

# Extra Credit

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Linear Optimization - Dr. Tom Asaki

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## Extra Credit Challenge

Construct linear constraints that model the given situation (where all variables are binary)

② 1

$$x = 1 \text{ or } y = 1$$

This is an **OR** operation, so our constraints are:

*Linear constraint:*

$$x + y \geq 1$$

② 2

$$x = 1 \text{ xor } y = 1$$

*Linear constraint:*

$$x + y = 1$$

② 3

Exactly 5 of  $\{x_1, x_2, \dots, x_n\}$  are 1.

*Linear constraint:*

$$x_1 + x_2 + \dots + x_n = 5$$

② 4

If  $x = 1$  then  $y = 1$

*Linear constraint:*

$$x = y$$

⑤

If  $x = 1$ , then  $y = 0$

*Linear constraint:*

$$x + y = 1$$

⑥

If  $x_1 = 1$  and  $x_2 = 1$  then  $x_3 = 0$

*Linear constraint:*

$$(x_1 + x_2) + x_3 \leq 2$$

⑦

If  $x_1 = 1$  and  $x_2 = 0$  then  $x_3 = 0$

*Linear constraint:*

$$x_1 - x_2 + x_3 \leq 1$$

⑧

$$x = y_1 \vee y_2$$

*Linear constraint:*

$$y_1 + y_2 \geq x$$

⑨

$$x = y_1 \wedge y_2$$

Linear constraint:

$$y_1 + y_2 \geq 2x$$

Table of Operations for Reference

For  $x, y$  binary variables,

Operation	
AND	$x + y = 2$
OR	$x + y \geq 1$
NAND	$x + y \leq 1$
NOR	$x + y = 0$
XOR	$x + y = 1$

Constructed By Observing the Sums of Binary Variables in a K-map

$y \backslash x$	0	1
0	AND: 0 OR: 0 NAND: 1 NOR: 1 XOR: 0	AND: 0 OR: 1 NAND: 1 NOR: 0 XOR: 1
1	AND: 0 OR: 1 NAND: 1 NOR: 0 XOR: 1	AND: 1 OR: 1 NAND: 0 NOR: 0 XOR: 0

