Written Homework 2.1

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1. Let

$$A = \begin{bmatrix} -1 & 2 \\ 5 & 1 \\ -4 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 3 & -2 \\ -2 & 4 \end{bmatrix}$$

a) Calculate the following if it is defined or write "undefined". Explain your work.

(a1) AB, (a2) BA, (a3) A^T, (a4) A^TB^T and (a5) B^TA^T

Since $A_{(3\times2)} \cdot B_{(2\times2)}$

(a1)
$$\begin{bmatrix} -1 & 2 \\ 5 & 1 \\ -4 & -3 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ -2 & 4 \end{bmatrix} = AB =$$

Defined because inner dimensions of A and B match

$$\begin{bmatrix} (-1)3 + (2)(-2) & -1(-2) + (2)(4) \\ 5(3) + 1(2) & 5(-2) + 1(4) \\ (-4)(3) + (-3)(-2) & -4(-2) - 3(4) \end{bmatrix}$$

$$= \begin{bmatrix} -7 & 10 \\ 13 & -6 \\ -6 & -4 \end{bmatrix}$$

2

(a2)

$$BA = \begin{bmatrix} 3 & -2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 5 & 1 \\ 4 & -3 \end{bmatrix}$$

2x2 · 3x2

B is a 2x2 matrix

A is a 3x2 matrix

inner dimensions don't match

you cannot multiply a 2x2 by a 3x2 since the amount of columns of the first matrix does not match the amount of rows in the second matrix

$2 \neq 3$ UNDEFINED

(a3) $A^T = \begin{bmatrix} -1 & 2 \\ 5 & 1 \\ 4 & -3 \end{bmatrix}^T = \begin{bmatrix} -1 & 5 & 4 \\ 2 & 1 & -3 \end{bmatrix}$

(a4) $A^T B^T$

$$A^T = \begin{bmatrix} -1 & 2 \\ 5 & 1 \\ -4 & -3 \end{bmatrix}^T = \begin{bmatrix} -1 & 5 & -4 \\ 2 & 1 & -3 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix}^T = \begin{bmatrix} 3 & -2 \\ -2 & 4 \end{bmatrix}$$

2x3 matrix cannot multiply with 2x2 in that order since colums of first matrix does not match rows of second $3 \neq 2$ UNDEFINED

(a5) Based on above calculations for

$$B^T \cdot A^T = \begin{bmatrix} 3 & -2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} -1 & 5 & -4 \\ 2 & 1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 3(-1)-2(2) & 3(5)-2(1) & 3(-4)-2(-3) \\ -2(-1)+4(2) & -2(5)+4(1) & -2(-4)-3(4) \end{bmatrix}$$

$$= \begin{bmatrix} -7 & 13 & -6 \\ 10 & -6 & -4 \end{bmatrix}$$

b) Is $AB = BA$? Explain your work,

NO

As solved earlier,

$$AB = \begin{bmatrix} -7 & 10 \\ 13 & -6 \\ -6 & -4 \end{bmatrix}$$

which is not equal to

BA = undefined

$\begin{bmatrix} -7 & 10 \\ 13 & -6 \\ -6 & -4 \end{bmatrix} \neq$ undefined

c) is $(AB)^T = A^T B^T$? Explain Your Work,

$$\left(\begin{bmatrix} -1 & 2 \\ 5 & 1 \\ -4 & -3 \end{bmatrix} \cdot \begin{bmatrix} 3 & -2 \\ -2 & 4 \end{bmatrix}\right)^T = \left(\begin{bmatrix} -7 & 10 \\ 13 & -6 \\ -6 & -4 \end{bmatrix}\right)^T$$

$(A \cdot B)^T$

NO

c)

$$= \begin{bmatrix} -7 & 13 & -6 \\ 10 & -6 & 4 \end{bmatrix}$$

$$A^T = \begin{bmatrix} -1 & 5 & -4 \\ 2 & 1 & -3 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 3 & -2 \\ -2 & 4 \end{bmatrix}$$

$$A^T B^T = \begin{bmatrix} -1 & 5 & -4 \\ 2 & 1 & -3 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ -2 & 4 \end{bmatrix}$$

UNDEFINED

2x3 matrix cannot be multiplied with 2x2 in that order

$3 \neq 2$ (columns of first matrix $\neq$ rows of second matrix)

$$\begin{bmatrix} -7 & 13 & -6 \\ 10 & -6 & 4 \end{bmatrix} \neq \text{undefined}$$

2. Dimension Check

Matrix A: 2x6

Matrix C: 2x4

Matrix D: 2x6

Matrix M: 4x6

a) $AB = C$ dimension of B: $\boxed{6 \times 4}$

must be same

$(2 \times 6)(m \times n) = (2 \times 4)$

must be same (Recall: $(m \times n)(n \times k) = (m \times k)$)

$m = 6 \quad n = 4$

b) $\boxed{\text{Undefined}}$

$(2 \times 6) + (2 \times 4) = B$

Adding matrices only works when both matrices are the same size

c) $A+D=B$ [B: 2×6]

$(2\times6)+(2\times6)=(m\times n)=(2\times6)$

Adding matrices results in a matrix of the same size

d) $BA=M$

$(m\times n)(2\times6)=(4\times6)$

$n=2$ (equal)

$m=4$ (equal)

B: (4×2) matrix

e) $BA=D^T$ $\quad D: 2\times6$ $\quad D^T=6\times2$

$(m\times n)\cdot(2\times6)=6\times2$

B is Undefined

not equal $6\neq2$

3. Let A and B be n×n matrices.
Suppose the second column of B is all zeroes. What can you say about the second column of B^2.

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & & & \vdots \\ \vdots & & & \\ a_{n1} & \cdots & & a_{nn} \end{bmatrix} \begin{bmatrix} b_{11} & 0 & b_{13} & \cdots & b_{1n} \\ b_{21} & 0 & & & \vdots \\ \vdots & \vdots & & & \\ b_{n1} & 0 & b_{n3} & \cdots & b_{nn} \end{bmatrix}$$

$$= \begin{bmatrix} (a_{11}b_{11} + a_{12}b_{21} \cdots a_{1n}b_{n1}) & 0 & \cdots & * \\ * & 0 & & * \\ \vdots & \vdots & & \vdots \\ * & 0 & \cdots & * \end{bmatrix}$$

*'s are values that could be zero

The second column is all zeroes.

(Second column of B) $= Ab_2$

B_2

$b_2 = 0 \Rightarrow B_2 = Ab_2 = A0 = 0$

Second column of $B = 0$