HW 6

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The sun and solar system formed from a low-density cloud of gas. The force of gravity attracted the gas to itself, causing it to shrink ("collapse") to smaller size and higher density. Basic aspects of this process can be understood using the principle of conservation of energy.

USEFUL FORMULAE

The gravitational potential energy of a uniform density sphere of mass M and radius R is $E_G = -\frac{3}{5}\frac{GM^2}{R}$

The orbital period for orbiting an object of mass M at distance R is $P = \sqrt{\frac{4\pi^2 R^3}{GM}}$

The gravitational constant is $G = 6.67 \times 10^{-11} \mathrm{N\ m^2\ kg^{-2}}$

luminosity $= \frac{\text{amount of energy radiated}}{\text{time it takes to radiate it}}$

The luminosity of a blackbody is $L = \sigma T^4 A$, where $\sigma = 5.7 \times 10^{-8}\ \mathrm{W\ m^{-2} K^{-4}}$, T is the surface temperature and A is the surface area.

PROBLEM SETUP

A cloud fragment that collapses to form a sun-like star might have a mass $M = M_\odot = 2 \times 10^{30}$ kg and radius $R_i = 10^{12}$km. It will collapse until it becomes dense and hot enough for pressure to resist gravity, when its radius has shrunk to a final radius, which we will estimate as $R_f = 10^7$ km.

A) The time it takes gravity to collapse the cloud, called the "free-fall time" or τ_{ff}, is roughly the same as the orbital period for an object at radius R_i to orbit an object of mass M. (For both orbiting and collapsing, we are talking about falling under the influence of gravity, so maybe it's not surprising that the time for both processes is similar.) Compute τ_{ff} for this cloud.

$$\tau_{ff} \approx P = \sqrt{\frac{4\pi^2(10^{12}\times10^3)^3}{6.67\times10^{-11}\times2\times10^{30}}} = 1.72\times10^{13}\text{s}$$

B) As the cloud fragment collapses, it sinks deeper into its gravitational potential, releasing gravitational potential energy. The amount of energy released ΔE is just the difference between the initial and final gravitational potential energy. Compute ΔE. Also, explain in your own words why energy is released rather than absorbed by the collapse.

$$\begin{aligned}\Delta E &= -\frac{3}{5}\frac{GM^2}{R_i} + \frac{3}{5}\frac{GM^2}{R_f} \\ &= -\frac{3}{5}\frac{6.67\times10^{-11}(2\times10^{30})^2}{10^{12}\times10^3} + \frac{3}{5}\frac{6.67\times10^{-11}(2\times10^{30})^2}{10^7\times10^3} \\ &= 1.6\times10^{40}\text{ Joules}\end{aligned}$$

C) Half of the ΔE is converted into radiation energy. (The other half goes into thermal energy of the gas.) Let us assume this energy is radiated at a constant rate over a time τ_{ff}. Under this assumption, compute the radiated energy per time (the luminosity, with units Joule per second = Watts). Look up the current luminosity of the sun. How does the luminosity of the collapsing cloud compare to the luminosity of the sun?

$$\text{luminosity} = \frac{0.5\times1.6\times10^{40}}{1.72\times10^{13}} = 4.65\times10^{26}\text{ watts}$$

Luminosity of the sun = 3.86×10^{26} watts

The luminosity of the collapsing cloud is larger than the luminosity of the sun, but of the same magnitude.

D) Once the gas reaches a radius R_f, it stops collapsing. It is not yet hot enough to have the nuclear reactions that would make it a star; it is called a "protostar". The protostar is a dense, opaque ball of gas with surface temperature of about $4000K$. The protostar will continue to shrink (just much more slowly) if it is losing energy. Explain the mechanism that will certainly carry energy away from the protostar. Compute how much gravitational potential energy must be reduced for it to shrink to the size of the sun:
$R_\odot = 1.4 \times 10^6$ km.

The protostar is certain to emit blackbody radiation. Since it is around $4000K$ (which is very hot), it will emit lots of infrared radiation due to blackbody radiation (which is dependent on temperature).

$$\Delta E = -\frac{3\times6.67\times10^{-11}\times(2\times10^{30})^2}{5\times10^7} + \frac{3\times6.67\times10^{-11}\times(2\times10^{30})^2}{5\times1.4\times10^6\times10^3} = 9.833 \times 10^{40} \text{ Joules}$$

E) Given the radiation laws you know, what will the spectrum of the protostar look like (continuous, emission line, or absorption line)? Explain your reasoning.

Since the object emits blackbody radiation since it's very hot, it will emit a continuous spectrum. However, since it's outer layers have cooled down, it will absorb certain light wavelengths and produce absorption lines.

F) As the protostar shrinks while its surface temperature stays the same, what do you think will happen to the luminosity? Explain. (Hint: shrinking means decreasing surface area.)

It's luminosity will go down. Luminosity is dependent on the surface temperature and surface area. If the surface temperature or surface area go up or down, the luminosity will go up or down respectively. Thus, if the surface temperature stays the same while the surface temperature goes down, then the luminosity will go down.

Recall: $L = \sigma T^4 A$

Citation

“Luminosity.” Encyclopædia Britannica. Accessed March 4, 2025. https://www.britannica.com/science/luminosity.