

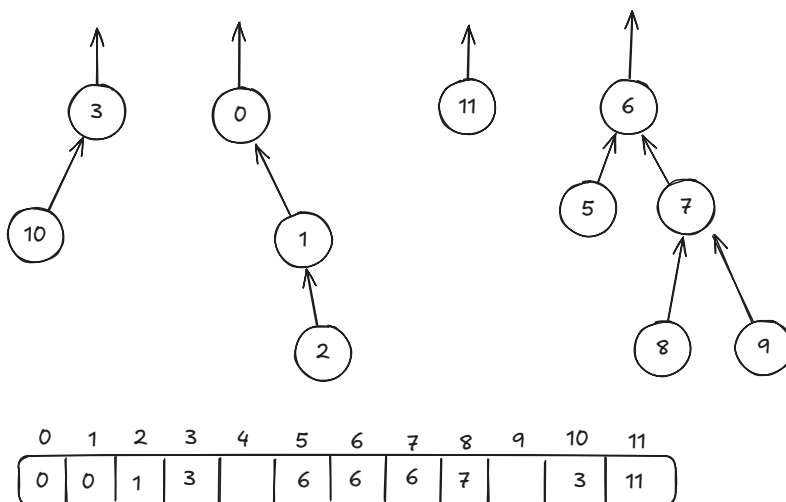
HW 5

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Advanced Data Structures - Dr. Subu Kandaswamy

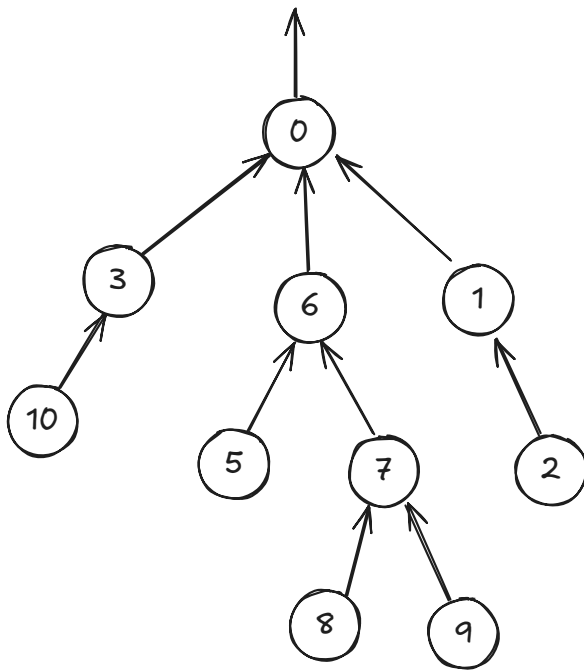
April 23rd, 2025

1. For the union-find data structure shown below, show the *array* representation



2. We are given two programs *A* and *B* that use two different implementations of the union-find data structure. Program *A* applies path compression when it performs each `find()` operation; whereas Program *B* *does not* apply path compression for its `find()` function calls.

a) Both programs start off with the same initial union-find data structure shown below.



Both programs perform the same sequence of `find()` operations (in the specified order):

`find(8) → find(7) → find(10) → find(7) → find(9) → find(9) → find(2)`

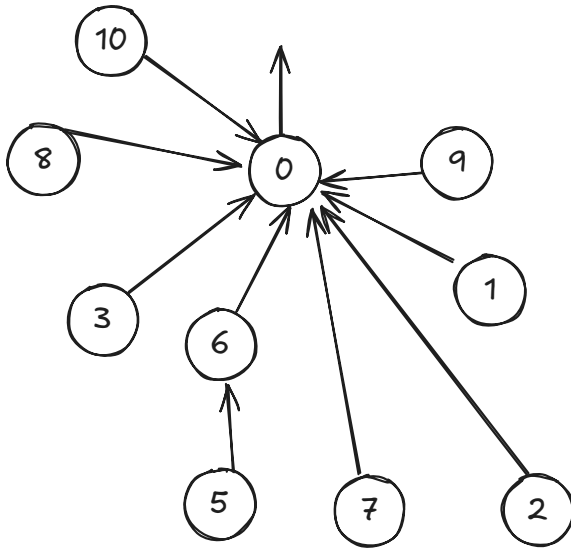
Calculate the number of *steps* each of the above `find()` operations takes to climb from the element being searched to the root node.

Give your answer by filling the number of steps for each `find()` operation in the table below:

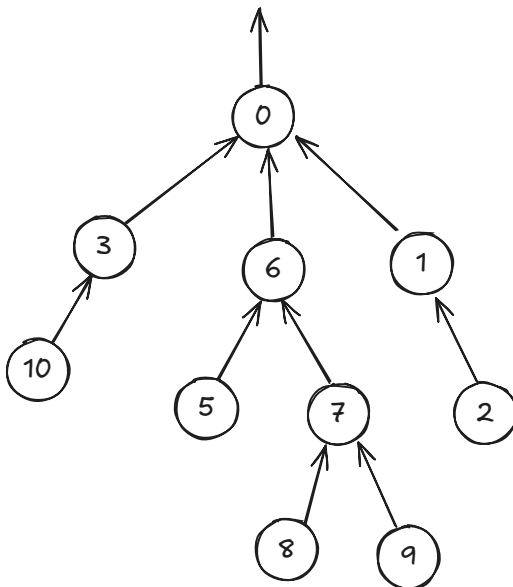
	Program A (w/ path compression)	Program B (w/o path compression)
1. <code>find(8)</code>	3	3
2. <code>find(7)</code>	2	2
3. <code>find(10)</code>	2	2
4. <code>find(7)</code>	1	2
5. <code>find(9)</code>	2	3
6. <code>find(2)</code>	2	2
Total Steps	12	14

b) Also in the empty space provided below, show the final trees resulting from both programs, after the last `find()` operation (i.e., `find(2)`)

With path compressions



Without path compressions

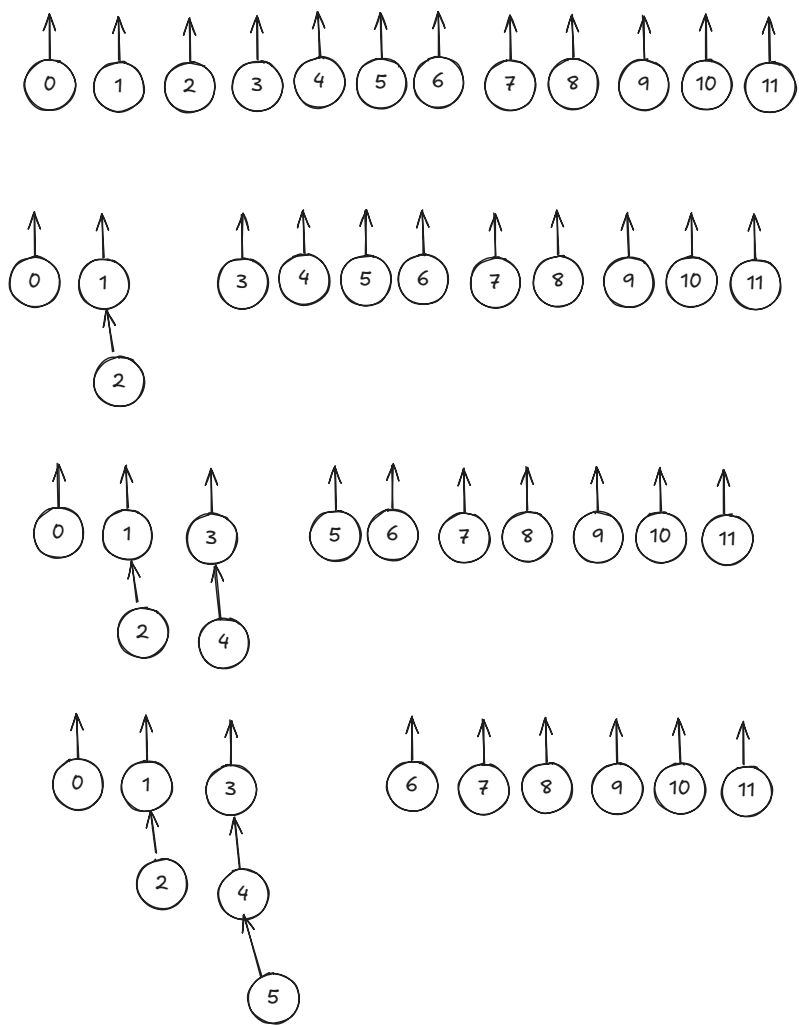


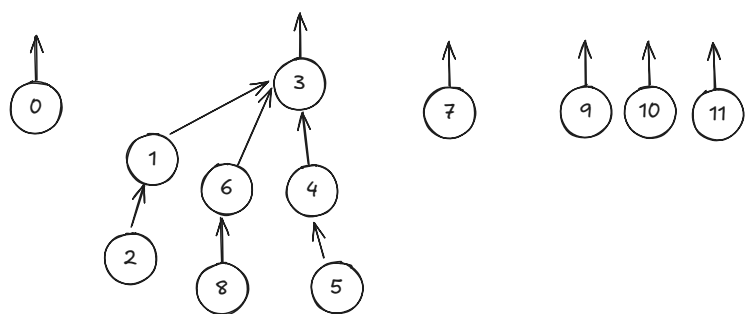
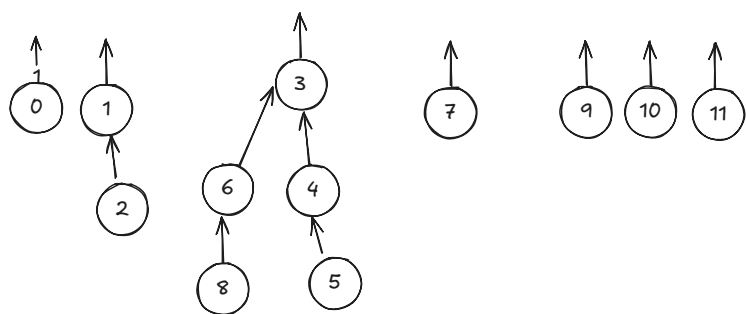
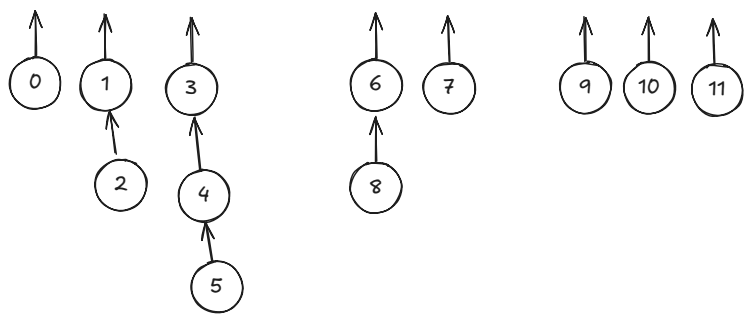
3. Starting with the union-find data structure shown below, show the *sequence* of union-find data structures that result from applying the following operations (in that order):

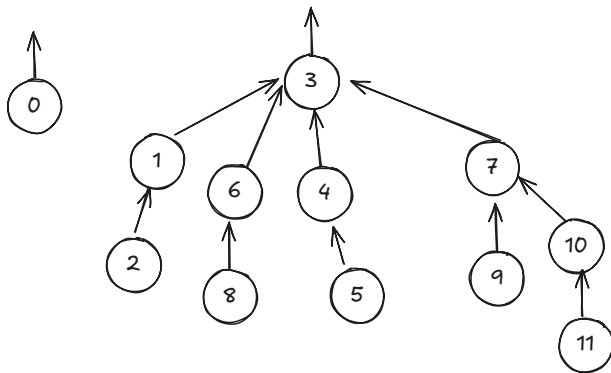
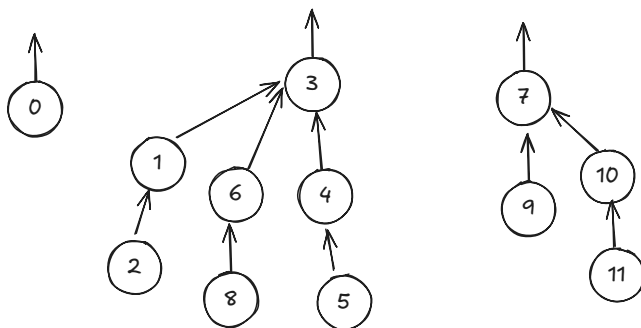
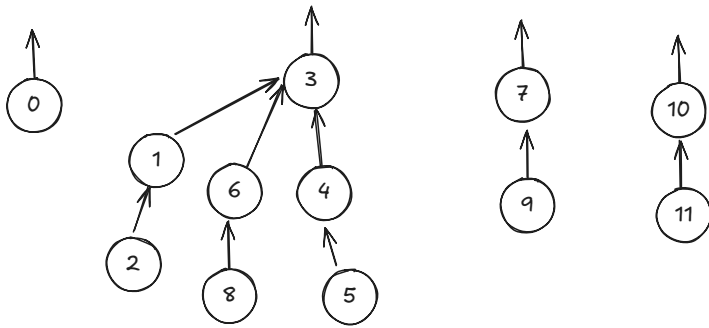
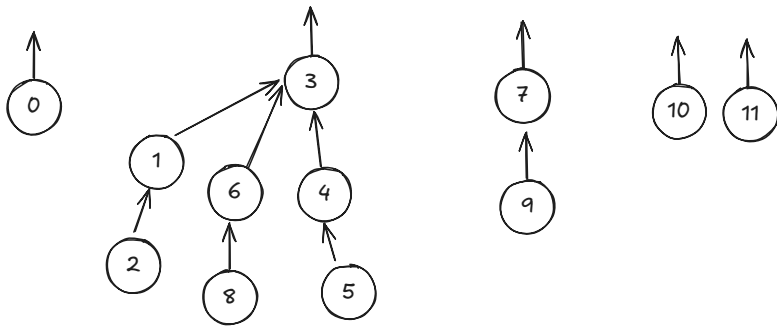
$\text{union}(1, 2) \rightarrow \text{union}(3, 4) \rightarrow \text{union}(4, 5) \rightarrow \text{union}(6, 8) \rightarrow \text{union}(5, 8)$
 $\rightarrow \text{union}(1, 6) \rightarrow \text{union}(7, 9) \rightarrow \text{union}(10, 11) \rightarrow \text{union}(11, 9)$
 $\rightarrow \text{union}(1, 11)$

Answer this question for each of the three following parts separately:

a) The `union()` s are performed by size and `find()` s are simple;







b) The `union()` s are performed by size and the `find()` s use path-compression.

