Written HW 3.1

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1. (8 pts) Consider the matrix A,

$$A = \begin{bmatrix} 2 & 0 & 3 \\ -1 & 1 & -2 \\ 3 & 0 & 1 \end{bmatrix}$$

Find the determinant of A by expanding about a row or column in 3 different ways. Please indicate which row/column you are expanding about before showing your work.

Solution.

- **(1) 2nd Column (Best Approach)**

> $$C_{22} = (-1)^{2+2} = 1$$
>
> $$C_{12} * 0 * \begin{vmatrix} -1 & -2 \\ 3 & 1 \end{vmatrix} + C_{22} * 1 * \begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} + C_{32} * 0 * \begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} = 0 + \begin{vmatrix} 2 & 3 \\ -1 & -2 \end{vmatrix} + 0$$
>
> $$= 2 * 1 - 3 * 3 = 2 - 9 = -7$$

- **(2) 1st Row**

> $$C_{11} = (-1)^{1+1} = 1\,;\, C_{13} = (-1)^{1+3} = 1$$
>
> $$C_{11} * 2 * \begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} + C_{12} * 0 * \begin{vmatrix} -1 & -2 \\ 3 & 1 \end{vmatrix} + C_{13} * 3 * \begin{vmatrix} -1 & 1 \\ 3 & 0 \end{vmatrix}$$
>
> $$= 2 * \begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} + 0 + 3 * \begin{vmatrix} -1 & 1 \\ 3 & 0 \end{vmatrix}$$
>
> $$= 2 * (1 * 1 - (-2) * 0) + 3 * ((-1) * 0 - 1 * 3) = 2 * 1 + 3 * (-3) = 2 - 9 = -7$$

- **(2) 3rd Row**

> $$C_{31} = (-1)^{3+1} = 1\,;\, C_{33} = (-1)^{3+3} = 1$$
>
> $$C_{31} * 3 * \begin{vmatrix} 0 & 3 \\ 1 & -2 \end{vmatrix} + C_{32} * 0 * \begin{vmatrix} 2 & 3 \\ -1 & -2 \end{vmatrix} + C_{33} * 1 * \begin{vmatrix} 2 & 0 \\ -1 & 1 \end{vmatrix}$$
>
> $$= 3 * \begin{vmatrix} 0 & 3 \\ 1 & -2 \end{vmatrix} + 0 + 1 * \begin{vmatrix} 2 & 0 \\ -1 & 1 \end{vmatrix}$$
>
> $$= 3 * (0 * (-2) - 3 * 1) + 1 * (2 * 1 - 0 * (-1)) = 3 * (-3) + 1 * 2 = -9 + 2 = -7$$

2. (14 pts) Consider the same matrix A,

$$A = \begin{bmatrix} 2 & 0 & 3 \\ -1 & 1 & -2 \\ 3 & 0 & 1 \end{bmatrix}$$

(a) (4 pts) Multiply the first row by a nonzero constant, a. What is the determinant?

Cofactor expansion across 2nd column

$$M = \begin{bmatrix} 2a & 0 & 3a \\ -1 & 1 & -2 \\ 3 & 0 & 1 \end{bmatrix}$$

$$C_{22} = (-1)^{2+2} = 1$$

$$C_{12} * 0 * \begin{vmatrix} -1 & -2 \\ 3 & 1 \end{vmatrix} + C_{22} * 1 * \begin{vmatrix} 2a & 3a \\ 3 & 1 \end{vmatrix} + C_{32} * 0 * \begin{vmatrix} 2a & 3a \\ -1 & -2 \end{vmatrix}$$

$$= 0 + \begin{vmatrix} 2a & 3a \\ 3 & 1 \end{vmatrix} + 0$$

$$= 2a * 1 - 3a * 3 = 2a - 9a = -7a$$

(b) (2 pts) Suppose we multiplied the entire matrix A by a. Without doing any calculations, what do you think the answer will be? (no justification necessary).

$-7a^3$

(c) (4 pts) Calculate the determinant of aA.

$$aA = \begin{bmatrix} 2a & 0 & 3a \\ -1a & 1a & -2a \\ 3a & 0 & 1a \end{bmatrix}$$

$$C_{22} = (-1)^{2+2} = 1$$

$$C_{12} * 0 * \begin{vmatrix} -1a & -2a \\ 3a & 1a \end{vmatrix} + C_{22} * 1a * \begin{vmatrix} 2a & 3a \\ 3a & 1a \end{vmatrix} + C_{32} * 0 * \begin{vmatrix} 2a & 3a \\ -1a & -2a \end{vmatrix}$$

$$= 0 + a * \begin{vmatrix} 2a & 3a \\ 3a & 1a \end{vmatrix} + 0$$

$$= a * (2a * 1a - 3a * 3a) = 2a^2 - 9a^2 = -7a^3$$

(d) (4 pts)
(i) Suppose B is a 4×4 matrix and the determinant of B is 6. What is the $\det(3B)$?
$\det(3B) = \det(B) * 3^4 = 6 * 3^4 = 6 * 81 = 486$

(ii) Suppose C is a 10 × 10 matrix and the determinant of C is 32. What is the $\det(\frac{1}{2}C)$?

$\det(\frac{1}{2}C) = det(C) * (\frac{1}{2})^{10} = 32 * (\frac{1}{2})^{10} = 32 * \frac{1}{1024} = \frac{1}{32}$
