Written HW 1.5

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1. Suppose A is the 3X3 zero matrix (with all zero entries). Describe the solution set of the equation $Ax=0$.

$$\underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}_{0}$$

Augmented Matrix:

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This means

$$x = \begin{cases} x_1 \text{ is free} \\ x_2 \text{ is free} \\ x_3 \text{ is free} \end{cases}$$

So the solution set is

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x = x_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

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1) Continued...

or $\text{Span}\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}\begin{bmatrix}0\\1\\0\end{bmatrix}\begin{bmatrix}0\\0\\1\end{bmatrix}\right\} = \mathbb{R}^3$

2. Suppose A is a 3×2 matrix with two pivot positions

a) Does the equation $Ax=0$ have a nontrivial solution?

No. The augmented matrix has a pivot column in each column. This means its solution is all basic variables and has no free variables (unique solution). If a homogenous system has no free variables, it must only have the trivial solution, and no non-trivial solutions.

2 b) For matrix A, does the equation $Ax=b$ have at least one solution for every possible b?

No. In order for $Ax=b$ to have a solution, A must span $\mathbb{R}^3$, and thus have a pivot in every row (since it is a 3x2 matrix), but it does not since there are only two pivots and 3 rows.

3. Let

$$A = \begin{bmatrix} 1 & 5 & 2 & -6 & 0 \\ 0 & 0 & 1 & -7 & -8 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

a) Solve $Ax=b$ where

$$b = \begin{bmatrix} -3 \\ -10 \\ 2 \\ 0 \end{bmatrix}$$

and write the solution in parametric form. How many solutions are there?

3 a) continued...

$$\begin{bmatrix} 1 & 5 & 2 & -6 & 0 \\ 0 & 0 & 1 & -7 & -8 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 3 \\ -10 \\ 2 \\ 0 \end{bmatrix}$$

Augmented Matrix

$$\begin{bmatrix} 1 & 5 & 2 & -6 & 0 & 3 \\ 0 & 0 & 1 & -7 & -8 & -10 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_2 \to R_3 \cdot (8) + R_2}$$

$$\begin{bmatrix} 1 & 5 & 2 & -6 & 0 & 3 \\ 0 & 0 & 1 & -7 & 0 & 6 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_1 \to R_2 \cdot (-2) + R_1}$$

$$\begin{bmatrix} 1 & 5 & 0 & 8 & 0 & -9 \\ 0 & 0 & 1 & -7 & 0 & 6 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Reduced Echelon Form

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So: $x_5 = 2$

x_2 and x_4 are free

$x_3 = 6 + 7x_4$

$x_1 = -9 - 5x_2 - 8x_4$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -9 - 5x_2 - 8x_4 \\ x_2 \\ 6 + 7x_4 \\ x_4 \\ 2 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} -9 \\ 0 \\ 6 \\ 0 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} -5 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -8 \\ 0 \\ 7 \\ 1 \\ 0 \end{bmatrix}}$$

Parametric Form

There are infinite solutions, since there are free variables.

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3. b) Solve $Ax=0$ where 0 is the zero vector and write the solution in parametric form.

Since the solution of the non-homogenous system is:

$$X = \begin{bmatrix} \text{Solution of homogenous} \\ \text{in parametric form} \end{bmatrix} + \begin{bmatrix} \text{particular} \\ \text{solution} \end{bmatrix},$$

and we solved for the non-homogenous system previously

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \underbrace{\begin{bmatrix} -9 \\ 0 \\ 6 \\ 0 \\ 2 \end{bmatrix}}_{\text{particular solution}} + \underbrace{x_2 \begin{bmatrix} -5 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -8 \\ 0 \\ 7 \\ 1 \\ 0 \end{bmatrix}}_{\text{Solution to homogenous system}}$$

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3 b) continued...

The solution of $AX=0$ is

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{bmatrix} = X_2 \begin{bmatrix} -5 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + X_4 \begin{bmatrix} -8 \\ 0 \\ 7 \\ 1 \\ 0 \end{bmatrix}$$

3. b) Solve to ensure relation

$$\begin{bmatrix} 1 & 5 & 2 & -6 & 0 & 0 \\ 0 & 0 & 1 & -7 & -8 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \longrightarrow \overset{RREF}{\begin{bmatrix} 1 & 5 & 0 & 8 & 0 & 0 \\ 0 & 0 & 1 & -7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}}$$

$X_5 = 0$ X_4 is free $X_3 = 7X_4$ free

X_2 is free $X_1 = -5X_2 - 8X_4$

$$X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{bmatrix} = X_2 \begin{bmatrix} -5 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + X_4 \begin{bmatrix} -8 \\ 0 \\ 7 \\ 1 \\ 0 \end{bmatrix}$$

3 c) The solution will be the parametric form of the solution of the non-homogenous system $Ax=b$ without the particular solution.