Finding correlation for acmPRS

November 9, 2016

Goals:

- Find $Var(acmPRS)$
- Find $Cov(Y, acmPRS)$

0.1 Finding $Var(acmPRS)$:

Define

$$\hat{S}_i = \sum_{j=1}^{m_i} \hat{\beta}\mathbf{G_j}$$
$$\hat{S}_{acm} = \sum_{i=1}^{c} n_i r_i \sqrt{\frac{h_i}{h_1}} \hat{S}_i$$

Theorem 1. $E[\hat{S}_{acm}] = 0$

Proof.

$$\begin{aligned} E[\hat{S}_{acm}] &= E[\sum_{i=1}^{c} n_i r_i \sqrt{\frac{h_i}{h_1}} \hat{S}_i] \\ &= \sum_{i=1}^{c} E[n_i r_i \sqrt{\frac{h_i}{h_1}} \hat{S}_i] \\ &= \sum_{i=1}^{c} n_i r_i \sqrt{\frac{h_i}{h_1}} E[\hat{S}_i] \end{aligned}$$

$$\because E[\hat{S}_i] = 0$$
$$\therefore E[\hat{S}_{acm}] = 0$$

□

Theorem 2. $Var(\hat{S}_{acm}) = \sum_{i=1}^{c} \frac{m_i n_i^2 r_i^2 h_i}{h_1} Var(\hat{\beta}_i) + 2\sum_{i\neq j}^{c} n_i n_j r_i r_j \sqrt{\frac{h_i h_j}{h_1^2}} Cov(\hat{S}_i, \hat{S}_j)]$

Proof.

$$Var(\sum_{i=1}^{c} n_i r_i \sqrt{\frac{h_i}{h_1}} \hat{S}_i) = \sum_{i=1}^{c} Var\left(n_i r_i \sqrt{\frac{h_i}{h_1}} \hat{S}_i\right) + \sum_{i \neq j} Cov\left(n_i r_i \sqrt{\frac{h_i}{h_1}} \hat{S}_i, n_j r_j \sqrt{\frac{h_j}{h_1}} \hat{S}_j\right)$$

$$\begin{aligned}
\sum_{i=1}^{c} Var\left(n_i r_i \sqrt{\frac{h_i}{h_1}} \hat{S}_i\right) &= \sum_{i=1}^{c} n_i r_i \sqrt{\frac{h_i}{h_1}} Var(\hat{S}_i) \\
&= \sum_{i=1}^{c} n_i r_i \sqrt{\frac{h_i}{h_1}} m_i Var(\hat{\beta}_i) \\
\sum_{i \neq j} Cov\left(n_i r_i \sqrt{\frac{h_i}{h_1}} \hat{S}_i, n_j r_j \sqrt{\frac{h_j}{h_1}} \hat{S}_j\right) &= 2 \sum_{i \neq j} E\left[\left(n_i r_i \sqrt{\frac{h_i}{h_1}} \hat{S}_i\right), \left(n_j r_j \sqrt{\frac{h_j}{h_1}} \hat{S}_j\right)\right]
\end{aligned}$$

□