HW 4

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Linear Optimization - Dr. Tom Asaki
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1. A farmer is planning an orchard of mixed apple, pear and cherry trees that can hold a maximum of 345 trees. The seasonal cost of labor per tree is \$150, \$200 and \$240, respectively.. The seasonal cost of materials per tree is \$275, \$180 and \$125, respectively. Planting logistics require that IF any apple trees are planted then at least 150 must be planted. Similar limits on pear and cherry trees are 50 and 80, respectively. Write and solve an optimization model that maximizes the number of trees which the farmer can plant when labor costs are limited to a maximum of \$50,000 and materials costs are limited to a maximum of \$60,000.

$$\max a + p + c$$

$$y_1 = \begin{cases} 1 & a > 0 \\ 0 & a = 0 \end{cases}$$
$$y_2 = \begin{cases} 1 & p > 0 \\ 0 & p = 0 \end{cases}$$
$$y_3 = \begin{cases} 1 & c > 0 \\ 0 & c = 0 \end{cases}$$

$$a + p + c \leq 345$$
$$150y_1 \leq a \leq 345y_1$$
$$50y_2 \leq p \leq 345y_2$$
$$80y_3 \leq c \leq 345y_3$$

$$150a + 200p + 240c \leq 50000$$
$$275a + 180p + 125c \leq 60000$$

To maximize planting, we plant 172 apple trees, 0 pear trees, and 100 cherry trees for a total of 272 trees.

2. Radio station KOPT is studying where to place radio towers in the hills around Dennis County. Four tower sites are under consideration. The costs of building on sites A, B, C and D are $3500, $4000, $4500 and $2700, respectively. A tower on each site can reach one or more of ten districts of varying populations as shown in the table below, where populations are shown in thousands. For example, sites B and D can both broadcast to district 7 which has a population of 12,000. Construct and solve an integer program the provides the maximum radio coverage, by population, if KOPT has $10,000 for construction. Be careful to count any covered district only once.

Site\District	D1	D2	D2	D4	D5	D6	D7	D8	D9	D
A	30	12		21	13	4				
B		12	28		13	4	12	20		
C				21	13	4		20	11	2
D						4	12	20	11	

$$\max D = yp$$

$$p = [30 \quad 12 \quad 28 \quad 21 \quad 13 \quad 4 \quad 12 \quad 20 \quad 11 \quad 25]^T$$

$$y = [y_1 \quad y_2 \quad y_3 \quad y_4 \quad y_5 \quad y_6 \quad y_7 \quad y_8 \quad y_9 \quad y_{10} \quad]$$

Where

$$y_i = \begin{cases} 1 & P_i x \geq 1 \\ 0 & P_i x = 0 \end{cases}$$

$$P = [P_1 \quad P_2 \quad P_3 \quad P_4 \quad P_5 \quad P_6 \quad P_7 \quad P_8 \quad P_9 \quad P_{10} \quad]$$

$$= \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$x = [A \quad B \quad C \quad D]$$

$$3500A + 4000B + 4500C + 2700D \leq 10000$$

Putting this through software, we get that we should buy towers at B and C, getting to $146,000$ people.