§1. 练习总结.

1000.1.10.

$f(x)$ 连续、$f(0)=1$ 且 $x\to 0$ 时 $\int_0^{x-\tan x} f(t)dt$ 与 $(1+\sin^a x)^b-1$ 等价无穷小，$a,b=$?

①. $\lim\limits_{x\to 0}\int_0^{x-\tan x} f(t)dt=\lim\limits_{x\to 0}\int_0^{-\frac{x^3}{3}} f(t)dt=\lim\limits_{x\to 0} f(x)\cdot\frac{-x^3}{3}\sim\frac{-x^3}{3}$

$$\boxed{\begin{array}{l}\text{若 } f(x)\sim ax^m,\ g(x)\sim bx^n,\quad f(x),g(x),a,b \neq 0.\\ \text{则 } f[g(x)]\sim a(bx^n)^m=ab^m x^{mn}\end{array}}$$

② $\lim\limits_{x\to 0}(1+\sin^a x)^b-1=\lim\limits_{x\to 0} b\cdot\sin^a x=\lim\limits_{x\to 0} b\cdot x^a$

$\therefore\ b=-\frac{1}{3},\ a=3$

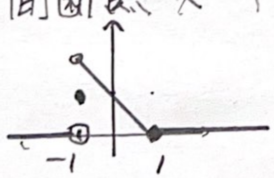
1000.1.14.

$f(x)=\lim\limits_{n\to\infty}\frac{1-x}{1+x^{2n}}$ 则 $f(x)$ (　　).

(A). 无间断点　(B) 间断点、$x=1$

(C). 间断点、$x=0$　(D) 间断点、$x=-1$

$$f(x)=\begin{cases}0, & x=1\\ 1, & x=-1\\ 0, & |x|>1\\ 1-x, & |x|<0\end{cases}$$



$\Rightarrow$ D.

1000.1.23

$$\lim_{x\to 0}\frac{\int_{\sin x}^{x}\sqrt{3+t^2}\,dt}{x(e^{x^2}-1)}=\lim_{x\to 0}\frac{F(x)-F(\sin x)}{x^3}\quad =F'(\xi)(x-\sin x)$$

$$\left(\int\sqrt{3+t^2}\,dt=F(x)\right)\quad=\lim_{x\to 0}\frac{\sqrt{3+\xi}\,(x-\sin x)}{x^3}=\frac{\sqrt{3}}{6}.\qquad \frac{x^3}{6}$$

1000.1.24

$$\lim_{x\to 0}\frac{(1+x)^{\frac{2}{x}}-e^2}{x}=\lim_{x\to 0}\frac{e^{\frac{2}{x}\ln(1+x)}-e^2}{x}=e^2\lim_{x\to 0}\frac{e^{\frac{2\ln(1+x)}{x}-2}-1}{x}$$

$$=e^2\lim_{x\to 0}\frac{2\ln(1+x)-2x}{x^2}\overset{洛}{=}e^2\lim_{x\to 0}\frac{2(1+x)^{-1}}{2x}\overset{洛}{=}-e^2\lim_{x\to 0}(1+x)^{-2}=-e^2$$

1000.1.28.(1).

$$\lim_{x\to+\infty}\left(\sqrt{x^2-5x}-\sqrt{x^2+7}+\frac{\sin^4 x}{\sqrt{x}}\right).$$

$$=\lim_{x\to+\infty}\frac{(\sqrt{x^2-5x}-\sqrt{x^2+7})(\sqrt{x^2-5x}+\sqrt{x^2+7})}{\sqrt{x^2-5x}+\sqrt{x^2+7}}+\lim_{x\to+\infty}\frac{1}{\sqrt{x}}\cdot\sin^4 x$$

$$=\lim_{x\to+\infty}\frac{x^2-5x-x^2-7}{\sqrt{x^2-5x}+\sqrt{x^2+7}}$$

$$=\lim_{x\to+\infty}\frac{-5-\frac{7}{x}}{\sqrt{1-\frac{5}{x}}+\sqrt{1+\frac{7}{x}}}=-\frac{5}{2}$$

1000.1.28(4).

$$\lim_{x\to 2}\frac{\sqrt{5x-1}-\sqrt{2x+5}}{x^2-4}=\lim_{x\to 2}\frac{3x-6}{x^2-4}\cdot\frac{1}{\sqrt{5x-1}+\sqrt{2x+5}}=\frac{1}{8}$$

1000.1.28(5) [换元]

$$\lim_{x\to\infty} e^{-x}(1+\frac{1}{x})^{x^2}$$ $\cancel{\ln(1+\frac{1}{x}) = \ln\frac{x+1}{x}}$

$$= \lim_{x\to\infty} e^{-x + x^2\ln(1+\frac{1}{x})} = \lim_{x\to\infty} e^{\frac{\ln(1+\frac{1}{x})-\frac{1}{x}}{\frac{1}{x^2}}}$$

$$\xlongequal[\text{t}\to 0]{\text{令}t=\frac{1}{x}} \lim_{t\to 0} e^{\frac{\ln(1+t)-t}{t^2}} \overset{\text{洛}}{=} \lim_{t\to 0} e^{\frac{\frac{1}{1+t}-1}{2t}} = e^{-\frac{1}{2}}$$

1000.1.28(6). [洛+中值].

$$\lim_{x\to 3^+}\frac{\cos x\ \ln(x-3)}{\ln(e^x-e^3)} = \cos 3\lim_{x\to 3^+}\frac{\ln(x-3)}{\ln(e^x-e^3)}$$

$$\overset{\text{洛}}{=} \lim_{x\to 3^+}\frac{e^x-e^3}{x-3}\cdot\lim_{x\to 3^+}\frac{1}{e^x}\cdot\cos 3$$

$$= \frac{\cos 3}{e^3}\cdot\lim_{x\to 3^+}\frac{e^x-e^3}{x-3},$$ $f(x)=e^x$, $f(x)-f(3)=f'(\xi)(x-3)$

$$= \frac{\cos 3}{e^3}e^3 = \cos 3$$

1000.1.28.17

$$\lim_{x\to 0}\left(\frac{a_1^x+a_2^x+\cdots a_n^x}{n}\right)^{\frac{1}{x}}$$, $a_i>0$, 且 $a_i\neq 1$, $i=1\sim n$, $n\geq 2$

[solution].

$$\lim_{x\to 0}\left(\frac{a_1^x+a_2^x+\cdots a_n^x}{n}\right)^{\frac{1}{x}} = \lim_{x\to 0}\left(1+\frac{a_1^x-1+a_2^x-1+\cdots+a_n^x-1}{n}\right)^{\frac{1}{x}}$$

$$\neq \triangle \lim_{x\to 0}\frac{a_1^x-1+a_2^x-1+\cdots+a_n^x-1}{nx}$$

$$= \frac{1}{n}\left(\lim_{x\to 0}\frac{a_1^x-1}{x}+\lim_{x\to 0}\frac{a_2^x-1}{x}+\cdots+\lim_{x\to 0}\frac{a_n^x-1}{x}\right)$$

$$= \frac{1}{n}(\ln a_1+\ln a_2+\cdots \ln a_n)$$

$$= \frac{1}{n}\ln(a_1a_2\cdots a_n) = k$$

$$\therefore \text{原} = \lim_{x\to 0}\left[(1+kx)^{\frac{1}{kx}}\right]^k = e^{\frac{1}{n}\ln(a_1\cdots a_n)} = \sqrt[n]{a_1a_2\cdots a_n}$$