Written Homework 1.7
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1. How many pivot columns must a 6×5 matrix have if it's columns are linearly independent?

It must have 5 pivot columns, One for each column. This ensures a unique solution to the system $Ax=0$, where A is the 6×5 matrix, meaning it must only have the trivial solution, meaning the Column vectors of A, or the 6×5 column, are linearly independent.

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2. How many pivot columns must a 4x6 matrix have if its columns span $\mathbb{R}^4$? Why?

It must have 4 pivot columns, Since there is a pivot in every row (4 pivots) If there is 4 pivot columns, and pivot in every row implies the columns span $\mathbb{R}^m$ (m = number of columns), if there are 4 pivot columns, the columns must span $\mathbb{R}^4$.

3. Construct 3x2 matrices A and B so that Ax=0 has only the trivial solution, but Bx=0 has non-trivial solutions.

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

3 continued:

$Ax=0$ only has the trivial solution

$\Rightarrow$ the columns of A are linearly independent

$Bx=0$ has nontrivial solutions

$\Rightarrow$ the columns of B are linearly dependent

$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ $\quad \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \neq$ zero vector or multiples of each other

therefore, $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ are linearly independent, and $Ax=0$ only has the trivial solution

$B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\quad \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}_{\text{zero vector}}$

Since one of the column vectors of B is the zero vector, they are linearly dependent and $Bx=0$ has non-trivial solutions.