SI152 Final Project : Simplex Linear Solver

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1. Intro

The project is to implement a primal simplex method for solving the *standard form* of *linear programming* problem which can be formulated as :

$$\min_{x \in \mathbb{R}^n} \quad c^T x \quad \text{s.t. } Ax = b, x \geq 0$$

where $A \in \mathbb{R}^{m \times n}, c \in \mathbb{R}^n$ and $b \in \mathbb{R}^m$

The input provided for the solver should be A, b and c and the output of the solver will be the optimal objective f and the values of variables x^*.

2. Usage

The solver is implemented in python.The workspace contains *simplex.py* where the solver is implemented and *data* folder. To use the solver in the command line, user should prepare the A, b and c in .csv format respectively and put them in the same path.

A typical test data should look like:

```
—— workspace
    - simplex.py
    —— datafolder
        - A.csv
        - b.csv
        - c.csv
```

The command to use the solver:

python3 simplex.py -p path_to_datafolder

e.g. *python3 simplex.py -p ./data/test1*

The result will be printed in the terminal.

Notice: The dependency library of this solver is Numpy (make sure Numpy is installed)

3. Implementation Details

What the program mainly does is to simulate how the simplex table functions. The simplex table can be printed in any step of the whole algorithm (if you want). I implemented a two-stage simplex method.The first stage is used to identify whether the constraints can be satisfiable and whether there is redundant constraints. After the first stage, a basic solution will be achieved and this initial setting will used to solve the origin problem.

The main structure of the code:

- The class *stdLP* is implemented to store the information of the problem. The *auxiliaryProblem* method is provided to generate the auxiliary problem of the origin linear programming problem. The auxiliary problem is solved in the first stage.
- The *simplexTable* function is implemented to generate the simplex tableau according to the input of an *stdLP* class.
- The *solve* function is the one of the core parts of the whole program. It will keep selecting pivot and perform gaussian elimination according to the pivot until all reduce costs are non-negative.
- The *interpret* function is implemented to interpret the simplex table after *solve* the simplex table. The function will find the index and value of decision variables and the optimal objective value according to the table.
- The *firstPhase* function is implemented to perform the first stage of two-stage method. It will solve the auxiliary problem of the origin problem at first and keep elimination according whether there is still auxiliary variables in the bases. If the constraints of the origin problem are satisfiable, the firstPhase will return the simplex table which contains a basic solution and the redundant constraints and auxiliary variables will be removed.
- The *simplex* function is the main function of the program which call *firstPhase* function and *solve* the simplex table returned by the *firstPhase.*

There are some trivial functions which I didn't mention in the code. Please read the code.

4. Experiment Result

Test case 1:

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 3 & 3 & 1 \end{bmatrix} \; B = \begin{bmatrix} 4 \\ 3 \end{bmatrix} \; C = [4 \quad 1 \quad 1]$$

Result:

```
The solution is  [0.  0.4 1.8]
The optimal is  2.1999999999999993
```

Test case 2（Case from HW3）:

$$A = \begin{bmatrix} -1 & 2 & 1 \\ -4 & 4 & -1 \\ -5 & 6 & 0 \\ 1 & 0 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 2 \\ 4 \\ 6 \\ 0 \end{bmatrix} \quad C = [1 \quad -1 \quad 0]$$

Result:
The simplex table generated by the first phase:

```
[[ 0.  1.  0.  1.]
 [ 1.  0.  0.  0.]
 [-0. -0.  1. -0.]
 [ 1. -1.  0.  0.]]
```

The solver can remove the redundant constraints correctly.

```
The solution is  [ 0.  1. -0.]
The optimal is  -1.0
```

5. Comment

Because of my poor understanding of simplex algorithm, the implementation is some kind awkward. The program is just to simulate the manual process of operating a simplex table. But it works very well.