Lecture 9
Inference Statistics
Hypothesis Testing

Aug 08

Content

- Hypothesis Testing - Introduction
- Population Proportions
  - One Proportion
  - Two Proportions

- Population Mean
  - One Mean

Hypothesis Testing - Introduction

- We have a prior knowledge about our data against it.
- Basically, a precise statement about the population of interest is made,
    - then we aim to falsify the statement.

Hypothesis Testing - One Proportions

- Question: Could the parameter of interest be ___ ?
- How to support or argue against this?
  - Use data.

Research Question

In previous years 52% of parents believed that electronics and social media was the cause of their teenager's lack of sleep. Do more parents today believe that their teenager's lack of sleep is caused due to electronics and social media?

Hypothesis Testing - One Proportions - Example

Research Question

In previous years 52% of parents believed that electronics and social media was the cause of their teenager's lack of sleep. Do more parents today believe that their teenager's lack of sleep is caused due to electronics and social media?

Hypothesis Testing - One Proportions - Example -2

Hypotheses

$H_0 : p = 0.52$

$H_a : p > 0.52$

Where p is the population proportion of parents with a teenager who believe that electronics and social media is the cause of their teenager's lack of sleep

Hypothesis Testing - One Proportions - Example -3

Survey Results

A random sample of **1018** parents with a teenager was taken and **56%** said they believe electronics and social media was the cause of their teenager's lack of sleep.

Hypothesis Testing - One Proportions - Example -4

Test Statistic

$$\frac{\text{Best estimate - Hypothesized estimate}}{\text{Standard error of estimate}}$$

$$\frac{\hat{p} - p_o}{s.e.}$$

$$s.e.(\hat{p}) = \sqrt{\frac{p \cdot (1-p)}{n}}$$

Hypothesis Testing - One Proportions - Example -5

Test Statistic

$$\frac{\hat{p} - p_o}{s.e.}$$

Null $s.e.(\hat{p}) = \sqrt{\frac{p_0 \cdot (1 - p_o)}{n}}$

Hypothesis Testing - One Proportions - Z test statistic

Test Statistic Distribution

- A Z test statistic is another random variable! It has a distribution.
- The Z test statistic will always follow a N(0,1)
- This is due to us centering and scaling our original data

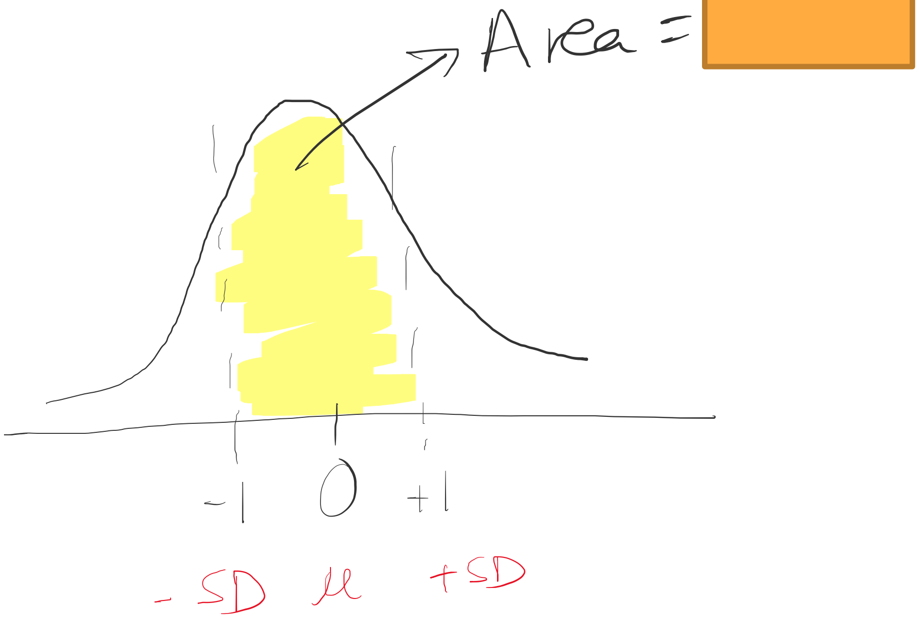
Scales Data

$$\frac{\hat{p}-p_o}{s.e.(\hat{p})}$$

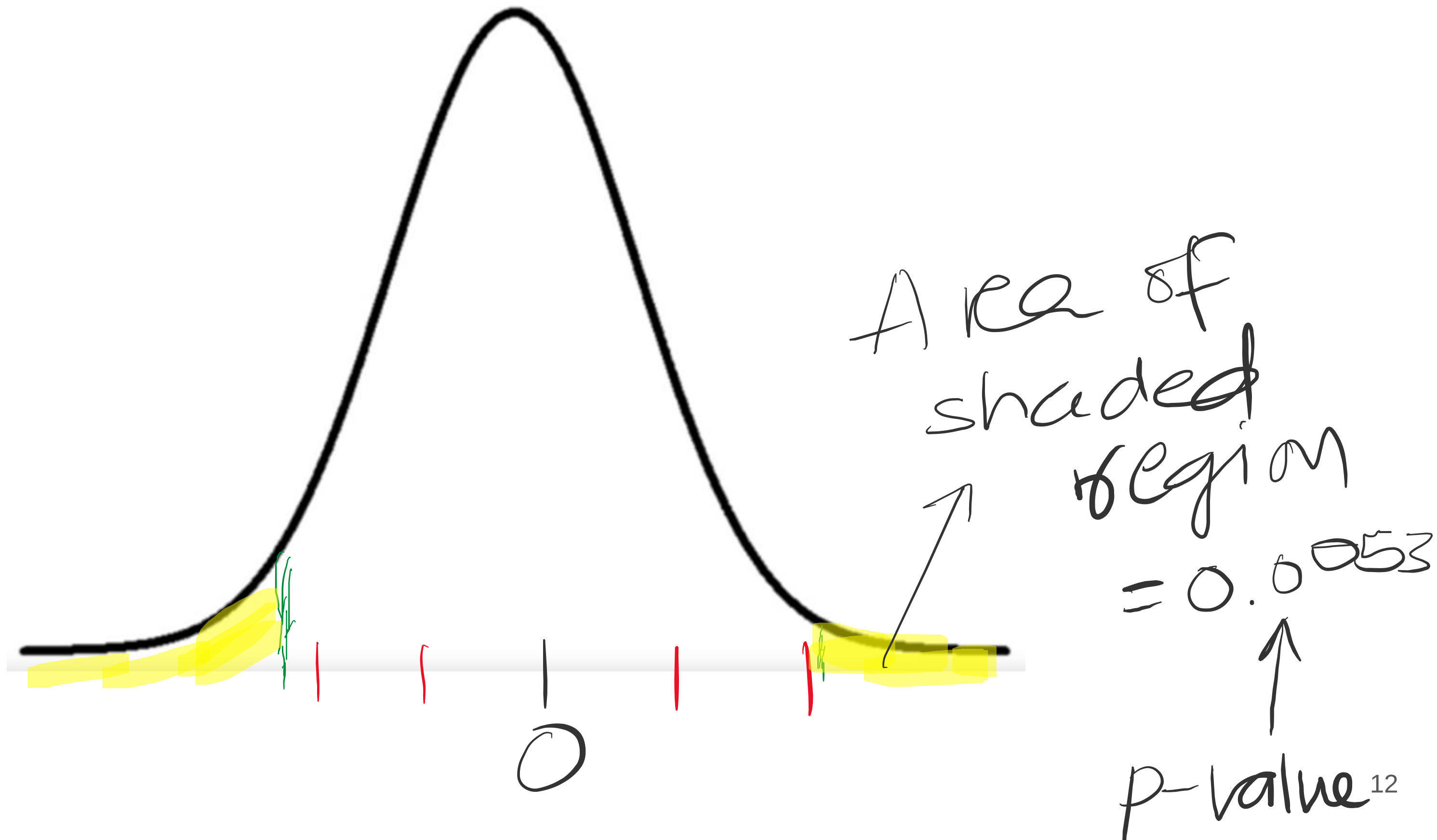
Centers Data

Normal Distribution

$N(0,1)$



Hypothesis Testing - One Proportions - Z test statistic



Conclusions

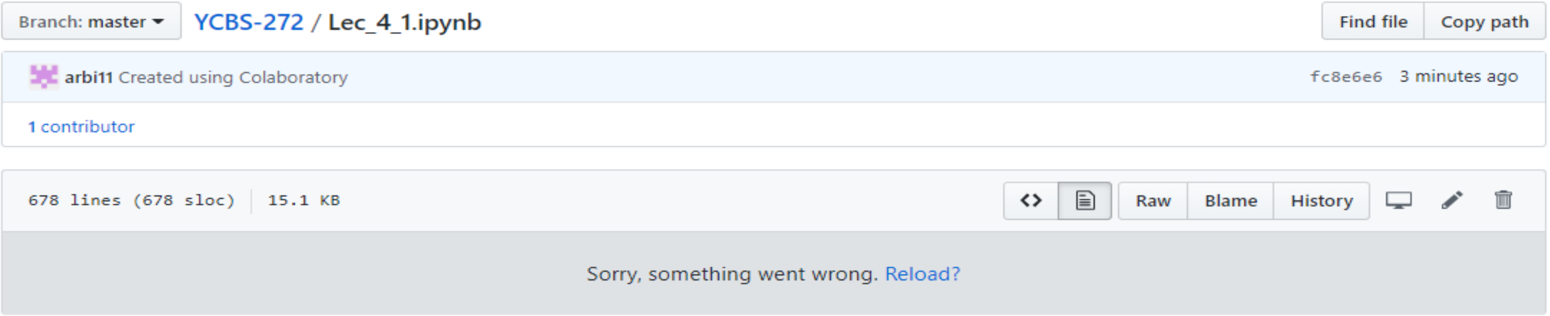
p-value = 0.0053 < α = 0.05

Reject the null hypothesis (H_0: p = 0.52)

Link for the notebook

https://github.com/arbi11/YCBS-272/blob/master/Lec_9_1_Introduction_to_Hypothesis_Testing_in_Python.ipynb

If you see this error on github



Copy the link (of github page) and paste here:

https://nbviewer.jupyter.org/

Hypothesis Testing - Two Proportions

Research Question

Is there a significant difference between the population proportions of parents of black children and parents of Hispanic children who report that their child has had some swimming lessons?

Hypothesis Testing - Two Proportions - Example

Hypotheses

$H_0 : p_1 - p_2 = 0$

$H_a : p_1 - p_2 \ ? \ 0$

$\hat{p} = (91+120)/(247+308) = 211/555 = 0.38$

$247(0.38) = 94$; $247(1-0.38) = 153$;

$308(0.38) = 117$; $308(1-0.38) = 191$

Hypothesis Testing - Two Proportions - Example -2

Best Estimate of the Parameter

$\hat{p}_1 = 91/247 = 0.37$

$\hat{p}_2 = 120/308 = 0.39$

$\hat{p}_1 - \hat{p}_2 = 0.37 - 0.39 = -0.02$

1 = black

2 = Hispanic

Hypothesis Testing - Two Proportions - Example -3

Test Statistic

$$\frac{\text{Best estimate - Hypothesized estimate}}{\text{Standard error of estimate}}$$

$$\frac{\hat{p}_1 - \hat{p}_2 - 0}{se(\hat{p})}$$

where $se(\hat{p}) = \sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$

$z = -0.02/0.041 = -0.48$

Decision & Conclusion

p-val = 0.63 > 0.10 = α → fail to reject null hypothesis

→ don't have evidence against equal population proportions

Hypothesis Testing - One Mean

Is the ***average*** cartwheel distance for adults more than 80 inches?

Population: All adults

Parameter of Interest: population mean cartwheel distance μ

Perform a one-sample test regarding the value for the mean cartwheel distance for the population of all such adults.

Hypothesis Testing - One Mean - Null & Alternative

- Null: Population mean CW distance (μ) is 80 inches
- Alternative: Population mean is _**greater than (>)**_ 80 inches

More compact notation:

- H_0: $\mu = 80$
- H_a: $\mu > 80$

where μ represents the population mean cartwheel distance (inches) for all adults

Hypothesis Testing - One Mean - Data Exploration

```
df.describe()["CWDistance"]
```

```
count     25.000000
mean      82.480000
std       15.058552
min       63.000000
25%       70.000000
50%       81.000000
75%       92.000000
max      115.000000
Name: CWDistance, dtype: float64
```

n = 25 observations

Minimum = 63 inches
Maximum = 115 inches

$\boxed{\text{Mean = 82.48 inches}}$
Standard Deviation = 15.06 inches

Hypothesis Testing - One Mean - SE

H_0: $\mu = 80$

H_a: $\mu > 80$

- Is sample mean of 82.48 inches *significantly greater than* hypothesized mean of 80 inches?

standard error of the sample mean $= \dfrac{\sigma}{\sqrt{n}}$

estimated **standard error of the sample mean** $= \dfrac{s}{\sqrt{n}}$

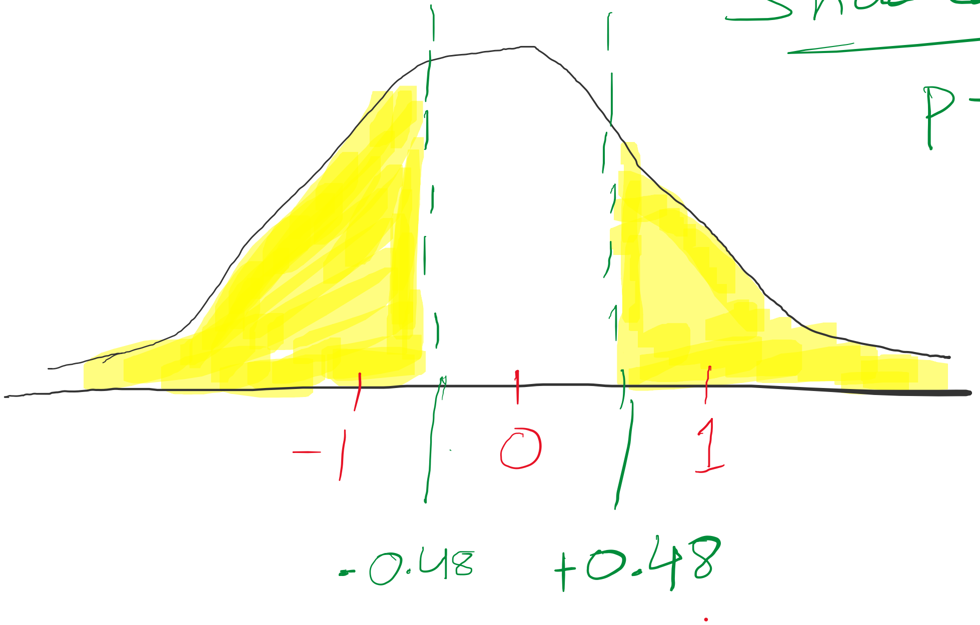
Hypothesis Testing - One Mean - Test Statistic

Test Statistic: Assuming sampling distribution of sample mean is normal,

$$t = \frac{best\ estimate - null\ value}{estimated\ std\ error} = \frac{\bar{x} - 80}{\frac{s}{\sqrt{n}}}$$

Our **sample** mean is **only 0.82**
(*estimated*) **standard errors**
above null value of 80 inches

Normal (0, 1)



Shaded Area

p-value

$\boxed{= 0.63}$

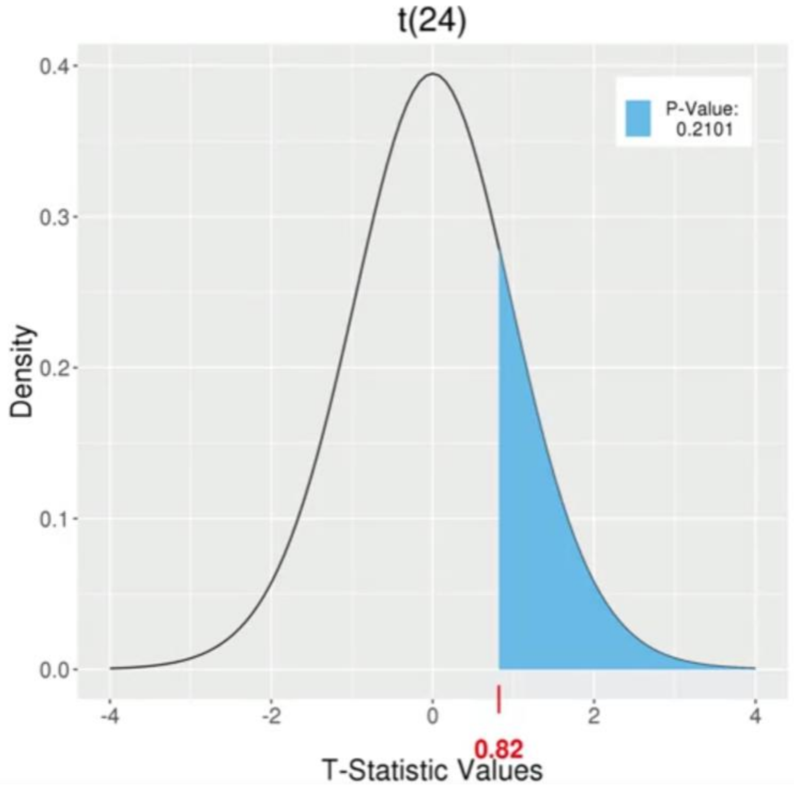
Hypothesis Testing - One Mean - P-value

- If null hypothesis was true, would a test statistic value of only t = 0.82 be unusual enough to reject the null?
- **P-value** = Probability of seeing test statistic of 0.82 or *more extreme* assuming the null hypothesis is true.

Hypothesis Testing - One Mean - P value

P-value = 0.21

If population mean CW distance
was really was 80 inches, then
observing a sample mean of 82.48 inches
(i.e. a t statistic of 0.82)
or larger is **quite likely**.



Hypothesis Testing - One Mean - Conclusion

Since our P-value is much bigger than 0.05 significance level,
weak evidence against the null
→ we **fail to reject the null!**

Based on estimated mean (82.48 inches),
we ***cannot support***
the population mean CW distance
is greater than 80 inches

Hypothesis Testing - Errors

- Type I
  - Reject null hypothesis, when it is TRUE.
  - Also called as False Positive.
  - Typically try to control probability of Type - I to below 5%.

- Type II
  - Accept null hypothesis when it is FALSE.
  - Also called False Negative.

Relation: Hypothesis Testing & Confidence Interval

- For single parameter estimation, Confidence Interval can be used to check for NULL Hypothesis.

90% Confidence Interval Estimate

Mean = 82.48 inches

Standard Deviation = 15.08 inches

n = 25 observations → t^* = 1.711

$$\bar{x} \quad \pm \quad t^* \left(\frac{s}{\sqrt{n}}\right)$$

$$82.48 \pm 1.711\left(\frac{15.06}{\sqrt{25}}\right)$$

$$82.48 \pm 1.711(3.012)$$

$$82.48 \pm 5.15$$

$$\boxed{(77.33 \text{ inches}, 87.63 \text{ inches})}$$

Note: 80 inches is IN confidence interval of reasonable values for population mean CW distance