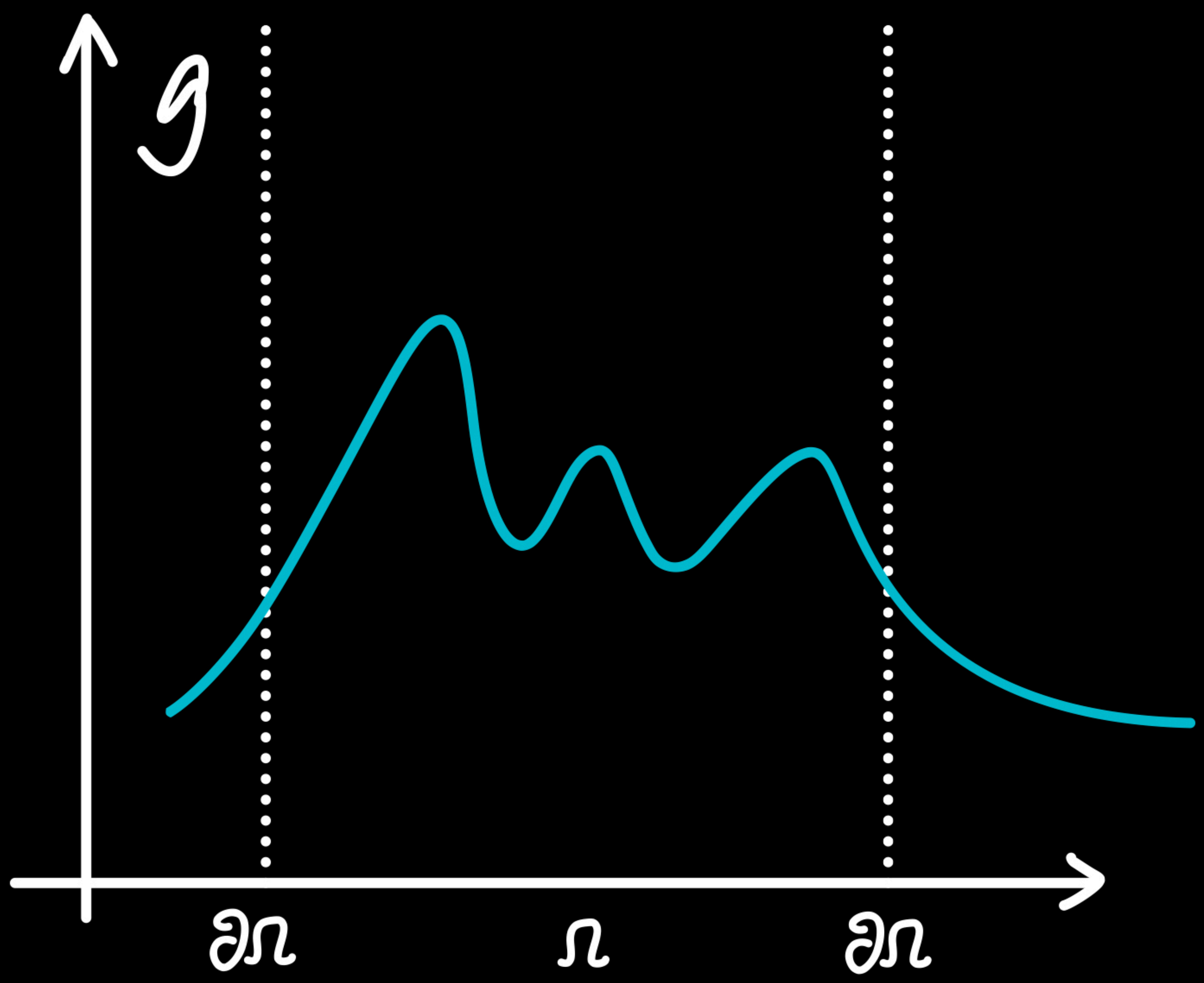
Alpha Blending

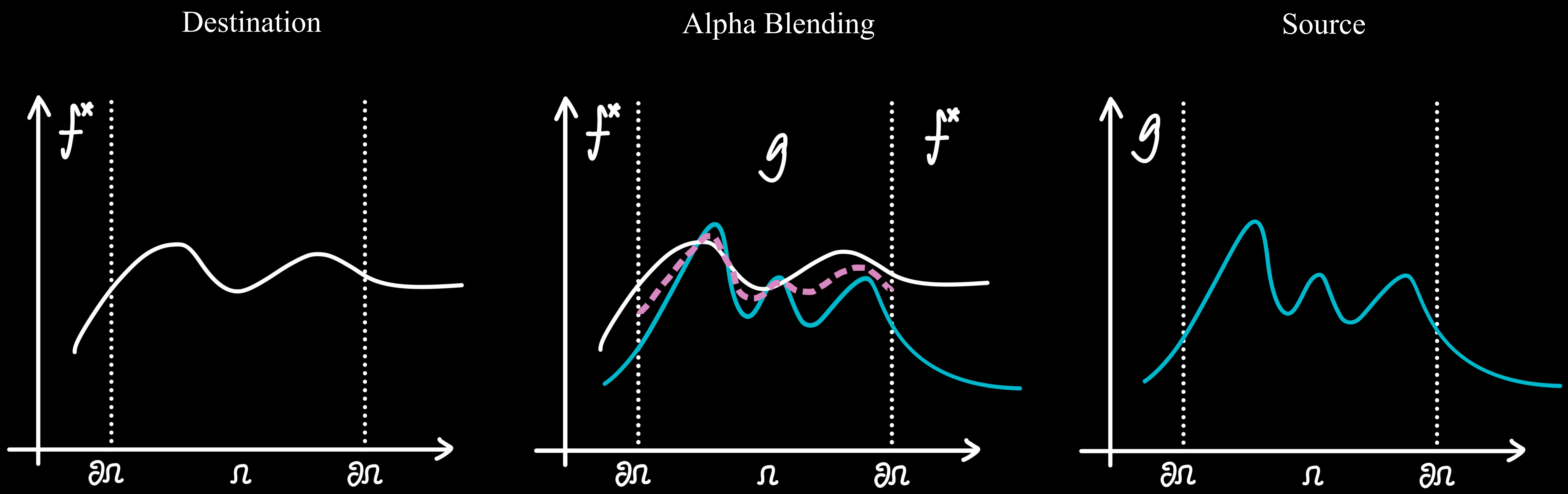
$$I = \alpha \cdot I_f + (1 - \alpha) \cdot I_b$$

Destination



Source




Destination
Alpha Blending
Source
f^*
g
f^*
g
$\partial\Omega$
Ω
$\partial\Omega$

Alpha Blending

$$I = \alpha \cdot I_f + (1 - \alpha) \cdot I_b$$

Alpha Blending

Smart Alpha Blending: Feathering

$$I = \alpha \cdot I_f + (1 - \alpha) \cdot I_b$$

Feathering Mask

Smart Alpha Blending: Feathering

$$I = \alpha \cdot I_f + (1 - \alpha) \cdot I_b$$

Feathering Mask

Smart Alpha Blending: Feathering

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Smart Alpha Blending: Feathering

Can we do better?

Recap

“Gradient”

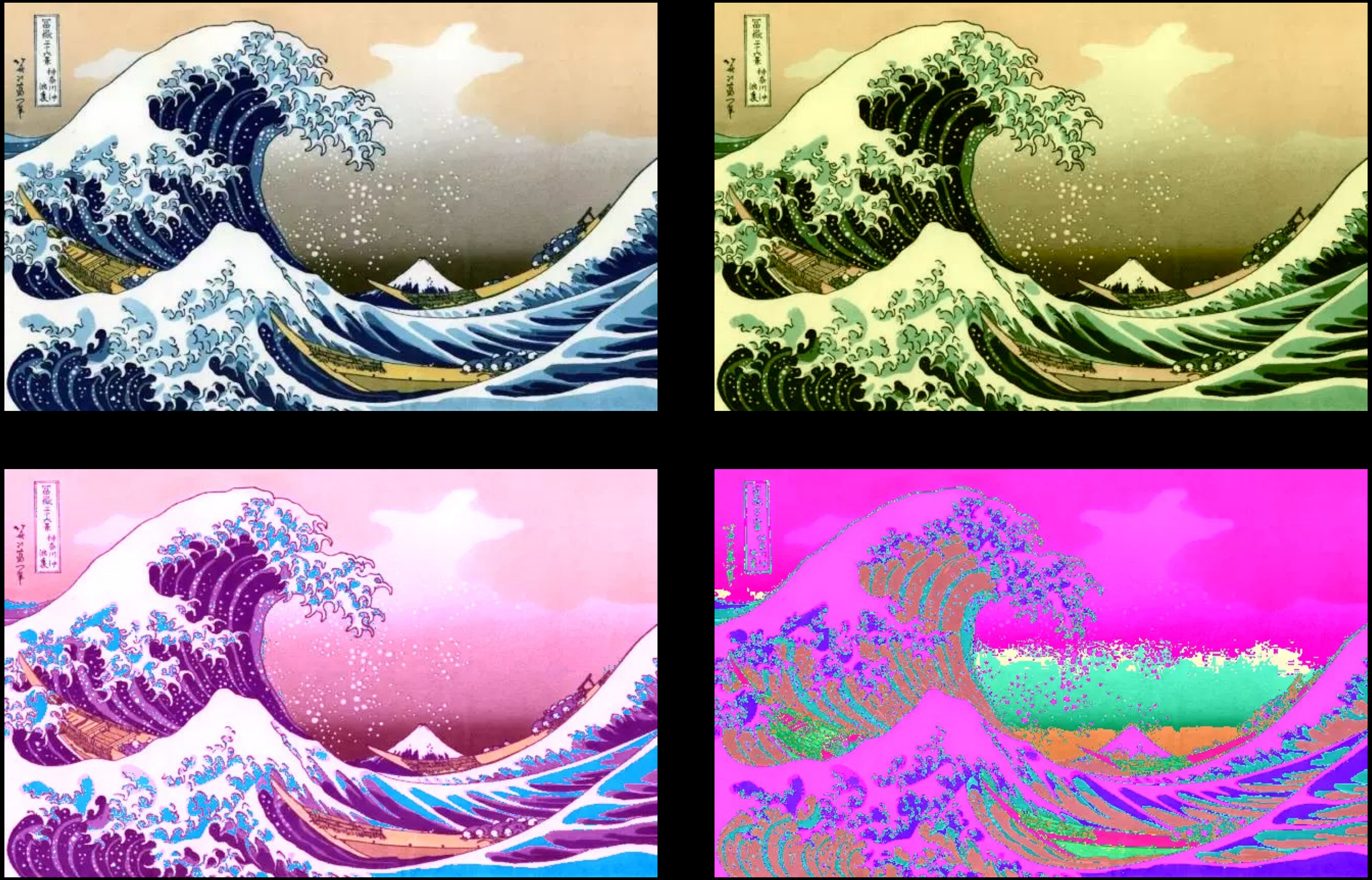
Recap

"Gradient"

Recap

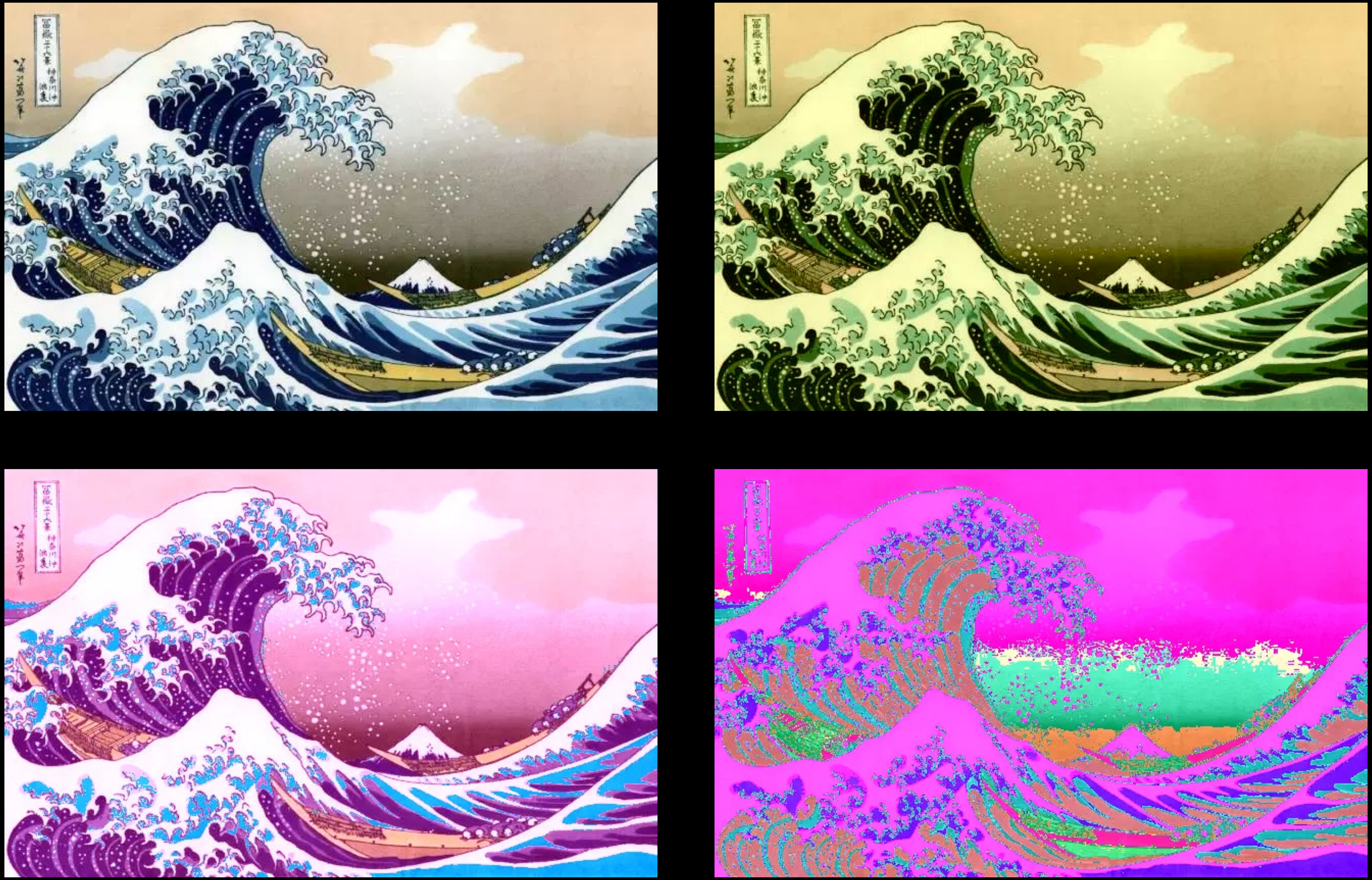
“Gradient”

- “Gradient captures everything important about shape and shading.”
- “It contains the microscope texture of the object.”
- “It encodes subtle changes of illumination.”

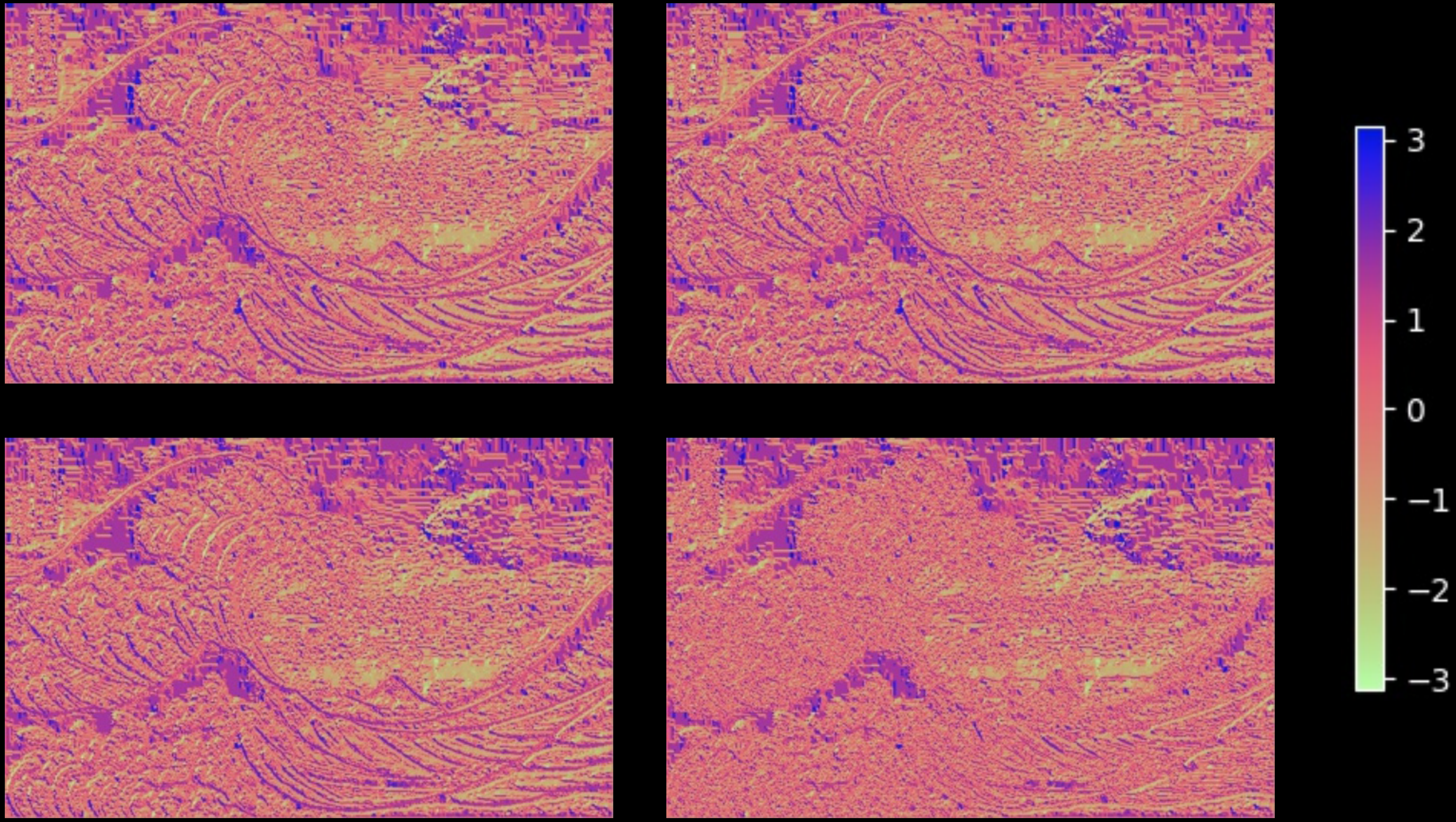


《神奈氕氘氚冲浪里》

Gradient Magnitude



《神奈气氚氚冲浪里》



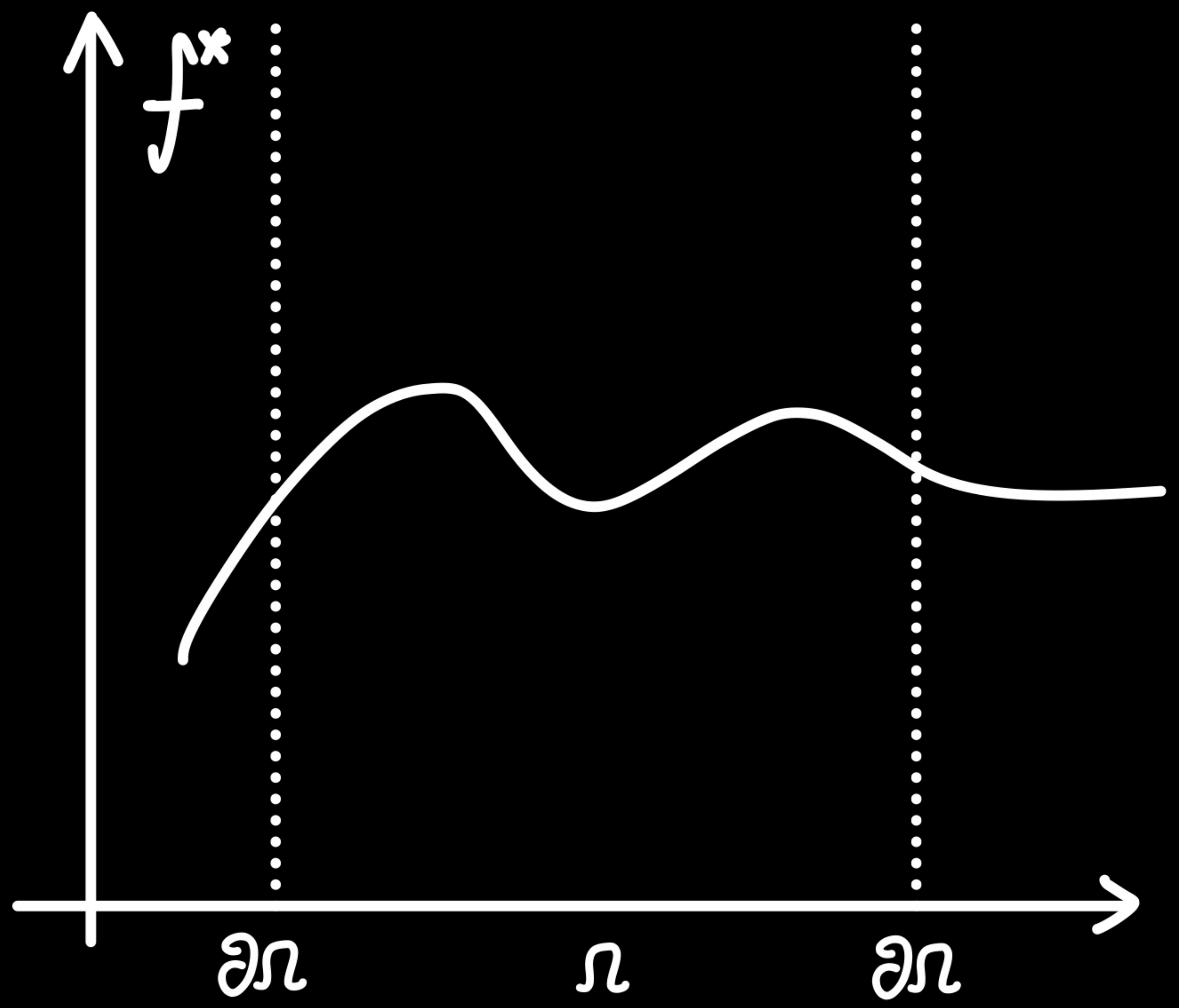
Gradient Angle

“Gradient Blending”

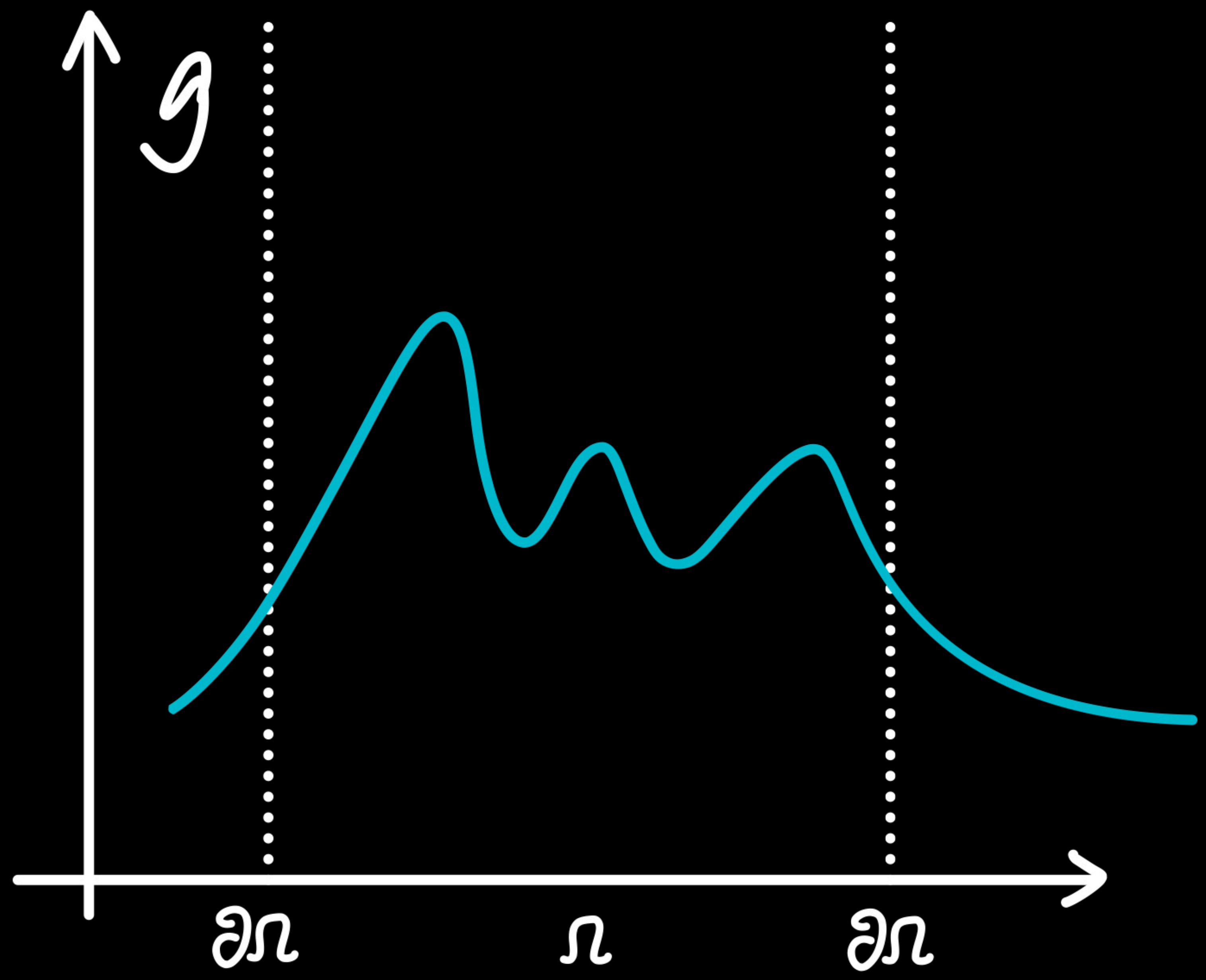
“Gradient Blending”

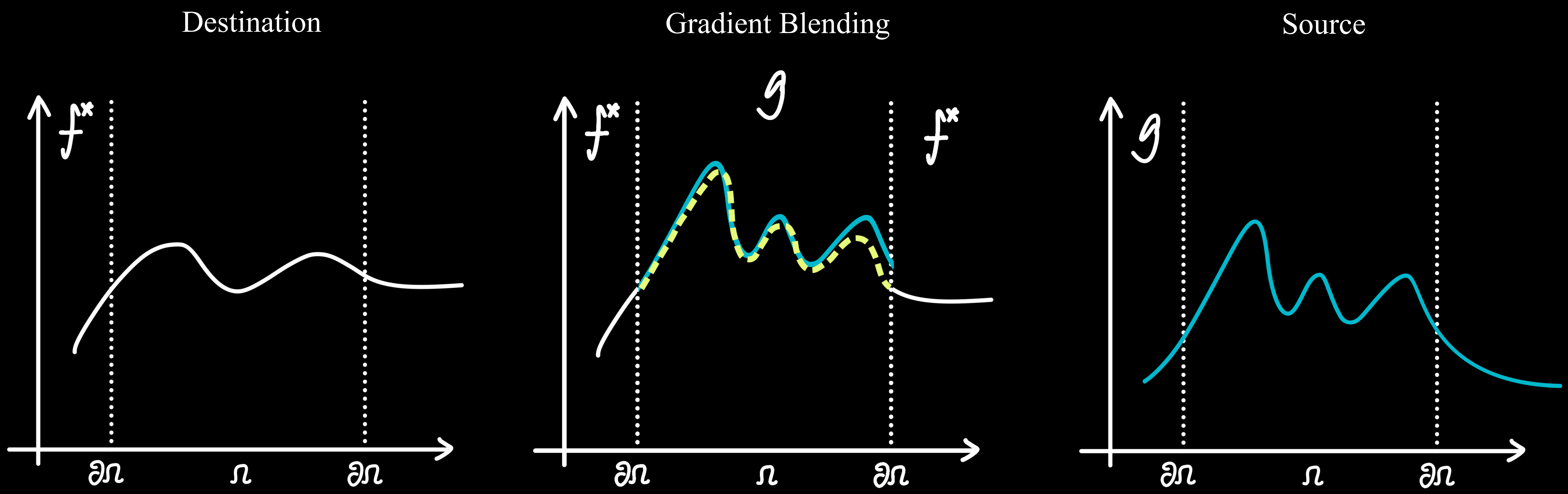
- We reconstruct the source image from its gradient field.
- The microscopic texture, shape of object boundary and the illumination changes of the source image will be retained.
- Keep the value on the boundary of blending area the same to achieve seamlessly blending.

Destination



Source




Destination
Gradient Blending
Source
f^*
g
f^*
g
$\partial\Omega$
Ω
$\partial\Omega$

How to paste ?

Gradient Blending

Gioconda, Monna Lisa

By Leonardo da Vinci

Monna Lusa

By Charles

Gioconda, Monna Lisa

By Leonardo da Vinci

Monna Lusa

By Charles

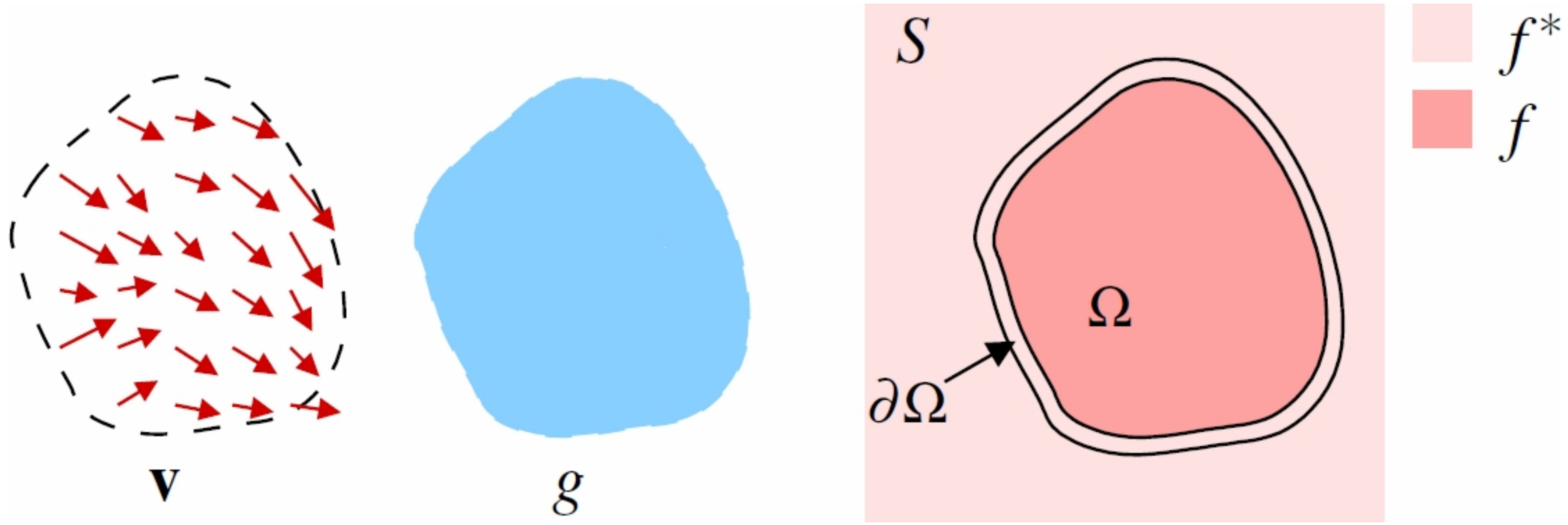
Gioconda, Monna Lisa

By Leonardo da Vinci

Monna Lusa

By Charles

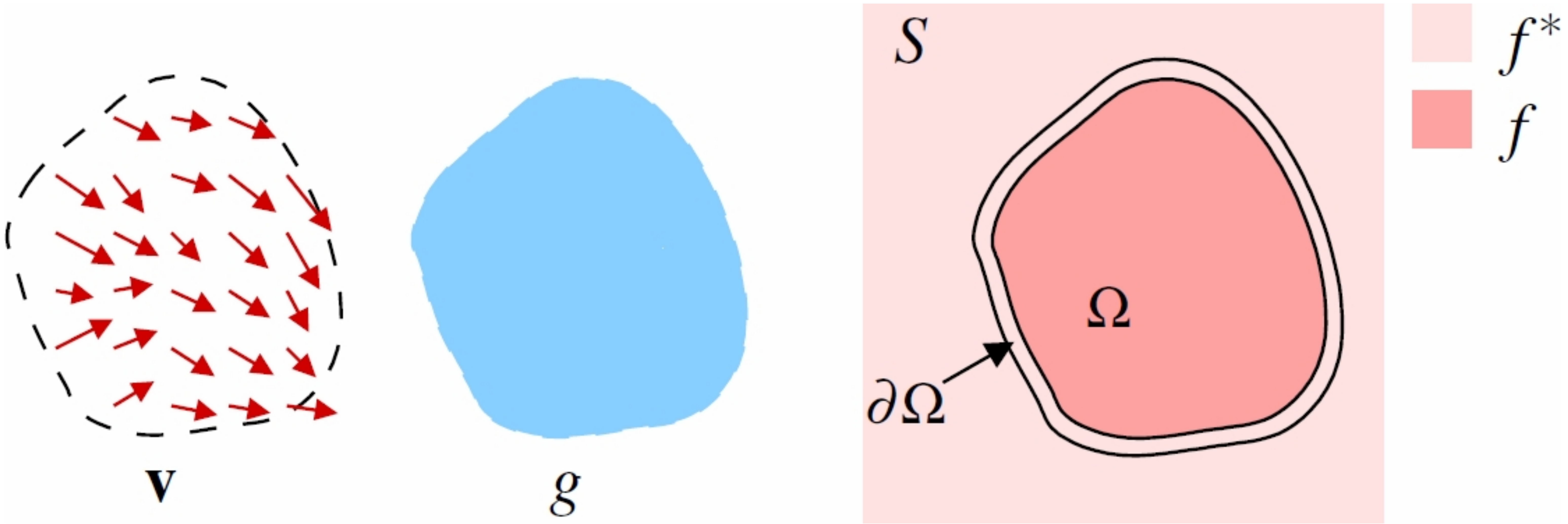
Problem Formulation



Problem Formulation

Notation

v	: *Guidance Vector*
g	: *Source Image*
S	: *Destination Image*
f	: *Interpolation*
f^*	: *Destination Image in Blending Area*
Ω	: *Blending Area*
$\partial\Omega$	: *Boundary of Blending Area*



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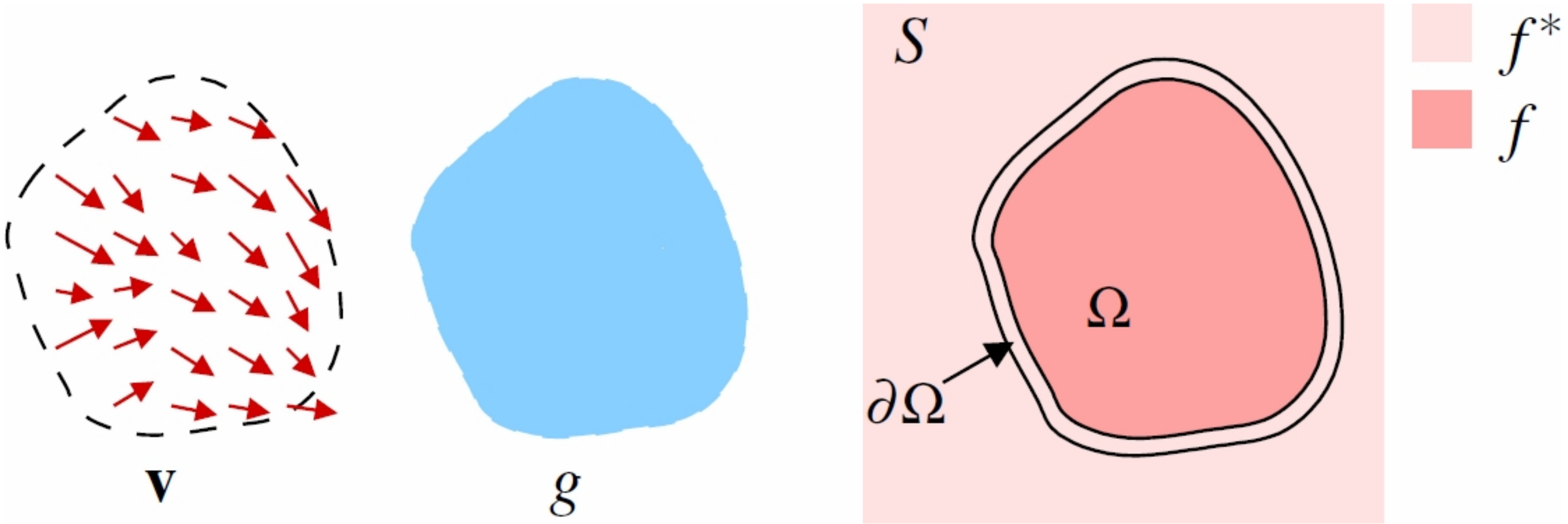

$$\min_f \iint_\Omega |\nabla f - v|^2 \quad \text{s.t. } f\big|_{\partial\Omega} = f^*\big|_{\partial\Omega}$$

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Boundary

The value on the boundary of the blending area must be the same.

Difference between gradient of interpolation and guidance vector

v : *Guidance Vector*
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Boundary

We want to minimize the difference.

The value on the boundary of the blending area must be the same.

∇ • Gradient Operator

Difference between gradient of interpolation and guidance vector

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∇ · Gradient Operator

We use gradient of source image as guidance vector : $v = \nabla g$

This is an optimization problem.

Difference between gradient of interpolation and guidance vector

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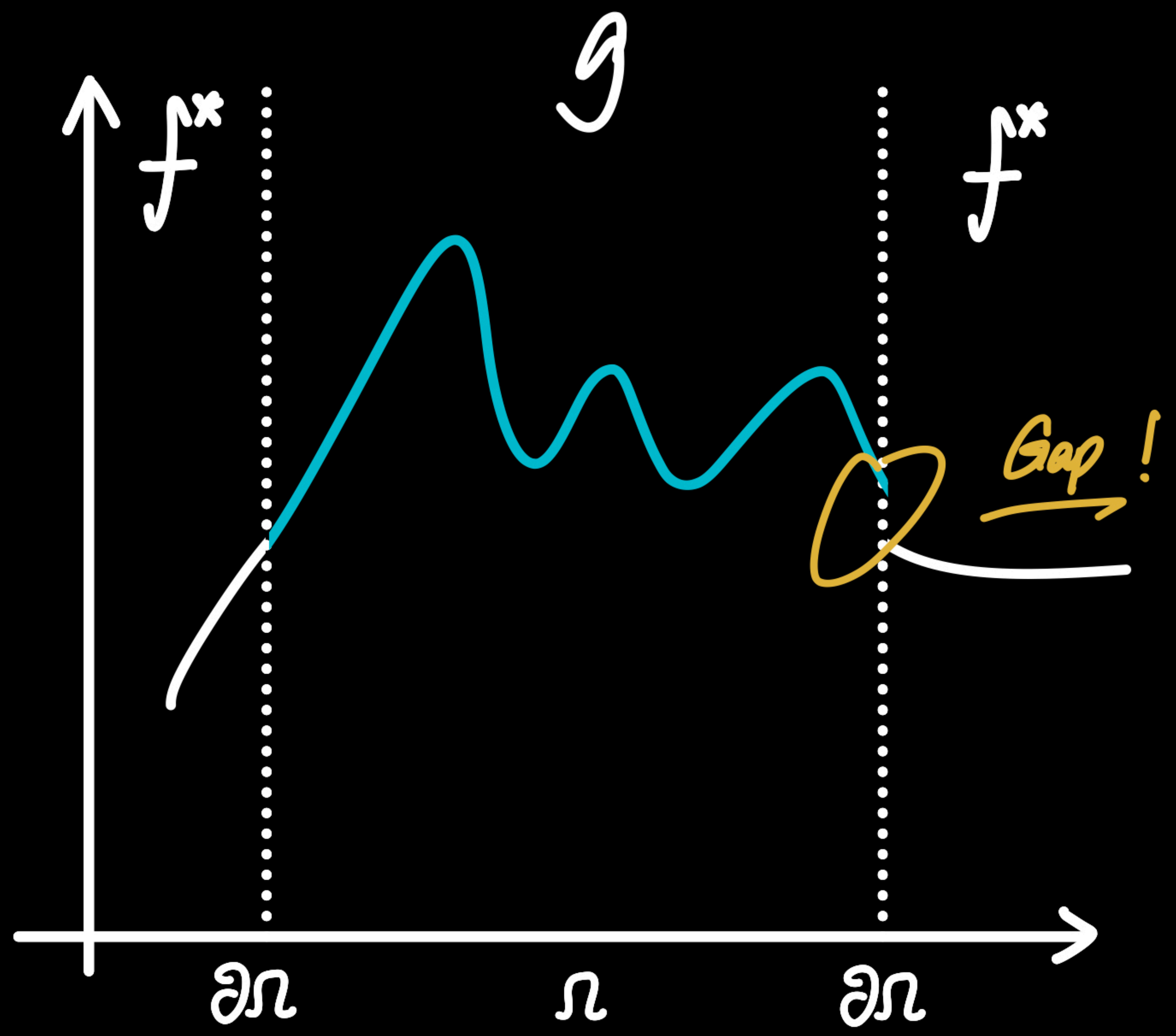
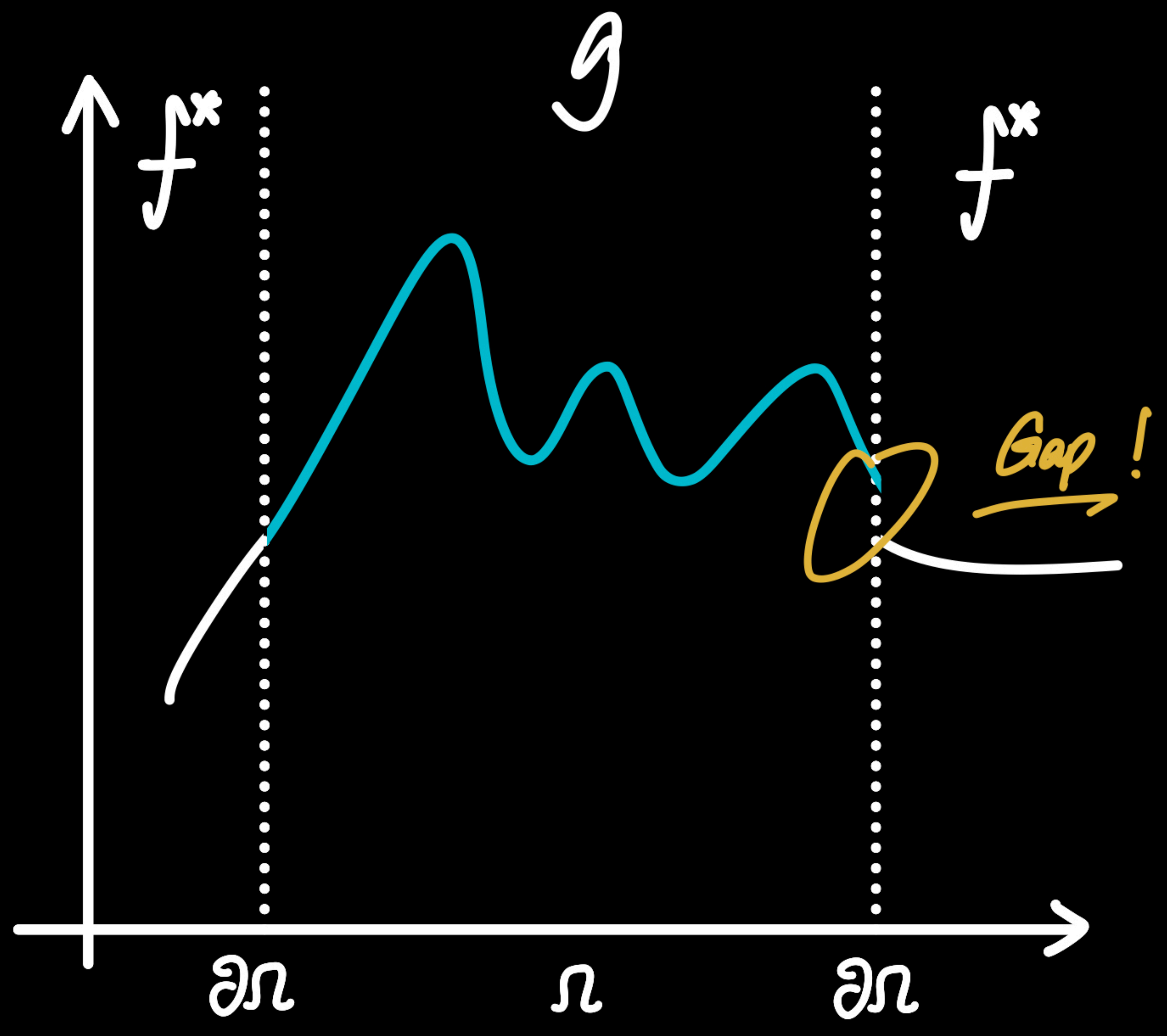
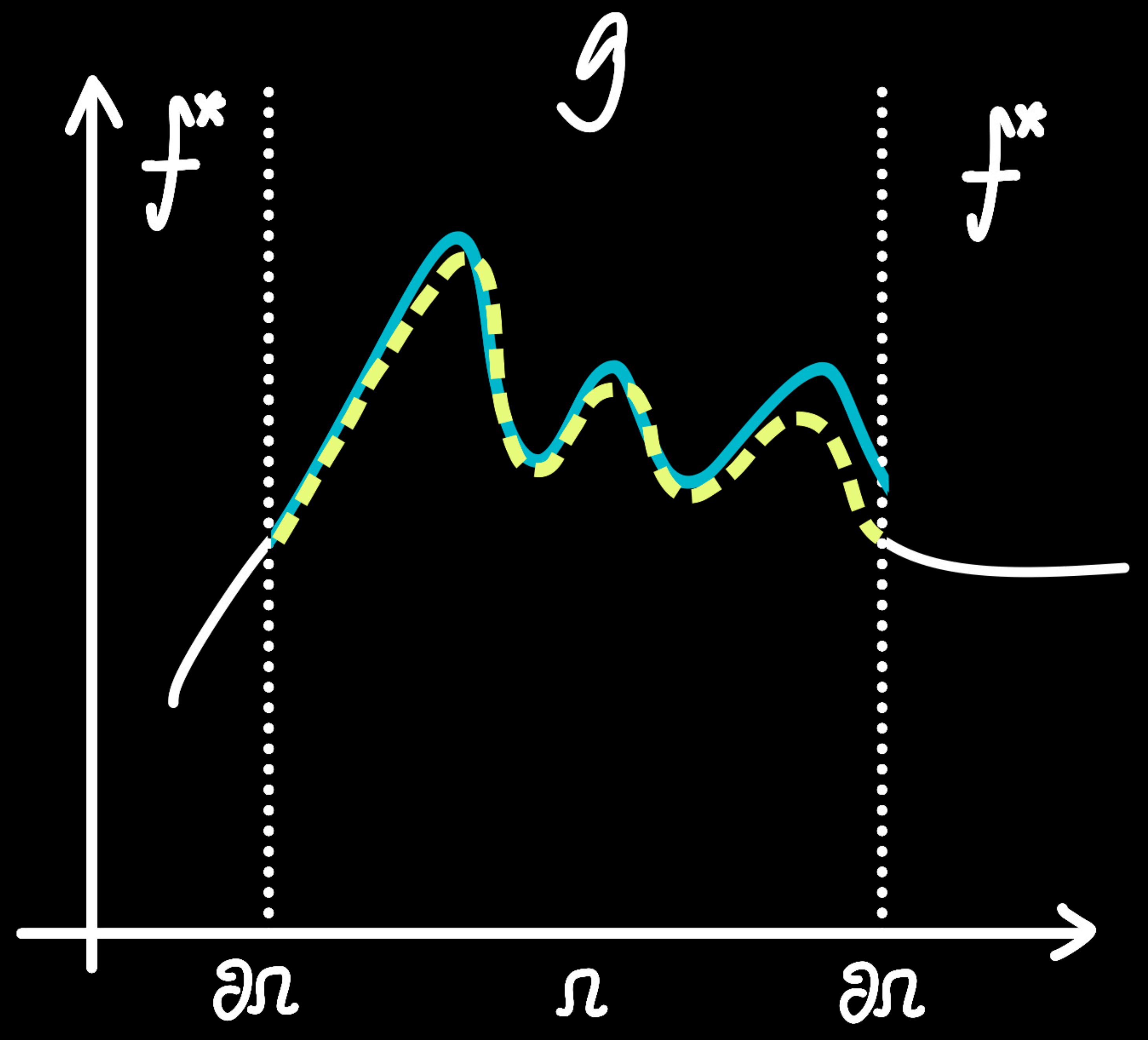
This constraint is difficult to meet.



$$\min_{f} \iint_{\Omega} |\nabla f - v|^2 \quad \text{s.t. } f\Big|_{\partial\Omega} = f^*\Big|_{\partial\Omega}$$

This constraint is difficult to meet. So we have to optimize the interpolation f.





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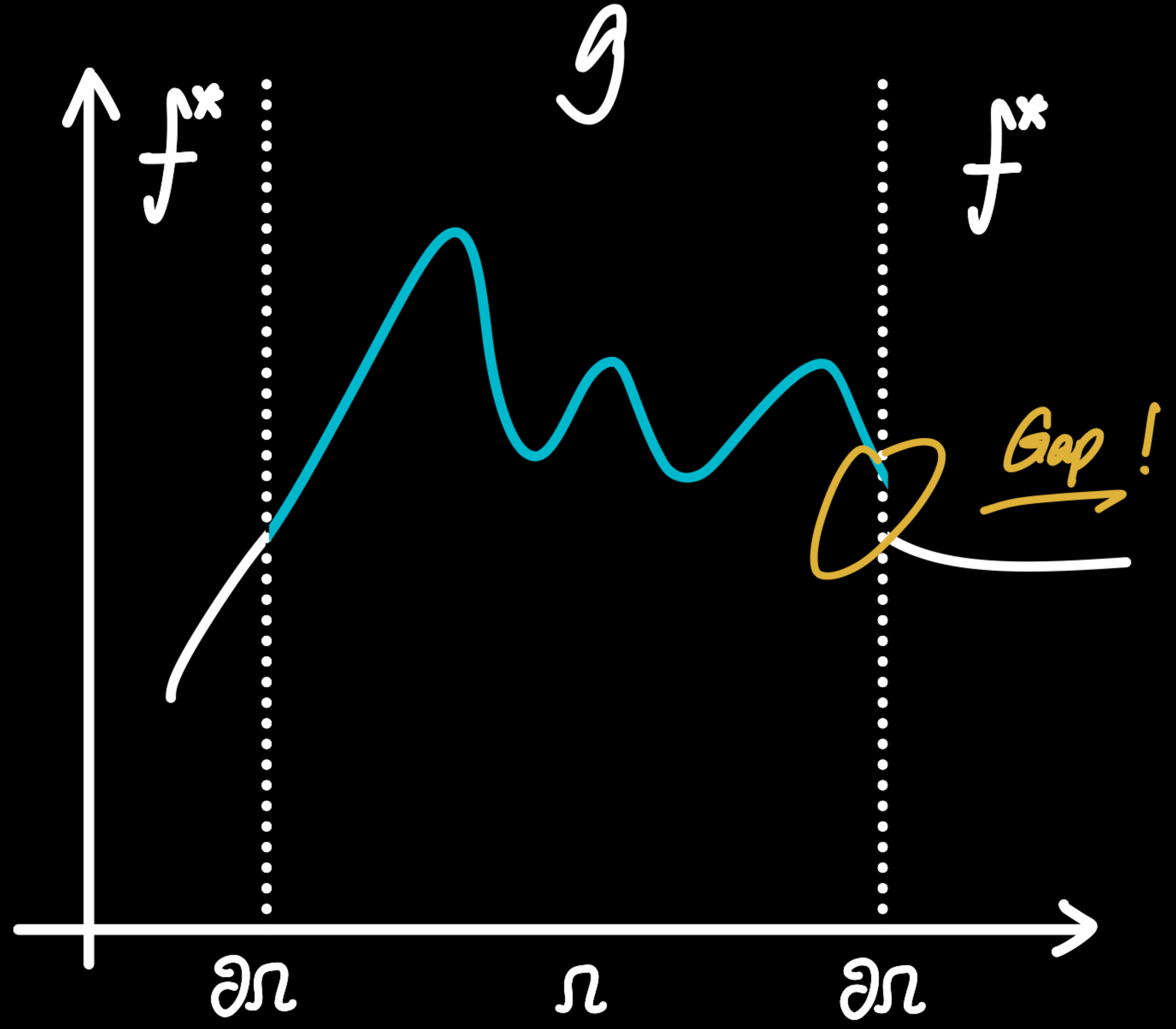

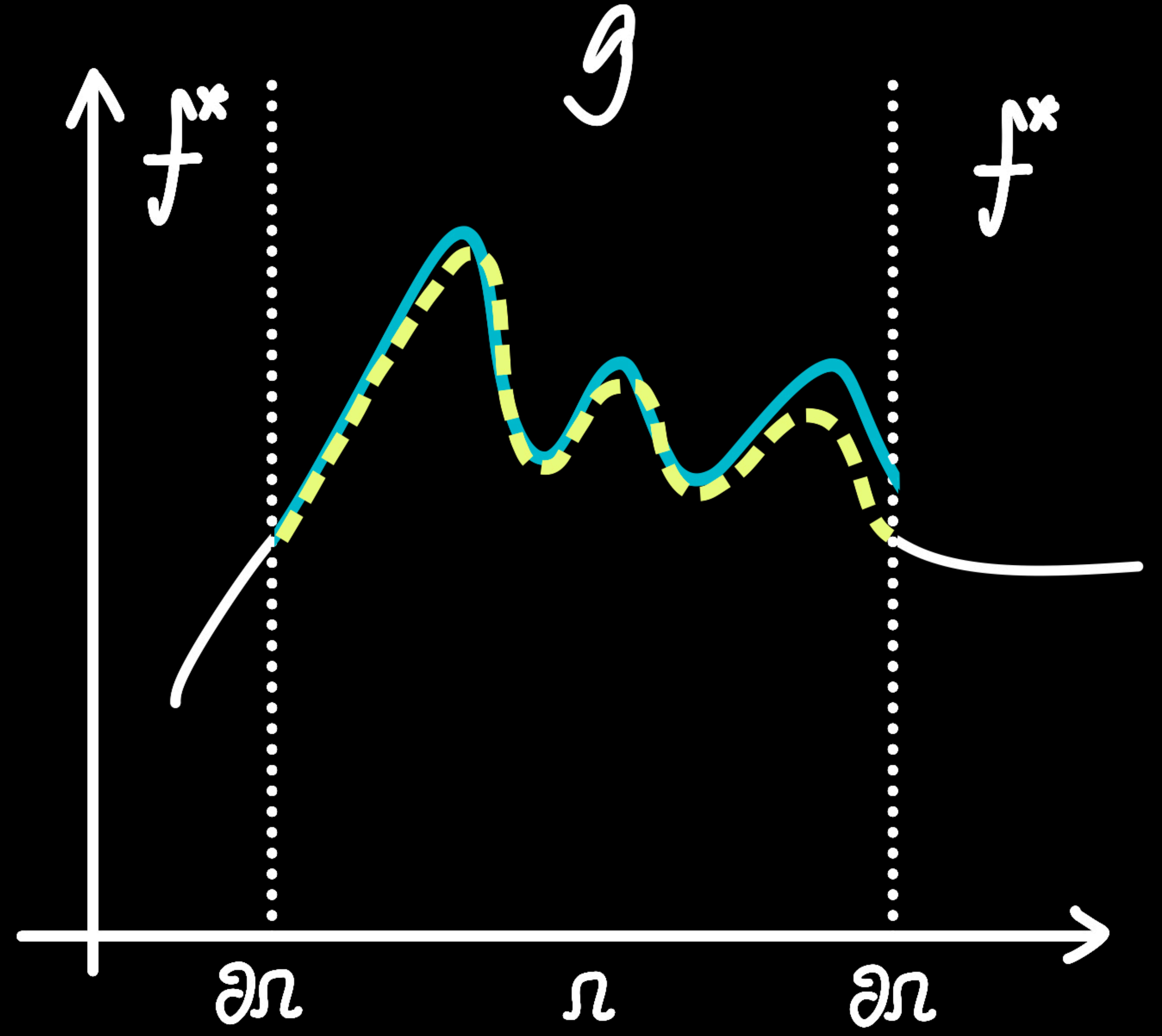

Luckily, the mathematicians have solved this optimization problem for us.

Poisson Equation

$$\min_f \iint_\Omega |\nabla f - v|^2 \quad \text{s.t. } f\big|_{\partial\Omega} = f^*\big|_{\partial\Omega}$$

The solution to this optimization problem is the solution to the following Poisson equation with Dirichlet boundary condition:

$$\Delta f = \text{div } v \text{ over } \Omega \text{ with } f\big|_{\partial\Omega} = f^*\big|_{\partial\Omega}$$

Gradient Blending is also called Poisson Blending.

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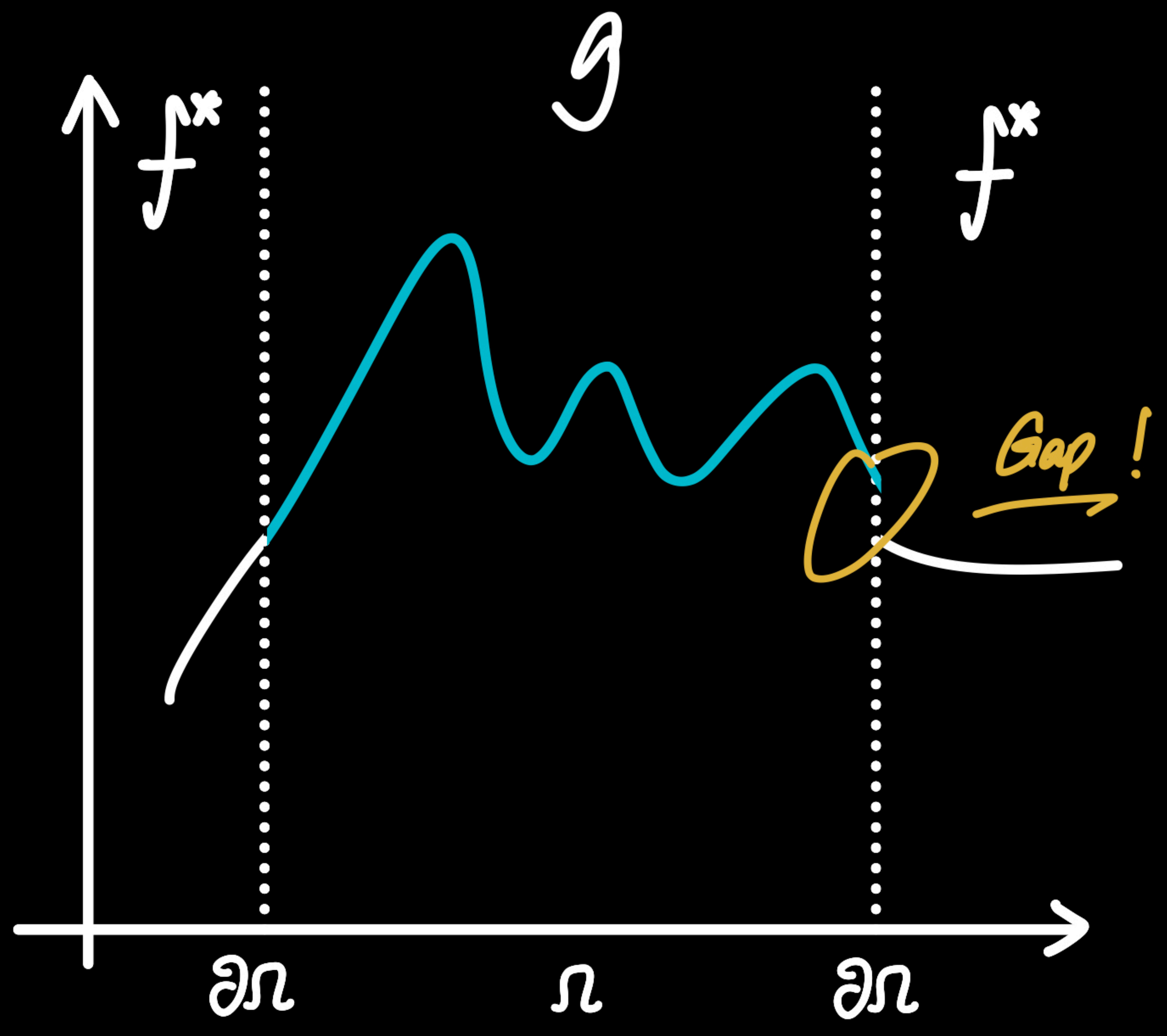

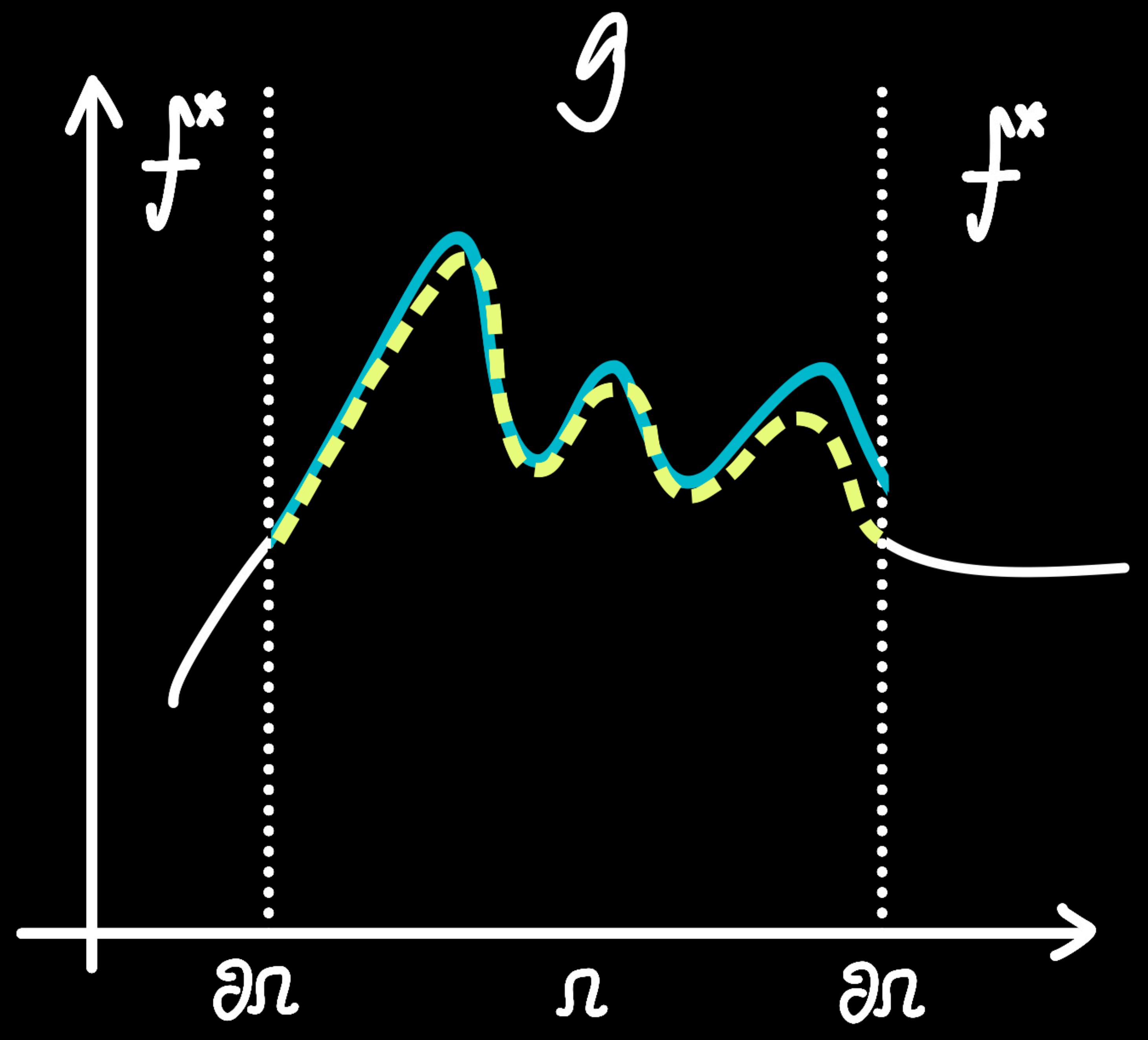

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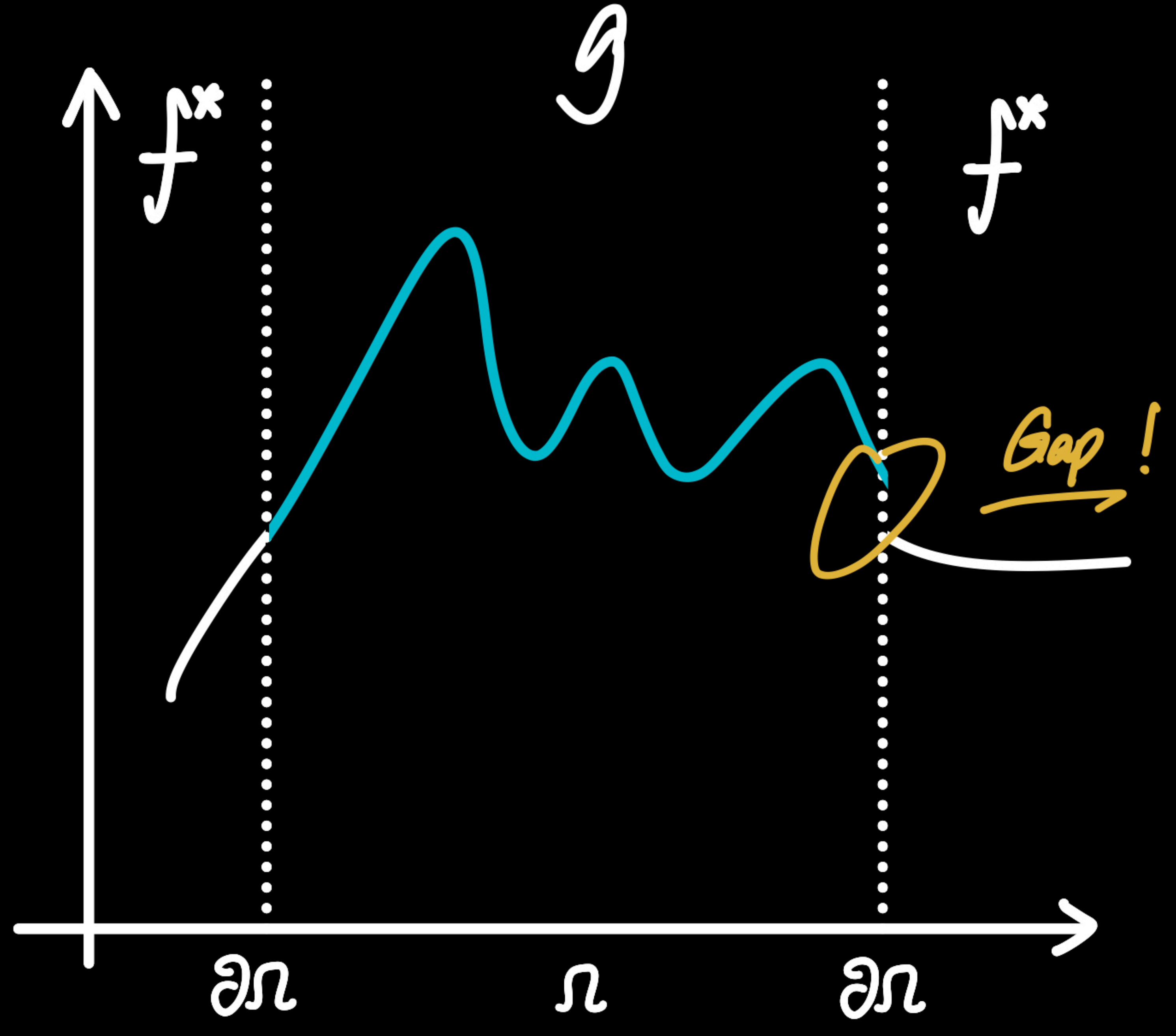

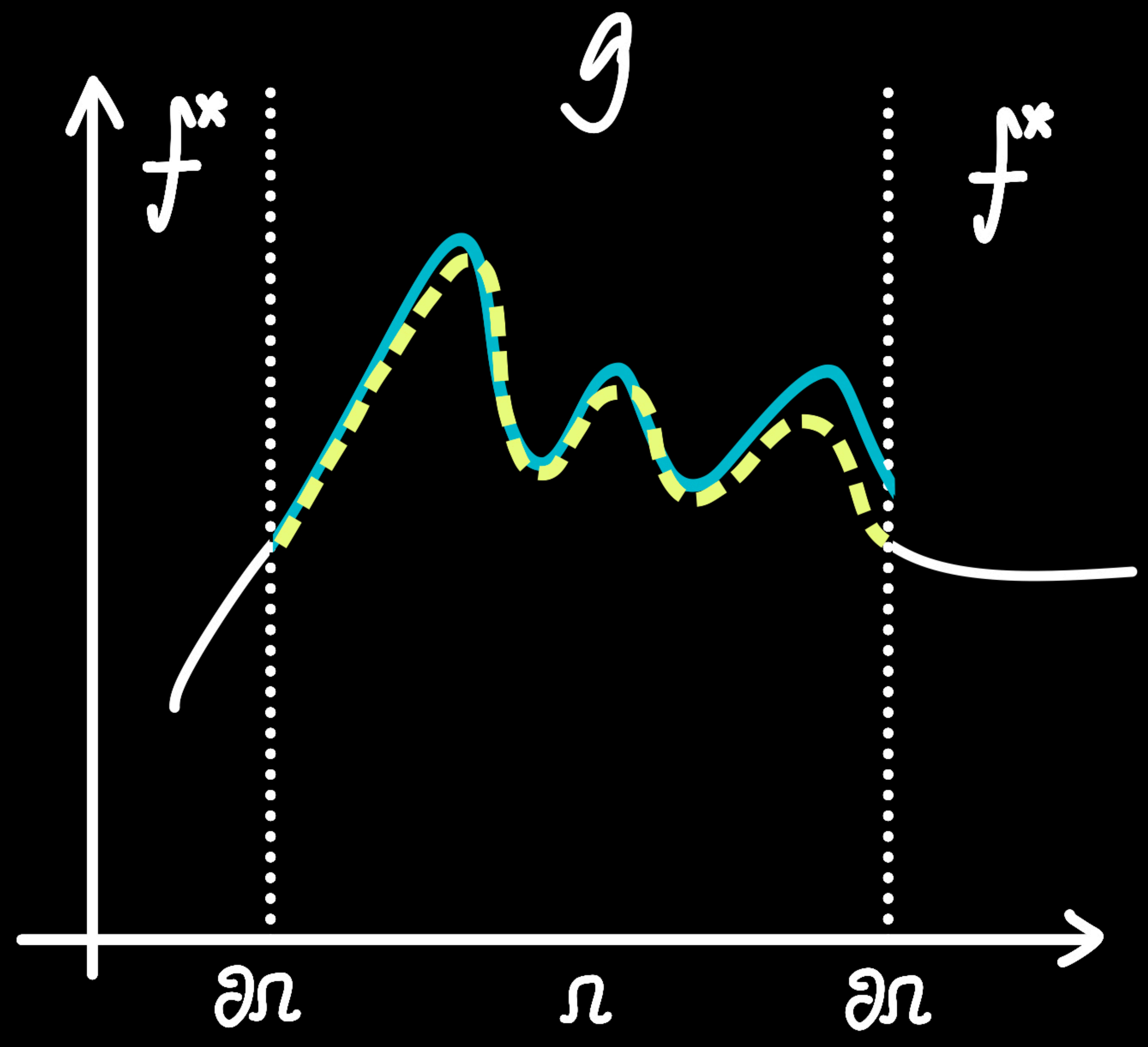

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$$\Delta f = \operatorname{div} v$$

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$$\Delta f = \operatorname{div} v \text{ over } \Omega \text{ with } f\Big|_{\partial\Omega} = f^*\Big|_{\partial\Omega}$$

Δ : Laplacian Operator

$$\Delta f = \underbrace{\operatorname{div}}_{\text{Divergence}}\ v$$

Divergence

We use gradient of source image as guidance vector : $v = \nabla g$

Poisson Equation

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∇ · Gradient Operator $= [\frac{\partial}{\partial x}, \frac{\partial}{\partial y}]$

Δ · Laplacian Operator $= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$

Δ · Laplacian Operator

$$\Delta f = \text{div } v$$

Divergence

$$\text{div } \vec{u} = \nabla \vec{u}$$

$$\text{div } v = \nabla^2 g = \Delta g$$

We use gradient of source image as guidance vector : $v = \nabla g$

Poisson Equation

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The solution to this optimization problem is the solution to the following Poisson equation with Dirichlet boundary condition:

$$\Delta f = \operatorname{div} v \text{ over } \Omega \text{ with } f\big|_{\partial\Omega} = f^*\big|_{\partial\Omega}$$

So we only need to solve this :

$$\Delta f = \Delta g \text{ over } \Omega \text{ with } f\big|_{\partial\Omega} = f^*\big|_{\partial\Omega}$$

Δ • Laplacian Operator

$$\Delta f = \operatorname{div} v$$

Divergence

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We use gradient of source image as guidance vector : $v = \nabla g$

$\Delta f = \Delta g$ over Ω with $f\big|_{\partial\Omega} = f^*\big|_{\partial\Omega}$ The mathematical symbols may look scary. But please do not panic.

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HANG ON! PLEASE!

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HANG ON! PLEASE!

$$\frac{\partial}{\partial x} f(x,y) = \frac{f(x+h,y) - f(x,y)}{h}$$

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Everything in computer is discrete, which sometimes makes life much easier.

For an image f, the pixel value on (x, y) is $f(x, y)$.

$$\nabla_x f(x, y) = \frac{\partial}{\partial x} f(x, y) = f(x+1, y) - f(x, y) \qquad \Delta_x f(x, y) = \frac{\partial^2}{\partial x^2} f(x, y) = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

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For an image f, the pixel value on (x, y) is $f(x, y)$.

$$\nabla_x f(x,y) = \frac{\partial}{\partial x} f(x,y) = f(x+1,y) - f(x,y) \qquad \Delta_x f(x,y) = \frac{\partial^2}{\partial x^2} f(x,y) = f(x+1,y) + f(x-1,y) - 2f(x,y)$$

$\Delta f = \Delta g$ over Ω with $f\big|_{\partial\Omega} = f^*\big|_{\partial\Omega}$

The mathematical symbols may look scary. But please do not panic.

HANG ON! PLEASE!

∇ • Gradient Operator $= [\frac{\partial}{\partial x}, \frac{\partial}{\partial y}]$

$$\frac{\partial}{\partial x} f(x,y) = \frac{f(x+h,y) - f(x,y)}{h}$$

Δ • Laplacian Operator $= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$

$$\frac{\partial^2}{\partial x^2} f(x,y) = \frac{\frac{\partial}{\partial x} f(x,y) - \frac{\partial}{\partial x} f(x-h,y)}{h} = \frac{\frac{f(x+h)-f(x)}{h} - \frac{f(x)-f(x-h)}{h}}{h} = \frac{f(x+h)+f(x-h)-2f(x)}{h^2}$$

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Laplacian in discrete case:

$$\Delta f(x,y) = \Delta_x f(x,y) + \Delta_y f(x,y) = -4f(x,y) + f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)$$

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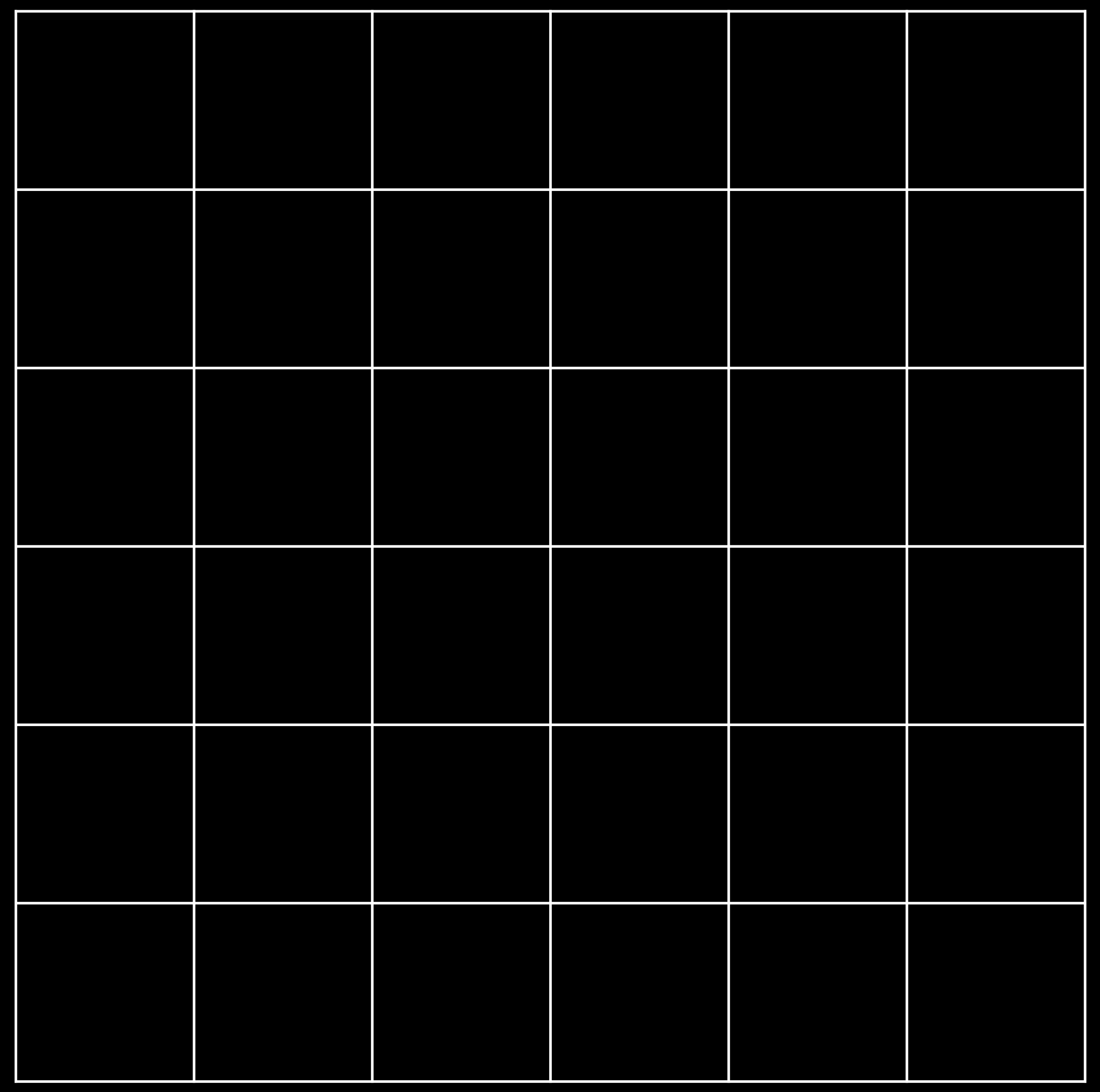
f



$\otimes$

	-1	
-1	4	-1
	-1	

Convolutional kernel for Laplacian Operator

$\Delta f = \Delta g$ over Ω with $f\big|_{\partial\Omega} = f^*\big|_{\partial\Omega}$

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Laplacian in discrete case:

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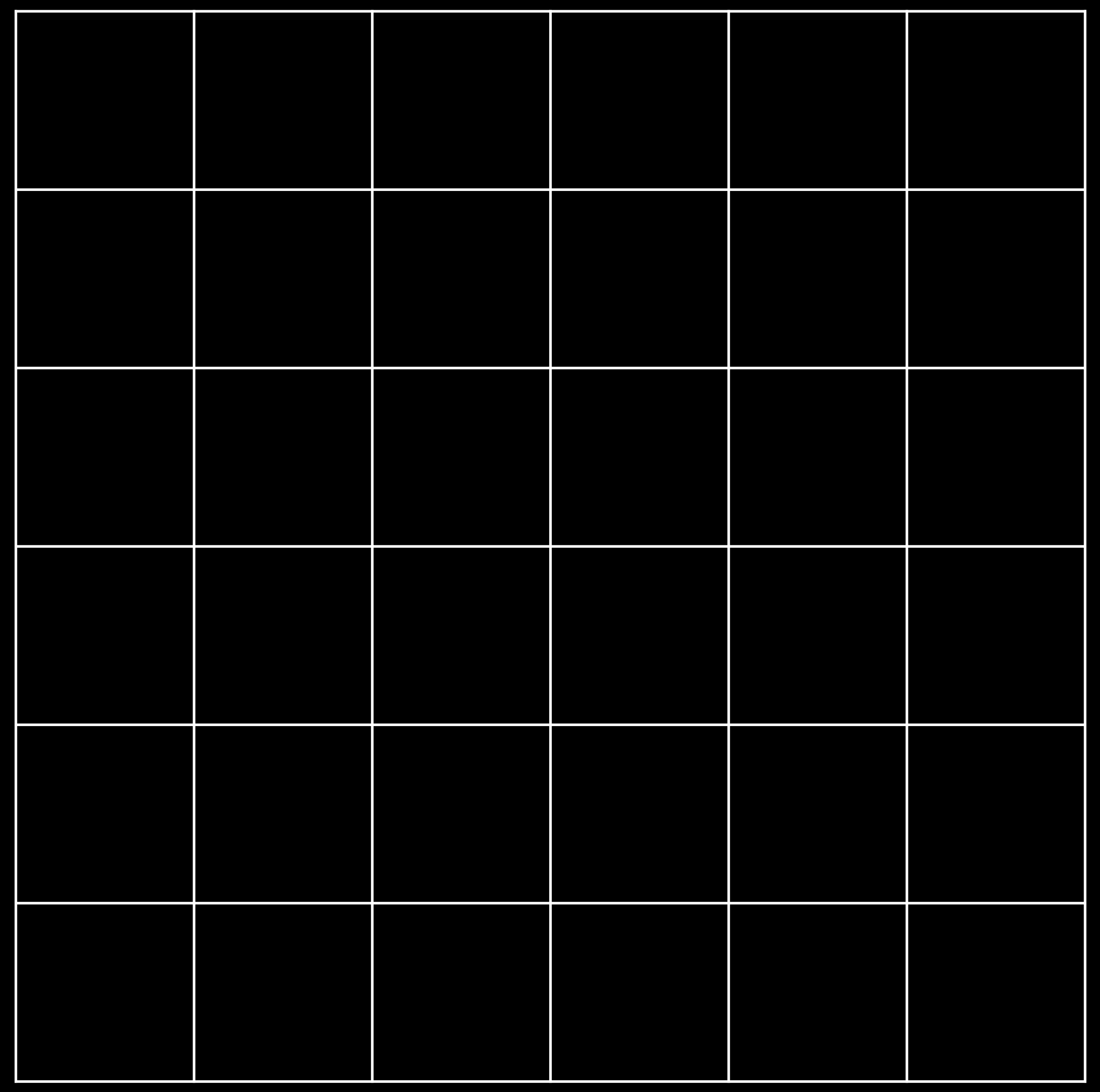
f



$\otimes$

	-1	
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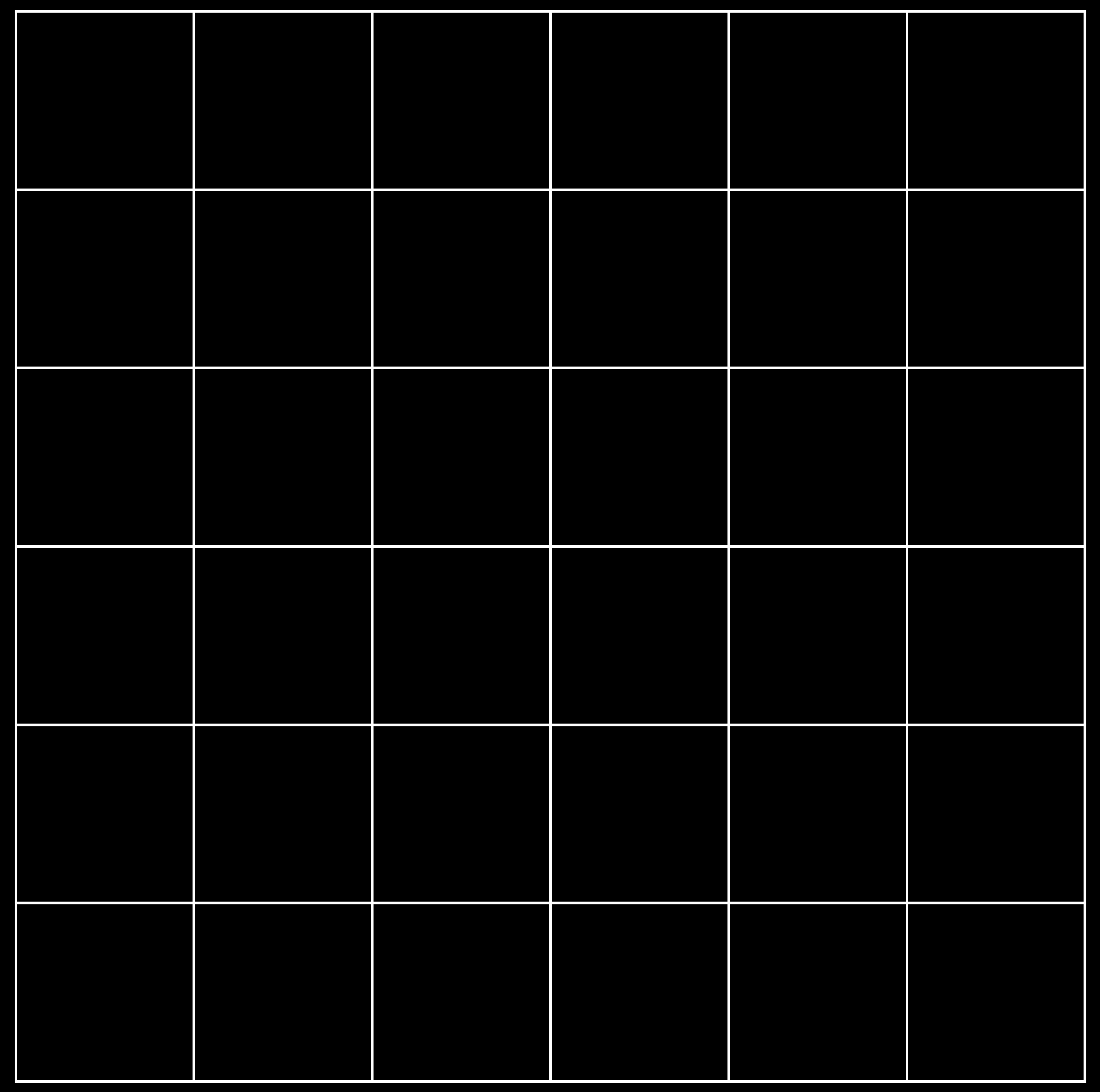
f



$$\otimes \quad \begin{array}{|c|c|c|} \hline & -1 & \\ \hline -1 & 4 & -1 \\ \hline & -1 & \\ \hline \end{array}$$

Convolutional kernel for Laplacian Operator

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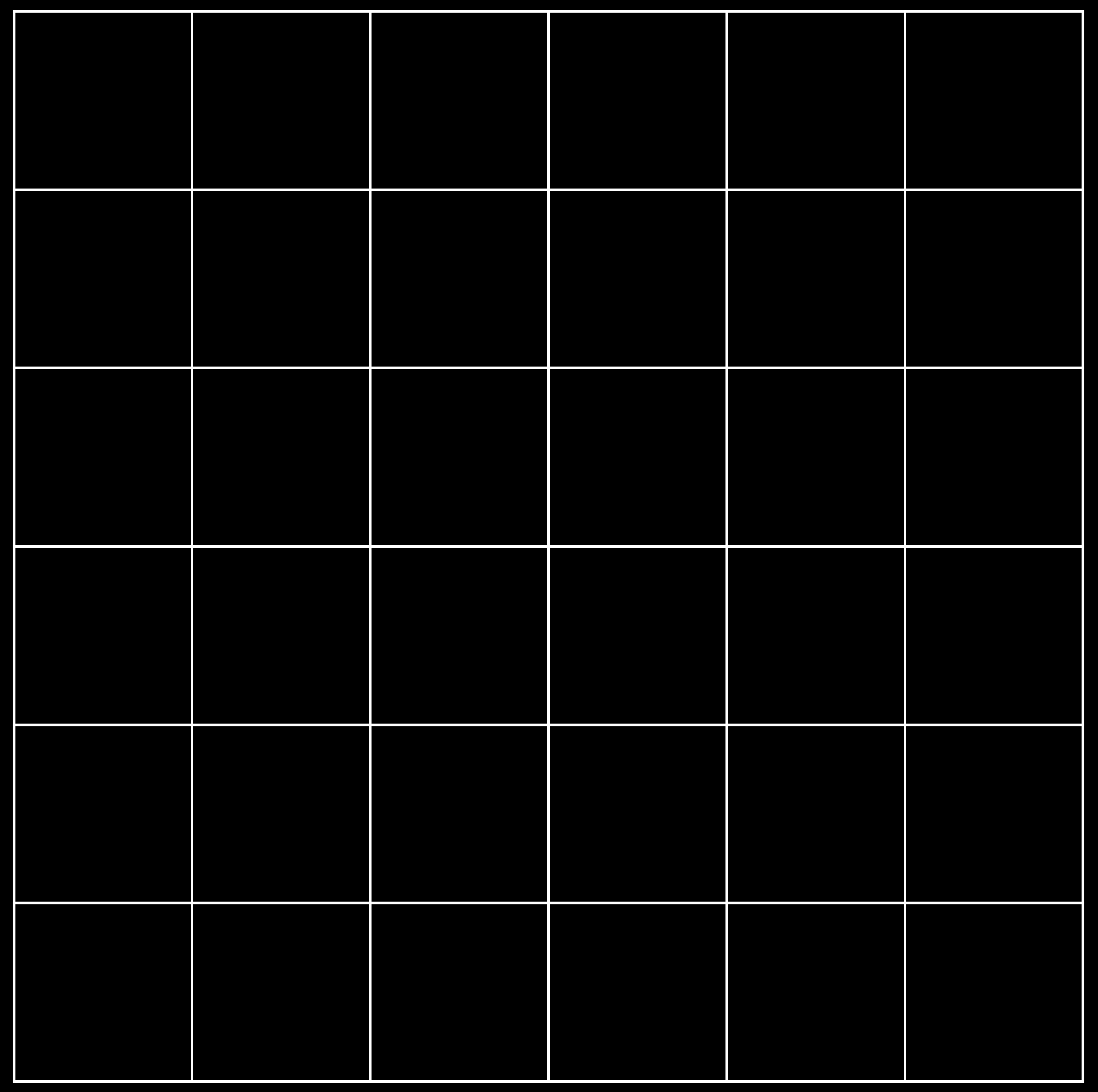
f



$$\otimes \quad \begin{array}{|c|c|c|} \hline & -1 & \\ \hline -1 & 4 & -1 \\ \hline & -1 & \\ \hline \end{array}$$

Convolutional kernel for Laplacian Operator

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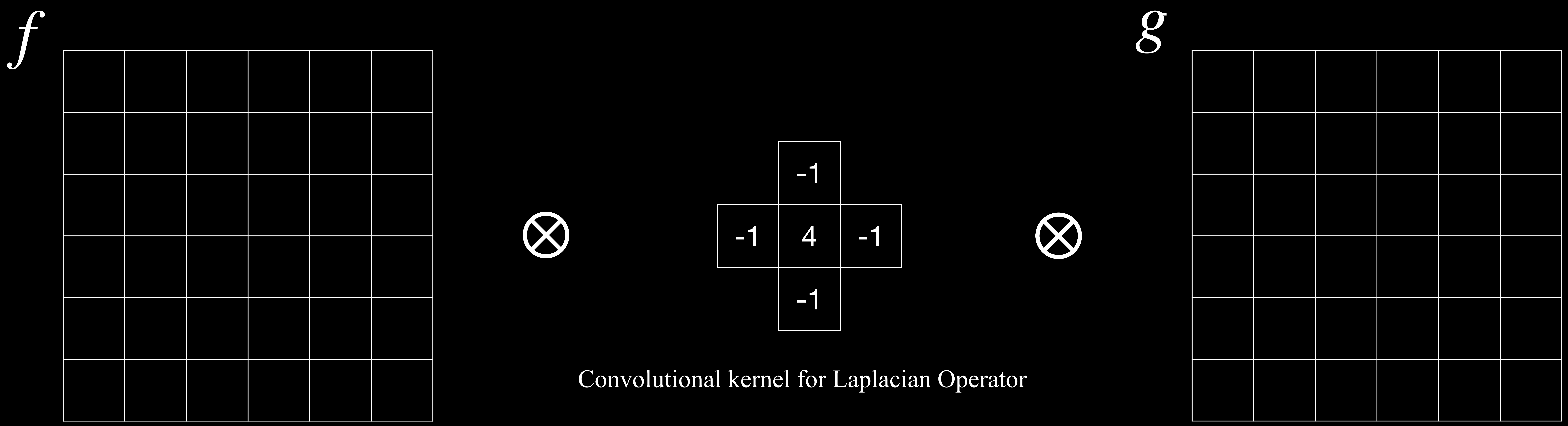

Convolutional kernel for Laplacian Operator

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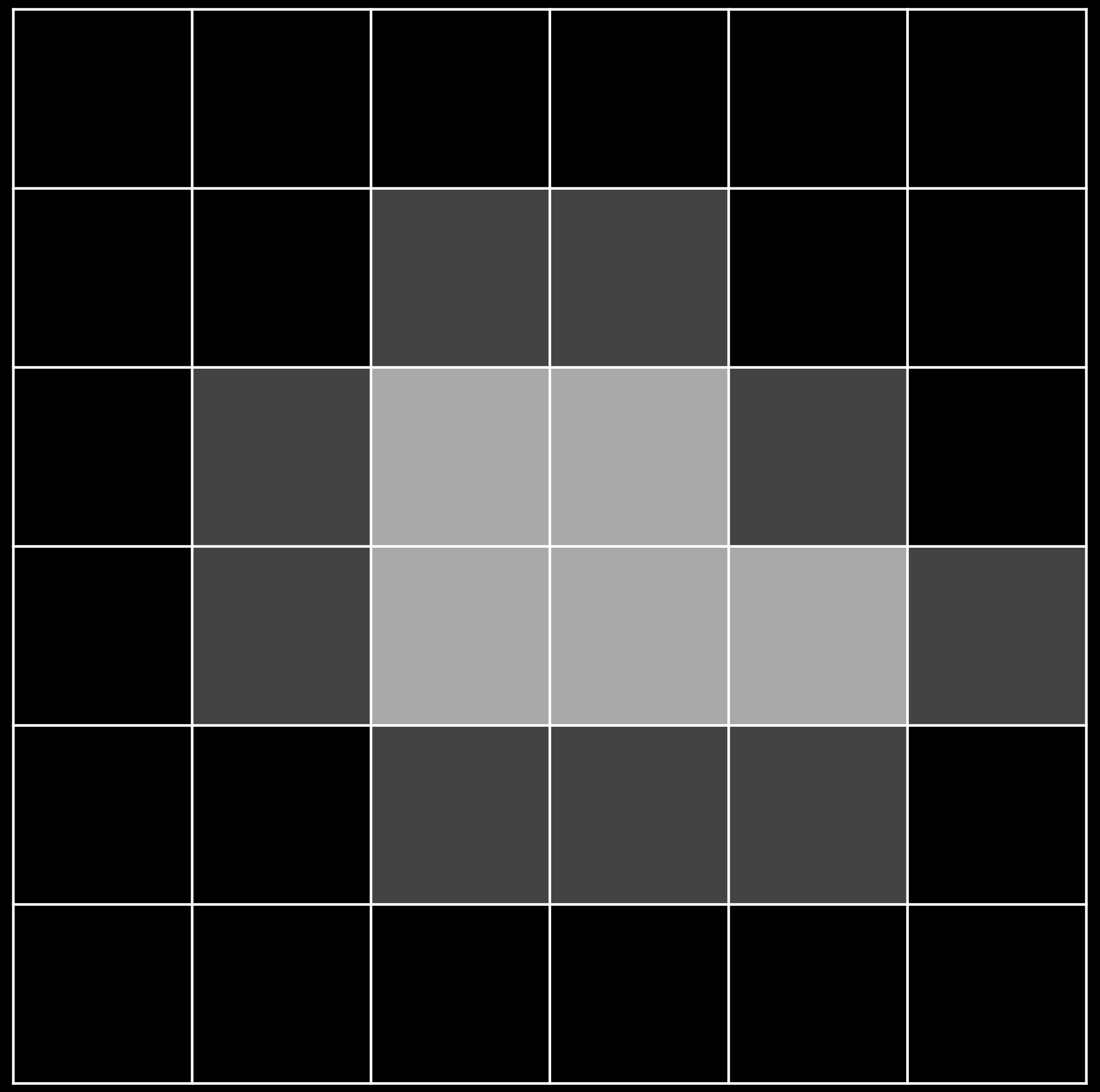
f



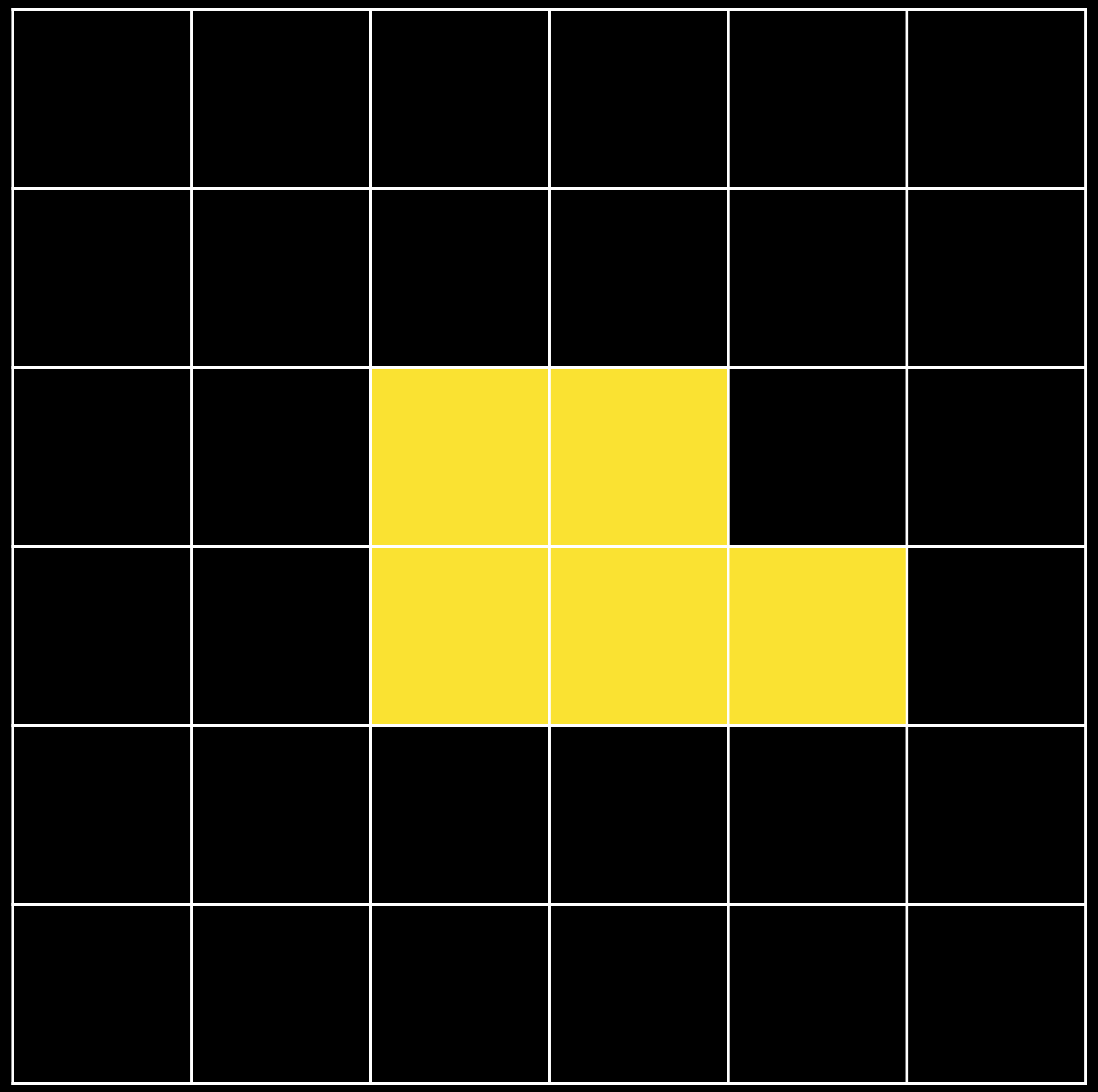
$\otimes$

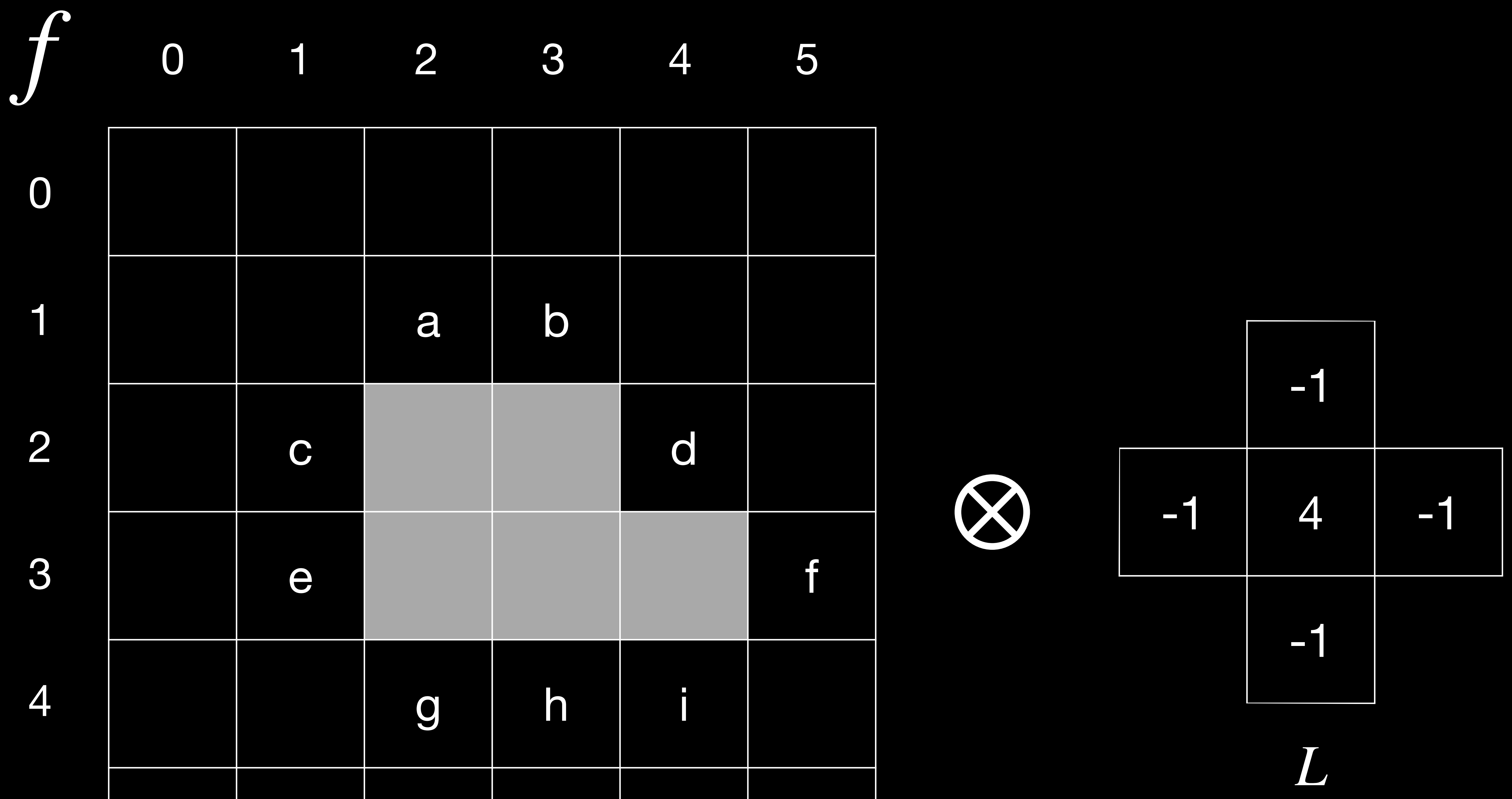
	-1	
-1	4	-1
	-1	

Convolutional kernel for Laplacian Operator

$\otimes$

g

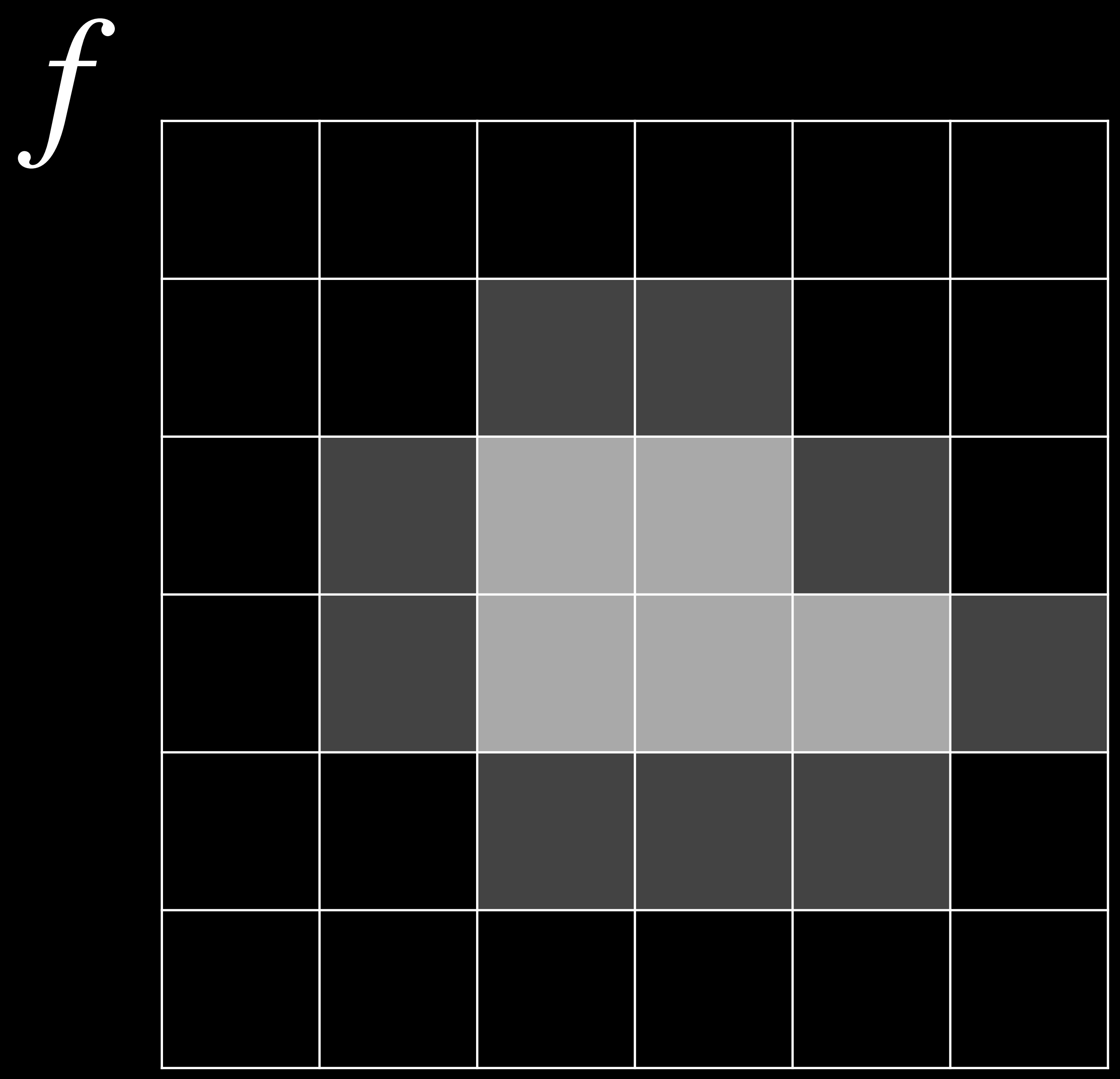


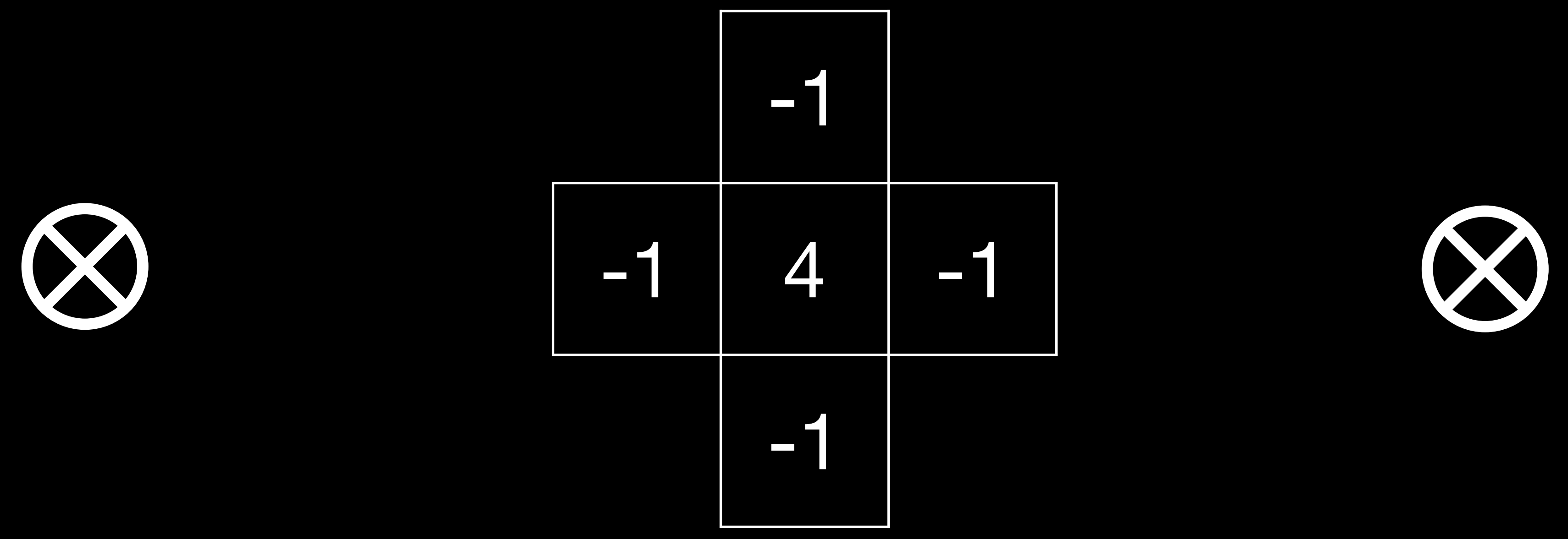


Convolutional kernel for Laplacian Operator

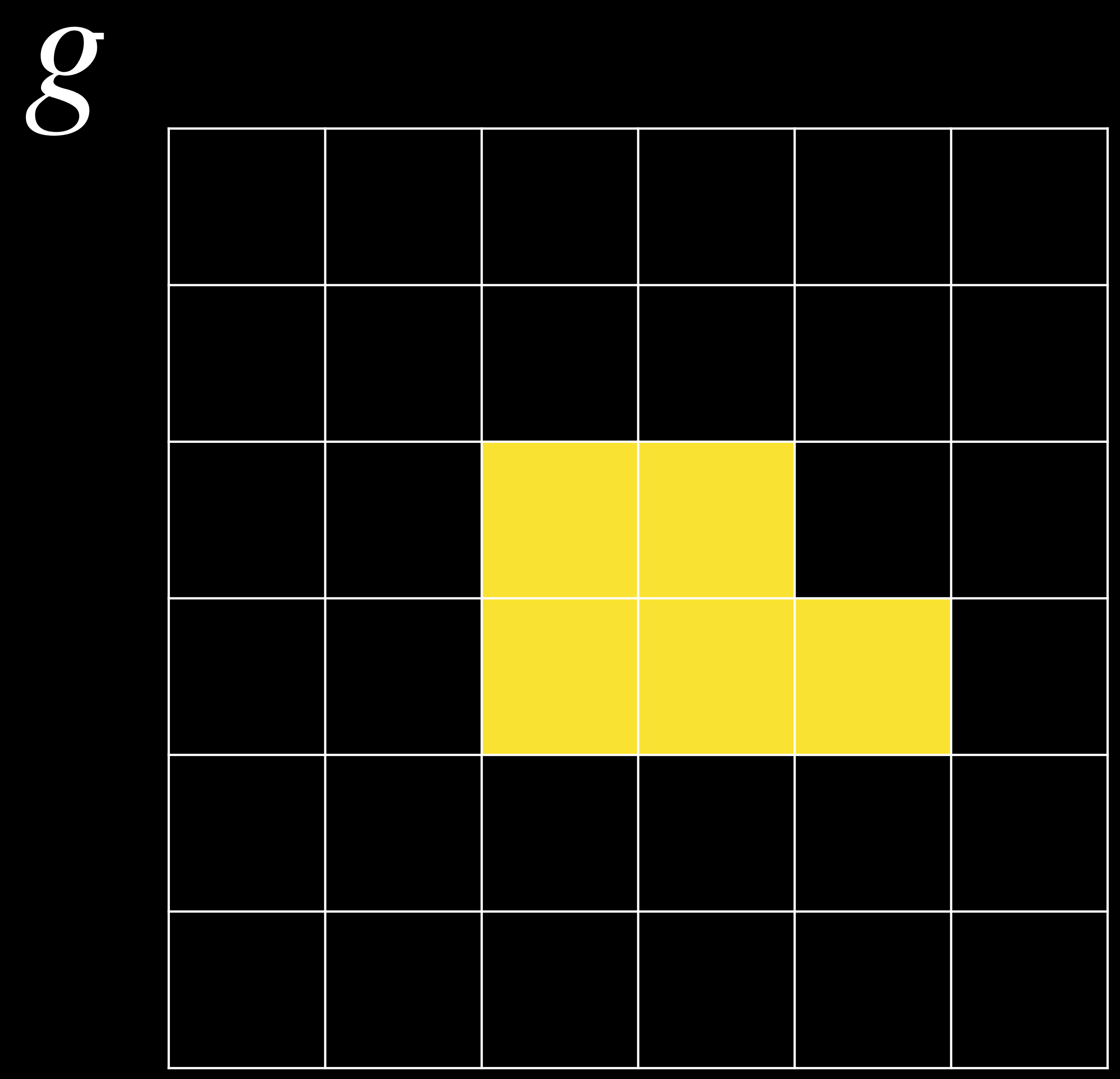
$$f(2,2) \otimes L = 4 \times f(2,2) - a - c - f(2,3) - f(3,2)$$
$$f(2,3) \otimes L = 4 \times f(2,3) - b - d - f(2,2) - f(3,3)$$
$$f(3,2) \otimes L = 4 \times f(3,2) - e - g - f(2,2) - f(3,3)$$
$$f(3,3) \otimes L = 4 \times f(3,3) - h - f(3,2) - f(2,3) - f(3,4)$$
$$f(3,4) \otimes L = 4 \times f(3,4) - d - f - i - f(3,3)$$

$\Delta f = \Delta g$ over Ω with $f\big|_{\partial\Omega} = f^*\big|_{\partial\Omega}$ The mathematical symbols may look scary. But please do not panic.





Convolutional kernel for Laplacian Operator



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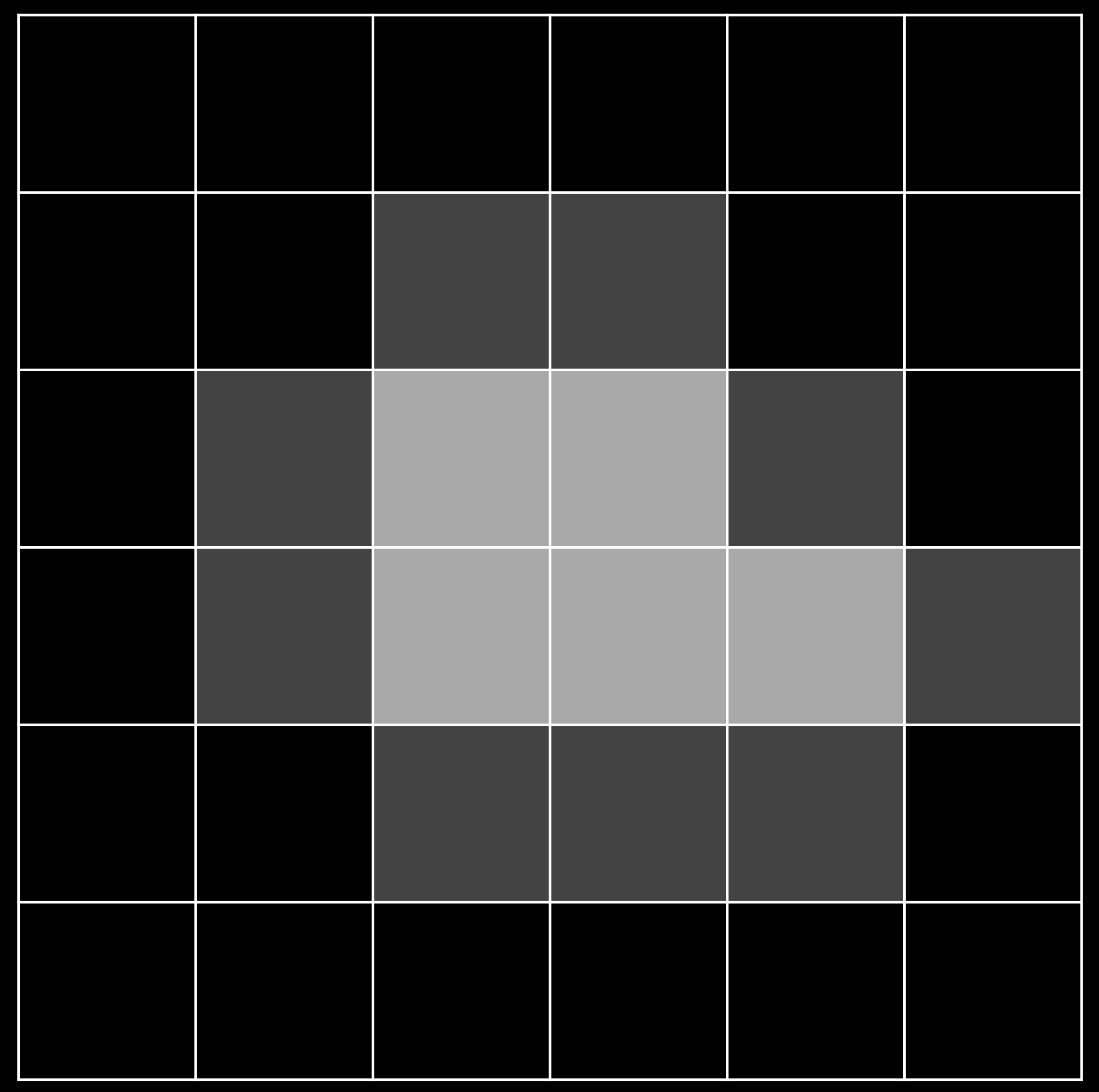
In discrete case, this equation can be rewritten in a more elegant way.

HANG ON! PLEASE!

f



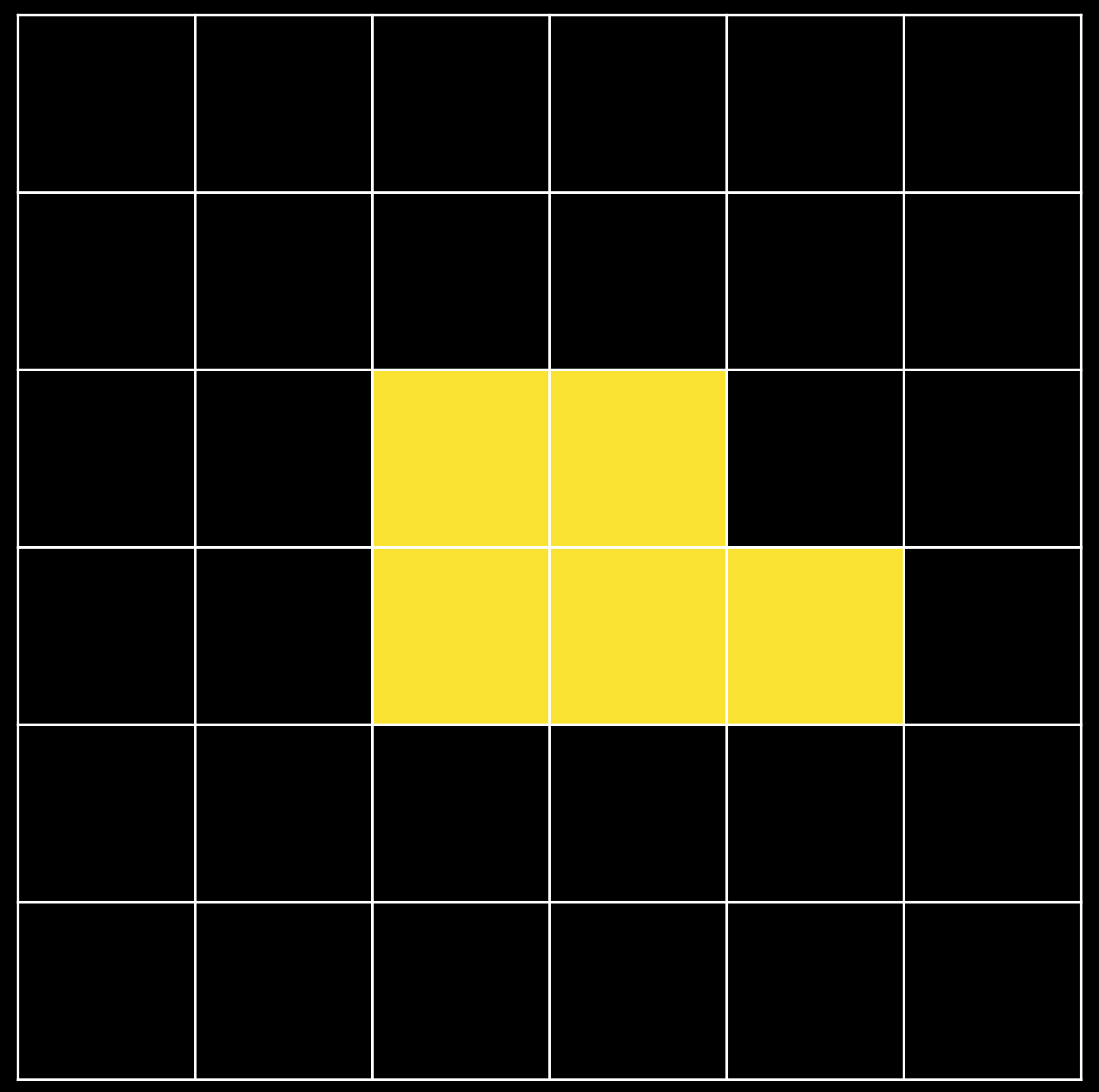
$\otimes$

	-1	
-1	4	-1
	-1	

Convolutional kernel for Laplacian Operator

$\otimes$

g



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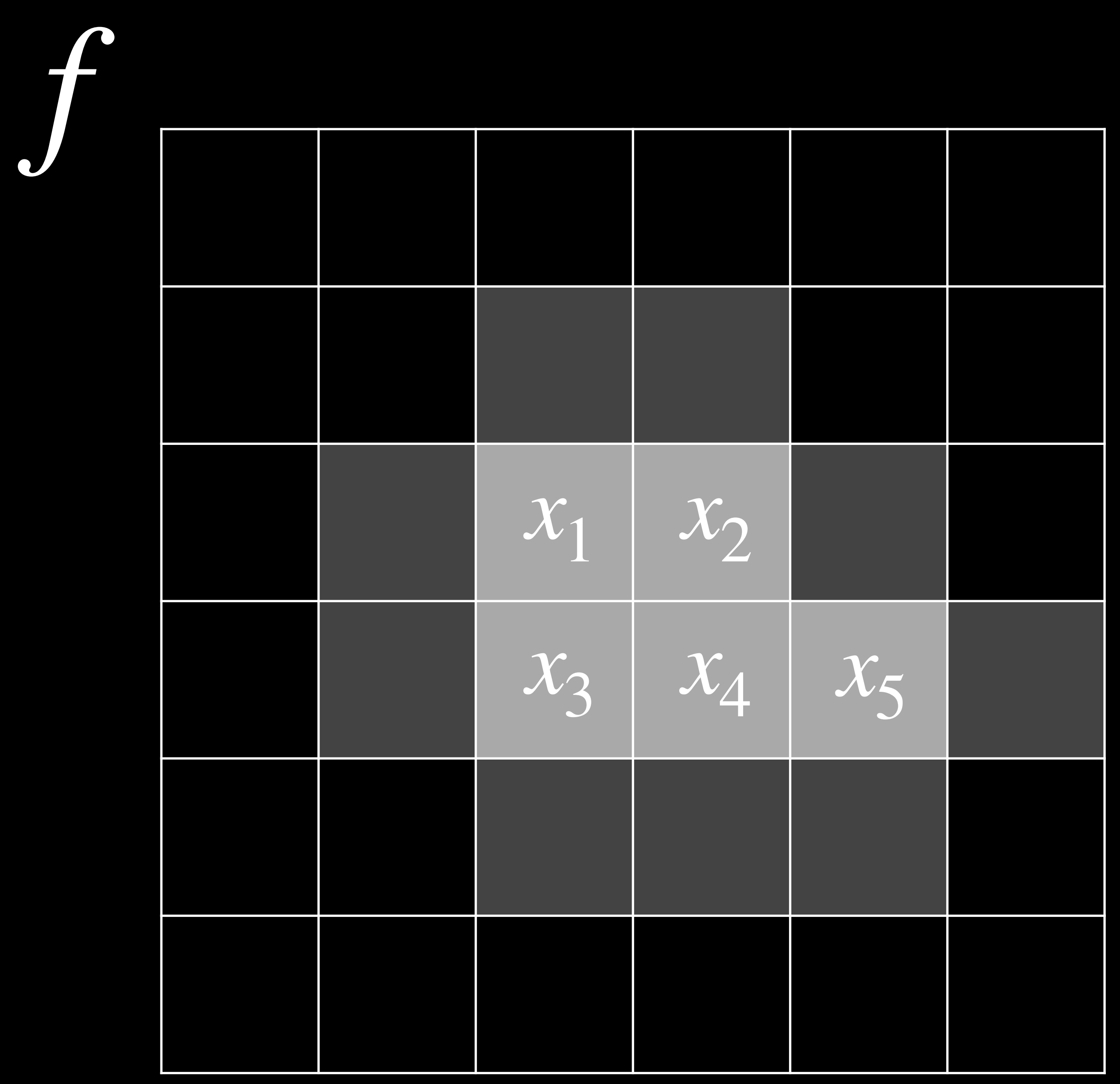
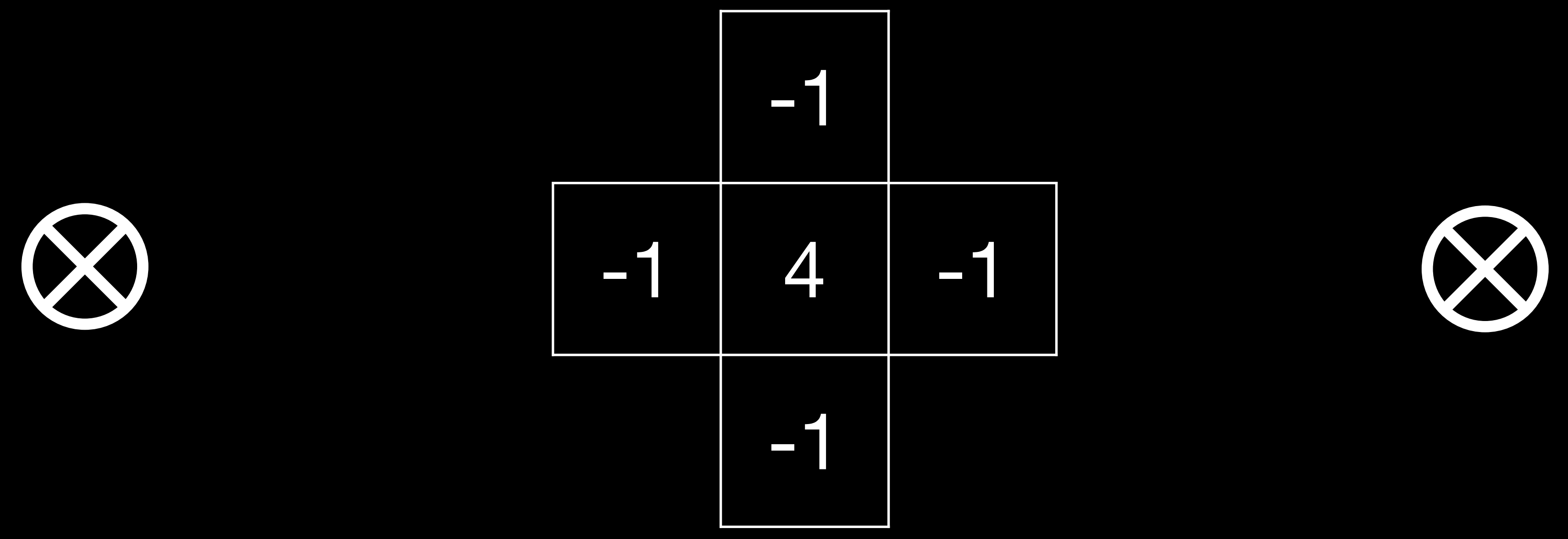
In discrete case, this equation can be rewritten in a more elegant way.

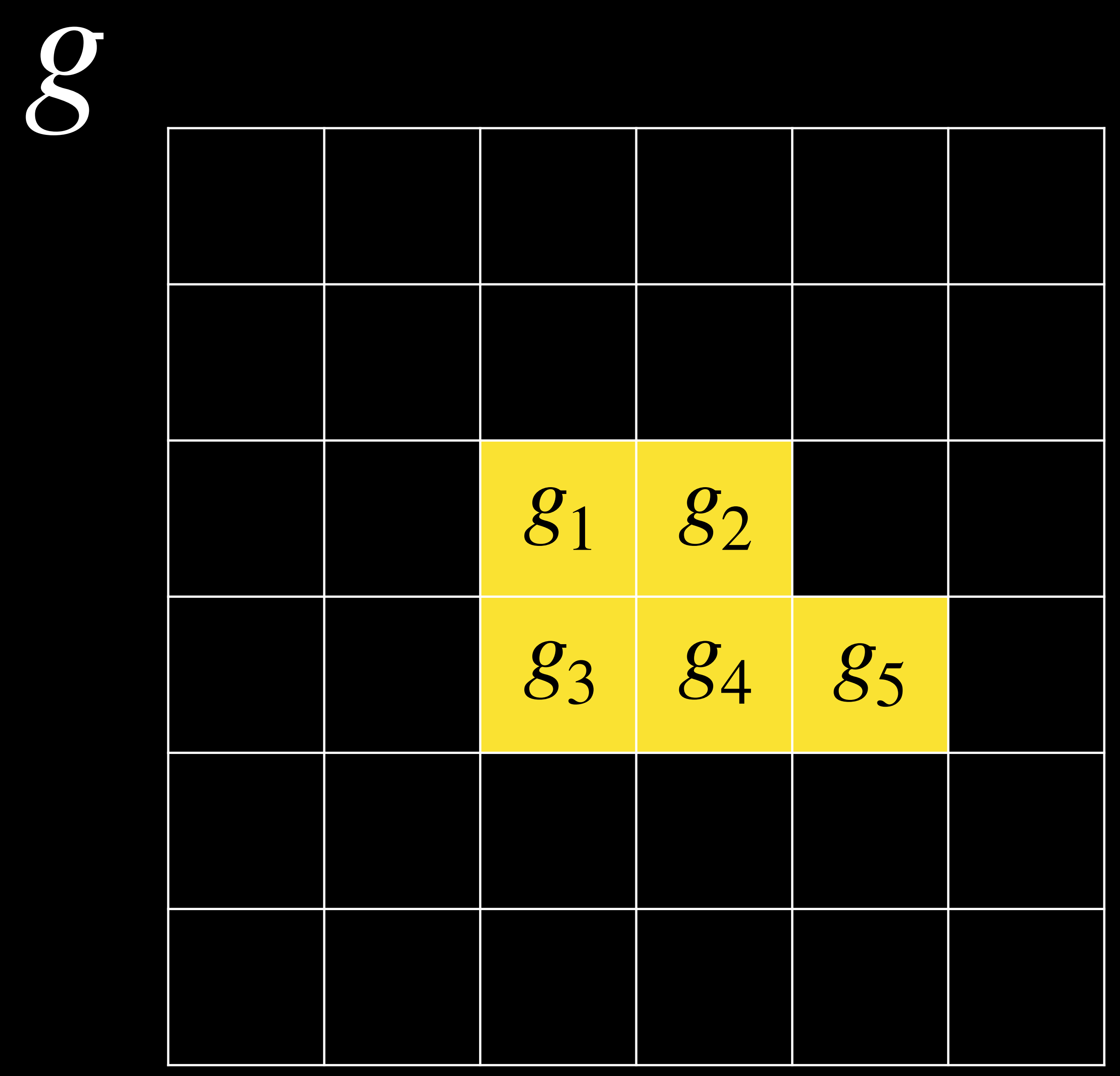
$$Ax = b \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \qquad b = \begin{bmatrix} \Delta g_1 \\ \Delta g_2 \\ \Delta g_3 \\ \Delta g_4 \\ \Delta g_5 \end{bmatrix}$$

HANG ON! PLEASE!





Convolutional kernel for Laplacian Operator



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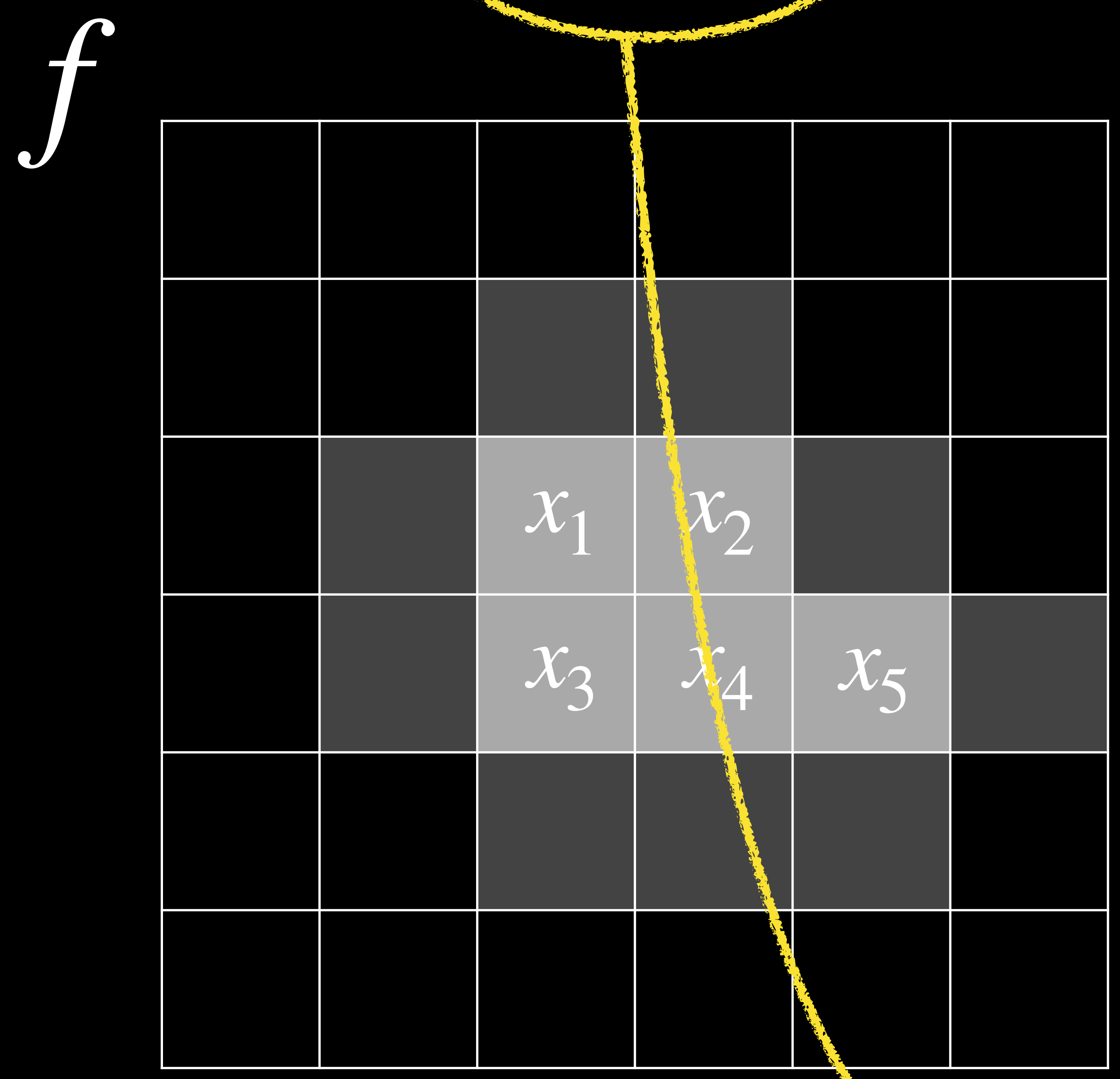
HANG ON! PLEASE!

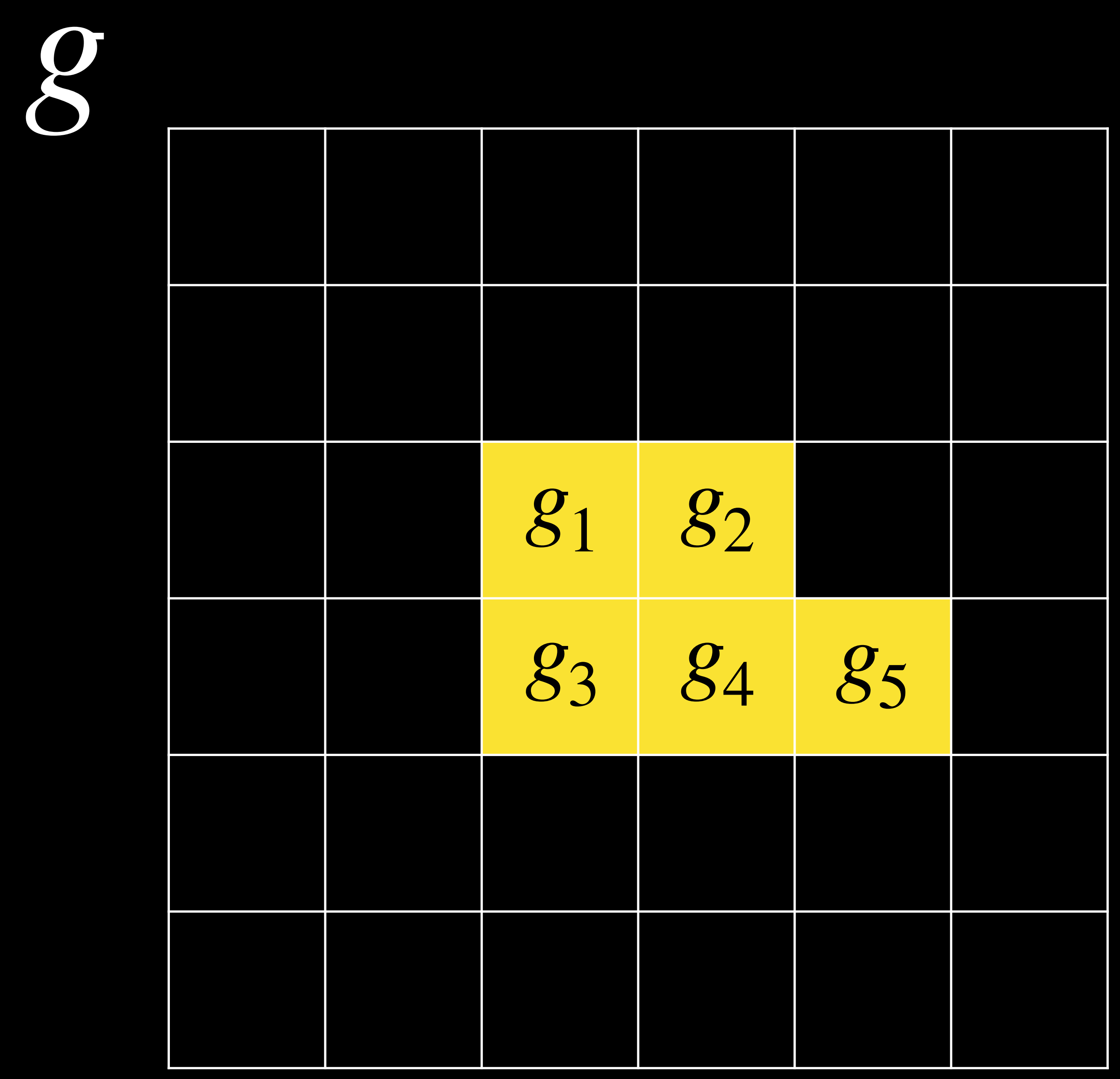


$\otimes$

	-1	
-1	4	-1
	-1	

Convolutional kernel for Laplacian Operator

$\otimes$



We have tons of tools to solve linear equation.

$$\Delta f = \Delta g \ \text{ over } \ \Omega \ \text{ with } \ f\big|_{\partial\Omega} = f^*\big|_{\partial\Omega}$$

f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	4	0	3	9
2	0	5	?	?	2	1
3	5	2	?	?	?	2
4	1	3	0	2	3	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
3	2	1	4	1	2	6
4	5	3	5	2	4	3
5	6	4	3	1	5	7

$$\Delta f = \Delta g \quad \begin{cases} \Delta f(2,2) = 4f(2,2) - f(1,2) - f(2,1) - f(3,2) - f(2,3) = \Delta g(2,2) = 4*3-4-2-5-4 = -3 \\ \Delta f(2,3) = 4f(2,3) - f(1,3) - f(2,2) - f(3,3) - f(2,4) = \Delta g(2,3) = 4*5-2-3-3-1 = 11 \\ \Delta f(3,2) = 4f(3,2) - f(2,2) - f(3,1) - f(3,3) - f(4,2) = \Delta g(3,2) = 4*4-3-1-1-5 = 6 \\ \Delta f(3,3) = 4f(3,3) - f(2,3) - f(3,2) - f(3,4) - f(4,3) = \Delta g(3,3) = 4*1-5-4-2-2 = -9 \\ \Delta f(3,4) = 4f(3,4) - f(2,4) - f(3,3) - f(3,5) - f(4,4) = \Delta g(3,4) = 4*2-1-3-6-4 = -6 \end{cases}$$

$$f\big|_{\partial\Omega} = f^*\big|_{\partial\Omega} \qquad f(1,2)=4,\ f(1,3)=0,\ f(2,1)=5,\ f(3,1)=2,\ f(4,2)=0,\ f(4,3)=2,\ f(4,4)=3,\ f(2,4)=2,\ f(3,5)=2$$

$\Delta f = \Delta g$ over Ω with $f\big|_{\partial\Omega} = f^*\big|_{\partial\Omega}$

f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	4	0	3	9
2	0	5	?	?	2	1
3	5	2	?	?	?	2
4	1	3	0	2	3	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
3	2	1	4	1	2	6
4	5	3	5	2	4	3
5	6	4	3	1	5	7

HANG ON! PLEASE!

$$\Delta f = \Delta g \quad \begin{cases} \Delta f(2,2) = 4f(2,2) - f(1,2) - f(2,1) - f(3,2) - f(2,3) = \Delta g(2,2) = 4*3-4-2-5-4 = -3 \\ \Delta f(2,3) = 4f(2,3) - f(1,3) - f(2,2) - f(3,3) - f(2,4) = \Delta g(2,3) = 4*5-2-3-3-1 = 11 \\ \Delta f(3,2) = 4f(3,2) - f(2,2) - f(3,1) - f(3,3) - f(4,2) = \Delta g(3,2) = 4*4-3-1-1-5 = 6 \\ \Delta f(3,3) = 4f(3,3) - f(2,3) - f(3,2) - f(3,4) - f(4,3) = \Delta g(3,3) = 4*1-5-4-2-2 = -9 \\ \Delta f(3,4) = 4f(3,4) - f(2,4) - f(3,3) - f(3,5) - f(4,4) = \Delta g(3,4) = 4*2-1-3-6-4 = -6 \end{cases}$$

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$$\Delta f = \Delta g \quad \text{over} \quad \Omega \quad \text{with} \quad f\big|_{\partial\Omega} = f^*\big|_{\partial\Omega}$$

f	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	4	0	3	9
2	0	5	?	?	2	1
3	5	2	?	?	?	2
4	1	3	0	2	3	5
5	2	6	7	1	2	6

g	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
3	2	1	4	1	2	6
4	5	3	5	2	4	3
5	6	4	3	1	5	7

HANG ON! PLEASE!

$$\Delta f = \Delta g \quad \begin{cases} \Delta f(2,2) = 4f(2,2) - f(1,2) - f(2,1) - f(3,2) - f(2,3) = \Delta g(2,2) = 4*3-4-2-5-4 = -3 \\ \Delta f(2,3) = 4f(2,3) - f(1,3) - f(2,2) - f(3,3) - f(2,4) = \Delta g(2,3) = 4*5-2-3-3-1 = 11 \\ \Delta f(3,2) = 4f(3,2) - f(2,2) - f(3,1) - f(3,3) - f(4,2) = \Delta g(3,2) = 4*4-3-1-1-5 = 6 \\ \Delta f(3,3) = 4f(3,3) - f(2,3) - f(3,2) - f(3,4) - f(4,3) = \Delta g(3,3) = 4*1-5-4-2-2 = -9 \\ \Delta f(3,4) = 4f(3,4) - f(2,4) - f(3,3) - f(3,5) - f(4,4) = \Delta g(3,4) = 4*2-1-3-6-4 = -6 \end{cases}$$

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$$\Delta f = \Delta g \quad \text{over} \quad \Omega \quad \text{with} \quad f\big|_{\partial\Omega} = f^*\big|_{\partial\Omega}$$

f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	4	0	3	9
2	0	5	?	?	2	1
3	5	2	?	?	?	2
4	1	3	0	2	3	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
3	2	1	4	1	2	6
4	5	3	5	2	4	3
5	6	4	3	1	5	7

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$$\Delta f = \Delta g \quad \begin{cases} \Delta f(2,2) = 4f(2,2) - f(1,2) - f(2,1) - f(3,2) - f(2,3) = \Delta g(2,2) = 4*3-4-2-5-4 = -3 \\ \Delta f(2,3) = 4f(2,3) - f(1,3) - f(2,2) - f(3,3) - f(2,4) = \Delta g(2,3) = 4*5-2-3-3-1 = 11 \\ \Delta f(3,2) = 4f(3,2) - f(2,2) - f(3,1) - f(3,3) - f(4,2) = \Delta g(3,2) = 4*4-3-1-1-5 = 6 \\ \Delta f(3,3) = 4f(3,3) - f(2,3) - f(3,2) - f(3,4) - f(4,3) = \Delta g(3,3) = 4*1-5-4-2-2 = -9 \\ \Delta f(3,4) = 4f(3,4) - f(2,4) - f(3,3) - f(3,5) - f(4,4) = \Delta g(3,4) = 4*2-1-3-6-4 = -6 \end{cases}$$

$$f\big|_{\partial\Omega} = f^*\big|_{\partial\Omega} \quad f(1,2) = 4,\ f(1,3) = 0,\ f(2,1) = 5,\ f(3,1) = 2,\ f(4,2) = 0,\ f(4,3) = 2,\ f(4,4) = 3,\ f(2,4) = 2,\ f(3,5) = 2$$

$$\Delta f = \Delta g \quad \begin{cases} \Delta f(2,2) = 4f(2,2) - f(1,2) - f(2,1) - f(3,2) - f(2,3) = \Delta g(2,2) = 4*3 - 4 - 2 - 5 - 4 = -3 \\ \Delta f(2,3) = 4f(2,3) - f(1,3) - f(2,2) - f(3,3) - f(2,4) = \Delta g(2,3) = 4*5 - 2 - 3 - 3 - 1 = 11 \\ \Delta f(3,2) = 4f(3,2) - f(2,2) - f(3,1) - f(3,3) - f(4,2) = \Delta g(3,2) = 4*4 - 3 - 1 - 1 - 5 = 6 \\ \Delta f(3,3) = 4f(3,3) - f(2,3) - f(3,2) - f(3,4) - f(4,3) = \Delta g(3,3) = 4*1 - 5 - 4 - 2 - 2 = -9 \\ \Delta f(3,4) = 4f(3,4) - f(2,4) - f(3,3) - f(3,5) - f(4,4) = \Delta g(3,4) = 4*2 - 1 - 3 - 6 - 4 = -6 \end{cases}$$

$$f\Big|_{\partial\Omega} = f^*\Big|_{\partial\Omega} \quad f(1,2) = 4,\ f(1,3) = 0,\ f(2,1) = 5,\ f(3,1) = 2,\ f(4,2) = 0,\ f(4,3) = 2,\ f(4,4) = 3,\ f(2,4) = 2,\ f(3,5) = 2$$

$$\begin{cases} \Delta f(2,2) = 4f(2,2) - f(3,2) - f(2,3) = \Delta g(2,2) + f(1,2) + f(2,1) \\ \Delta f(2,3) = 4f(2,3) - f(2,2) - f(3,3) = \Delta g(2,3) + f(1,3) + f(2,4) \\ \Delta f(3,2) = 4f(3,2) - f(2,2) - f(3,3) = \Delta g(3,2) + f(3,1) + f(4,2) \\ \Delta f(3,3) = 4f(3,3) - f(2,3) - f(3,2) - f(3,4) = \Delta g(3,3) + f(4,3) \\ \Delta f(3,4) = 4f(3,4) - f(3,3) = \Delta g(3,4) + f(3,1) + f(3,5) + f(4,4) \end{cases}$$

$$\Delta f = \Delta g \quad \begin{cases} \Delta f(2,2) = 4f(2,2) - f(1,2) - f(2,1) - f(3,2) - f(2,3) = \Delta g(2,2) = 4*3 - 4 - 2 - 5 - 4 = -3 \\ \Delta f(2,3) = 4f(2,3) - f(1,3) - f(2,2) - f(3,3) - f(2,4) = \Delta g(2,3) = 4*5 - 2 - 3 - 3 - 1 = 11 \\ \Delta f(3,2) = 4f(3,2) - f(2,2) - f(3,1) - f(3,3) - f(4,2) = \Delta g(3,2) = 4*4 - 3 - 1 - 1 - 5 = 6 \\ \Delta f(3,3) = 4f(3,3) - f(2,3) - f(3,2) - f(3,4) - f(4,3) = \Delta g(3,3) = 4*1 - 5 - 4 - 2 - 2 = -9 \\ \Delta f(3,4) = 4f(3,4) - f(2,4) - f(3,3) - f(3,5) - f(4,4) = \Delta g(3,4) = 4*2 - 1 - 3 - 6 - 4 = -6 \end{cases}$$

$$f\Big|_{\partial\Omega} = f^*\Big|_{\partial\Omega} \quad f(1,2) = 4,\ f(1,3) = 0,\ f(2,1) = 5,\ f(3,1) = 2,\ f(4,2) = 0,\ f(4,3) = 2,\ f(4,4) = 3,\ f(2,4) = 2,\ f(3,5) = 2$$

$$\begin{cases} \Delta f(2,2) = 4f(2,2) - f(3,2) - f(2,3) = \Delta g(2,2) + f(1,2) + f(2,1) \\ \Delta f(2,3) = 4f(2,3) - f(2,2) - f(3,3) = \Delta g(2,3) + f(1,3) + f(2,4) \\ \Delta f(3,2) = 4f(3,2) - f(2,2) - f(3,3) = \Delta g(3,2) + f(3,1) + f(4,2) \\ \Delta f(3,3) = 4f(3,3) - f(2,3) - f(3,2) - f(3,4) = \Delta g(3,3) + f(4,3) \\ \Delta f(3,4) = 4f(3,4) - f(3,3) = \Delta g(3,4) + f(3,1) + f(3,5) + f(4,4) \end{cases}$$

$$Ax = \begin{bmatrix} 4 & -1 & -1 & 0 & 0 \\ -1 & 4 & 0 & -1 & 0 \\ -1 & 0 & 4 & -1 & 0 \\ 0 & -1 & -1 & 4 & -1 \\ 0 & 0 & 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} f(2,2) \\ f(2,3) \\ f(3,2) \\ f(3,3) \\ f(3,4) \end{bmatrix} = \begin{bmatrix} 4f(2,2) - f(2,3) - f(3,2) \\ 4f(2,3) - f(2,2) - f(3,3) \\ 4f(3,2) - f(2,2) - f(3,3) \\ 4f(3,3) - f(2,3) - f(3,2) - f(3,4) \\ 4f(3,4) - f(3,3) \end{bmatrix} = \begin{bmatrix} -3+4+5 \\ 11+0+2 \\ 6+2+0 \\ -9+2 \\ -6+2+2+3 \end{bmatrix} = \begin{bmatrix} 6 \\ 13 \\ 8 \\ -7 \\ 1 \end{bmatrix} = b$$

$$\Delta f = \Delta g \ \text{ over } \ \Omega \ \text{ with } \ f\Big|_{\partial\Omega} = f^*\Big|_{\partial\Omega}$$

f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	4	0	3	9
2	0	5	?	?	2	1
3	5	2	?	?	?	2
4	1	3	0	2	3	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
3	2	1	4	1	2	6
4	5	3	5	2	4	3
5	6	4	3	1	5	7

$$\begin{cases} \Delta f(2,2) = 4f(2,2) - f(3,2) - f(2,3) = \Delta g(2,2) + f(1,2) + f(2,1) \\ \Delta f(2,3) = 4f(2,3) - f(2,2) - f(3,3) = \Delta g(2,3) + f(1,3) + f(2,4) \\ \Delta f(3,2) = 4f(3,2) - f(2,2) - f(3,3) = \Delta g(3,2) + f(3,1) + f(4,2) \\ \Delta f(3,3) = 4f(3,3) - f(2,3) - f(3,2) - f(3,4) = \Delta g(3,3) + f(4,3) \\ \Delta f(3,4) = 4f(3,4) - f(3,3) = \Delta g(3,4) + f(3,1) + f(3,5) + f(4,4) \end{cases}$$

$$Ax = \begin{bmatrix} 4 & -1 & -1 & 0 & 0 \\ -1 & 4 & 0 & -1 & 0 \\ -1 & 0 & 4 & -1 & 0 \\ 0 & -1 & -1 & 4 & -1 \\ 0 & 0 & 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} f(2,2) \\ f(2,3) \\ f(3,2) \\ f(3,3) \\ f(3,4) \end{bmatrix} = \begin{bmatrix} 4f(2,2) - f(2,3) - f(3,2) \\ 4f(2,3) - f(2,2) - f(3,3) \\ 4f(3,2) - f(2,2) - f(3,3) \\ 4f(3,3) - f(2,3) - f(3,2) - f(3,4) \\ 4f(3,4) - f(3,3) \end{bmatrix} = \begin{bmatrix} -3+4+5 \\ 11+0+2 \\ 6+2+0 \\ -9+2 \\ -6+2+2+3 \end{bmatrix} = \begin{bmatrix} 6 \\ 13 \\ 8 \\ -7 \\ 1 \end{bmatrix} = b$$

$$\Delta f = \Delta g \quad \text{over} \quad \Omega \quad \text{with} \quad f\Big|_{\partial\Omega} = f^*\Big|_{\partial\Omega}$$

f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	?	?	3	9
2	0	?	?	?	?	1
3	2	?	?	?	3	2
4	1	3	0	2	4	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
3	2	3	4	1	2	6
4	5	3	5	2	4	3
5	6	4	3	1	5	7

$$Ax = \begin{bmatrix} \quad \end{bmatrix} \begin{bmatrix} f(1,2) \\ f(1,3) \\ f(2,1) \\ f(2,2) \\ f(2,3) \\ f(2,4) \\ f(3,1) \\ f(3,2) \\ f(3,3) \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix} = b$$

$$\Delta f = \Delta g \;\text{ over }\; \Omega \;\text{ with }\; f\Big|_{\partial\Omega} = f^*\Big|_{\partial\Omega}$$

f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	?	?	3	9
2	0	?	?	?	?	1
3	2	?	?	?	3	2
4	1	3	0	2	4	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
3	2	3	4	1	2	6
4	5	3	5	2	4	3
5	6	4	3	1	5	7

$$\Delta f(1,2) = 4f(1,2) - f(1,1) - f(0,2) - f(2,2) - f(1,3) = \Delta g(1,2)$$

$$Ax = \begin{bmatrix} \; \end{bmatrix} \begin{bmatrix} f(1,2) \\ f(1,3) \\ f(2,1) \\ f(2,2) \\ f(2,3) \\ f(2,4) \\ f(3,1) \\ f(3,2) \\ f(3,3) \end{bmatrix} = \begin{bmatrix} \; \end{bmatrix} = \begin{bmatrix} \; \end{bmatrix} = b$$

$$\Delta f = \Delta g \text{ over } \Omega \text{ with } f\Big|_{\partial\Omega} = f^*\Big|_{\partial\Omega}$$

f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	?	?	3	9
2	0	?	?	?	?	1
3	2	?	?	?	3	2
4	1	3	0	2	4	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
3	2	3	4	1	2	6
4	5	3	5	2	4	3
5	6	4	3	1	5	7

$$\Delta f(1,2) = 4f(1,2) - f(1,1) - f(0,2) - f(2,2) - f(1,3) = \Delta g(1,2)$$

$$Ax = \begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ & & & & & & & & \\ & & & & & & & & \end{bmatrix} \begin{bmatrix} f(1,2) \\ f(1,3) \\ f(2,1) \\ f(2,2) \\ f(2,3) \\ f(2,4) \\ f(3,1) \\ f(3,2) \\ f(3,3) \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix} = b$$

$$\Delta f = \Delta g \;\text{ over }\; \Omega \;\text{ with }\; f\Big|_{\partial\Omega} = f^*\Big|_{\partial\Omega}$$

f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	?	?	3	9
2	0	?	?	?	?	1
3	2	?	?	?	3	2
4	1	3	0	2	4	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
3	2	3	4	1	2	6
4	5	3	5	2	4	3
5	6	4	3	1	5	7

$$\Delta f(1,2) = 4f(1,2) - f(1,1) - f(0,2) - f(2,2) - f(1,3) = \Delta g(1,2)$$

$$Ax = \begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \end{bmatrix} \begin{bmatrix} f(1,2) \\ f(1,3) \\ f(2,1) \\ f(2,2) \\ f(2,3) \\ f(2,4) \\ f(3,1) \\ f(3,2) \\ f(3,3) \end{bmatrix} = \begin{bmatrix} \Delta g(1,2) + f(1,1) + f(0,2) \\ \\ \\ \\ \\ \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \end{bmatrix} = b$$

$$\Delta f = \Delta g \ \text{ over } \ \Omega \ \text{ with } \ f\Big|_{\partial\Omega} = f^*\Big|_{\partial\Omega}$$

f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	?	?	3	9
2	0	?	?	?	?	1
3	2	?	?	?	3	2
4	1	3	0	2	4	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
3	2	3	4	1	2	6
4	5	3	5	2	4	3
5	6	4	3	1	5	7

$$\Delta f(1,2) = 4f(1,2) - f(1,1) - f(0,2) - f(2,2) - f(1,3) = \Delta g(1,2)$$

$$Ax = \begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ & & & & & & & & \\ & & & & & & & & \end{bmatrix} \begin{bmatrix} f(1,2) \\ f(1,3) \\ f(2,1) \\ f(2,2) \\ f(2,3) \\ f(2,4) \\ f(3,1) \\ f(3,2) \\ f(3,3) \end{bmatrix} = \begin{bmatrix} \Delta g(1,2) + f(1,1) + f(0,2) \\ \\ \\ \end{bmatrix} = \begin{bmatrix} 8 \\ \\ \\ \end{bmatrix} = b$$

$$\Delta f = \Delta g \ \text{ over } \ \Omega \ \text{ with } \ f\Big|_{\partial\Omega} = f^*\Big|_{\partial\Omega}$$

f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	?	?	3	9
2	0	?	?	?	?	1
3	2	?	?	?	3	2
4	1	3	0	2	4	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
3	2	3	4	1	2	6
4	5	3	5	2	4	3
5	6	4	3	1	5	7

$$\Delta f(1,2) = 4f(1,2) - f(1,1) - f(0,2) - f(2,2) - f(1,3) = \Delta g(1,2)$$
$$\Delta f(1,3) = 4f(1,3) - f(1,2) - f(0,3) - f(2,3) - f(1,4) = \Delta g(1,3)$$

$$Ax = \begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{bmatrix} \begin{bmatrix} f(1,2) \\ f(1,3) \\ f(2,1) \\ f(2,2) \\ f(2,3) \\ f(2,4) \\ f(3,1) \\ f(3,2) \\ f(3,3) \end{bmatrix} = \begin{bmatrix} \Delta g(1,2) + f(1,1) + f(0,2) \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{bmatrix} = \begin{bmatrix} 8 \\ \\ \\ \\ \\ \\ \\ \\ \end{bmatrix} = b$$

$$\Delta f = \Delta g \ \text{ over } \ \Omega \ \text{ with } \ f\big|_{\partial\Omega} = f^*\big|_{\partial\Omega}$$

f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	?	?	3	9
2	0	?	?	?	?	1
3	2	?	?	?	3	2
4	1	3	0	2	4	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
3	2	3	4	1	2	6
4	5	3	5	2	4	3
5	6	4	3	1	5	7

$$\Delta f(1,2) = 4f(1,2) - f(1,1) - f(0,2) - f(2,2) - f(1,3) = \Delta g(1,2)$$
$$\Delta f(1,3) = 4f(1,3) - f(1,2) - f(0,3) - f(2,3) - f(1,4) = \Delta g(1,3)$$

$$Ax = \begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \end{bmatrix} \begin{bmatrix} f(1,2) \\ f(1,3) \\ f(2,1) \\ f(2,2) \\ f(2,3) \\ f(2,4) \\ f(3,1) \\ f(3,2) \\ f(3,3) \end{bmatrix} = \begin{bmatrix} \Delta g(1,2) + f(1,1) + f(0,2) \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{bmatrix} = \begin{bmatrix} 8 \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{bmatrix} = b$$

$$\Delta f = \Delta g \ \text{ over } \ \Omega \ \text{ with } \ f\big|_{\partial\Omega} = f^*\big|_{\partial\Omega}$$

f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	?	?	3	9
2	0	?	?	?	?	1
3	2	?	?	?	3	2
4	1	3	0	2	4	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
3	2	3	4	1	2	6
4	5	3	5	2	4	3
5	6	4	3	1	5	7

$$\Delta f(1,2) = 4f(1,2) - f(1,1) - f(0,2) - f(2,2) - f(1,3) = \Delta g(1,2)$$
$$\Delta f(1,3) = 4f(1,3) - f(1,2) - f(0,3) - f(2,3) - f(1,4) = \Delta g(1,3)$$

$$Ax = \begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ & & & & & & & & \end{bmatrix} \begin{bmatrix} f(1,2) \\ f(1,3) \\ f(2,1) \\ f(2,2) \\ f(2,3) \\ f(2,4) \\ f(3,1) \\ f(3,2) \\ f(3,3) \end{bmatrix} = \begin{bmatrix} \Delta g(1,2) + f(1,1) + f(0,2) \\ \Delta g(1,3) + f(0,3) + f(1,4) \\ \end{bmatrix} = \begin{bmatrix} 8 \\ \\ \end{bmatrix} = b$$

$$\Delta f = \Delta g \ \text{ over } \ \Omega \ \text{ with } \ f\Big|_{\partial\Omega} = f^*\Big|_{\partial\Omega}$$

f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	?	?	3	9
2	0	?	?	?	?	1
3	2	?	?	?	3	2
4	1	3	0	2	4	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
3	2	3	4	1	2	6
4	5	3	5	2	4	3
5	6	4	3	1	5	7

$$\Delta f(1,2) = 4f(1,2) - f(1,1) - f(0,2) - f(2,2) - f(1,3) = \Delta g(1,2)$$
$$\Delta f(1,3) = 4f(1,3) - f(1,2) - f(0,3) - f(2,3) - f(1,4) = \Delta g(1,3)$$

$$Ax = \begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \end{bmatrix} \begin{bmatrix} f(1,2) \\ f(1,3) \\ f(2,1) \\ f(2,2) \\ f(2,3) \\ f(2,4) \\ f(3,1) \\ f(3,2) \\ f(3,3) \end{bmatrix} = \begin{bmatrix} \Delta g(1,2) + f(1,1) + f(0,2) \\ \Delta g(1,3) + f(0,3) + f(1,4) \\ \\ \\ \\ \\ \\ \\ \\ \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ \\ \\ \\ \\ \\ \\ \\ \end{bmatrix} = b$$

$$\Delta f = \Delta g \text{ over } \Omega \text{ with } f\big|_{\partial\Omega} = f^*\big|_{\partial\Omega}$$

f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	?	?	3	9
2	0	?	?	?	?	1
3	2	?	?	?	3	2
4	1	3	0	2	4	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
3	2	3	4	1	2	6
4	5	3	5	2	4	3
5	6	4	3	1	5	7

$$\Delta f(1,2) = 4f(1,2) - f(1,1) - f(0,2) - f(2,2) - f(1,3) = \Delta g(1,2)$$
$$\Delta f(1,3) = 4f(1,3) - f(1,2) - f(0,3) - f(2,3) - f(1,4) = \Delta g(1,3)$$
$$\Delta f(2,1) = 4f(2,1) - f(2,0) - f(1,1) - f(2,2) - f(3,1) = \Delta g(2,1)$$

$$Ax = \begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \end{bmatrix} \begin{bmatrix} f(1,2) \\ f(1,3) \\ f(2,1) \\ f(2,2) \\ f(2,3) \\ f(2,4) \\ f(3,1) \\ f(3,2) \\ f(3,3) \end{bmatrix} = \begin{bmatrix} \Delta g(1,2) + f(1,1) + f(0,2) \\ \Delta g(1,3) + f(0,3) + f(1,4) \\ \\ \\ \\ \\ \\ \\ \\ \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ \\ \\ \\ \\ \\ \\ \\ \end{bmatrix} = b$$

$$\Delta f = \Delta g \ \text{ over } \ \Omega \ \text{ with } \ f\big|_{\partial\Omega} = f^*\big|_{\partial\Omega}$$

f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	?	?	3	9
2	0	?	?	?	?	1
3	2	?	?	?	3	2
4	1	3	0	2	4	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
3	2	3	4	1	2	6
4	5	3	5	2	4	3
5	6	4	3	1	5	7

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$$Ax = \begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & -1 & 0 & 0 & -1 & 0 & 0 \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \end{bmatrix} \begin{bmatrix} f(1,2) \\ f(1,3) \\ f(2,1) \\ f(2,2) \\ f(2,3) \\ f(2,4) \\ f(3,1) \\ f(3,2) \\ f(3,3) \end{bmatrix} = \begin{bmatrix} \Delta g(1,2) + f(1,1) + f(0,2) \\ \Delta g(1,3) + f(0,3) + f(1,4) \\ \\ \\ \\ \\ \\ \\ \\ \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ \\ \\ \\ \\ \\ \\ \\ \end{bmatrix} = b$$

$$\Delta f = \Delta g \;\text{ over }\; \Omega \;\text{ with }\; f\big|_{\partial\Omega} = f^*\big|_{\partial\Omega}$$

f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	?	?	3	9
2	0	?	?	?	?	1
3	2	?	?	?	3	2
4	1	3	0	2	4	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
3	2	3	4	1	2	6
4	5	3	5	2	4	3
5	6	4	3	1	5	7

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$$Ax = \begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & -1 & 0 & 0 & -1 & 0 & 0 \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \end{bmatrix} \begin{bmatrix} f(1,2) \\ f(1,3) \\ f(2,1) \\ f(2,2) \\ f(2,3) \\ f(2,4) \\ f(3,1) \\ f(3,2) \\ f(3,3) \end{bmatrix} = \begin{bmatrix} \Delta g(1,2) + f(1,1) + f(0,2) \\ \Delta g(1,3) + f(0,3) + f(1,4) \\ \Delta g(2,1) + f(2,0) + f(1,1) \\ \\ \\ \\ \\ \\ \\ \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ \\ \\ \\ \\ \\ \\ \\ \end{bmatrix} = b$$

$$\Delta f = \Delta g \ \text{ over } \ \Omega \ \text{ with } \ f\Big|_{\partial\Omega} = f^*\Big|_{\partial\Omega}$$

f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	?	?	3	9
2	0	?	?	?	?	1
3	2	?	?	?	3	2
4	1	3	0	2	4	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
3	2	3	4	1	2	6
4	5	3	5	2	4	3
5	6	4	3	1	5	7

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$$Ax = \begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & -1 & 0 & 0 & -1 & 0 & 0 \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \end{bmatrix} \begin{bmatrix} f(1,2) \\ f(1,3) \\ f(2,1) \\ f(2,2) \\ f(2,3) \\ f(2,4) \\ f(3,1) \\ f(3,2) \\ f(3,3) \end{bmatrix} = \begin{bmatrix} \Delta g(1,2) + f(1,1) + f(0,2) \\ \Delta g(1,3) + f(0,3) + f(1,4) \\ \Delta g(2,1) + f(2,0) + f(1,1) \\ \\ \\ \\ \\ \\ \\ \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ -1 \\ \\ \\ \\ \\ \\ \\ \end{bmatrix} = b$$

$$\Delta f = \Delta g \text{ over } \Omega \text{ with } f\big|_{\partial\Omega} = f^*\big|_{\partial\Omega}$$

f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	?	?	3	9
2	0	?	?	?	?	1
3	2	?	?	?	3	2
4	1	3	0	2	4	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
3	2	3	4	1	2	6
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$$Ax = \begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & -1 & 0 & 0 & -1 & 0 & 0 \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \end{bmatrix} \begin{bmatrix} f(1,2) \\ f(1,3) \\ f(2,1) \\ f(2,2) \\ f(2,3) \\ f(2,4) \\ f(3,1) \\ f(3,2) \\ f(3,3) \end{bmatrix} = \begin{bmatrix} \Delta g(1,2) + f(1,1) + f(0,2) \\ \Delta g(1,3) + f(0,3) + f(1,4) \\ \Delta g(2,1) + f(2,0) + f(1,1) \\ \\ \\ \\ \\ \\ \\ \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ -1 \\ \\ \\ \\ \\ \\ \\ \end{bmatrix} = b$$

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f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	?	?	3	9
2	0	?	?	?	?	1
3	2	?	?	?	3	2
4	1	3	0	2	4	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
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$$Ax = \begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & -1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 & 0 \\ \\ \\ \\ \\ \\ \end{bmatrix} \begin{bmatrix} f(1,2) \\ f(1,3) \\ f(2,1) \\ f(2,2) \\ f(2,3) \\ f(2,4) \\ f(3,1) \\ f(3,2) \\ f(3,3) \end{bmatrix} = \begin{bmatrix} \Delta g(1,2) + f(1,1) + f(0,2) \\ \Delta g(1,3) + f(0,3) + f(1,4) \\ \Delta g(2,1) + f(2,0) + f(1,1) \\ \\ \\ \\ \\ \\ \\ \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ -1 \\ \\ \\ \\ \\ \\ \\ \end{bmatrix} = b$$

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f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	?	?	3	9
2	0	?	?	?	?	1
3	2	?	?	?	3	2
4	1	3	0	2	4	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
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$$Ax = \begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & -1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 & 0 \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \end{bmatrix} \begin{bmatrix} f(1,2) \\ f(1,3) \\ f(2,1) \\ f(2,2) \\ f(2,3) \\ f(2,4) \\ f(3,1) \\ f(3,2) \\ f(3,3) \end{bmatrix} = \begin{bmatrix} \Delta g(1,2) + f(1,1) + f(0,2) \\ \Delta g(1,3) + f(0,3) + f(1,4) \\ \Delta g(2,1) + f(2,0) + f(1,1) \\ \Delta g(2,2) \\ \\ \\ \\ \\ \\ \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ -1 \\ \\ \\ \\ \\ \\ \\ \end{bmatrix} = b$$

$$\Delta f = \Delta g \ \text{ over } \ \Omega \ \text{ with } \ f\Big|_{\partial\Omega} = f^*\Big|_{\partial\Omega}$$

f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	?	?	3	9
2	0	?	?	?	?	1
3	2	?	?	?	3	2
4	1	3	0	2	4	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
3	2	3	4	1	2	6
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$$Ax = \begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & -1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 & 0 \\ & & & & & & & & \\ & & & & & & & & \end{bmatrix} \begin{bmatrix} f(1,2) \\ f(1,3) \\ f(2,1) \\ f(2,2) \\ f(2,3) \\ f(2,4) \\ f(3,1) \\ f(3,2) \\ f(3,3) \end{bmatrix} = \begin{bmatrix} \Delta g(1,2) + f(1,1) + f(0,2) \\ \Delta g(1,3) + f(0,3) + f(1,4) \\ \Delta g(2,1) + f(2,0) + f(1,1) \\ \Delta g(2,2) \\ \\ \\ \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ -1 \\ -3 \\ \\ \\ \end{bmatrix} = b$$

$$\Delta f = \Delta g \;\text{ over }\; \Omega \;\text{ with }\; f\Big|_{\partial\Omega} = f^*\Big|_{\partial\Omega}$$

f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	?	?	3	9
2	0	?	?	?	?	1
3	2	?	?	?	3	2
4	1	3	0	2	4	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
3	2	3	4	1	2	6
4	5	3	5	2	4	3
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$$Ax = \begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & -1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 & 0 \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \end{bmatrix} \begin{bmatrix} f(1,2) \\ f(1,3) \\ f(2,1) \\ f(2,2) \\ f(2,3) \\ f(2,4) \\ f(3,1) \\ f(3,2) \\ f(3,3) \end{bmatrix} = \begin{bmatrix} \Delta g(1,2) + f(1,1) + f(0,2) \\ \Delta g(1,3) + f(0,3) + f(1,4) \\ \Delta g(2,1) + f(2,0) + f(1,1) \\ \Delta g(2,2) \\ \\ \\ \\ \\ \\ \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ -1 \\ -3 \\ \\ \\ \\ \\ \\ \end{bmatrix} = b$$

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f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	?	?	3	9
2	0	?	?	?	?	1
3	2	?	?	?	3	2
4	1	3	0	2	4	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
3	2	3	4	1	2	6
4	5	3	5	2	4	3
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$$Ax = \begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & -1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \end{bmatrix} \begin{bmatrix} f(1,2) \\ f(1,3) \\ f(2,1) \\ f(2,2) \\ f(2,3) \\ f(2,4) \\ f(3,1) \\ f(3,2) \\ f(3,3) \end{bmatrix} = \begin{bmatrix} \Delta g(1,2) + f(1,1) + f(0,2) \\ \Delta g(1,3) + f(0,3) + f(1,4) \\ \Delta g(2,1) + f(2,0) + f(1,1) \\ \Delta g(2,2) \\ \\ \\ \\ \\ \\ \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ -1 \\ -3 \\ \\ \\ \\ \\ \\ \end{bmatrix} = b$$

$$\Delta f = \Delta g \text{ over } \Omega \text{ with } f\big|_{\partial\Omega} = f^*\big|_{\partial\Omega}$$

f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	?	?	3	9
2	0	?	?	?	?	1
3	2	?	?	?	3	2
4	1	3	0	2	4	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
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4	5	3	5	2	4	3
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$$\Delta f(2,2) = 4f(2,2) - f(2,1) - f(1,2) - f(2,3) - f(3,2) = \Delta g(2,2)$$
$$\Delta f(2,3) = 4f(2,3) - f(2,2) - f(1,3) - f(3,3) - f(2,4) = \Delta g(2,3)$$

$$Ax = \begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & -1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \end{bmatrix} \begin{bmatrix} f(1,2) \\ f(1,3) \\ f(2,1) \\ f(2,2) \\ f(2,3) \\ f(2,4) \\ f(3,1) \\ f(3,2) \\ f(3,3) \end{bmatrix} = \begin{bmatrix} \Delta g(1,2) + f(1,1) + f(0,2) \\ \Delta g(1,3) + f(0,3) + f(1,4) \\ \Delta g(2,1) + f(2,0) + f(1,1) \\ \Delta g(2,2) \\ \Delta g(2,3) \\ \\ \\ \\ \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ -1 \\ -3 \\ \\ \\ \\ \\ \end{bmatrix} = b$$

$$\Delta f = \Delta g \text{ over } \Omega \text{ with } f\big|_{\partial\Omega} = f^*\big|_{\partial\Omega}$$

f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	?	?	3	9
2	0	?	?	?	?	1
3	2	?	?	?	3	2
4	1	3	0	2	4	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
3	2	3	4	1	2	6
4	5	3	5	2	4	3
5	6	4	3	1	5	7

$$\Delta f(1,2) = 4f(1,2) - f(1,1) - f(0,2) - f(2,2) - f(1,3) = \Delta g(1,2)$$
$$\Delta f(1,3) = 4f(1,3) - f(1,2) - f(0,3) - f(2,3) - f(1,4) = \Delta g(1,3)$$
$$\Delta f(2,1) = 4f(2,1) - f(2,0) - f(1,1) - f(2,2) - f(3,1) = \Delta g(2,1)$$
$$\Delta f(2,2) = 4f(2,2) - f(2,1) - f(1,2) - f(2,3) - f(3,2) = \Delta g(2,2)$$
$$\Delta f(2,3) = 4f(2,3) - f(2,2) - f(1,3) - f(3,3) - f(2,4) = \Delta g(2,3)$$

$$Ax = \begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & -1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \end{bmatrix} \begin{bmatrix} f(1,2) \\ f(1,3) \\ f(2,1) \\ f(2,2) \\ f(2,3) \\ f(2,4) \\ f(3,1) \\ f(3,2) \\ f(3,3) \end{bmatrix} = \begin{bmatrix} \Delta g(1,2) + f(1,1) + f(0,2) \\ \Delta g(1,3) + f(0,3) + f(1,4) \\ \Delta g(2,1) + f(2,0) + f(1,1) \\ \Delta g(2,2) \\ \Delta g(2,3) \\ \\ \\ \\ \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ -1 \\ -3 \\ 11 \\ \\ \\ \\ \end{bmatrix} = b$$

$$\Delta f = \Delta g \ \text{ over } \ \Omega \ \text{ with } \ f\Big|_{\partial\Omega} = f^*\Big|_{\partial\Omega}$$

f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	?	?	3	9
2	0	?	?	?	?	1
3	2	?	?	?	3	2
4	1	3	0	2	4	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
3	2	3	4	1	2	6
4	5	3	5	2	4	3
5	6	4	3	1	5	7

$$\Delta f(1,2) = 4f(1,2) - f(1,1) - f(0,2) - f(2,2) - f(1,3) = \Delta g(1,2)$$
$$\Delta f(1,3) = 4f(1,3) - f(1,2) - f(0,3) - f(2,3) - f(1,4) = \Delta g(1,3)$$
$$\Delta f(2,1) = 4f(2,1) - f(2,0) - f(1,1) - f(2,2) - f(3,1) = \Delta g(2,1)$$
$$\Delta f(2,2) = 4f(2,2) - f(2,1) - f(1,2) - f(2,3) - f(3,2) = \Delta g(2,2)$$
$$\Delta f(2,3) = 4f(2,3) - f(2,2) - f(1,3) - f(3,3) - f(2,4) = \Delta g(2,3)$$
$$\Delta f(2,4) = 4f(2,4) - f(2,3) - f(1,4) - f(2,5) - f(3,4) = \Delta g(2,4)$$

$$Ax = \begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & -1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 \\ \\ \\ \\ \\ \end{bmatrix} \begin{bmatrix} f(1,2) \\ f(1,3) \\ f(2,1) \\ f(2,2) \\ f(2,3) \\ f(2,4) \\ f(3,1) \\ f(3,2) \\ f(3,3) \end{bmatrix} = \begin{bmatrix} \Delta g(1,2) + f(1,1) + f(0,2) \\ \Delta g(1,3) + f(0,3) + f(1,4) \\ \Delta g(2,1) + f(2,0) + f(1,1) \\ \Delta g(2,2) \\ \Delta g(2,3) \\ \\ \\ \\ \\ \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ -1 \\ -3 \\ 11 \\ \\ \\ \\ \\ \end{bmatrix} = b$$

$$\Delta f = \Delta g \;\text{ over }\; \Omega \;\text{ with }\; f\big|_{\partial\Omega} = f^*\big|_{\partial\Omega}$$

f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	?	?	3	9
2	0	?	?	?	?	1
3	2	?	?	?	3	2
4	1	3	0	2	4	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
3	2	3	4	1	2	6
4	5	3	5	2	4	3
5	6	4	3	1	5	7

$$\Delta f(1,2) = 4f(1,2) - f(1,1) - f(0,2) - f(2,2) - f(1,3) = \Delta g(1,2)$$
$$\Delta f(1,3) = 4f(1,3) - f(1,2) - f(0,3) - f(2,3) - f(1,4) = \Delta g(1,3)$$
$$\Delta f(2,1) = 4f(2,1) - f(2,0) - f(1,1) - f(2,2) - f(3,1) = \Delta g(2,1)$$
$$\Delta f(2,2) = 4f(2,2) - f(2,1) - f(1,2) - f(2,3) - f(3,2) = \Delta g(2,2)$$
$$\Delta f(2,3) = 4f(2,3) - f(2,2) - f(1,3) - f(3,3) - f(2,4) = \Delta g(2,3)$$
$$\Delta f(2,4) = 4f(2,4) - f(2,3) - f(1,4) - f(2,5) - f(3,4) = \Delta g(2,4)$$

$$Ax = \begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & -1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 4 & 0 & 0 & 0 \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \end{bmatrix} \begin{bmatrix} f(1,2) \\ f(1,3) \\ f(2,1) \\ f(2,2) \\ f(2,3) \\ f(2,4) \\ f(3,1) \\ f(3,2) \\ f(3,3) \end{bmatrix} = \begin{bmatrix} \Delta g(1,2) + f(1,1) + f(0,2) \\ \Delta g(1,3) + f(0,3) + f(1,4) \\ \Delta g(2,1) + f(2,0) + f(1,1) \\ \Delta g(2,2) \\ \Delta g(2,3) \\ \\ \\ \\ \\ \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ -1 \\ -3 \\ 11 \\ \\ \\ \\ \\ \end{bmatrix} = b$$

$$\Delta f = \Delta g \ \text{ over } \ \Omega \ \text{ with } \ f\big|_{\partial\Omega} = f^*\big|_{\partial\Omega}$$

f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	?	?	3	9
2	0	?	?	?	?	1
3	2	?	?	?	3	2
4	1	3	0	2	4	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
3	2	3	4	1	2	6
4	5	3	5	2	4	3
5	6	4	3	1	5	7

$$\Delta f(1,2) = 4f(1,2) - f(1,1) - f(0,2) - f(2,2) - f(1,3) = \Delta g(1,2)$$
$$\Delta f(1,3) = 4f(1,3) - f(1,2) - f(0,3) - f(2,3) - f(1,4) = \Delta g(1,3)$$
$$\Delta f(2,1) = 4f(2,1) - f(2,0) - f(1,1) - f(2,2) - f(3,1) = \Delta g(2,1)$$
$$\Delta f(2,2) = 4f(2,2) - f(2,1) - f(1,2) - f(2,3) - f(3,2) = \Delta g(2,2)$$
$$\Delta f(2,3) = 4f(2,3) - f(2,2) - f(1,3) - f(3,3) - f(2,4) = \Delta g(2,3)$$
$$\Delta f(2,4) = 4f(2,4) - f(2,3) - f(1,4) - f(2,5) - f(3,4) = \Delta g(2,4)$$

$$Ax = \begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & -1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 4 & 0 & 0 & 0 \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \end{bmatrix} \begin{bmatrix} f(1,2) \\ f(1,3) \\ f(2,1) \\ f(2,2) \\ f(2,3) \\ f(2,4) \\ f(3,1) \\ f(3,2) \\ f(3,3) \end{bmatrix} = \begin{bmatrix} \Delta g(1,2) + f(1,1) + f(0,2) \\ \Delta g(1,3) + f(0,3) + f(1,4) \\ \Delta g(2,1) + f(2,0) + f(1,1) \\ \Delta g(2,2) \\ \Delta g(2,3) \\ \Delta g(2,4) + f(1,4) + f(2,5) + f(3,4) \\ \\ \\ \\ \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ -1 \\ -3 \\ 11 \\ \\ \\ \\ \\ \end{bmatrix} = b$$

$$\Delta f = \Delta g \;\text{ over }\; \Omega \;\text{ with }\; f\Big|_{\partial\Omega} = f^*\Big|_{\partial\Omega}$$

f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	?	?	3	9
2	0	?	?	?	?	1
3	2	?	?	?	3	2
4	1	3	0	2	4	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
3	2	3	4	1	2	6
4	5	3	5	2	4	3
5	6	4	3	1	5	7

$$\Delta f(1,2) = 4f(1,2) - f(1,1) - f(0,2) - f(2,2) - f(1,3) = \Delta g(1,2)$$
$$\Delta f(1,3) = 4f(1,3) - f(1,2) - f(0,3) - f(2,3) - f(1,4) = \Delta g(1,3)$$
$$\Delta f(2,1) = 4f(2,1) - f(2,0) - f(1,1) - f(2,2) - f(3,1) = \Delta g(2,1)$$
$$\Delta f(2,2) = 4f(2,2) - f(2,1) - f(1,2) - f(2,3) - f(3,2) = \Delta g(2,2)$$
$$\Delta f(2,3) = 4f(2,3) - f(2,2) - f(1,3) - f(3,3) - f(2,4) = \Delta g(2,3)$$
$$\Delta f(2,4) = 4f(2,4) - f(2,3) - f(1,4) - f(2,5) - f(3,4) = \Delta g(2,4)$$

$$Ax = \begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & -1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 4 & 0 & 0 & 0 \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \end{bmatrix} \begin{bmatrix} f(1,2) \\ f(1,3) \\ f(2,1) \\ f(2,2) \\ f(2,3) \\ f(2,4) \\ f(3,1) \\ f(3,2) \\ f(3,3) \end{bmatrix} = \begin{bmatrix} \Delta g(1,2) + f(1,1) + f(0,2) \\ \Delta g(1,3) + f(0,3) + f(1,4) \\ \Delta g(2,1) + f(2,0) + f(1,1) \\ \Delta g(2,2) \\ \Delta g(2,3) \\ \Delta g(2,4) + f(1,4) + f(2,5) + f(3,4) \\ \\ \\ \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ -1 \\ -3 \\ 11 \\ 12 \\ \\ \\ \end{bmatrix} = b$$

$$\Delta f = \Delta g \ \text{ over } \ \Omega \ \text{ with } \ f\Big|_{\partial\Omega} = f^*\Big|_{\partial\Omega}$$

f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	?	?	3	9
2	0	?	?	?	?	1
3	2	?	?	?	3	2
4	1	3	0	2	4	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
3	2	3	4	1	2	6
4	5	3	5	2	4	3
5	6	4	3	1	5	7

$$\Delta f(1,2) = 4f(1,2) - f(1,1) - f(0,2) - f(2,2) - f(1,3) = \Delta g(1,2)$$
$$\Delta f(1,3) = 4f(1,3) - f(1,2) - f(0,3) - f(2,3) - f(1,4) = \Delta g(1,3)$$
$$\Delta f(2,1) = 4f(2,1) - f(2,0) - f(1,1) - f(2,2) - f(3,1) = \Delta g(2,1)$$
$$\Delta f(2,2) = 4f(2,2) - f(2,1) - f(1,2) - f(2,3) - f(3,2) = \Delta g(2,2)$$
$$\Delta f(2,3) = 4f(2,3) - f(2,2) - f(1,3) - f(3,3) - f(2,4) = \Delta g(2,3)$$
$$\Delta f(2,4) = 4f(2,4) - f(2,3) - f(1,4) - f(2,5) - f(3,4) = \Delta g(2,4)$$
$$\Delta f(3,1) = 4f(3,1) - f(3,0) - f(2,1) - f(3,2) - f(4,1) = \Delta g(3,1)$$

$$Ax = \begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & -1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 4 & 0 & 0 & 0 \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \end{bmatrix} \begin{bmatrix} f(1,2) \\ f(1,3) \\ f(2,1) \\ f(2,2) \\ f(2,3) \\ f(2,4) \\ f(3,1) \\ f(3,2) \\ f(3,3) \end{bmatrix} = \begin{bmatrix} \Delta g(1,2) + f(1,1) + f(0,2) \\ \Delta g(1,3) + f(0,3) + f(1,4) \\ \Delta g(2,1) + f(2,0) + f(1,1) \\ \Delta g(2,2) \\ \Delta g(2,3) \\ \Delta g(2,4) + f(1,4) + f(2,5) + f(3,4) \\ \\ \\ \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ -1 \\ -3 \\ 11 \\ 12 \\ \\ \\ \end{bmatrix} = b$$

$$\Delta f = \Delta g \ \text{ over } \ \Omega \ \text{ with } \ f\Big|_{\partial\Omega} = f^*\Big|_{\partial\Omega}$$

f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	?	?	3	9
2	0	?	?	?	?	1
3	2	?	?	?	3	2
4	1	3	0	2	4	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
3	2	3	4	1	2	6
4	5	3	5	2	4	3
5	6	4	3	1	5	7

$$\Delta f(1,2) = 4f(1,2) - f(1,1) - f(0,2) - f(2,2) - f(1,3) = \Delta g(1,2)$$
$$\Delta f(1,3) = 4f(1,3) - f(1,2) - f(0,3) - f(2,3) - f(1,4) = \Delta g(1,3)$$
$$\Delta f(2,1) = 4f(2,1) - f(2,0) - f(1,1) - f(2,2) - f(3,1) = \Delta g(2,1)$$
$$\Delta f(2,2) = 4f(2,2) - f(2,1) - f(1,2) - f(2,3) - f(3,2) = \Delta g(2,2)$$
$$\Delta f(2,3) = 4f(2,3) - f(2,2) - f(1,3) - f(3,3) - f(2,4) = \Delta g(2,3)$$
$$\Delta f(2,4) = 4f(2,4) - f(2,3) - f(1,4) - f(2,5) - f(3,4) = \Delta g(2,4)$$
$$\Delta f(3,1) = 4f(3,1) - f(3,0) - f(2,1) - f(3,2) - f(4,1) = \Delta g(3,1)$$

$$Ax = \begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & -1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 4 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 4 & -1 & 0 \\ & & & & & & & & \end{bmatrix} \begin{bmatrix} f(1,2) \\ f(1,3) \\ f(2,1) \\ f(2,2) \\ f(2,3) \\ f(2,4) \\ f(3,1) \\ f(3,2) \\ f(3,3) \end{bmatrix} = \begin{bmatrix} \Delta g(1,2) + f(1,1) + f(0,2) \\ \Delta g(1,3) + f(0,3) + f(1,4) \\ \Delta g(2,1) + f(2,0) + f(1,1) \\ \Delta g(2,2) \\ \Delta g(2,3) \\ \Delta g(2,4) + f(1,4) + f(2,5) + f(3,4) \\ \\ \\ \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ -1 \\ -3 \\ 11 \\ 12 \\ \\ \\ \end{bmatrix} = b$$

$$\Delta f = \Delta g \;\text{ over }\; \Omega \;\text{ with }\; f\Big|_{\partial\Omega} = f^*\Big|_{\partial\Omega}$$

f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	?	?	3	9
2	0	?	?	?	?	1
3	2	?	?	?	3	2
4	1	3	0	2	4	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
3	2	3	4	1	2	6
4	5	3	5	2	4	3
5	6	4	3	1	5	7

$$\Delta f(1,2) = 4f(1,2) - f(1,1) - f(0,2) - f(2,2) - f(1,3) = \Delta g(1,2)$$
$$\Delta f(1,3) = 4f(1,3) - f(1,2) - f(0,3) - f(2,3) - f(1,4) = \Delta g(1,3)$$
$$\Delta f(2,1) = 4f(2,1) - f(2,0) - f(1,1) - f(2,2) - f(3,1) = \Delta g(2,1)$$
$$\Delta f(2,2) = 4f(2,2) - f(2,1) - f(1,2) - f(2,3) - f(3,2) = \Delta g(2,2)$$
$$\Delta f(2,3) = 4f(2,3) - f(2,2) - f(1,3) - f(3,3) - f(2,4) = \Delta g(2,3)$$
$$\Delta f(2,4) = 4f(2,4) - f(2,3) - f(1,4) - f(2,5) - f(3,4) = \Delta g(2,4)$$
$$\Delta f(3,1) = 4f(3,1) - f(3,0) - f(2,1) - f(3,2) - f(4,1) = \Delta g(3,1)$$

$$Ax = \begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & -1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 4 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 4 & -1 & 0 \\ & & & & & & & & \\ & & & & & & & & \end{bmatrix} \begin{bmatrix} f(1,2) \\ f(1,3) \\ f(2,1) \\ f(2,2) \\ f(2,3) \\ f(2,4) \\ f(3,1) \\ f(3,2) \\ f(3,3) \end{bmatrix} = \begin{bmatrix} \Delta g(1,2) + f(1,1) + f(0,2) \\ \Delta g(1,3) + f(0,3) + f(1,4) \\ \Delta g(2,1) + f(2,0) + f(1,1) \\ \Delta g(2,2) \\ \Delta g(2,3) \\ \Delta g(2,4) + f(1,4) + f(2,5) + f(3,4) \\ \Delta g(3,1) + f(3,0) + f(4,1) \\ \\ \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ -1 \\ -3 \\ 11 \\ 12 \\ \\ \\ \end{bmatrix} = b$$

$$\Delta f = \Delta g \ \text{ over } \ \Omega \ \text{ with } \ f\Big|_{\partial\Omega} = f^*\Big|_{\partial\Omega}$$

f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	?	?	3	9
2	0	?	?	?	?	1
3	2	?	?	?	3	2
4	1	3	0	2	4	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
3	2	3	4	1	2	6
4	5	3	5	2	4	3
5	6	4	3	1	5	7

$$\Delta f(1,2) = 4f(1,2) - f(1,1) - f(0,2) - f(2,2) - f(1,3) = \Delta g(1,2)$$
$$\Delta f(1,3) = 4f(1,3) - f(1,2) - f(0,3) - f(2,3) - f(1,4) = \Delta g(1,3)$$
$$\Delta f(2,1) = 4f(2,1) - f(2,0) - f(1,1) - f(2,2) - f(3,1) = \Delta g(2,1)$$
$$\Delta f(2,2) = 4f(2,2) - f(2,1) - f(1,2) - f(2,3) - f(3,2) = \Delta g(2,2)$$
$$\Delta f(2,3) = 4f(2,3) - f(2,2) - f(1,3) - f(3,3) - f(2,4) = \Delta g(2,3)$$
$$\Delta f(2,4) = 4f(2,4) - f(2,3) - f(1,4) - f(2,5) - f(3,4) = \Delta g(2,4)$$
$$\Delta f(3,1) = 4f(3,1) - f(3,0) - f(2,1) - f(3,2) - f(4,1) = \Delta g(3,1)$$

$$Ax = \begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & -1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 4 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 4 & -1 & 0 \\ & & & & & & & & \\ & & & & & & & & \end{bmatrix} \begin{bmatrix} f(1,2) \\ f(1,3) \\ f(2,1) \\ f(2,2) \\ f(2,3) \\ f(2,4) \\ f(3,1) \\ f(3,2) \\ f(3,3) \end{bmatrix} = \begin{bmatrix} \Delta g(1,2) + f(1,1) + f(0,2) \\ \Delta g(1,3) + f(0,3) + f(1,4) \\ \Delta g(2,1) + f(2,0) + f(1,1) \\ \Delta g(2,2) \\ \Delta g(2,3) \\ \Delta g(2,4) + f(1,4) + f(2,5) + f(3,4) \\ \Delta g(3,1) + f(3,0) + f(4,1) \\ \\ \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ -1 \\ -3 \\ 11 \\ 12 \\ 6 \\ \\ \end{bmatrix} = b$$

$$\Delta f = \Delta g \text{ over } \Omega \text{ with } f\Big|_{\partial\Omega} = f^*\Big|_{\partial\Omega}$$

f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	?	?	3	9
2	0	?	?	?	?	1
3	2	?	?	?	3	2
4	1	3	0	2	4	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
3	2	3	4	1	2	6
4	5	3	5	2	4	3
5	6	4	3	1	5	7

$$\Delta f(1,2) = 4f(1,2) - f(1,1) - f(0,2) - f(2,2) - f(1,3) = \Delta g(1,2)$$
$$\Delta f(1,3) = 4f(1,3) - f(1,2) - f(0,3) - f(2,3) - f(1,4) = \Delta g(1,3)$$
$$\Delta f(2,1) = 4f(2,1) - f(2,0) - f(1,1) - f(2,2) - f(3,1) = \Delta g(2,1)$$
$$\Delta f(2,2) = 4f(2,2) - f(2,1) - f(1,2) - f(2,3) - f(3,2) = \Delta g(2,2)$$
$$\Delta f(2,3) = 4f(2,3) - f(2,2) - f(1,3) - f(3,3) - f(2,4) = \Delta g(2,3)$$
$$\Delta f(2,4) = 4f(2,4) - f(2,3) - f(1,4) - f(2,5) - f(3,4) = \Delta g(2,4)$$
$$\Delta f(3,1) = 4f(3,1) - f(3,0) - f(2,1) - f(3,2) - f(4,1) = \Delta g(3,1)$$
$$\Delta f(3,2) = 4f(3,2) - f(3,1) - f(2,2) - f(3,3) - f(4,2) = \Delta g(3,2)$$

$$Ax = \begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & -1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 4 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 4 & -1 & 0 \\ & & & & & & & & \end{bmatrix} \begin{bmatrix} f(1,2) \\ f(1,3) \\ f(2,1) \\ f(2,2) \\ f(2,3) \\ f(2,4) \\ f(3,1) \\ f(3,2) \\ f(3,3) \end{bmatrix} = \begin{bmatrix} \Delta g(1,2) + f(1,1) + f(0,2) \\ \Delta g(1,3) + f(0,3) + f(1,4) \\ \Delta g(2,1) + f(2,0) + f(1,1) \\ \Delta g(2,2) \\ \Delta g(2,3) \\ \Delta g(2,4) + f(1,4) + f(2,5) + f(3,4) \\ \Delta g(3,1) + f(3,0) + f(4,1) \\ \\ \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ -1 \\ -3 \\ 11 \\ 12 \\ 6 \\ \\ \end{bmatrix} = b$$

$$\Delta f = \Delta g \ \text{ over } \ \Omega \ \text{ with } \ f\big|_{\partial\Omega} = f^*\big|_{\partial\Omega}$$

f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	?	?	3	9
2	0	?	?	?	?	1
3	2	?	?	?	3	2
4	1	3	0	2	4	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
3	2	3	4	1	2	6
4	5	3	5	2	4	3
5	6	4	3	1	5	7

$$\Delta f(1,2) = 4f(1,2) - f(1,1) - f(0,2) - f(2,2) - f(1,3) = \Delta g(1,2)$$
$$\Delta f(1,3) = 4f(1,3) - f(1,2) - f(0,3) - f(2,3) - f(1,4) = \Delta g(1,3)$$
$$\Delta f(2,1) = 4f(2,1) - f(2,0) - f(1,1) - f(2,2) - f(3,1) = \Delta g(2,1)$$
$$\Delta f(2,2) = 4f(2,2) - f(2,1) - f(1,2) - f(2,3) - f(3,2) = \Delta g(2,2)$$
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$$\Delta f(3,1) = 4f(3,1) - f(3,0) - f(2,1) - f(3,2) - f(4,1) = \Delta g(3,1)$$
$$\Delta f(3,2) = 4f(3,2) - f(3,1) - f(2,2) - f(3,3) - f(4,2) = \Delta g(3,2)$$

$$Ax = \begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & -1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 4 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 4 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & -1 & 4 & -1 \\ & & & & & & & & \end{bmatrix} \begin{bmatrix} f(1,2) \\ f(1,3) \\ f(2,1) \\ f(2,2) \\ f(2,3) \\ f(2,4) \\ f(3,1) \\ f(3,2) \\ f(3,3) \end{bmatrix} = \begin{bmatrix} \Delta g(1,2) + f(1,1) + f(0,2) \\ \Delta g(1,3) + f(0,3) + f(1,4) \\ \Delta g(2,1) + f(2,0) + f(1,1) \\ \Delta g(2,2) \\ \Delta g(2,3) \\ \Delta g(2,4) + f(1,4) + f(2,5) + f(3,4) \\ \Delta g(3,1) + f(3,0) + f(4,1) \\ \\ \\ \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ -1 \\ -3 \\ 11 \\ 12 \\ 6 \\ \\ \end{bmatrix} = b$$

$$\Delta f = \Delta g \ \text{ over } \ \Omega \ \text{ with } \ f\Big|_{\partial\Omega} = f^*\Big|_{\partial\Omega}$$

f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	?	?	3	9
2	0	?	?	?	?	1
3	2	?	?	?	3	2
4	1	3	0	2	4	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
3	2	3	4	1	2	6
4	5	3	5	2	4	3
5	6	4	3	1	5	7

$$\Delta f(1,2) = 4f(1,2) - f(1,1) - f(0,2) - f(2,2) - f(1,3) = \Delta g(1,2)$$
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$$\Delta f(3,2) = 4f(3,2) - f(3,1) - f(2,2) - f(3,3) - f(4,2) = \Delta g(3,2)$$

$$Ax = \begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & -1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 4 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 4 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & -1 & 4 & -1 \\ & & & & & & & & \end{bmatrix} \begin{bmatrix} f(1,2) \\ f(1,3) \\ f(2,1) \\ f(2,2) \\ f(2,3) \\ f(2,4) \\ f(3,1) \\ f(3,2) \\ f(3,3) \end{bmatrix} = \begin{bmatrix} \Delta g(1,2) + f(1,1) + f(0,2) \\ \Delta g(1,3) + f(0,3) + f(1,4) \\ \Delta g(2,1) + f(2,0) + f(1,1) \\ \Delta g(2,2) \\ \Delta g(2,3) \\ \Delta g(2,4) + f(1,4) + f(2,5) + f(3,4) \\ \Delta g(3,1) + f(3,0) + f(4,1) \\ \Delta g(3,2) + f(4,2) \\ \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ -1 \\ -3 \\ 11 \\ 12 \\ 6 \\ \\ \end{bmatrix} = b$$

$$\Delta f = \Delta g \;\text{ over }\; \Omega \;\text{ with }\; f\Big|_{\partial\Omega} = f^*\Big|_{\partial\Omega}$$

f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	?	?	3	9
2	0	?	?	?	?	1
3	2	?	?	?	3	2
4	1	3	0	2	4	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
3	2	3	4	1	2	6
4	5	3	5	2	4	3
5	6	4	3	1	5	7

$$\Delta f(1,2) = 4f(1,2) - f(1,1) - f(0,2) - f(2,2) - f(1,3) = \Delta g(1,2)$$
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$$\Delta f(2,1) = 4f(2,1) - f(2,0) - f(1,1) - f(2,2) - f(3,1) = \Delta g(2,1)$$
$$\Delta f(2,2) = 4f(2,2) - f(2,1) - f(1,2) - f(2,3) - f(3,2) = \Delta g(2,2)$$
$$\Delta f(2,3) = 4f(2,3) - f(2,2) - f(1,3) - f(3,3) - f(2,4) = \Delta g(2,3)$$
$$\Delta f(2,4) = 4f(2,4) - f(2,3) - f(1,4) - f(2,5) - f(3,4) = \Delta g(2,4)$$
$$\Delta f(3,1) = 4f(3,1) - f(3,0) - f(2,1) - f(3,2) - f(4,1) = \Delta g(3,1)$$
$$\Delta f(3,2) = 4f(3,2) - f(3,1) - f(2,2) - f(3,3) - f(4,2) = \Delta g(3,2)$$

$$Ax = \begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & -1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 4 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 4 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & -1 & 4 & -1 \\ & & & & & & & & \end{bmatrix} \begin{bmatrix} f(1,2) \\ f(1,3) \\ f(2,1) \\ f(2,2) \\ f(2,3) \\ f(2,4) \\ f(3,1) \\ f(3,2) \\ f(3,3) \end{bmatrix} = \begin{bmatrix} \Delta g(1,2) + f(1,1) + f(0,2) \\ \Delta g(1,3) + f(0,3) + f(1,4) \\ \Delta g(2,1) + f(2,0) + f(1,1) \\ \Delta g(2,2) \\ \Delta g(2,3) \\ \Delta g(2,4) + f(1,4) + f(2,5) + f(3,4) \\ \Delta g(3,1) + f(3,0) + f(4,1) \\ \Delta g(3,2) + f(4,2) \\ \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ -1 \\ -3 \\ 11 \\ 12 \\ 6 \\ 4 \\ \end{bmatrix} = b$$

$$\Delta f = \Delta g \ \text{ over } \ \Omega \ \text{ with } \ f\Big|_{\partial\Omega} = f^*\Big|_{\partial\Omega}$$

f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	?	?	3	9
2	0	?	?	?	?	1
3	2	?	?	?	3	2
4	1	3	0	2	4	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
3	2	3	4	1	2	6
4	5	3	5	2	4	3
5	6	4	3	1	5	7

$$\Delta f(1,2) = 4f(1,2) - f(1,1) - f(0,2) - f(2,2) - f(1,3) = \Delta g(1,2)$$
$$\Delta f(1,3) = 4f(1,3) - f(1,2) - f(0,3) - f(2,3) - f(1,4) = \Delta g(1,3)$$
$$\Delta f(2,1) = 4f(2,1) - f(2,0) - f(1,1) - f(2,2) - f(3,1) = \Delta g(2,1)$$
$$\Delta f(2,2) = 4f(2,2) - f(2,1) - f(1,2) - f(2,3) - f(3,2) = \Delta g(2,2)$$
$$\Delta f(2,3) = 4f(2,3) - f(2,2) - f(1,3) - f(3,3) - f(2,4) = \Delta g(2,3)$$
$$\Delta f(2,4) = 4f(2,4) - f(2,3) - f(1,4) - f(2,5) - f(3,4) = \Delta g(2,4)$$
$$\Delta f(3,1) = 4f(3,1) - f(3,0) - f(2,1) - f(3,2) - f(4,1) = \Delta g(3,1)$$
$$\Delta f(3,2) = 4f(3,2) - f(3,1) - f(2,2) - f(3,3) - f(4,2) = \Delta g(3,2)$$
$$\Delta f(3,3) = 4f(3,3) - f(3,2) - f(2,3) - f(3,4) - f(4,3) = \Delta g(3,3)$$

$$Ax = \begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & -1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 4 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 4 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & -1 & 4 & -1 \\ & & & & & & & & \end{bmatrix} \begin{bmatrix} f(1,2) \\ f(1,3) \\ f(2,1) \\ f(2,2) \\ f(2,3) \\ f(2,4) \\ f(3,1) \\ f(3,2) \\ f(3,3) \end{bmatrix} = \begin{bmatrix} \Delta g(1,2) + f(1,1) + f(0,2) \\ \Delta g(1,3) + f(0,3) + f(1,4) \\ \Delta g(2,1) + f(2,0) + f(1,1) \\ \Delta g(2,2) \\ \Delta g(2,3) \\ \Delta g(2,4) + f(1,4) + f(2,5) + f(3,4) \\ \Delta g(3,1) + f(3,0) + f(4,1) \\ \Delta g(3,2) + f(4,2) \\ \\ \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ -1 \\ -3 \\ 11 \\ 12 \\ 6 \\ 4 \\ \\ \end{bmatrix} = b$$

$$\Delta f = \Delta g \ \text{ over } \ \Omega \ \text{ with } \ f\Big|_{\partial\Omega} = f^*\Big|_{\partial\Omega}$$

f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	?	?	3	9
2	0	?	?	?	?	1
3	2	?	?	?	3	2
4	1	3	0	2	4	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
3	2	3	4	1	2	6
4	5	3	5	2	4	3
5	6	4	3	1	5	7

$$\Delta f(1,2) = 4f(1,2) - f(1,1) - f(0,2) - f(2,2) - f(1,3) = \Delta g(1,2)$$
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$$\Delta f(2,1) = 4f(2,1) - f(2,0) - f(1,1) - f(2,2) - f(3,1) = \Delta g(2,1)$$
$$\Delta f(2,2) = 4f(2,2) - f(2,1) - f(1,2) - f(2,3) - f(3,2) = \Delta g(2,2)$$
$$\Delta f(2,3) = 4f(2,3) - f(2,2) - f(1,3) - f(3,3) - f(2,4) = \Delta g(2,3)$$
$$\Delta f(2,4) = 4f(2,4) - f(2,3) - f(1,4) - f(2,5) - f(3,4) = \Delta g(2,4)$$
$$\Delta f(3,1) = 4f(3,1) - f(3,0) - f(2,1) - f(3,2) - f(4,1) = \Delta g(3,1)$$
$$\Delta f(3,2) = 4f(3,2) - f(3,1) - f(2,2) - f(3,3) - f(4,2) = \Delta g(3,2)$$
$$\Delta f(3,3) = 4f(3,3) - f(3,2) - f(2,3) - f(3,4) - f(4,3) = \Delta g(3,3)$$

$$Ax = \begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & -1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 4 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 4 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & -1 & 4 & -1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} f(1,2) \\ f(1,3) \\ f(2,1) \\ f(2,2) \\ f(2,3) \\ f(2,4) \\ f(3,1) \\ f(3,2) \\ f(3,3) \end{bmatrix} = \begin{bmatrix} \Delta g(1,2) + f(1,1) + f(0,2) \\ \Delta g(1,3) + f(0,3) + f(1,4) \\ \Delta g(2,1) + f(2,0) + f(1,1) \\ \Delta g(2,2) \\ \Delta g(2,3) \\ \Delta g(2,4) + f(1,4) + f(2,5) + f(3,4) \\ \Delta g(3,1) + f(3,0) + f(4,1) \\ \Delta g(3,2) + f(4,2) \\ \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ -1 \\ -3 \\ 11 \\ 12 \\ 6 \\ 4 \\ \end{bmatrix} = b$$

$$\Delta f = \Delta g \ \text{ over } \ \Omega \ \text{ with } \ f\Big|_{\partial\Omega} = f^*\Big|_{\partial\Omega}$$

f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	?	?	3	9
2	0	?	?	?	?	1
3	2	?	?	?	3	2
4	1	3	0	2	4	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
3	2	3	4	1	2	6
4	5	3	5	2	4	3
5	6	4	3	1	5	7

$$\Delta f(1,2) = 4f(1,2) - f(1,1) - f(0,2) - f(2,2) - f(1,3) = \Delta g(1,2)$$
$$\Delta f(1,3) = 4f(1,3) - f(1,2) - f(0,3) - f(2,3) - f(1,4) = \Delta g(1,3)$$
$$\Delta f(2,1) = 4f(2,1) - f(2,0) - f(1,1) - f(2,2) - f(3,1) = \Delta g(2,1)$$
$$\Delta f(2,2) = 4f(2,2) - f(2,1) - f(1,2) - f(2,3) - f(3,2) = \Delta g(2,2)$$
$$\Delta f(2,3) = 4f(2,3) - f(2,2) - f(1,3) - f(3,3) - f(2,4) = \Delta g(2,3)$$
$$\Delta f(2,4) = 4f(2,4) - f(2,3) - f(1,4) - f(2,5) - f(3,4) = \Delta g(2,4)$$
$$\Delta f(3,1) = 4f(3,1) - f(3,0) - f(2,1) - f(3,2) - f(4,1) = \Delta g(3,1)$$
$$\Delta f(3,2) = 4f(3,2) - f(3,1) - f(2,2) - f(3,3) - f(4,2) = \Delta g(3,2)$$
$$\Delta f(3,3) = 4f(3,3) - f(3,2) - f(2,3) - f(3,4) - f(4,3) = \Delta g(3,3)$$

$$Ax = \begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & -1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 4 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 4 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & -1 & 4 & -1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} f(1,2) \\ f(1,3) \\ f(2,1) \\ f(2,2) \\ f(2,3) \\ f(2,4) \\ f(3,1) \\ f(3,2) \\ f(3,3) \end{bmatrix} = \begin{bmatrix} \Delta g(1,2) + f(1,1) + f(0,2) \\ \Delta g(1,3) + f(0,3) + f(1,4) \\ \Delta g(2,1) + f(2,0) + f(1,1) \\ \Delta g(2,2) \\ \Delta g(2,3) \\ \Delta g(2,4) + f(1,4) + f(2,5) + f(3,4) \\ \Delta g(3,1) + f(3,0) + f(4,1) \\ \Delta g(3,2) + f(4,2) \\ \Delta g(3,3) + f(4,3) + f(3,4) \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ -1 \\ -3 \\ 11 \\ 12 \\ 6 \\ 4 \\ \ \end{bmatrix} = b$$

$$\Delta f = \Delta g \;\text{ over }\; \Omega \;\text{ with }\; f\Big|_{\partial\Omega} = f^*\Big|_{\partial\Omega}$$

f

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	0	?	?	3	9
2	0	?	?	?	?	1
3	2	?	?	?	3	2
4	1	3	0	2	4	5
5	2	6	7	1	2	6

g

	0	1	2	3	4	5
0	7	6	5	3	5	2
1	5	0	4	2	0	1
2	3	2	3	5	3	0
3	2	3	4	1	2	6
4	5	3	5	2	4	3
5	6	4	3	1	5	7

$$\begin{aligned}
\Delta f(1,2) &= 4f(1,2) - f(1,1) - f(0,2) - f(2,2) - f(1,3) = \Delta g(1,2)\\
\Delta f(1,3) &= 4f(1,3) - f(1,2) - f(0,3) - f(2,3) - f(1,4) = \Delta g(1,3)\\
\Delta f(2,1) &= 4f(2,1) - f(2,0) - f(1,1) - f(2,2) - f(3,1) = \Delta g(2,1)\\
\Delta f(2,2) &= 4f(2,2) - f(2,1) - f(1,2) - f(2,3) - f(3,2) = \Delta g(2,2)\\
\Delta f(2,3) &= 4f(2,3) - f(2,2) - f(1,3) - f(3,3) - f(2,4) = \Delta g(2,3)\\
\Delta f(2,4) &= 4f(2,4) - f(2,3) - f(1,4) - f(2,5) - f(3,4) = \Delta g(2,4)\\
\Delta f(3,1) &= 4f(3,1) - f(3,0) - f(2,1) - f(3,2) - f(4,1) = \Delta g(3,1)\\
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\Delta f(3,3) &= 4f(3,3) - f(3,2) - f(2,3) - f(3,4) - f(4,3) = \Delta g(3,3)
\end{aligned}$$

$$Ax = \begin{bmatrix}
4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0\\
-1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 & 0\\
0 & 0 & 4 & -1 & 0 & 0 & -1 & 0 & 0\\
-1 & 0 & -1 & 4 & -1 & 0 & 0 & -1 & 0\\
0 & -1 & 0 & -1 & 4 & -1 & 0 & 0 & -1\\
0 & 0 & 0 & 0 & -1 & 4 & 0 & 0 & 0\\
0 & 0 & -1 & 0 & 0 & 0 & 4 & -1 & 0\\
0 & 0 & 0 & -1 & 0 & 0 & -1 & 4 & -1\\
0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 4
\end{bmatrix}
\begin{bmatrix} f(1,2)\\ f(1,3)\\ f(2,1)\\ f(2,2)\\ f(2,3)\\ f(2,4)\\ f(3,1)\\ f(3,2)\\ f(3,3) \end{bmatrix}
= \begin{bmatrix}
\Delta g(1,2) + f(1,1) + f(0,2)\\
\Delta g(1,3) + f(0,3) + f(1,4)\\
\Delta g(2,1) + f(2,0) + f(1,1)\\
\Delta g(2,2)\\
\Delta g(2,3)\\
\Delta g(2,4) + f(1,4) + f(2,5) + f(3,4)\\
\Delta g(3,1) + f(3,0) + f(4,1)\\
\Delta g(3,2) + f(4,2)\\
\Delta g(3,3) + f(4,3) + f(3,4)
\end{bmatrix}
= \begin{bmatrix} 8\\ 7\\ -1\\ -3\\ 11\\ 12\\ 6\\ 4\\ -4 \end{bmatrix} = b$$

	0	1	2	3	4	5
0	2	3	2	8	5	0
1	1	?	4	0	3	9
2	0	?	?	?	?	1
3	5	2	?	?	?	2
4	1	3	0	?	3	5
5	2	6	7	1	2	6

	0	1	2	3	4	5
0	5	3	7	6	0	1
1	1	2	4	3	2	3
2	2	1	3	5	1	6
3	1	3	4	1	2	5
4	3	5	4	7	6	4
5	2	3	6	5	4	5

Recap

Difference between gradient of interpolation and guidance vector

v : *Guidance Vector*
g : *Source Image*
f : *Interpolation*
f^* : *Destination Image in Blending Area*
Ω : *Blending Area*
$\partial\Omega$: *Boundary of Blending Area*

$$\min_f \iint_\Omega |\nabla f - v|^2 \quad \text{s.t.} \quad f\big|_{\partial\Omega} = f^*\big|_{\partial\Omega}$$

Boundary

We want to minimize the difference.

The value on the boundary of the blending area must be the same.

∇ : Gradient Operator

We use gradient of source image as guidance vector : $v = \nabla g$

Recap

Can we use other guidance vectors to achieve better blending effect?

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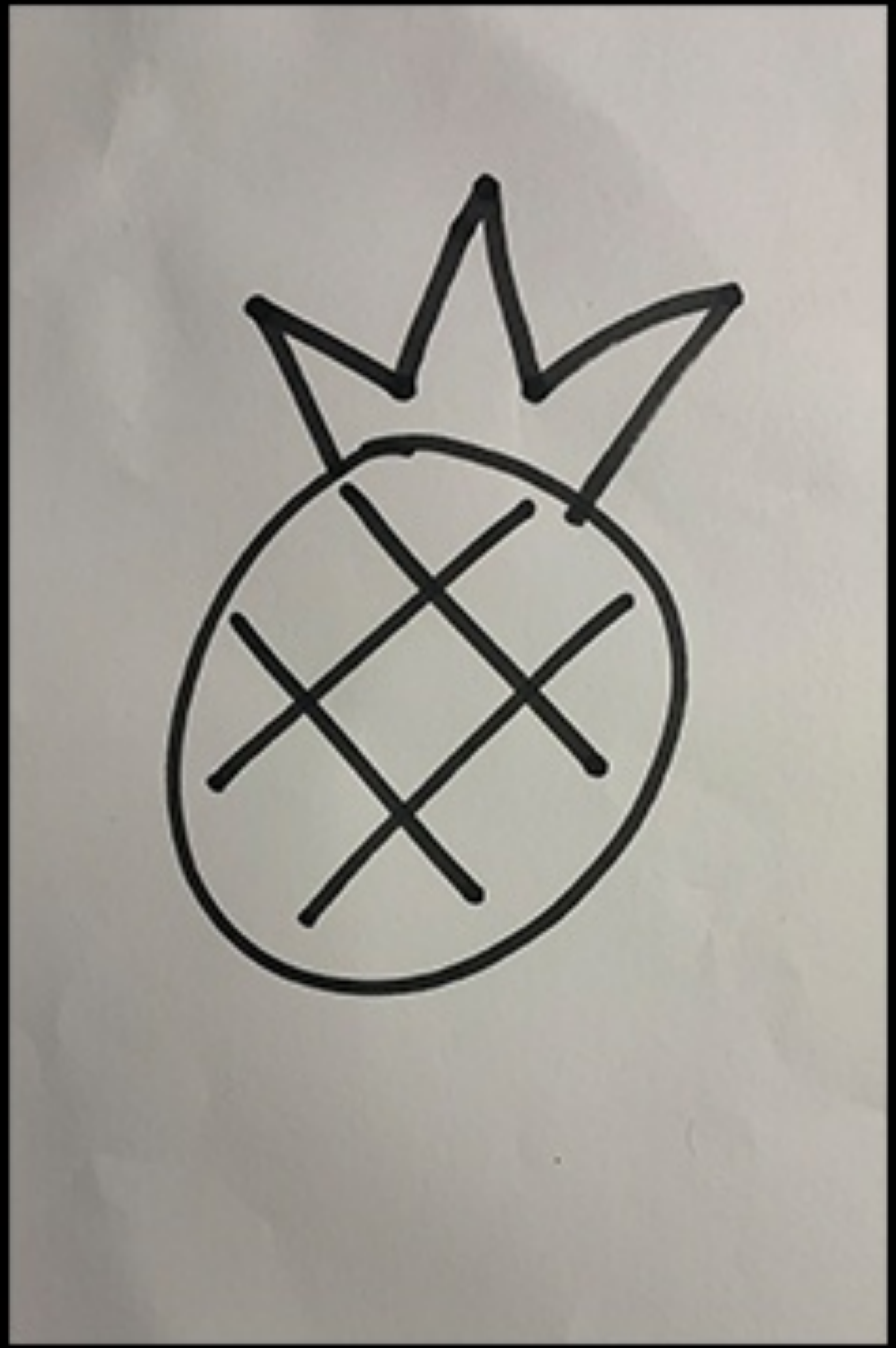


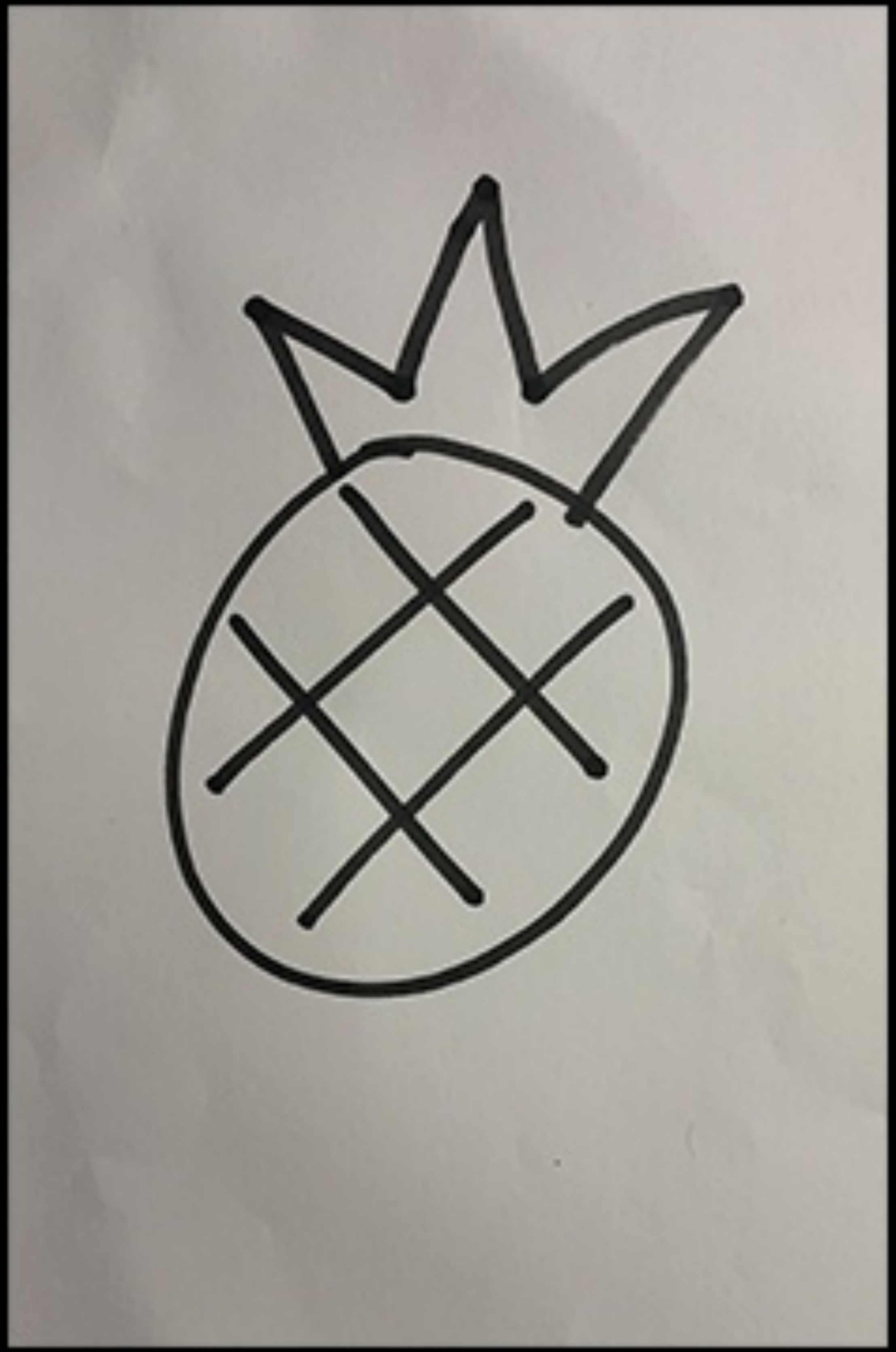


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Can we use other guidance vectors to achieve better blending effect?

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$$\min_f \iint_\Omega |\nabla f - v|^2 \quad \text{s.t. } f\big|_{\partial\Omega} = f^*\big|_{\partial\Omega}$$

Boundary

We want to minimize the difference.

The value on the boundary of the blending area must be the same.

∇ : Gradient Operator

$$v = \nabla g$$

We can use the mixture of gradient of both source and destination image as guidance vector so that we can keep the feature of background.

Recap

Can we use other guidance vectors to achieve better blending effect?

Difference between gradient of interpolation and guidance vector

v	: *Guidance Vector*
g	: *Source Image*
f	: *Interpolation*
f^*	: *Destination Image in Blending Area*
Ω	: *Blending Area*
$\partial\Omega$	: *Boundary of Blending Area*

$$\min_f \iint_\Omega |\nabla f - v|^2 \quad \text{s.t.} \quad f\big|_{\partial\Omega} = f^*\big|_{\partial\Omega}$$

Boundary

We want to minimize the difference.

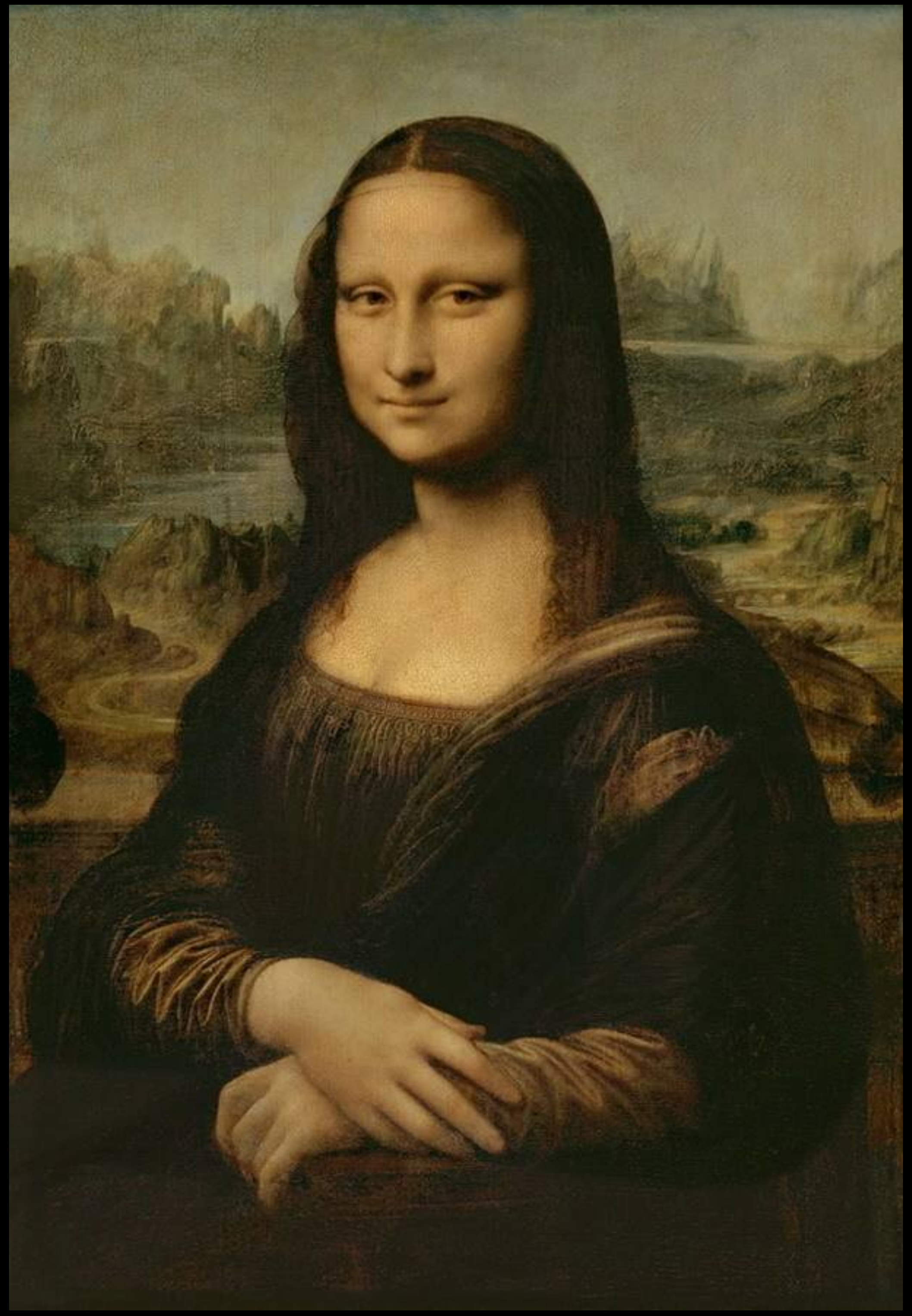
The value on the boundary of the blending area must be the same.

∇ : Gradient Operator

$$v = \max\{\nabla f^*, \nabla g\}$$

We can use the mixture of gradient of both source and destination image as guidance vector so that we can keep the feature of background.


Let’s BLEND !



Use *Monna Lisa* as destination image and create your mysterious smile.

Unleash your creativity and create your interesting blending result.