HW 2

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Linear Optimization - Dr. Tom Asaki
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Consider the linear program given below. Use software to find optimal solutions for values of δ from 0.3 to 1.0 in steps of 0.1. Compare the solutions and make some observations regarding how solutions change with respect to δ. Repeat the solutions for the corresponding integer program $(x \in \mathbb{Z}^4)$ making some observations about any significant relationships between the two sets of solutions (other than integer/non-integer).

$$
\begin{aligned}
\max_x \ & \delta x_1 + \delta x_2 + x_3 + x_4 \\
\text{s.t } & x_1 + 2x_2 + x_3 + 2x_4 \leq 70 \\
& 2x_1 + x_2 + 2x_3 + x_4 \leq 50 \\
& x_1 + x_2 + x_3 + 3x_4 \leq 62 \\
& x \geq 0 \\
& x \in \mathbb{R}^4
\end{aligned}
$$

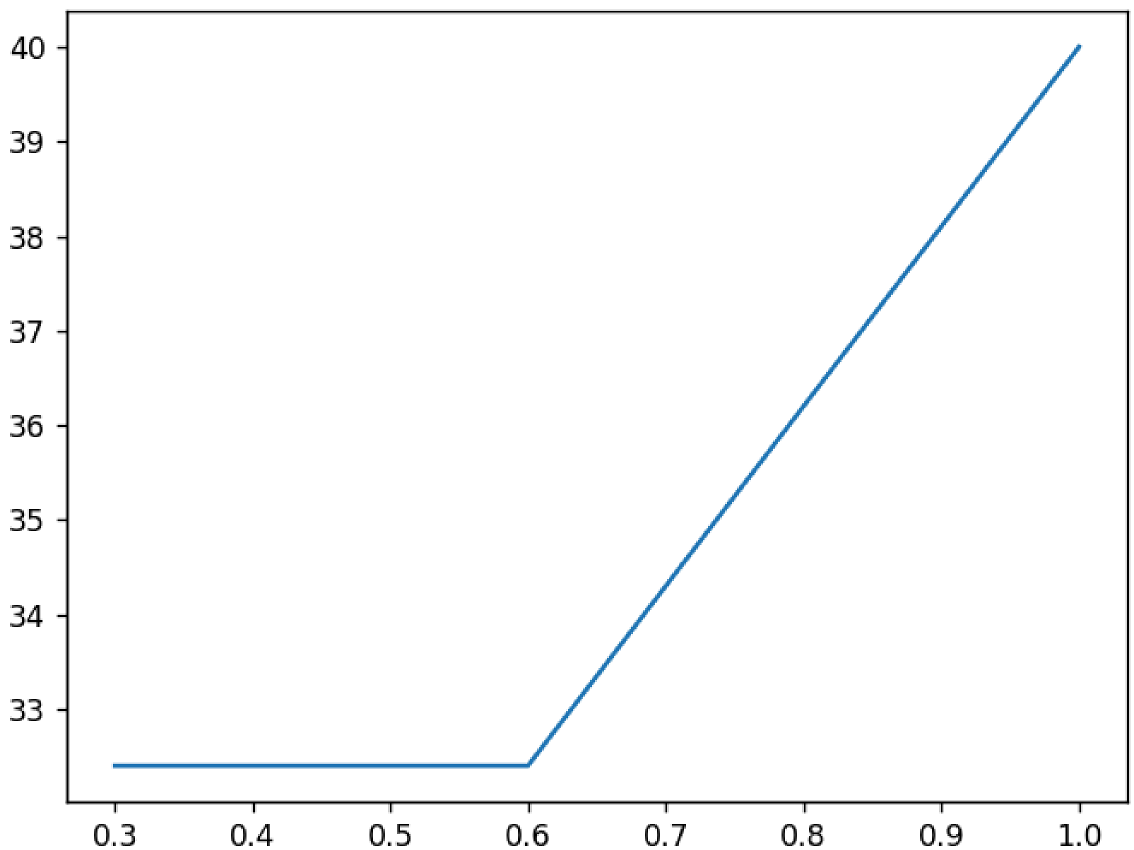
Linear Program

Result from software:

> $\delta = 0.3,\ x = [0, 0, 17.6, 14.8]$
>
> $\delta = 0.4,\ x = [0, 0, 17.6, 14.8]$
>
> $\delta = 0.5,\ x = [0, 0, 17.6, 14.8]$
>
> $\delta = 0.6,\ x = [0, 19, 10, 11]$
>
> $\delta = 0.7,\ x = [0, 19, 10, 11]$
>
> $\delta = 0.8,\ x = [0, 19, 10, 11]$
>
> $\delta = 0.9,\ x = [0, 19, 10, 11]$

> $\delta = 1,\ x = [10, 19, 0, 11]$

We can plug values of $x = (x_1, x_2, x_3, x_4)$ into the objective function to get the optimized values and plot those values on the y-axis. With δ on the x-axis, we get the following plot:



Dissecting this result:

- $0.3 \leq \delta \leq 0.6$

We can see that the optimized value of the objective function remains *constant* (32.4) here. This is because x remains constant and $x_1 = x_2 = 0$. Since δ is multiplied by x_1 and x_2 in the objective function, the function doesn't grow as δ grows since $x_1 = x_2 = 0$. Therefore, the value of the objective function is only reliant on x_3 and x_4, which remain constant, explaining the observed behavior.

- $0.6 \leq \delta \leq 1$

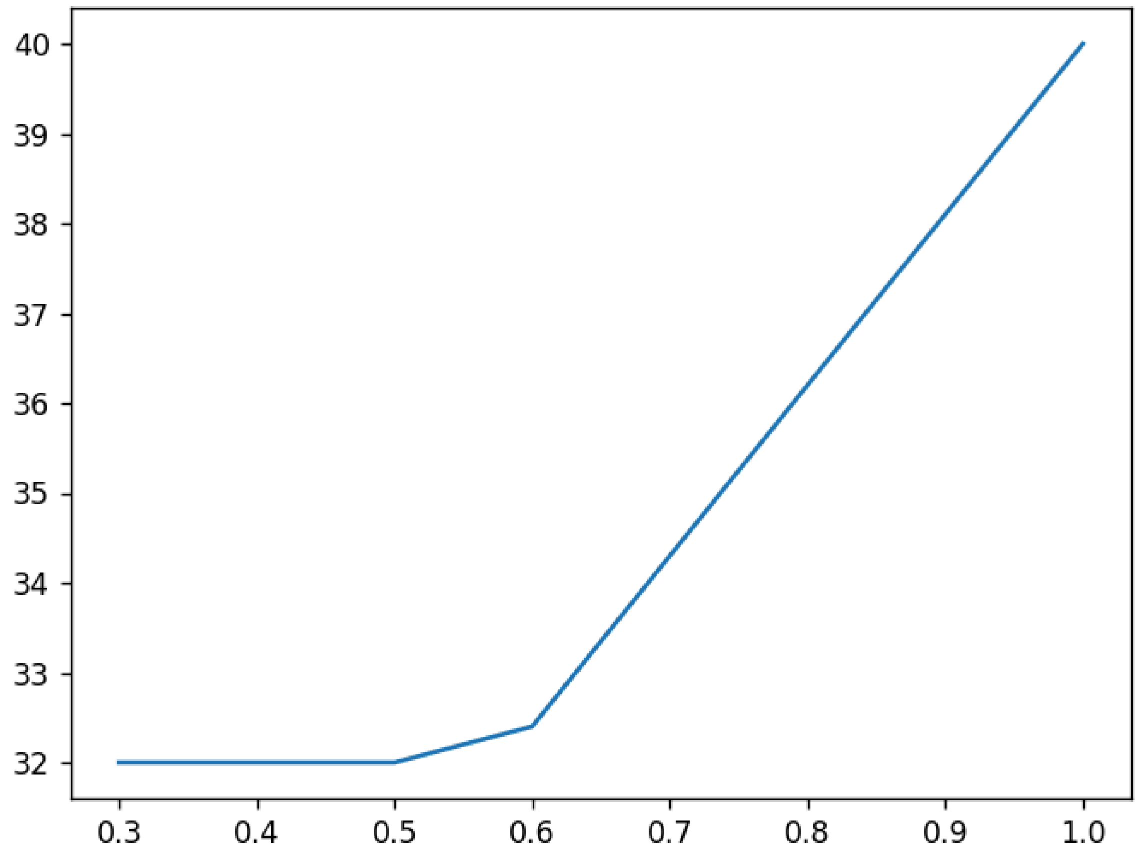
We can see that the optimized value of the objective function *grows linearly* here. This is because x_2 becomes a non-zero constants and remain that value for the remainder of the interval. Since δ is multiplied by x_2 in the objective function, δ will be proportional to the growth of the graph in this interval. Specifically, the objective function grows linearly at $19 \times 0.1 = 1.9$.

Mixed Integer Program

Result from software:

> $\delta = 0.3,\ x = [0, 0, 18, 14]$
>
> $\delta = 0.4,\ x = [0, 0, 18, 14]$
>
> $\delta = 0.5,\ x = [0, 4, 16, 14]$
>
> $\delta = 0.6,\ x = [0, 19, 10, 11]$
>
> $\delta = 0.7,\ x = [0, 19, 10, 11]$
>
> $\delta = 0.8,\ x = [0, 19, 10, 11]$
>
> $\delta = 0.9,\ x = [0, 19, 10, 11]$
>
> $\delta = 1,\ x = [0, 19, 10, 11]$

We can plug values of $x = (x_1, x_2, x_3, x_4)$ into the objective function to get the optimized values and plot those values on the y-axis. With δ on the x-axis, we get the following plot:



Dissecting this result:

- $0.3 \leq \delta \leq 0.5$

We can see that the optimized value of the objective function remains *constant* (32) here.

For $0.3 \leq \delta \leq 0.4$, the value is constant because x remains constant and $x_1 = x_2 = 0$. Since δ is multiplied by x_1 and x_2 in the objective function, the function doesn't grow as δ grows since $x_1 = x_2 = 0$. Therefore, the value of the objective function is only reliant on x_3 and x_4, which remain constant, explaining the observed behavior.

For $\delta = 0.5$, there is a change in the values of x. x_2 goes from 0 to 4 and x_3 decreases by 2. Doing calculations by plugging into the objective function, we can see these changes counteract each other as $4 \times \delta = 4 \times 0.5 = 2$ and x_3 decreases by 2. Therefore, the optimized value also remains the same.

- $0.6 \leq \delta \leq 1$

We can see that the optimized value of the objective function *grows linearly* here. This is because x_2 becomes a non-zero constants and remain that value for the remainder of the interval. Since δ is multiplied by x_2 in the objective function, δ will be proportional to the growth of the graph in this interval. Specifically, the objective function grows linearly at $19 \times 0.1 = 1.9$.

Comparison Between LP and MIP

From the interval $0.3 \le \delta \le 1$, the main difference between the linear program and mixed integer program is the optimized value of the objective function on $0.3 \le \delta \le 0.5$.

In the *linear program*, the values of x aren't restricted to integers so there is no rounding/integer conforming behavior.

In contrast, in the *mixed-integer program*, values of x are restricted to integers so the optimized value which is constant on the interval $0.3 \le \delta \le 0.5$ is different. Specifically, since δ is irrelevant in this interval as $x_1 = x_2 = 0$, the optimized value is an integer (32), which differs from the linear program where it is 32.7.

In the remaining interval $0.6 \le \delta \le 1$, both programs yield the same result for optimized objective values. This is because in the linear program, the values for x are already integers, the mixed-integer program doesn't need to do additional conforming.
