

HW 3

- I. A technician for a large animal shelter must formulate a diet from two commercial products, food A and food B, for the reptiles. In 200 grams of food A there are 16 grams of fat, 28 grams of carbohydrate, and 6 grams of protein. In 200 grams of food B there are 8 grams of fat, 50 grams of carbohydrate, and 10 grams of protein. The minimum daily requirements are 176 grams of fat, 980 grams of carbohydrate, and 200 grams of protein. Food A costs 8 cents per 100 grams and food B costs 18 cents per 100 grams. Compose and solve an optimization model to answer the following question: What is the minimum cost diet that meets all daily requirements.

$$\min M$$

$$8A + 18B = M$$

$$8A + 4B \geq 176$$

$$14A + 25B \geq 980$$

$$3A + 5B \geq 200$$

Where A, B are in units of 100 grams.

$$M = 560, \text{ or } \$5.60.$$

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1  # Programmer: Charles Liu
2  # Date: 1/19/2025
3  # Class: Linear Optimization
4
5  import numpy as np
6  import scipy.optimize as opt
7  import matplotlib.pyplot as plt
8
9  # coefficient vector for the objective function
10 c=np.array([8,18])
11
12 #coefficient array for inequality constraints (Ax <= b)
13 A = np.array([[ -8, -4],\
14               [-14, -25],\
15               [-3, -5]])
16
17 b = np.array([-176 , -980, -200])
18
19 bounds=((0, None),(0,None))
20 isint = [1,1,1,1]
21 res=opt.linprog(c,A_ub=A,b_ub=b,bounds=bounds)
22 if res.success:
23     #print(f"d = {d}: {res.fun}\n")
24     print(f"{res.fun} \n")
25     print(f"{res.x}\n")
26 else:
27     print("Optimization failed. \n")

```

2. A producer of trail mix has year-end left-over stock of peanuts, chocolate, raisins, pretzels and dried papayas. They would like to make and sell a holiday mix from this stock. The goal is to create a mix of these ingredients that has as close to 235 calories per bag as possible. Processing machinery dispenses each ingredient in integer unit amounts. The following table describes the stock.

Item	Weight (grams per unit)	Calories (calories per unit)
peanuts	6	35
chocolate	5	26
raisins	7	21
pretzels	4	15
papaya	8	4

Any optimal recipe for trail mix must contain at least one unit of each of the five items. The total bag weight cannot exceed 57 grams (about 2 oz.). Construct a linear optimization model whose solution provides the optimal trail mix recipe. Solve using software and present the solution.

Least Deviation Method

$$\min z = 0x_1 + 0x_2 + 0x_3 + 0x_4 + 0x_5 + \delta$$

s.t.

$$\delta \geq 35x_1 + 26x_2 + 21x_3 + 15x_4 + 4x_5$$

$$\delta \geq -35x_1 - 26x_2 - 21x_3 - 15x_4 - 4x_5$$

$$6x_1 + 5x_2 + 7x_3 + 4x_4 + 8x_5 \leq 57$$

$$x_1, x_2, x_3, x_4, x_5 \in \mathbb{N}$$

$$\delta \in \mathbb{Z}.$$

The optimal trail mix recipe uses 4 peanuts, 1 chocolate, 1 raisin, 3 pretzels, and 1 papaya contributing to a 56 gram bag for a total of 236 calories.

```
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3  # Class: Linear Optimization
4
5  import numpy as np
6  import scipy.optimize as opt
7  import matplotlib.pyplot as plt
8
9  # coefficient vector for the objective function
10 c=np.array([0,0,0,0,0,1])
11
12
13
14 #coefficient array for inequality constraints (Ax <= b)
15 A = np.array([[35,26,21,15,4, -1], [-35,-26,-21,-15,-4, -1], [6, 5, 7, 4, 8, 0]])
16
17 b = np.array([235, -235, 57])
18
19 bounds=((1,None),(1,None),(1,None),(1,None),(1,None), (None,None))
20 isint = [1,1,1,1,1,1]
21 res=opt.linprog(c,A_ub=A,b_ub=b,bounds=bounds, integrality=isint)
22
23 if res.success:
24     #print(f"d = {d}: {res.fun}\n")
25     print(f"{res.fun} \n")
26     print(f"{res.x}\n\n")
27 else:
28     print("Optimization failed. \n")
```