HW 11

Charles Liu
Linear Optimization - Dr. Tom Asaki
April 8th, 2025

Consider the LP

$$\min_x f(x) = x_1 + 2x_2 + 3x_3$$
$$\text{s.t. } x_1 + x_2 + x_3 = 3$$
$$x \geq 0$$
$$x \in \mathbb{R}^3$$

A. Using a logarithmic barrier interior point approach, show that as $\mu \to 0, x(\mu)$ converges to the unique optimal solution. [You do not need to analytically solve for the central path.]

$$\min\ f(x) - \mu \sum_{j=1}^{3} \log(x_j)$$

As μ goes to 0, we get

$$\min\ f(x)$$

so it just becomes the original problem. The solution of this is the unique optimal solution, so we are done.

B. Find the analytic center of the feasible region by carefully considering the limit $\mu \to \infty$.

Taking $\mu \to \infty$, the problem then becomes

$$\min -\mu \sum_{j=1}^{3} \log(x_j)$$

as $f(x)$ is no longer relevant (the μ portion simply is weighted much higher)

We can solve a problem with the equivalent solution

$$\max \log(x_1 x_2 x_3)$$

This is minimized when $x_j = 1$ for $j = 1, 2, 3$. This satisfies the constraints as $x_1 + x_2 + x_3 = 1 + 1 + 1 = 3$, and $x_1, x_2, x_3 \geq 0$.

The intuition is that any value less than 1 for x_j would negatively contribute overall, and any value greater than 1 forces at least one other value to be less than 1.

Thus, the analytic center is $(1, 1, 1)$