# 第7讲 — 物理应用 1000题

**1000.7.1**

球半径 5cm/s 增长, $r=50$ cm时

$V_{S表}$ 与 $V_{体积}$?

$S_表 = 4\pi r^2$  $\frac{dS}{dt}=\frac{dS}{dr}\cdot\frac{dr}{dt}$

$V=\frac{4}{3}\pi r^3$  $= 8\pi r\cdot 5 = 40\pi r\big|_{r=50} = 2000\pi$ cm²/s

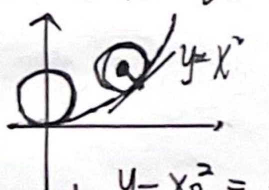
$\frac{dV}{dt}=\frac{dV}{dr}\cdot\frac{dr}{dt} = 4\pi r^2\cdot 5\big|_{r=50} = 50000\pi$ cm³/s

**1000.7.2**

半径为 $\frac12$, 圆在抛物线 $x=\sqrt{y}$ ($y=x^2$) 凹侧滚动.

(1). 圆心 $(\xi,\eta)$ 轨迹方程

(2). 圆心 $v_0$ 与上升, 横坐标 $\xi$ 增长速度?




$y=x^2$ 切点 $(x_0, x_0^2)$.

$y'=2x$ 斜率 $2x_0 \to -\frac{1}{2x_0}$

$y-x_0^2 = -\frac{1}{2x_0}(x-x_0)$   $x_0>0$.

$x_0=0$时 $(x,y)=(0,\frac12)$.

~~$x^2+y^2=\frac14$~~

$(x-x_0)^2+(y-x_0^2)^2=\frac14$

$$\begin{cases} x = -\sqrt{\frac{x_0^2}{4x_0^2+1}}+x_0 \\ y = x_0^2+\frac12\sqrt{\frac{1}{4x_0^2+1}} \end{cases} \qquad \begin{cases} x = t-\left(1-\frac{1}{4t^2+1}\right)^{\frac12} \\ y = t^2+\frac12\left(\frac{1}{4t^2+1}\right)^{\frac12} \end{cases}$$

取了个[illegible]

(2) $\frac{d\xi}{d\eta} = \frac{d\xi/dt}{d\eta/dt} = \frac{1}{2x}$   $\frac{d\eta}{dt}=v_0$   $\frac{d\xi}{dt}=\frac{d\xi}{d\eta}\cdot\frac{d\eta}{dt}=\frac{v_0}{2x}$.

**1000.7t.1**

甲乙港口距 1000 km, 从甲处出发, $v$, → 乙

$y$燃料 $A = kv^{\frac32}$, $k=1$. ($A=v^{\frac32}$)

$v_水 = 20$ km/h.  $v=?$ $y$燃料 min.

[solution].

~~$y=\int_0^t A\,dt = \int_0^t$~~ ~~$v^{\frac32}\cdot\frac{1000}{v+20}$~~

$y = v^{\frac32}\frac{1000}{v+20}$

$y' = \frac{\frac32\cdot 1000\cdot v^{\frac12}(v+20)^2 - v^{\frac32}\cdot 1000\cdot 2(v+20)}{(v+20)^2} \triangleq 0$.

$\to v_0=60$.

$A(60)=3000\sqrt{15}$.

**1000.7t.2**

质点从单位时间内 $v=0$, → B 静止 $d_{AB}=1$.

证明: 该质点在$(0,1)$内总有一时刻加速度 $\geq 4$.

[solution]. $v(t)$

~~$1=\int_0^1 v(t)dt \Rightarrow 1=\frac{\int_0^1 v(t)dt}{1-0} = v(\xi)$  $\xi\in(0,1)$~~

二阶用泰勒

$y(0)=y(1)=0$.  $y'(0)=y'(1)=0$.

$y(\frac12)=y(0)+y'(0)(\frac12-0)+\frac{1}{2!}y''(\xi_1)(\frac12-0)^2=\frac{y''(\xi_1)}{8}$  $0<\xi_1<\frac12$.

$y(\frac12)=y(1)+y'(1)(\frac12-1)+\frac{1}{2!}y''(\xi_2)(\frac12-1)^2=1+\frac18 y''(\xi_2)$  $\frac12<\xi_2<1$.

若 $y(\frac12)\geq\frac12$  $y''(\xi_1)\geq 4$   若 $y(\frac12)<\frac12$  $y''(\xi_2)<-4$