Written Homework 2.8

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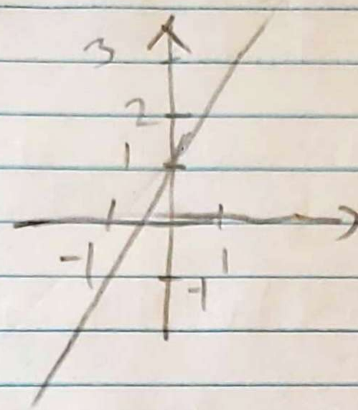
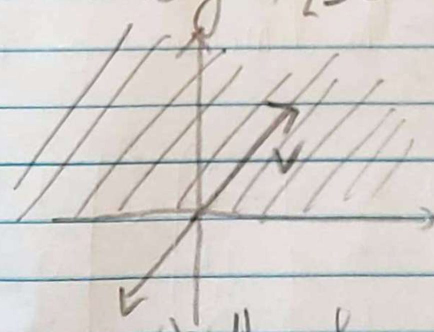
1. Sketch the following sets that are in $\mathbb{R}^2$. Using the definition of a subspace, show that the following sets are not subspaces.

a) The set containing all points (x_1, x_2) that satisfy the equation $x_2 = 2x_1 + 1$



The set does not contain the origin, so it CANNOT be a subspace

b) The set containing all points (x_1, x_2) that satisfy $x_2 \geq 0$



The set does not follow closed under scalar multiplication

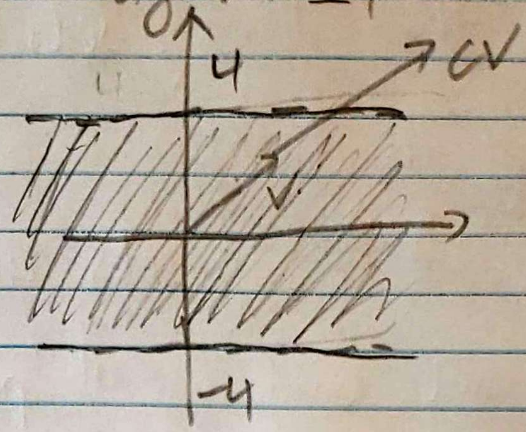
v is in the set

$cv \notin$ the set

if $c<0$, cv is not in the set

Therefore, the set CANNOT be a subspace

c) The set containing all points (x_1, x_2) that satisfy $|x_2| \le 4$



The set does not follow closed under Scalar multiplication.

In the above example, v is in the set, but cv is not.

Therefore it CANNOT be a subspace

2. Consider the matrix A given below and a row echelon form of the matrix (also given). Determine a basis for the column space and a basis for the null space. then state the dimensions of these two subspaces.

$$A = \begin{bmatrix} 4 & 5 & 9 & -2 \\ 6 & 5 & 1 & 12 \\ 3 & 4 & 8 & -3 \end{bmatrix} \sim \underbrace{\begin{bmatrix} 1 & 2 & 6 & -5 \\ 0 & 1 & 5 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{EF}$$

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2. Basis of Column Space of A (Col A)

= Pivot columns of A

$$B = \left\{ \begin{bmatrix} 4 \\ 6 \\ 3 \end{bmatrix} \begin{bmatrix} 5 \\ 5 \\ 4 \end{bmatrix} \right\}$$ forms basis of Col(A)

Basis of Null Space of A (Nul (A))

$$\begin{bmatrix} 1 & 2 & 6 & -5 \\ 0 & 1 & 5 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_1 \to R_2 \cdot (-2) + R_1}$$

$$\begin{bmatrix} 1 & 0 & -4 & 7 \\ 0 & 1 & 5 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{cases} x_1 = 4x_3 - 7x_4 \\ x_2 = -5x_3 + 6x_4 \\ x_3 \text{ is free} \\ x_4 \text{ is free} \end{cases}$$

$$\underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} -4 \\ -5 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -7 \\ 6 \\ 0 \\ 1 \end{bmatrix}}_{\text{parametric vector form}}$$

$$B = \left\{ \begin{bmatrix} 4 \\ -5 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} -7 \\ 6 \\ 0 \\ 1 \end{bmatrix} \right\}$$ forms basis of Nul (A)

2. cont.

dimension of column space is $\boxed{2}$ since there are 2 elements in the basis for it

dimension for null space is $\boxed{2}$ since there are 2 elements in its basis.