§12. 一元函数积分学应用

例12.1

$v = t\sin t^2$ m/s.

$t_1=\sqrt{\frac{\pi}{2}}$ 到 $t_2=\sqrt{\pi}$. $S=?$

$$\int_{\sqrt{\frac{\pi}{2}}}^{\sqrt{\pi}} t\sin t^2\,dt = \frac{1}{2}\int_{\sqrt{\frac{\pi}{2}}}^{\sqrt{\pi}} \sin t^2\,dt^2 = \frac{1}{2}m.$$

变力沿直线做功. $W=\int_a^b F(x)\,dx$

例12.2 (12')

打地基, 阻力做功, $f \sim x$ (深度), ① $f=kx$.

打第一次入 a cm, ② 每次W. 之比 $= r$.

$W_1=\int_{x=0}^{x_1} kx\,dx$, $x_1=a$ $\quad W_2=\int_{x_1}^{x_2} kx\,dx = rW_1$

$\Rightarrow \frac{k}{2}a^2 = W_1$

$\Rightarrow \frac{k}{2}a^2\cdot r = \frac{k}{2}(x_2^2-a^2)$

$\Rightarrow \frac{k}{2}(x_2^2-a^2)r = \frac{k}{2}(x_3^2-x_2^2)$

$W_1+W_2+W_3 = \frac{k}{2}x_3^2$

$W_1(1+r+r^2) = \frac{k}{2}x_3^2 \Rightarrow x_3=\sqrt{1+r+r^2}\cdot a$ (1).

$\Rightarrow x_n=\sqrt{1+r+r^2+\cdots+r^n}\cdot a = \frac{a}{\sqrt{1-r}}$

提取物体做功. (第2).

$W=\rho g h = \rho g\int_a^b x\,A(x)\,dx$. — $A(x)$ 水平截面面积.

例12.3 ~ 习12.2

清除井底污泥, 井深30m, 抓斗400N, 缆绳每米50N

抓起污泥2000N, 提升v 3m/s.

在提升过程, 污泥以 20N/s 漏掉. 做功

(1) 常力功: $W_1=400\cdot 30$ (抓斗).

绳 $W_2=\int_0^{30} \underbrace{50(30-x)\,dx}_{绳重\cdot 绳高=绳总重}$

★污泥 $\int_0^{30/3=10} \underbrace{(2000-20t)}_{重力}\underbrace{3dt}_{位移}$

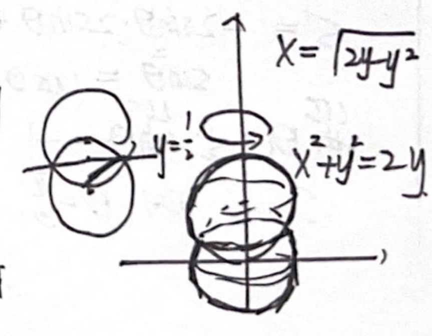
[考: 物体漏掉, 水的蒸发化].

例12.4

(1) $V=2\int_{\frac{1}{2}}^{2}\int_0^{\sqrt{2y-y^2}} \pi x^2\,dx\,dy$

$=2\int_{\frac{1}{2}}^{2} (\pi (\sqrt{2y-y^2})^2)\,dy$

$=2\pi\int_{\frac{1}{2}}^{2} 2y-y^2\,dy =$ [illegible]



(2). $W_1=\int_{\frac{1}{2}}^{2} \rho g\, A_1(y)\, dy\, (2-y)$.

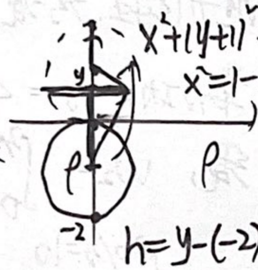
$=\int_{\frac{1}{2}}^{2} 10^3 g\pi\,(2y-y^2)\, dy\, (2-y)$

$=\int_{\frac{1}{2}}^{2} 10^3\cdot g\cdot\pi(2y-y^2)(2-y)\, dy$.

$W_2=\int_{-1}^{\frac{1}{2}} \rho g A_2(y)\, dy(2-y)=\int_{-1}^{\frac{1}{2}}10^3 g\pi(1-y^2)(2-y)\, dy$

$W=\frac{27}{8}\times10^3\pi g$ J.

例12.5 打捞、做功.



$x^2+(y+1)^2=1$

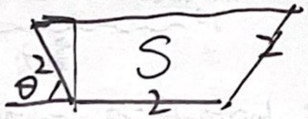
$x^2=1-(y+1)^2$

$W=\int_{-2}^{0}\rho g\cdot \underbrace{\pi[1-(y+1)^2]}_{r^2}\cdot dy\,\underbrace{(y+2)}_{h}$

$h=y-(-2)=y+2$

例12.6 ★★★.

(1). θ? S_{max}.



$S=(2+2\cos\theta)\,2\sin\theta$

$S'=-2\sin\theta\cdot2\sin\theta+(2+2\cos\theta)\,2\cos\theta\overset{令}{=}0$.

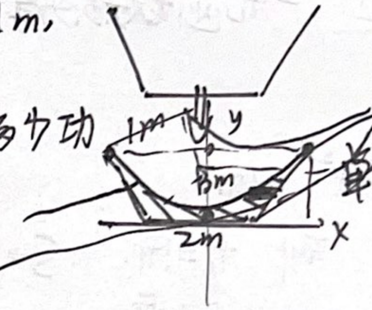
$\sin^2\theta=\cos\theta\cdot(1+\cos\theta)$

$\cos\theta_1=\frac{1}{2}$ $\cos\theta_2=-1$ $\theta_1=\frac{\pi}{3}$ $\theta_2=2\pi$

S_{max} 时 $\theta=\frac{\pi}{3}$.

(3) 若排水沟长1m,

淤泥搬出,至少做多少功



单位体积淤泥重.

ρ N/m³

$x=\sqrt{\frac{4y}{\sqrt{3}}}$

$x=\frac{y}{\sqrt{3}}+1$

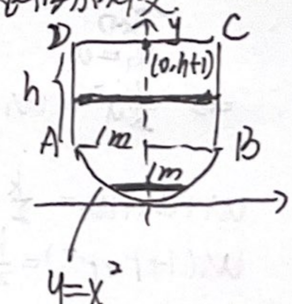
$\left[\underset{\rho g}{\rho}\int_{0}^{\sqrt{3}}\underbrace{\left(\frac{y}{\sqrt{3}}+1-\sqrt{\frac{4y}{\sqrt{3}}}\right)dy\cdot1}_{V}\cdot\underbrace{(\sqrt{3}-y)}_{h}\right]\times2$

4 静水压力

$P=\rho g h=\rho g\int_a^b x\,\underbrace{[f(x)-h(x)]}_{矩形宽度}\,dx$

例12.7

矩形部分压力：下部压力=5:4



$\frac{\int_1^{h+1}\rho g\,(h+1-y)\cdot2\,dy}{\int_0^1\rho g\,(h+1-y)\cdot2\sqrt{y}\,dy}=\frac{\rho g h^2}{4\rho g(\frac{1}{3}h+\frac{2}{15})}=\frac{4}{5}$

$\Rightarrow h=2$

• 细杆质心

$$\bar{x} = \frac{\int_a^b x\,\rho(x)\,dx}{\int_a^b \rho(x)\,dx}$$ (← 形心的横坐标.)

• eg. 长度为1. $[0,1]$. $\rho(x) = -x^2+2x+1$.

$$\bar{x} = \frac{\int_0^1 x(-x^2+2x+1)\,dx}{\int_0^1 (-x^2+2x+1)\,dx} = \frac{11}{20}$$

• (1) 距市中心2km内人数. $P(r) = \frac{4}{r^2+20}$.

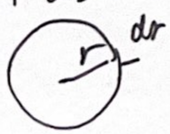
单位 10万人/km^2

$\boxed{2\pi r dr}$ 微元.

$$\int_0^2 P(r)\cdot 2\pi r\,dr$$

• 习12.5. 流量问题 ☆

eg. $10(1-r^2)$



$$\int_0^1 P(r)\ 2\pi r\,dr$$